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Effective Theory for Heavy Quarks

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Abstract

In this series of lectures the basic ideas of the $1/m_Q$ expansion in QCD (m_Q is the mass of a heavy quark) are outlined. Applications to exclusive and inclusive decays are given.

Contents

1 Introduction

With the precise formulation of the $1/m_Q$ expansion in QCD using effective field theory and operator product expansion heavy quark physics has been based on model independent ground, which allows to reduce the uncertainties due to the QCD bound state problem drastically. From the point of view of weak interactions the main interest in processes with heavy quarks lies in the exploration and the test of the CKM sector of the standard model (SM) describing the masses and the mixing of quarks. From the experimental side the running experiments as well as the ones planned in the near future constitute a large effort to explore this part of the SM, which is not yet tested with an accuracy comparable with the one of the coupling of the Z_0 to the fermions.

The main theoretical progress in the description of systems involving a single heavy quark is based on the infinite mass limit of QCD [1], in which two additional symmetries appear that are not present in full QCD [2]. This limit may be regarded as the leading term of a $1/m_Q$ expansion and a systematic approximation to full QCD may be constructed using the methods of effective field theory, the so-called Heavy Quark Effective Theory (HQET) [3]. The heavy mass limit and applications of HQET have been extensively studied, and the development of the field is documented in more or less extensive reviews [4].

The corrections to the heavy mass limit are characterized by two small parameters, namely the strong coupling constant, taken at the scale of the heavy quark $\alpha_s(m_Q)$ and the ratio $\bar{\Lambda}/m_Q$ of the scale of the light degrees of freedom $\bar{\Lambda}$ and the heavy quark mass. While the first kind of corrections may be calculated perturbatively in terms of Feynman diagrams, the second type needs additional non-perturbative input encoded in the matrix elements of higher dimensional operators.

However, the non-perturbative input is to be taken at a small scale, where the symmetries of HQET may be applied and hence the non-perturbative physics is constrained by these symmetries. In particular, for heavy to heavy decays these additional symmetries restrict the number of independent form factors to only a single one, of which the absolute normalization at a specific kinematic point is fixed by the symmetries.

By combining the method of the $1/m_Q$ expansion with the short distance expansion one may obtain a heavy mass expansion also for inclusive decay rates [5]-[7],[8]-[13]. The heavy quark mass sets a scale that is large compared to Λ_{QCD}, and one may use a similar setup as in deep inelastic scattering for the description of inclusive decays. In this way one may not only study total rates, but also differential distributions such as the lepton energy spectra in inclusive semileptonic decays.

This series of three lectures tries to summarize the basic ideas of the heavy mass expansion. In the first lecture the formulation of HQET as an effective field theory is described and the additional symmetries of heavy quarks are introduced. The second and third lecture deal with applications to exclusive and inclusive decays.

2 Effective Field Theory

Effective field theories [14] have become a widely used tool in modern elementary particle physics. An effective theory treatment is convenient if the problem under consideration involves very disparate mass scales such that the physics that is to be described happens at much lower energies than the scale set by some heavy particles in the theory. In such a case it is useful to switch to an effective theory in which the heavy degrees of freedom do not appear explicitly;

they only reappear in the effective theory as higher dimensional operators, which are multiplied by coupling constants with negative mass dimension. The scale of the coupling constants is set by the large mass and thus these contributions are small, if the scale of the physics described with the help of the effective theory is small compared to this large mass.

An effective theory is always valid only in a limited region of scales, a natural cut-off is given by the mass of the particle which has been removed by switching from the full to the effective theory. As mentioned above an effective theory involves interactions which would lead to a non-renormalizable theory, if one would consider the theory to all orders in these higher dimensional operators. However, working to a definite order of the expansion in inverse powers of the large scale one does not face any problem concerning renormalization. Starting from the renormalizable dimension-4 piece of the effective theory Lagrangian we may use its renormalization group properties to study the cut-off dependence of the effective theory, which is determined by the short distance properties of the effective theory.

Applying effective theory methods corresponds to an expansion of the Greens functions of the full theory in inverse powers of the large mass scale; such an expansion is only possible up to logarithmic dependences on this large scale. These logarithms may be accessed using a properly constructed effective theory, where these logarithms correspond to the renormalization group logarithms of the cut off. In this way one may even achieve a resummation of the logarithmic terms using renormalization group methods in the effective theory.

In the case at hand the large scale is the mass m_Q of the heavy quark, and the leading term of this expansion is the static limit. This effective theory (Heavy Quark Effective Theory, HQET) is a a powerful tool, allowing for numerous purely QCD based calculations. Renormalization in this effective theory implies a factorization theorem for the Greens function of full QCD, which means that to any order in the $1/m_Q$ expansion one may factorize the short distance physics from the long distance effects. Explicitly this means for a Greens function G_{full} calculated in full QCD that one may rewrite it as

$$G_{full}(p_1 \cdots p_n, m_Q, \mu_0 = m_Q) = \sum_j \left(\frac{1}{m_Q}\right)^j Z^{(j)}(m_Q, \mu) G^{(j)}_{eff}(p_1 \cdots p_n, \mu). \tag{1}$$

where μ_0 is the renormalization point of the full QCD function. Here the constants $Z^{(j)}(m_Q, \mu)$ depend on the factorization scale μ and on logarithms of m_Q; it contains all the short distance effects, which may be calculated perturbatively. Using the renormalization group of the effective theory, one may perform a systematic resummation of logarithmic dependences on the heavy quark mass.

The Greens functions $G^{(j)}_{eff}$ are calculated in HQET and do not depend on the heavy mass any more; they contain the long distance effects which are not calcualable via perturbation theory. However, as we shall see below, they are constrained by heavy quark symmetries. Consequently one gains complete control over the mass dependence by switching from full QCD to HQET, the effective theory obtained in the heavy mass limit.

In this section we shall discuss the formulation of this effective theory. First we shall review and compare some of the different possibilities to formulate the infinite mass limit. In this limit new symmetries appear which are the key to various model independent statements concerning weak decay matrix elements; these symmetries are reviewed in paragraph 3.1. Finally, we shall consider the systematic approach to the calculation of corrections to the infinite mass limit.

2.1 Infinite Quark Mass Limit as an Effective Field Theory

This issue has been discussed repeatedly and various formulations of the infinite mass limit are available. Of course, as far as physical quantities are concerned, all approaches yield the same result. However, for some special applications one approach may be more convenient than another.

The equivalence of all different approaches is ensured by a theorem well established in the field of effective theories. It has been shown that in an effective theory involving a field ϕ one may perform redefinitions of the fields , such that

$$\phi \to P(\phi, \partial\phi) \tag{2}$$

where P is an arbitrary polynomial function [15, 16].

Such a redefinition will not change the S-matrix, although the Lagrangian (and Greens function) expressed in the redefined fields may look completely different. In this sense the different formulations of HQET are equivalent.

In the following we shall consider two formulations of HQET. The first one nicely exhibits the fact that HQET is an effective theory in the sense that one integrates out a heavy degree of freedom and performs an expansion of the remaining action functional in the large mass scale, which for the case at hand is the heavy quark mass. The process of integrating out the heavy degree of freedom may be performed explicitly as a Gaussian functional integral, and one may construct a formulation of HQET be expanding the result in powers of $1/m_Q$. In this way one obtains a $1/m_Q$ expansion of both the heavy quark field as well as for the Lagrangian.

The second formulation is based on the standard way of separating "upper" and "lower" components of the spinor fields by performing a sequence of Foldy Wouthuysen transformations. These transformations lead to an expansion in $1/m_Q$ for the heavy quark field and for the Lagrangian, which is different for each of these quantities from the other formulation. However, if one calculates a physical quantity, both approaches will yield the same answer, because the $1/m_Q$ expansion has to be unique for an observable.

Integrating out heavy degrees of freedom

One may obtain a formulation of the heavy mass limit by integrating out heavy degrees of freedom from the functional integral of QCD Greens functions [17]. This integration may in fact be done explicitly, since for the case at hand it amounts to a Gaussian functional integration. Formulating the heavy mass limit in this way clearly exhibits that it corresponds to an effective theory in the usual sense. We start from the generating functional of the QCD Greens functions

$$Z(\eta, \bar\eta, \lambda) = \int [dQ][d\bar Q][d\phi_\lambda] \exp\left\{ iS + iS_\lambda + i \int d^4x \, (\bar\eta Q + \bar Q \eta + \phi_\lambda \lambda) \right\}, \tag{3}$$

where $\phi_\lambda = q$, A^a_μ denotes the light degrees of freedom (light quarks q and gluons A_μ) with the action S_λ, while S denotes the piece of the action for the heavy quark Q including its coupling to the gluons

$$S = \int d^4x \, \bar Q (i\not{D} - m_Q) Q. \tag{4}$$

where

$$D_\mu = \partial_\mu + igA_\mu, \tag{5}$$

is the covariant derivative of QCD. We have introduced source terms η for the heavy quark and λ for the light degrees of freedom.

We shall consider hadrons containing a single heavy quark, and we assume that this heavy hadron moves with a velocity v

$$v = \frac{p_{\text{hadron}}}{m_{\text{hadron}}}, \quad v^2 = 1, \quad v_0 > 0 \tag{6}$$

This velocity vector may be used to split the heavy quark field Q into an "upper" component ϕ and a "lower" one χ

$$\phi_v = \frac{1}{2}(1+\not{v})Q, \ \ \not{v}\phi_v = \phi, \tag{7}$$

$$\chi_v = \frac{1}{2}(1-\not{v})Q, \ \ \not{v}\chi_v = -\chi, \tag{8}$$

and to define a decomposition of the covariant derivative into a "longitudinal" and a "transverse" ($\perp$) part

$$D_\mu = v_\mu(v \cdot D) + D_\mu^\perp, \quad D_\mu^\perp = (g_{\mu\nu} - v_\mu v_\nu)D^\nu, \quad \left\{\not{D}^\perp, \not{v}\right\} = 0. \tag{9}$$

Using (7-9) the action (4) of the heavy quark field takes the form

$$S = \int d^4x \left[\bar{\phi}\{i(v\cdot D) - m_Q\}\phi - \bar{\chi}\{i(v\cdot D) + m_Q\}\chi + \bar{\phi}i\not{D}^\perp\chi + \bar{\chi}i\not{D}^\perp\phi\right]. \tag{10}$$

The heavy quark in the meson is very close to being on shell, and thus the space time dependence of the heavy quark field is mainly that of a free particle moving with velocity v. This suggests a reparametrization of the fields by removing the space time dependence of a solution of the free Dirac equation. We shall choose the "particle-type" parametrization corresponding to the "positive energy solution" of the Dirac equation

$$\phi_v = e^{-im_Q(v\cdot x)}h_v\,, \qquad \chi_v = e^{-im_Q(v\cdot x)}H_v, \tag{11}$$

such that the space time dependence of the remaining fields h_v and H_v is determined by the residual momentum $k = p - m_Q v$, which is due to binding effects of the heavy quark inside the heavy hadron, and which is a "small" quantity of order Λ_{QCD}.

Expressed in these fields the action of the heavy quark becomes

$$S = \int d^4x \left[\bar{h}_v i(v\cdot D)h_v - \bar{H}_v\{i(v\cdot D) + 2m_Q\}H_v + \bar{h}_v i\not{D}^\perp H_v + \bar{H}_v i\not{D}^\perp h_v\right], \tag{12}$$

The term containing the sources is also rewritten in terms of the fields h_v and H_v

$$\int d^4x\,(\bar{\eta}\psi + \bar{\psi}\eta) = \int d^4x\,(\bar{\rho}_v h_v + \bar{h}_v\rho_v + \bar{R}_v H_v + \bar{H}_v R_v), \tag{13}$$

where ρ_v and R_v are now source terms for the upper component field h_v and the lower componenent part H_v respectively.

In terms of the new variables the generating functional reads

$$Z(\rho_v, \bar{\rho}_v, R_v, \bar{R}_v, \lambda) = \int [dh_v][d\bar{h}_v][dH_v][d\bar{H}_v][d\phi_\lambda] \tag{14}$$
$$\exp\left\{iS + S_\lambda + i\int d^4x\,(\bar{\rho}_v h_v + \bar{h}_v\rho_v + \bar{R}_v H_v + \bar{H}_v R_v + \phi_\lambda\lambda)\right\},$$

where the action S for the heavy quark is given in eq.(12).

From (12) it is obvious that the heavy degree of freedom is the lower component field H_v, since it has a mass term $2m_Q$, while the upper component field h_v is a massless field describing the static heavy quark. In the heavy mass limit only the Greens functions involving the field h_v have to be calculated, and hence we integrate over H_v in the functional integral (14) with the sources of the lower component field R_v and $\bar{R}_v$ set to zero. This can be done explicitly, since it is a Gaussian integration

$$Z(\rho_v, \bar{\rho}_v, \lambda) = \int [dh_v][d\bar{h}_v][d\lambda]\Delta \exp\left\{iS + S_\lambda + i\int d^4x\, (\bar{\rho}_v^+ h_v^+ + \bar{h}_v^+ \rho_v^+ + \phi_\lambda \lambda)\right\}, \tag{15}$$

where now the action functional for the heavy quark becomes a non-local object

$$S = \int d^4x \left[\bar{h}_v^+ i(v\cdot D) h_v^+ - \bar{h}_v^+ \not{D}^\perp \left(\frac{1}{i(v\cdot D) + 2m_Q - i\epsilon}\right) \not{D}^\perp h_v^+\right]. \tag{16}$$

This Gaussian integration corresponds to the replacement

$$H_v = \left(\frac{1}{2m_Q + ivD}\right) i\not{D}_\perp h_v \tag{17}$$

for the lower compponent field. Furthermore, the Gaussian integration yields a determinant Δ. In the full theory one may also perform this Gaussian integration, and the determinant obtained contains all the closed loops of heavy quarks. After renormalization of the full theory their contribution starts at order $1/m^2$ with an Uehling potential like term. In the effective theory one may take the determinant Δ to be a constant, if the terms of order $1/m_Q^2$ and higher comming from the closed heavy quark loops are included by matching to the full theory. Since we shall discuss only the leading term of the $1/m_Q$ expansion in this section, we may drop the determinant in what follows.

The non-locality of the action functional is connected to the large scale set by the heavy quark mass, and the non-local terms may be expanded in terms of an infinite series of local operators, which come with increasing powers of $1/m_Q$. In the context of a field theory this corresponds to a short distance expansion and hence these operators have to be renormalized. The tree level relations may be read off from the geometric series expansion of the non-local term in (16). In this way we obtain the expansion of the field and the Lagrangian

$$\begin{aligned} Q(x) &= e^{-im_Q vx}\left[1 + \left(\frac{1}{2m + ivD}\right) i\not{D}_\perp\right] h_v \qquad (18)\\ &= e^{-im_Q vx}\left[1 + \frac{1}{2m_Q}\not{D}_\perp + \left(\frac{1}{2m_Q}\right)^2 (-ivD)\not{D}_\perp + \cdots\right] h_v \\ \mathcal{L} &= \bar{h}_v (ivD) h_v + \bar{h}_v i\not{D}_\perp \left(\frac{1}{2m + ivD}\right) i\not{D}_\perp h_v \qquad (19)\\ &= \bar{h}_v (ivD) h_v + \frac{1}{2m}\bar{h}_v (i\not{D}_\perp)^2 i h_v + \left(\frac{1}{2m}\right) \bar{h}_v (i\not{D}_\perp)(-ivD)(i\not{D}_\perp) h_v + \cdots \end{aligned}$$

A Greens function with an operator insertion is treated in a similar way; the heavy quark fields entering the inserted operator are dealt with in the same way. The net effect of this is that the heavy quark fields in the operator insertion are replaced by the expansion (18).

Foldy Wouthuysen Transformation

A second way of formulating the heavy mass limit proceed along the well known steps performed in deriving the non-relativistic limit of the Dirac equation [18]. The reasoning used here is motivated by quantum mechanics; as a first step one rewrites the equation of motion in a Hamiltonian form

$$iv\partial Q = HQ, \quad H = \not{v}(\not{D}_\perp + m_Q + gvA) \tag{20}$$

where A is the gluon field. Note that in the rest frame we have $v = (1,0,0,0)$, and thus (20) takes the usual form, since $v\partial = \partial_0$ and $vA = A_0$.

In general the Hamiltonian couples the upper and the lower component of the heavy quark field Q, projected out by $P_\pm = (1 \pm \not{v})$. The Foldy Wouthuysen transformation is a transformation of the form

$$Q \to Q' = \exp(iF)Q \quad H \to H' = \exp(iF)\left[H - iv\partial\right]\exp(-iF) \tag{21}$$

where F is a hermitian matrix in the space of the Dirac spinors, such that $\exp(iF)$ is unitary in that space. This requirement is motivated by quantum mechanics where the spinor is interpreted as a wave function, but this interpretation becomes meaningless once we switch to a field theory.

The transformation (21) is designed such that the resulting Hamiltonian H' does not couple upper and lower components of the field Q any more. The generator F of this transformation may be expanded in powers of $1/m_Q$, from which one may construct a $1/m_Q$ expansion of the Hamiltonian and the transformed fields. Removing the mass term of the Hamiltonian by a phase redefinition as in (18), one obtains for the fields and the Lagrangian

$$Q(x) = e^{-im_Q v\cdot x}\left[1 + \frac{1}{2m_Q}(i\not{D}_\perp) + \frac{1}{4m_Q^2}\left(v\cdot D\not{D}_\perp - \frac{1}{2}\not{D}_\perp^2\right) + \cdots\right]h_v(x)\,, \tag{22}$$

$$\mathcal{L} = \bar{h}_v\left[iv\cdot D - \frac{1}{2m_Q}\not{D}_\perp^2 + \frac{i}{4m_Q^2}\left(-\frac{1}{2}\not{D}_\perp^2 v\cdot D + \not{D}_\perp v\cdot D\not{D}_\perp - \frac{1}{2}v\cdot D\not{D}_\perp^2\right) + \cdots\right]h_v\,. \tag{23}$$

We note that the leading order term as well as the terms of order $1/m_Q$ are identical in the two approaches. Differences start to appear at order $1/m_Q^2$, which are terms involving a factor which would vanish by leading order equations of motion. In the Fouldy Wouthuysen formulation the Lagrangian does not contain such terms, while these terms appear in the approach of integrating out the lower components of the heavy quark field. However, subleading terms of a physical matrix element will consist of local contributions originating from the expansion of the field Q as well as of non-local pieces involving time-ordered products of the leading order currents with the subleading terms of the Lagrangian (see below). If the time-ordered products are taken with a term that would vanish by a naive application of the leading-order equation of motion, these will lead to a contact terms, i.e. effectively to a local contribution. In this way the terms in the first approach rearrange in such a way that the final result for an observable quantity is the same in both cases. For some practical applications the Foldy Wouthuysen approach has an advantage that all the time-ordered product with the Lagrangian are truly non-local contributions, in other words, none of the contributions will lead to a contact term.

2.2 Corrections to the Heavy Mass Limit

Tree Level Considerations

Corrections to the infinite mass limit may be considered in a systematic way. Starting from the tree level expressions given in the last sections one may use the expansion of the Lagrangian and the fields as given in section 2.1. to construct the $1/m_Q$ expansion of full QCD matrix elements. In doing this it will not matter which representation of HQET (e.g. the one that is obtained from integrating out the heavy quark or the one constructed from the Foldy Wouthuysen transformation), since the matrix elements have to have a unique $1/m_Q$ expansion.

As an example we shall consider a matrix element of a current $\bar{q}\Gamma Q$ mediating a transition between a heavy meson and some arbitrary state $|A\rangle$. The full QCD Lagrangian $\mathcal{L}$ and the fields Q are expanded in terms of a power series in $1/m_Q$ in the way described in section 2.1, and the matrix element under consideration up to order $1/m_Q$ takes the form:

$$\begin{aligned}\langle A|\bar{q}\Gamma Q|M(v)\rangle &= \langle A|\bar{q}\Gamma h_v|H(v)\rangle \\ &+\frac{1}{2m_Q}\langle A|\bar{q}\Gamma P_- i\not{D} h_v|H(v)\rangle - i\int d^4x \langle A|T\{L_1(x)\bar{q}\Gamma h_v\}|H(v)\rangle + \mathcal{O}(1/m^2)\end{aligned} \tag{24}$$

where L_1 are the first-order corrections to the Lagrangian as given in (19) or (23). Furthermore, $|M(v)\rangle$ is the state of the heavy meson in full QCD, including all its mass dependence, while $|H(v)\rangle$ is the corresponding state in the infinite mass limit.

Expression (24) displays the generic structure of the higher-order corrections as they appear in any HQET calculation. There will be local contributions coming from the expansion of the full QCD field; these may be interpreted as the corrections to the currents. The non-local contributions, i.e. the time-ordered products, are the corresponding corrections to the states and thus in the r.h.s. of (24) only the states of the infinite-mass limit appear. If one switches to another representation of HQET, one reshuffles terms from the fields into the Lagrangian; in this way the Lagrangian picks up operators which are proportional to the equations of motion.

As an example for the kinds of matrix elements appearing in subleading orders of the $1/m_Q$ expansion we consider the mass of a heavy hadron. In the infinite mass limit this mass is given in terms of the quark mass plus some "binding energy" $\bar{\Lambda}$. The corrections are of order $1/m_Q$ and are given by the matrix elements of the leading correction term of the Lagrangian. One obtains [19]

$$m_H = m_Q\left(1+\frac{\bar{\Lambda}}{m_Q}+\frac{1}{2m_Q^2}(\lambda_1+d_H\lambda_2)+\mathcal{O}(1/m_Q^3)\right) \tag{25}$$

where $d_H = 3$ for the 0^- and $d_H = -1$ for the 1^- meson. Up to this order no non-local terms appear; such terms show up the first time at order $1/m_Q^3$. The parameters $\bar{\Lambda}$, λ_1 and λ_2 correspond to matrix elements involving higher order terms that appear in the effective theory Lagrangian

$$\bar{\Lambda} = \frac{\langle 0|q\, iv\overleftarrow{D}\,\gamma_5 h_v|H(v)\rangle}{\langle 0|q\gamma_5 h_v|H(v)\rangle} \tag{26}$$

$$\lambda_1 = \frac{\langle H(v)|\bar{h}_v(iD)^2 h_v|H(v)\rangle}{2M_H} \tag{27}$$

$$\lambda_2 = \frac{\langle H(v)|\bar{h}_v\sigma_{\mu\nu}iD^\mu iD^\nu h_v|H(v)\rangle}{2M_H} \tag{28}$$

where the normalization of the states is chosen to be $\langle H(v)|\bar{h}_v h_v|H(v)\rangle = 2M_H = 2(m_Q + \bar{\Lambda})$. These parameters may be interpreted as the binding energy of the heavy meson in the infinite mass limit ($\bar{\Lambda}$), the expectation value of the kinetic energy of the heavy quark (λ_1) and its energy due to the chromomagnetic moment of the heavy quark (λ_2) inside the heavy meson. The latter two parameters play an important role since they parametrize the non-perturbative input needed in the subleading order of the $1/m_Q$ expansion.

The only parameter which is easy to access is λ_2, since it is related to the mass splitting between $H(v)$ and $H^*(v,\epsilon)$. From the B-meson system we obtain

$$\lambda_2(m_b) = \frac{1}{4}(M_{H^*} - M_H) = 0.12\ \mathrm{GeV}^2; \tag{29}$$

from the charm system the same value is obtained. This shows that indeed the spin-symmetry partners are degenerate in the infinite mass limit and the splitting between them scales as $1/m_Q$.

The other parameters appearing in (25) are not simply related to the hadron spectrum. Using the pole mass for m_Q in (25), QCD sum rules yield for a value of $\bar{\Lambda} = 570 \pm 70$ MeV [4]. More problematic is the parameter λ_1; from its definition one is led to assume $\lambda_1 < 0$; a more restrictive inequality

$$-\lambda_1 > 3\lambda_2 \tag{30}$$

has been derived in a quantum mechanical framework in [62] and using heavy-flavour sum rules [21]. Furthermore, there exists also a sum rule estimate [22] for this parameter:

$$\lambda_1 = -0.52 \pm 0.12\ \mathrm{GeV}^2. \tag{31}$$

This value is compatible with the bounds; however, it is unexpectedly large since it corresponds to a rms-momentum of the heavy quark inside the meson of

$$\sqrt{\langle \vec{p}^{\,2}\rangle} \sim 720\ \mathrm{MeV} \tag{32}$$

which is large compared to the naive guess of $-\lambda_1 \sim (\Lambda_{QCD})^2$ This is the reason why also smaller values of λ_1 have been used in the literature.

Recently there has been an attempt [23] to extract $\bar{\Lambda}$ and λ_1 from the shape of the lepton energy spectrum in inclusive semileptonic B decays (see sections 4.3 and 4.4). The values obtained from this analysis are $\bar{\Lambda} = 0.39 \pm 0.11$ GeV and $-\lambda_1 = 0.19 \pm 0.10$ GeV^2, where the $\overline{\mathrm{MS}}$ definition of the mass has been used. The uncertainties quoted are only the 1σ statistical ones; the systematical uncertainties of this approach are difficult to estimate.

Beyond Tree Level

Going beyond tree level will induce corrections of order $\alpha_s^n(m_Q)$, $n = 1, \ldots$. These may be calculated in terms of Feynman diagrams which may be evaluated using the Feynman rules of HQET. Only two Feynmal rules are modified compared to full QCD:

	full QCD		HQET	
Propagator of the heavy quark	$\dfrac{i}{\not{p} - m_Q + i\epsilon}$	$\longrightarrow$	$\dfrac{i}{vk + i\epsilon}$,	$p = mv + k$
Heavy quark gluon vertex	$ig\gamma_\mu T^a$	$\longrightarrow$	$igv_\mu T^a$	

For the sake of clarity we shall stick to our example of a heavy light current considered above. To leading order in the $1/m_Q$ expansion one may evaluate the radiative corrections to such a matrix element using the above Feynman rules and finds a divergent result with a divergence related to the short distance behaviour. Since HQET is an effective theory, the machinery of effective theory guarantees the factorization of long distance effects from the short distance ones, which are related to the large mass m_Q. Neglecting $1/m_Q$ corrections, this factorization takes the form

$$\langle A|\bar{q}\Gamma Q|M(v)\rangle = Z\left(\frac{m_Q}{\mu}\right)\langle A|\bar{q}\Gamma h_v|H(v)\rangle|_\mu + \mathcal{O}(1/m_Q) \tag{33}$$

From Feynman rule calculation one obtains the perturbative expansion of the renormalization constant Z which generically looks like

$$\begin{aligned} Z\left(\frac{m_Q}{\mu}\right) &= a_{00} \\ &+ a_{11}\left(\alpha_s \ln\left(\frac{m_Q}{\mu}\right)\right) + a_{10}\alpha_s \\ &+ a_{22}\left(\alpha_s \ln\left(\frac{m_Q}{\mu}\right)\right)^2 + a_{21}\alpha_s\left(\alpha_s \ln\left(\frac{m_Q}{\mu}\right)\right) + a_{20}\alpha_s^2 \\ &+ a_{33}\left(\alpha_s \ln\left(\frac{m_Q}{\mu}\right)\right)^3 + a_{32}\alpha_s\left(\alpha_s \ln\left(\frac{m_Q}{\mu}\right)\right)^2 + a_{31}\alpha_s^2\left(\alpha_s \ln\left(\frac{m_Q}{\mu}\right)\right) \\ &\qquad + a_{30}\alpha_s^3 + \cdots \end{aligned} \tag{34}$$

where $\alpha_s = g^2/(4\pi)$.

This factorization theorem corresponds to the statement that the ultraviolet divergencies in the effective theory have to match the logarithmic mass dependences of full QCD. The factorization scale μ is an arbitrary parameter, and the physical quantity $\langle A|\bar{q}\Gamma Q|M(v)\rangle$ does not depend on this parameter. However, calculating the matrix element of this operator in the effective theory and studying its ultraviolet behaviour allows us to access the mass dependence of the matrix element $\langle A|\bar{q}\Gamma Q|M(v)\rangle$.

The ultraviolet behaviour of the effective theory is investigated by the renormalization group equations. Differentiating (33) with respect to the factorization scale μ yields the renormalization group equation

$$\frac{d}{d\ln\mu}\left\{Z\left(\frac{m_Q}{\mu}\right)\langle A|\bar{q}\Gamma h_v|H(v)\rangle|_\mu\right\} = 0 \tag{35}$$

from which we may obtain an equation which determines the change of the coefficient Z when the scale is changed

$$\begin{aligned} \left(\frac{d}{d\ln\mu} + \gamma_J(\mu)\right) Z\left(\frac{m_Q}{\mu}\right) &= 0 \\ \gamma_J(\mu) = \frac{d}{d\ln\mu}&\ln(\langle A|\bar{q}\Gamma h_v|H(v)\rangle|_\mu). \end{aligned} \tag{36}$$

The quantity γ_J is called the anomalous dimension of the operator $J = \bar{q}\Gamma h_v$ which is universal for all matrix elements of this operator, since it is connected with the short distance behaviour of the insertion of J.

Eq.(36) describes the renormalization group scaling in the effective theory. It allows to shift logarithms of the large mass scale from the matrix element of J into the coefficient Z: If the matrix element is renormalized at the large scale m_Q the logarithms of the type $\ln m_Q$ will apear in the matrix element of J while the coefficient Z at this scale will simply be

$$Z(1) = a_{00} + a_{10}\alpha_s(m_Q) + a_{20}\alpha_s^2(m_Q) + a_{30}\alpha_s^3(m_Q) + \cdots \tag{37}$$

The renormalization group equation (36) allows to lower the renormalization point from m_Q to μ; the matrix element renormalized at μ will not contain any logarithms of m_Q any more, they will appear in the coefficient Z in the way shown in (34).

In all cases relevant in the present context the matrix elements will be matrix elements involving hadronic states, which are in most cases impossible to calculate from first principles. However, eq.(36) allows to extract the short distance piece, i.e. the logarithms of the large mass m_Q and to separate it into the Wilson coefficients.

The anomalous dimension may be calculated in perturbation theory in powers of the coupling constant g of the theory. In general, in a renormalizable theory the coupling constant depends on the scale μ at which the theory is renormalized. The scale dependence of the coupling constant is determined by the β function

$$\frac{d}{d\ln\mu} g(\mu) = \beta(\mu). \tag{38}$$

In a mass independent scheme the renormalization group functions γ_O and β will depend on the scale μ only through their dependence on the coupling constant

$$\beta = \beta(g(\mu)) \qquad \gamma_J = \gamma_J(g(\mu)). \tag{39}$$

Hence we may rewrite the renormalization group equation (36) as

$$\left(\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} + \gamma_J(g)\right) Z\left(\frac{m_Q}{\mu}, g\right) = 0. \tag{40}$$

The renormalization group functions β and γ_O are calculated in perturbation theory; the first term of the β function on QCD is obtained from a one-loop calculation and is given by

$$\beta(g) = -\frac{1}{(4\pi)^2}\left(11 - \frac{2}{3}n_f\right) g^3 + \cdots, \tag{41}$$

where n_f is the number of active flavors, i.e. the number of flavors with a mass less than m_Q.

With this input the renormalization group equation may be solved to yield

$$Z\left(\frac{m_Q}{\mu}\right) = a_{00}\left(\frac{\alpha_s(\mu)}{\alpha_s(m_Q)}\right)^{-\frac{48\pi^2}{33-2n_f}\gamma_1} \tag{42}$$

where γ_1 is the first coefficient in the perturbative expansion of the anomalous dimension $\gamma_J = \gamma_1 g^2 + \cdots$ and $\alpha_s(\mu)$ is the one loop expression for the running coupling constant of QCD

$$\alpha_s(\mu) = \frac{12\pi}{(33 - 2n_f)\ln(\mu^2/\Lambda_{QCD}^2)} \tag{43}$$

which is obtained from solving (38) using (41). This expression corresponds to a summation of the leading logarithms $(\alpha_s \ln m_Q)^n$ which is achieved by a one-loop calculation of the renormalization group functions β and γ_Q; in other words, in this way a resummation of the first column of the expansion (34) is obtained.

In a similar way one may also resum the second column of (34), if the renormalization group functions β and γ are calculated to two loops and the finite terms of the one loop expression are included.

Finally, the case we have considered as an example is indeed very simple; in general all operators of a given dimension may mix under renormalization, i.e. instead of a simple anomalous dimension a matrix of anomalous dimensions may occur. For more details on this I refer the reader to a textbook discussion of these issues as given e.g. in [24].

2.3 Heavy Quark Symmetries

The main impact of the heavy quark limit is due to two additional symmetries which are not present in full QCD; the first is a heavy flavour symmetry and the second one is the so-called spin symmetry. The presence of these symmetries implies Wigner-Eckart theorems for transition matrix elements which have far-reaching phenomenological consequences.

We shall first study the heavy flavour symmetry. The interaction of the quarks with the gluons is flavour independent; all flavour dependence in QCD is only due to the different quark masses. In the $1/m_Q$ expansion the leading order Lagrangian is mass independent and hence a flavour symmetry appears relating heavy quarks moving with the same velocity.

For the case of two heavy flavours b and c one has to leading order the Lagrangian [3]

$$\mathcal{L}_{heavy} = \bar{b}_v(v \cdot D)b_v + \bar{c}_v(v \cdot D)c_v, \tag{44}$$

where b_v (c_v) is the field operator h_v for the b (c) quark moving with velocity v and $D = \partial + igA$ is the QCD covariant derivative. This Lagrangian is obviously invariant under the $SU(2)_{HF}$ rotations

$$\begin{pmatrix} b_v \\ c_v \end{pmatrix} \to U_v \begin{pmatrix} b_v \\ c_v \end{pmatrix} \quad U_v \in SU(2)_{HF}. \tag{45}$$

We have put a subscript v for the transformation matrix U, since this symmetry only relates heavy quarks moving with the same velocity.

The second symmetry is the heavy-quark spin symmetry. As is clear form the Lagrangian in the heavy-mass limit, both spin degrees of freedom of the heavy quark couple in the same way to the gluons; we may rewrite the leading-order Lagrangian as

$$\mathcal{L} = \bar{h}_v^{+s}(ivD)h_v^{+s} + \bar{h}_v^{-s}(ivD)h_v^{-s}, \tag{46}$$

where $h_v^{\pm s}$ are the projections of the heavy quark field on a definite spin direction s

$$h_v^{\pm s} = \frac{1}{2}(1 \pm \gamma_5 \not{s})h_v, \quad s \cdot v = 0. \tag{47}$$

This Lagrangian has a symmetry under the rotations of the heavy quark spin and hence all the heavy hadron states moving with the velocity v fall into spin-symmetry doublets as $m_Q \to \infty$. In Hilbert space this symmetry is generated by operators $S_v(\epsilon)$ as

$$[h_v, S_v(\epsilon)] = i\not{\epsilon}\not{v}\gamma_5 h_v \tag{48}$$

where ϵ with $\epsilon^2 = -1$ is the rotation axis. The simplest spin-symmetry doublet in the mesonic case consists of the pseudoscalar meson $H(v)$ and the corresponding vector meson $H^*(v, \epsilon)$, since a spin rotation yields

$$\exp\left(iS_v(\epsilon)\frac{\pi}{2}\right)|H(v)\rangle = (-i)|H^*(v,\epsilon)\rangle, \tag{49}$$

where we have chosen an arbitrary phase to be $(-i)$.

In the heavy-mass limit the spin symmetry partners have to be degenerate and their splitting has to scale as $1/m_Q$. In other words, the quantity

$$\lambda_2 = \frac{1}{4}(M_{H^*}^2 - M_H^2) \tag{50}$$

has to be the same for all spin symmetry doublets of heavy ground state mesons. This is well supported by data: For both the (B, B^*) and the (D, D^*) doublets one finds a value of $\lambda_2 \sim 0.12$ GeV^2. This shows that the spin-symmetry partners become degenerate in the infinite mass limit and the splitting between them scales as $1/m_Q$.

In the infinite mass limit the symmetries imply relations between matrix elements involving heavy quarks. For a transition between heavy ground-state mesons H (either pseudoscalar or vector) with heavy flavour f (f') moving with velocities v (v'), one obtains in the heavy-quark limit

$$\langle H^{(f')}(v')|\bar{h}_{v'}^{(f')}\Gamma h_v^{(f)}|H^{(f)}(v)\rangle = \xi(vv') \text{ Tr } \left\{\overline{\mathcal{H}(v)}\Gamma\mathcal{H}(v)\right\}, \tag{51}$$

where Γ is some arbitrary Dirac matrix and $H(v)$ are the representation matrices for the two possibilities of coupling the heavy quark spin to the spin of the light degrees of freedom, which are in a spin-1/2 state for ground state mesons

$$\mathcal{H}(v) = \frac{\sqrt{M_H}}{2}\begin{cases}(1+\not{v})\gamma_5 & 0^-,\ (\bar{q}Q) \text{ meson}\\ (1+\not{v})\not{\epsilon} & 1^-,\ (\bar{q}Q) \text{ meson}\\ & \text{with polarization } \epsilon.\end{cases} \tag{52}$$

Due to the spin and flavour independence of the heavy mass limit the Isgur–Wise function ξ is the only non-perturbative information needed to describe all heavy to heavy transitions within a spin-flavour symmetry multiplet.

Excited mesons have been studied in [25]. They may be classified by the angular momentum of the light degrees of freedom j_l, which is coupled with the heavy quark spin S to the total angular momentum J of the meson. Furthermore, the orbital angular momentum ℓ determines the parity $P = (-1)^{\ell+1}$ of the meson. For a given $\ell > 0$ we can have $j_l = \ell \pm 1/2$ and the coupling of the heavy quark spin yields two spin symmetry doublets $(J = \ell - 1, J = \ell)$ and $(J = \ell, J = \ell + 1)$. For example, the lowest positive parity $\ell = 1$ mesons are two spin symmetry doublets $(0^+, 1^+)$ and $(1^+, 2^+)$. In the D meson system these states have been observed [26] and behave as predicted by heavy quark symmetry [27]

Similarly as for the mesons heavy-quark symmetries imply that only one form factor is needed to describe heavy to heavy transitions within a spin flavour symmetry multiplet; in other words, there is an Isgur Wise function for each multiplet.

The ground state baryons have been studied in [28, 29, 30]. According to the particle data group they are classified as follows

$$\Lambda_h = [(qq')_0 h]_{1/2} \qquad \Xi_h' = [(qs)_0 h]_{1/2} \tag{53}$$

$$\Sigma_h = [(qq')_1 h]_{1/2} \qquad \Xi_h = [(qs)_1 h]_{1/2} \qquad \Omega_h = [(ss)_1 h]_{1/2} \tag{54}$$

$$\Sigma_h^* = [(qq')_1 h]_{3/2} \qquad \Xi_h^* = [(qs)_1 h]_{3/2} \qquad \Omega_h^* = [(ss)_1 h]_{3/2}. \tag{55}$$

Here, q, q' refer to u and d quarks, $q \neq q'$ for the Λ_h, but q may be the same as q' for the Σ_h and Σ_h^*. The first subscript (0, 1) is the total spin of the light degrees of freedom, while the second subscript (1/2, 3/2) is the total spin of the baryon.

Spin symmetry forces these baryons into spin symmetry doublets. For the Λ-type baryons (53) the spin rotations are simply a subset of the Lorentz transformations, since the light degrees of freedom are in a spin-0 state. The corresponding spin symmetry doublet is in this case given by the two polarization directions of the heavy baryon. From the point of view of heavy quark symmetries the Λ-type baryons are the simplest hadrons, although from the quark model point of view they are composed of three quarks.

The baryons with the light degrees of freedom in a spin one state may be represented by a pseudovector-spinor object R^μ with $v_\mu R^\mu = 0$ [1]. In general $\gamma_\mu R^\mu \neq 0$ because R^μ contains spin 1/2 contributions as well as spin 3/2 parts. In other words, R^μ contains a Rarita-Schwinger field as well as a Dirac field. Under Lorentz transformations R^μ behaves as

$$R^\mu(v) \to \Lambda^\mu{}_\nu D(\Lambda) R^\nu(\Lambda v), \tag{56}$$

where $\Lambda_{\mu\nu}$ and $D(\Lambda)$ are the Lorentz transformations in the vector and spinor representation respectively, while under spin rotations we have

$$R^\mu(v) \to -\gamma_5 \not{v} \not{\epsilon} R^\mu(v). \tag{57}$$

The spin-3/2 component of the the pseudovector-spinor object corresponding to the Σ_h^* is projected out by contracting with γ_μ

$$\gamma_\mu R^\mu_{\Sigma_h^*} = 0. \tag{58}$$

The rest of the independent components of R correspond to Σ_h baryon:

$$R^\mu_{\Sigma_h} = \frac{1}{\sqrt{3}} (\gamma^\mu + v^\mu) \gamma_5 u_{\Sigma_h}, \tag{59}$$

where u_{Σ_h} is the Dirac spinor of the Σ_h state. Similar expressions hold for the non-strange baryons $\Xi_h^{(*)}$ and $\Omega_h^{(*)}$.

The spin rotation (57) transform the Σ-like baryons into the Σ^* states and vice versa. Thus the spin symmetry doublets for the ground state baryons are given by the two polarization directions of the baryons in (53), and by the two states with corresponding light quark flavour numbers in (54) and (55).

Similar to the case of mesons one may derive a Wigner-Eckart theorem for the spin symmetry doublets of the baryons

$$\langle \Lambda_h(v) | \bar{h} \Gamma h' | \Lambda_{h'}(v') \rangle = A(v \cdot v') \bar{u}_{\xi_h}(v) \Gamma u_{\xi_{h'}}(v'), \tag{60}$$

where we have allowed for the possibility of two heavy quark flavours h and h'. In the same way, one obtains two form factors for the $\Sigma_h^{(*)} \to \Sigma_{h'}^{(*)}$.

$$\begin{aligned} &\langle \Sigma_h^{(*)}(v) | \bar{h}_v \Gamma h_{v'} | \Sigma_h^{(*)} \rangle \\ &= \bar{R}^\mu_{\Sigma_h^{(*)}}(v) \Gamma R^\nu_{\Sigma_h^{(*)}}(v') \left[B(v \cdot v') g_{\mu\nu} + C(v \cdot v') v'_\mu v_\nu \right]. \end{aligned} \tag{61}$$

Finally, parity does not allow for transitions between Λ and $\Sigma^{(*)}$ type baryons

$$\langle \Sigma_h^{(*)}(v) | \bar{h}_v \Gamma h_{v'} | \Lambda_h(v) \rangle = 0, \tag{62}$$

[1] One could as well represent the light degrees of freedom by an antisymmetric tensor instead of a pseudovector; this is a completely equivalent formulation [30].

and hence these transitions are not only suppressed by the flavour symmetry of the light degrees of freedom, but additionally by heavy quark symmetry.

Excited baryons my be studied along the same lines as for the mesons. The spin symmetry doublets as well as the restrictions on transition matrix elements have been studied in [25].

Heavy quark symmetries thus lead to a strong reduction of the number of independent from factors that describe current induced transitions among heavy hadrons. In addition to that the symmetries even allow us to obtain the normalization of some of these form factors. Since the currents

$$J^{hh'} = \bar{h}_v \gamma_\mu h'_v = v_\mu \bar{h}_v h'_v \tag{63}$$

are the generators of heavy flavour symmetry in the velocity sector v, the normalization of the Wigner-Eckard theorems (51,60,61) is known at the non-recoil point $v = v'$. By standard arguments one obtains for the mesons

$$\xi(vv' = 1) = 1, \tag{64}$$

while the corresponding relation for the baryons is

$$\begin{aligned} A(vv' = 1) &= \sqrt{m_{\Lambda_h} m_{\Lambda_{h'}}} && (65) \\ B(vv' = 1) &= \sqrt{m_{\Sigma_h^{(*)}} m_{\Sigma_{h'}^{(*)}}}, && (66) \end{aligned}$$

where the factor involving the square root of the masses means that the hadron states in (60) are normalized relativistically.

Up to now we have considered only the consequences of heavy quark symmetries for the leading terms of the $1/m_Q$ expansion. However, the additional symmetries also restrict the subleading terms and one of these restrictions is called Lukes theorem [31]. It is a generalization of the Ademollo Gatto theorem [32], which states that in the presence of explicit symmetry breaking the matrix elements of the currents that generate the symmetry are still normalized up to terms which are second order in the symmetry breaking interaction.

For the case at hand the relevant symmetry is the heavy flavor symmetry. This symmetry is an $SU(2)$ symmetry and is generated by three operators $Q_\pm$ and Q_3 with

$$\begin{aligned} & Q_+ = \int d^3x\, \bar{b}_v(x) c_v(x) \quad Q_- = \int d^3x\, \bar{c}_v(x) b_v(x) \\ & Q_3 = \int d^3x\, (\bar{b}_v(x) b_v(x) - \bar{c}_v(x) c_v(x)) \\ & [Q_+, Q_-] = Q_3 \qquad [Q_+, Q_3] = -2Q_+ \qquad (Q_+)^\dagger = Q_- \end{aligned} \tag{67}$$

Let us denote the ground state flavour symmetry multiplet as $|B\rangle$ and $|D\rangle$. Then the operators act in the following way

$$\begin{aligned} & Q_3|B\rangle = |B\rangle \qquad Q_3|D\rangle = -|D\rangle \\ & Q_+|D\rangle = |B\rangle \qquad Q_-|B\rangle = |D\rangle \\ & Q_+|B\rangle = Q_-|D\rangle = 0. \end{aligned} \tag{68}$$

The Hamiltonian of this system has a $1/m_Q$ expansion of the form

$$H = H_0^{(b)} + H_0^{(c)} + \frac{1}{2m_b} H_1^{(b)} + \frac{1}{2m_c} H_1^{(c)} + \cdots \tag{69}$$

$$
\begin{aligned}
&= H_0^{(b)} + H_0^{(c)} + \frac{1}{2}\left(\frac{1}{2m_b} + \frac{1}{2m_c}\right)(H_1^{(b)} + H_1^{(c)}) \\
&\qquad\qquad + \frac{1}{2}\left(\frac{1}{2m_b} - \frac{1}{2m_c}\right)(H_1^{(b)} - H_1^{(c)}) + \cdots \\
&= H_{symm} + H_{break}.
\end{aligned}
$$

In the second equation, the first line is still symmetric under heavy flavour $SU(2)$ while the term in the second line does not commute any more with $Q_\pm$, but it still commutes with Q_3. In other words, to order $1/m_Q$ we still have common eigenstates of H and Q_3, which we shall denote as $|\tilde{B}\rangle$ and $|\tilde{D}\rangle$. Sandwiching the commutation relation we get

$$
\begin{aligned}
1 &= \langle\tilde{B}|Q_3|\tilde{B}\rangle = \langle\tilde{B}|[Q_+, Q_-]|\tilde{B}\rangle \\
&= \sum_n \left[\langle\tilde{B}|Q_+|\tilde{n}\rangle\langle\tilde{n}|Q_-|\tilde{B}\rangle - \langle\tilde{B}|Q_-|\tilde{n}\rangle\langle\tilde{n}|Q_+|\tilde{B}\rangle\right] \\
&= \sum_n \left[|\langle\tilde{B}|Q_+|\tilde{n}\rangle|^2 - |\langle\tilde{B}|Q_-|\tilde{n}\rangle|^2\right]
\end{aligned}
\tag{70}
$$

where $|\tilde{n}\rangle$ form a complete set of states of the Hamiltonian $H_{symm} + H_{break}$. The matrix elements may be written as

$$
\langle\tilde{B}|Q_\pm|\tilde{n}\rangle = \frac{1}{E_B - E_n}\langle\tilde{B}|[H_{break}, Q_\pm]|\tilde{n}\rangle \tag{71}
$$

where E_B and E_n are the energies of the states $|\tilde{B}\rangle$ and $|\tilde{n}\rangle$ respectively. In the case $|\tilde{n}\rangle = |\tilde{D}\rangle$ the matrix element will be of order unity, since both the numerator as well as the energy difference in the denominator are of the order of the symmetry breaking. For all other states the energy difference in the denominator is non-vanishing in the symmetry limit, and hence this difference is of order unity; thus the matrix element for these states will be of the order of the symmetry breaking. From this we conclude

$$
\langle\tilde{B}|Q_+|\tilde{D}\rangle = 1 + \mathcal{O}\left[\left(\frac{1}{2m_b} - \frac{1}{2m_c}\right)^2\right]. \tag{72}
$$

In particular, the weak transition currents at the non-recoil point $v = v'$ are proportional to these symmetry generators and hence we may conclude that for some of these matrix elements we only have corrections of the order $1/m_Q^2$.

Another restriction on the $1/m_Q$ expansion is imposed by the so-called reparametrization invariance [33] which is basically the remnant of the original Lorentz covariance of full QCD. The full theory depends only on the momentum of the heavy quark P, and the splitting of this momentum in an on-shell part $m_Q v$ and a residual momentum k corresponding to the covariant derivative acting on the heavy static field h_v is arbitrary. Formally this means that the Lagrangian does not depend on the velocity v, if all orders of the $1/m_Q$ expansion are included; the v dependence only enters once the expansion is truncated.

Using the representation (19) and (18) the Lagrangian is invariant under the transformation

$$
\begin{aligned}
&v \to v + \delta v \qquad v \cdot \delta v = 0 \\
&h_v \to h_v + \frac{\delta\!\!\!/ v}{2}\left(1 + P_- \frac{1}{2m + ivD} i D\!\!\!\!/\right) h_v \\
&iD \to -m\,\delta v.
\end{aligned}
\tag{73}
$$

This invariance is the so-called reparametrization invariance, which has non-trivial consequences, since it relates terms of different orders of the $1/m$ expansion.

3 Application to Exclusive Decays

The heavy mass limit and the resulting additional symmetries allow us to restrict the matrix elements which occur in weak transitions of heavy hadrons. We shall consider in the following in some detail the semileptonic $b \to c$ transition, which we shall treat as a heavy to heavy decay. In section 3.2 we investigate the consequences of heavy quark symmetries for transitions of the heavy to light type.

3.1 Transitions of the type Heavy $\to$ Heavy

For the case of a heavy to heavy transition the Wigner Eckart theorem (51) implies that there is only a single form factor which describe the weak decays of heavy hadrons; furthermore, the heavy mass limit yields the normalization of this form factor at the kinematic point $v = v'$.

Treating both the b and the c quark as heavy, the semileptonic decays $B \to D^{(*)}\ell\nu$ are the phenomenologically relevant examples. The matrix elements for these transitions are in general parametrized in terms of six form factors

$$\langle D(v')|\bar{c}\gamma_\mu b|B(v)\rangle = \sqrt{m_B m_D}\left[\xi_+(y)(v_\mu + v'_\mu) + \xi_-(y)(v_\mu - v'_\mu)\right] \tag{74}$$

$$\langle D^*(v',\epsilon)|\bar{c}\gamma_\mu b|B(v)\rangle = i\sqrt{m_B m_{D^*}}\xi_V(y)\varepsilon_{\mu\alpha\beta\rho}\epsilon^{*\alpha}v'^\beta v^\rho \tag{75}$$

$$\begin{aligned}\langle D^*(v',\epsilon)|\bar{c}\gamma_\mu\gamma_5 b|B(v)\rangle = \sqrt{m_B m_{D^*}}&\left[\xi_{A1}(y)(vv'+1)\epsilon^*_\mu - \xi_{A2}(y)(\epsilon^* v)v_\mu\right.\\ &\left.-\xi_{A2}(y)(\epsilon^* v)v'_\mu\right],\end{aligned} \tag{76}$$

where we have defined $y = vv'$. Due to the Wigner Eckart theorem (51) these six from factors are related to the Isgur Wise function by

$$\xi_i(y) = \xi(y) \text{ for } i = +, V, A1, A3, \qquad \xi_i(y) = 0 \text{ for } i = -, A2. \tag{77}$$

Since heavy quark symmetries also yield the normalization of the Isgur Wise function, we know the absolute value of the differential rate at the point $v = v'$ in terms of the meson masses and V_{cb}. Hence we may use this to extract V_{cb} from these decays in a model independent way by extrapolating the lepton spectrum to the kinematic endpoint $v = v'$. Using the mode $B \to D^{(*)}\ell\nu$ one obtains the relation

$$\lim_{v\to v'} \frac{1}{\sqrt{(vv')^2-1}}\frac{d\Gamma}{d(vv')} = \frac{G_F^2}{4\pi^3}|V_{cb}|^2(m_B - m_{D^*})^2 m_{D^*}^3|\xi_{A1}(1)|^2, \tag{78}$$

where ξ_{A1} is equal to the Isgur Wise function in the heavy mass limit, and hence $\xi_{A1}(1) = 1$.

Corrections to this relation have been calculated along the lines outlined above in leading and subleading order. A complete discussion may be found in more extensive review articles (see e.g. Neubert's review [4]), including reference to the original papers. Here we only state the final result

$$\begin{aligned}\xi_{A1}(1) = x^{6/25}&\left[1 + 1.561\frac{\alpha_s(m_c)-\alpha_s(m_b)}{\pi} - \frac{8\alpha_s(m_c)}{3\pi}\right.\\ &\left.+z\left\{\frac{25}{54} - \frac{14}{27}x^{-9/25} + \frac{1}{18}x^{-12/25} + \frac{8}{25}\ln x\right\} - \frac{\alpha_s(\bar{m})}{\pi}\frac{z^2}{1-z}\ln z\right] + \delta_{1/m^2},\end{aligned} \tag{79}$$

where we use the abbreviations

$$x = \frac{\alpha_s(m_c)}{\alpha_s(m_b)}, \quad z = \frac{m_c}{m_b}$$

and $\bar{m}$ is a scale somewhere between m_b and m_c.

Up to the term δ_{1/m^2} all these contributions may be calculated perturbatively, including the dependence on z. The quantity δ_{1/m^2} parametrizes the non-perturbative contributions, which enter here at order $1/m^2$. These corrections may be expressed in terms of the kinetic energy λ_1, the chromomagnetic moment λ_2, which are given in (27) and (28) respectively, and matrix elements involving time-ordered products between the current and the corrections of the Lagrangian

$$\begin{aligned}\delta_{1/m^2} = & -\left(\frac{1}{2m_c}\right)^2 \frac{1}{2}\Bigg(-\lambda_1 + \lambda_2 \\ & + (-i)^2 \frac{1}{2\sqrt{M_B M_D}} \int d^4x\, d^4y\, \langle B^*(v,\epsilon)| T\left[\mathcal{L}_b^{(1)}(x) \bar{b}_v c_v \mathcal{L}_c^{(1)}(y)\right] |D^*(v,\epsilon)\rangle\Bigg) \\ & + \mathcal{O}(1/m_c^3, 1/m_b^2, 1/(m_c m_b)),\end{aligned} \tag{80}$$

where $\mathcal{L}_Q^{(1)}$ is the first order Lagangian for the quark Q as given in (19) or (23) and M_B (M_D) are the masses of the B (D) meson in the heavy quark limit. Here we display only the largest contribution of order $1/m_c^2$; the complete expression, including the $1/m_b^2$ and $1/(m_c m_b)$ terms, may be found in [34, 35].

Thus the correction δ_{1/m^2} is given in terms of λ_1 defined in (27), λ_2 given in (28) and a non-local matrix element involving a time-ordered product. The problem concerning the determination of λ_1 has been considered already above; similarly it is not easy to obtain information on the matrix element involving the time-ordered product, and thus the corrections of order $1/m^2$ will finally limit our ability to determine the CKM matrix element V_{cb} in a model independent way, at least using the approach described here.

Various estimates for δ_{1/m^2} have been given in the literature. The first estimate of this correction has been given in [34] using the GISW model [36], which is based on a wave function for the light quark. In this work $\delta_{m^2} = -2\% \ldots - 3\%$ has been obtained. Another estimate with weaker assumptions yields $\delta_{m^2} = 0 \ldots - 5\%$ [35], but both estimates have been criticized recently as being too small. Based on heavy flavour sum rules it has been argued in [37] that the $1/m^2$ corrections can be quite large $\delta_{m^2} = 0\% \ldots - 8\%$ [37]. These various estimates indicate the size of the theoretical error involved in the determination of V_{cb} from the exclusive channel $B \to D^* \ell \bar{\nu}_\ell$; a generally accepted value for these corrections has been given recently [38]

$$\delta_{m^2} = -0.055 \pm 0.025 \tag{81}$$

from which one obtains

$$\xi_{A1}(1) = 0.91 \pm 0.03 \tag{82}$$

This result has been used to extract V_{cb} from CLEO [39] as well as from LEP data [40]. The values obtained are

$$|V_{cb}| = 0.0386 \pm 0.0019 \pm 0.0020 \pm 0.0014 \qquad \text{CLEO} \tag{83}$$

$$|V_{cb}| = 0.0392 \pm 0.0025 \pm 0.0027 \pm 0.0015 \qquad \text{ALEPH} \tag{84}$$

where (82) has been used. Note that the third error in $|V_{cb}|$ is due to the theoretical uncertainties, which by now almost match the experimental ones.

3.2 Transitions of the type Heavy $\to$ Light

Heavy quark symmetries may also be used to restrict the independent form factors appearing in heavy to light decays. For the decays of heavy mesons into light 0^- and 1^- particles heavy quark symmetries restrict the number of independent form factors to six, which is just the number needed to parametrize the semileptonic decays of this type. Furthermore, no absolute normalization of form factors may be obtained from heavy quark symmetries in the heavy to light case; only the relative normalization of B meson decays heavy to light transitions may be obtained from the corresponding D decays.

In general we shall discuss matrix elements of a heavy to light current which have the following structure

$$J = \langle A|\bar{\ell}\Gamma h_v|H(v)\rangle, \tag{85}$$

where Γ is an arbitrary Dirac matrix, ℓ is a light quark (u, d or s) and A is a state involving only light degrees of freedom.

Spin symmetry implies that the heavy quark index hooks directly the to the heavy quark index of the Dirac matrix of the current. Thus one may write for the transition matrix element (91)

$$\langle A|\bar{\ell}\Gamma h_v|H(v)\rangle = \text{ Tr } (\mathcal{M}_A \Gamma H(v)) \tag{86}$$

where the matrix $H(v)$ representing the heavy meson has been given in (52). The matrix $\mathcal{M}_A$ describes the light degrees of freedom and is the most general matrix which may be formed from the kinematical variables involved. Furthermore, if the energies of the particles in the state A are small, i.e. of the order of Λ_{QCD}, the matrix $\mathcal{M}_A$ does not depend on the heavy quark; in particular it does not depend on the heavy mass m_H. In the following we shall discuss some examples.

The first example is the heavy meson decay constant, where the state A is simply the vacuum state. The heavy meson decay constant is defined by

$$\langle 0|\bar{\ell}\gamma_\mu\gamma_5 h_v|H(v)\rangle = f_H m_H v_\mu, \tag{87}$$

and since $|A\rangle = |0\rangle$ the matrix $\mathcal{M}_0$ is simply the unit matrix times a dimensionful constant[2] and one has, using (86)

$$\langle 0|\bar{\ell}\gamma_\mu\gamma_5 h_v|H(v)\rangle = \kappa \text{ Tr } (\gamma\gamma_5 H(v)) = 2\kappa\sqrt{m_H} v_\mu. \tag{88}$$

As discussed above the constant κ does not depend on the heavy mass and thus one infers the well-known scaling law for the heavy meson decay constant from the last two equation

$$f_H \propto \frac{1}{\sqrt{m_H}} \tag{89}$$

Including the leading and subleading QCD radiative corrections one obtains a relation between f_B and f_D

$$f_B = \sqrt{\frac{m_c}{m_b}} \left(\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right)^{-6/25} \left[1 + 0.894 \frac{\alpha_s(m_c) - \alpha_s(m_b)}{\pi}\right] f_D \sim 0.69 f_D. \tag{90}$$

[2]Note that contributions proportional to $\not{v}$ may be eliminated using

$$H(v)\not{v} = -H(v).$$

The second example are transitions of a heavy meson into a light pseudoscalar meson, which we shall denote as π. The matrix element corresponding to (85) is

$$J_P = \langle \pi(p) | \bar{\ell} \Gamma h_v | H(v) \rangle, \tag{91}$$

where p is the momentum of the light quark,

The Dirac marix $\mathcal{M}_P$ for the light degrees of freedom appearing now in (86) depends on p and v. It may be expanded in terms of the sixteen independent Dirac matrices 1, γ_5, γ_μ, $\gamma_5\gamma_\mu$, and $\sigma_{\mu\nu}$ taking into account that it has to behave like a pseudoscalar. The form factors appearing in the decomposition of $\mathcal{M}_P$ depend on the variable $v \cdot p$, the energy of the light meson in the rest frame of the heavy one. In order to compare different heavy to light transition by employing heavy flavor symmetry this energy must be sufficiently small, since the typical scale for the light degrees of freedom has to be of the order of Λ_{QCD} to apply heavy quark symmetry[3]. For the case of a light pseudoscalar meson the most general decomposition of $\mathcal{M}_P$ is

$$\mathcal{M}_P = \sqrt{v \cdot p} A(\eta) \gamma_5 + \frac{1}{\sqrt{v \cdot p}} B(\eta) \gamma_5 \not{p}, \tag{92}$$

where we have defined the dimensionless variable

$$\eta = \frac{v \cdot p}{\Lambda_{QCD}}. \tag{93}$$

The form factors A and B are universal in the kinematic range of small energy of the light meson, i.e. where the momentum transfer to the light degrees of freedom is of the order Λ_{QCD}; in this region η is of order unity. This universality of the form factors may be used to relate various kinds of heavy to light transitions, e.g. the semileptonic decays like $D \to \pi e \nu$, $D \to K e \nu$ or $B \to \pi e \nu$ and also the rare decays like $B \to K \ell^+ \ell^-$ or $B \to \pi \ell^+ \ell^-$ where ℓ denotes an electron or a muon.

As an example we give the relations between exclusive semileptonic heavy to light decays. The relvant hadronic current for this case may be expressed in terms of two form factors

$$\begin{aligned} \langle \pi(p) | \bar{\ell} \gamma (1 - \gamma_5) h_v | H(v) \rangle &= F_1(v \cdot p) m_H v_\mu + F_2(v \cdot p) p_\mu \\ &= F_+(v \cdot p)(m_H v_\mu + p_\mu) + F_-(v \cdot p) q_\mu \end{aligned} \tag{94}$$

where

$$F_\pm(v \cdot p) = \frac{1}{2} \left(F_1(v \cdot p) \pm F_2(v \cdot p) \right) \tag{95}$$

Inserting this into (91) one may express $F_\pm$ in terms of the universal form factors A and B

$$F_1(v \cdot p) = F_+(v \cdot p) + F_-(v \cdot p) = -2\sqrt{\frac{v \cdot p}{m_H}} A(\eta) \tag{96}$$

$$F_2(v \cdot p) = F_+(v \cdot p) - F_-(v \cdot p) = -2\sqrt{\frac{m_H}{v \cdot p}} B(\eta) \tag{97}$$

From these relations one may read off the scaling of the form factors with the heavy mass which was already derived in [41].

[3]Note that in this case the variable $v \cdot p$ ranges between 0 and $m_H/2$ where we have neglected the pion mass. Thus at the upper end of phase space the variable $v \cdot p$ scales with the heavy mass and heavy quark symmetries are not applicable any more.

This may be used to normalize the semileptonic B decays into light mesons relative to the semileptonic D decays. One obtains

$$F_\pm^B(v\cdot p) = \frac{1}{2}\left(\sqrt{\frac{m_D}{m_B}} \pm \sqrt{\frac{m_B}{m_D}}\right) F_+^D(v\cdot p) + \frac{1}{2}\left(\sqrt{\frac{m_D}{m_B}} \mp \sqrt{\frac{m_B}{m_D}}\right) F_-^D(v\cdot p) \tag{98}$$

Note that F_+ for the B decay is expressed in terms of F_+ *and* F_- for the D decays. In the limit of vanishing fermion masses only F_+ contributes, which means that the F_- contribution to the rate is of the order of m_{lepton}/m_H. Thus it will be extremely difficult to determine experimentally.

The case of a heavy meson decaying into a light vector meson may be treated similarly. The matrix element for the transition of a heavy meson into a light vector meson (denoted generically as ρ in the following) is given again by (85) and is in this case

$$J_V = \langle \rho(p,\epsilon)|\bar\ell \Gamma h_v|H(v)\rangle. \tag{99}$$

Using (86) one has

$$\langle \rho(p,\epsilon)|\bar\ell \Gamma h_v|H(v)\rangle = \text{ Tr }\left(\mathcal{M}_V \Gamma H(v)\right), \tag{100}$$

where now the Dirac matrix $\mathcal{M}_V$ has to be a linear function of the polarization of the light vector meson.

The most general decomposition is given in terms of four dimensionless form factors

$$\mathcal{M}_V = \sqrt{v\cdot p}C(\eta)(v\cdot\epsilon) + \frac{1}{\sqrt{v\cdot p}}D(\eta)(v\cdot\epsilon)\not p + \sqrt{v\cdot p}E(\eta)\not\epsilon + \frac{1}{\sqrt{v\cdot p}}F(\eta)\not p\not\epsilon \tag{101}$$

where the variable η has been defined in (93).

Similar to the case of the decays into a light pseudoscalar meson (100) may be used to relate various exclusive heavy to light processes in the kinematic range where the energy of the outgoing vector meson is small. For example, the semileptonic decays $D\to\rho e\nu$, $D\to K^* e\nu$ and $B\to\rho e\nu$ are related among themselves and all of them may be related to the rare heavy to light decays $B\to K^*\ell^+\ell^-$ and $B\to\rho\ell^+\ell^-$ with $\ell = e,\mu$.

Data on these decays are still very sparse; there are first measurements of the decays $B\to\pi\ell\nu$ and $B\to\rho\ell\nu$ from CLEO [42], from which total rates may be obtained. From this one may extract a value of V_{ub} by employing form factor models, and the value given by CLEO is

$$|V_{ub}| = (3.3 \pm 0.2^{+0.3}_{-0.4} \pm 0.7)\times 10^{-3} \tag{102}$$

where the last uncertainty represents the variation of the result between different models. In order to perform a model independent determination along the lines discussed above a good measurement of the lepton energy spectra in these decays is needed.

Finally we comment on the heavy to light transitions of baryons. For the Λ-type heavy baryons (53) spin symmetry relates different polarizations of the same particle and thus imposes interesting constraints. Consider for example the matrix element of an operator $\bar\ell\Gamma h_v$ between a heavy Λ_Q and a light spin-1/2 baryon B_ℓ. It is described by only two form factors,

$$\langle B_\ell(p)|\bar\ell\Gamma h_v|\Lambda_Q(v)\rangle = \bar u_\ell(p)\{F_1(v\cdot p) + \not v F_2(v\cdot p)\}\Gamma u_{\Lambda_Q}(v). \tag{103}$$

Thus in this particular case spin symmetry greatly reduces the number of independent Lorentz-invariant amplitudes which describe the heavy to light transitions.

This has some interesting implications for exclusive semileptonic Λ_c decays. For the case of a left handed current $\Gamma = \gamma_\mu(1-\gamma_5)$, the semileptonic decay $\Lambda_c \to \Lambda \ell \bar{\nu}_\ell$ is in general parametrized in terms of six form factors

$$\begin{aligned}\langle \Lambda(p)|\bar{q}\gamma_\mu(1-\gamma_5)c|\Lambda_c(v)\rangle &= \bar{u}(p)\left[f_1\gamma_\mu + if_2\sigma_{\mu\nu}q^\nu + f_3 q^\mu\right]u(p') \\ &+ \bar{u}(p)\left[g_1\gamma_\mu + ig_2\sigma_{\mu\nu}q^\nu + g_3 q^\mu\right]\gamma_5 u(p'), \end{aligned} \tag{104}$$

where $p' = m_{\Lambda_c} v$ is the momentum of the Λ_c whereas $q = m_{\Lambda_c} v - p$ is the momentum transfer. From this one defines the ratio G_A/G_V by

$$\frac{G_A}{G_V} = \frac{g_1(q^2=0)}{f_1(q^2=0)}. \tag{105}$$

In the heavy c quark limit one may relate the six form factors f_i and g_i $(i = 1,2,3)$ to the two form factors F_j $(j = 1,2)$

$$f_1 = -g_1 = F_1 + \frac{m_\Lambda}{m_{\Lambda_c}} F_2 \tag{106}$$

$$f_2 = f_3 = -g_2 = -g_3 = \frac{1}{m_{\Lambda_c}} F_2 \tag{107}$$

from which one reads off $G_A/G_V = -1$. This ratio is acessible by measuring in semileptonic decays $\Lambda_c \to \lambda \ell \bar{\nu}_\ell$ the polarization variable α

$$\alpha = \frac{2G_A G_V}{G_A^2 + G_V^2} \tag{108}$$

which is predicted to be $\alpha = -1$ in the heavy c quark limit. The subleading corrections to the heavy c quark limit have been estimated and found to be small [43]

$$\alpha < -0.95, \tag{109}$$

and recent measurements yield

$$\alpha = -0.91 \pm 0.49 \quad \text{ARGUS [44]} \tag{110}$$

$$\alpha = -0.89^{+0.17+0.09}_{-0.11-0.05} \quad \text{CLEO[45]} \tag{111}$$

and are in satisfactory agreement with the theoretical predictions.

Recently the CLEO collaboration also measured the ratio of the form factors F_1 and F_2, averaged over phase space. Heavy quark symmetries do not fix this form factor ratio, at least not for a heavy to light decay, while for a heavy to heavy decay the form factor F_2 vanishes in the heavy mass limit for the final state quark. CLEO measures [46]

$$\left\langle \frac{F_2}{F_2} \right\rangle_{\text{phase space}} = -0.25 \pm 0.14 \pm 0.08 \tag{112}$$

which is in good agreement with model estimates [47].

4 The $1/m_Q$ Expansion in Inclusive Decays

For inclusive decays a $1/m_Q$ expansion is obtained for the rates by an approach similar to the one known from deep inelastic scattering [5]-[13]. The first step consists of an operator product expansion (OPE) which yields an infinite sum of operators with increasing dimension. The dimensions of the operators are compensated by inverse powers of a large scale, which is in general of the order of the heavy mass scale. The decay probability is then given as forward matrix elements of these operators between the state of the decaying heavy hadron; these matrix elements still have a mass dependence, which then may be extracted in terms of a $1/m_Q$ expansion using HQET as for exclusive decays.

The method described below also allows us to deal with inclusive non-leptonic processes and hence in principle opens the possibility for a calculation of lifetimes and branching fractions in the framework of the $1/m_Q$ expansion. This is remarkable, since non-leptonic processes are usually very hard to deal with, in particular the $1/m_Q$ expansion has not (yet ?) brought any success in the field of exclusive non-leptonic decays.

Applying the OPE to the energy spectra of the charged lepton in inclusive semileptonic decays of heavy mesons, the relevant expansion parameter is not $1/m_Q$, but rather $1/(m_Q-2E_\ell)$; the denominator is thus the energy release of the decay. In almost all phase space the energy release is of the order of the heavy mass; it is only in the endpoint region that it becomes small and hence the expansion breaks down. This problem may be fixed by a resummation of terms in the operator product expansion, which strongly resembles the summation corresponding to leading twist in deep inelastic scattering. Analogously to the parton-distribution function, a universal function appears, which determines all inclusive heavy-to-light decays.

4.1 Operator Product Expansion for Inclusive Decays

The effective Hamiltonian for a decay of a heavy (down-type) quark is in general linear in the decaying heavy flavoured quark

$$\mathcal{H}_{eff} = \bar{Q} R \tag{113}$$

where the operator R describes the decay products. In the following we shall consider semileptonic decays, for which

$$R_{sl} = \frac{G_F}{\sqrt{2}} V_{Qq}\, \gamma_\mu (1-\gamma_5) q\; (\bar{\nu}_\ell \gamma^\mu (1-\gamma_5) \ell), \tag{114}$$

where q is an up-type quark (c or u, since we shall consider b decays). Similarly, for non-leptonic decays the Cabbibo allowed contribution corresponds to

$$\begin{aligned} R_{nl} = \frac{G_F}{2\sqrt{2}} V_{Qq} V^*_{q'q''} \, & [(C_+(m_b) + C_-(m_b)) \gamma_\mu (1-\gamma_5) q \; (\bar{q}' \gamma^\mu (1-\gamma_5) q'') \\ & + (C_+(m_b) - C_-(m_b)) \gamma_\mu (1-\gamma_5) q'' \; (\bar{q}' \gamma^\mu (1-\gamma_5) q)] , \end{aligned} \tag{115}$$

where q' (q'') is a down-type (up-type) quark and V_{Qq} the corresponding CKM matrix element. The coefficients $C_\pm(m_b)$ are the QCD corrections obtained from the renormalization group running between M_W and m_b; in leading logarithmic approximation these coefficients are [48]

$$C_\pm(m_b) = [\frac{\alpha_S(M_W^2)}{\alpha_S(m_b^2)}]^{\gamma_\pm}, \text{ with } \gamma_+ = \frac{6}{33-2N_f} = -\frac{1}{2}\gamma_- \tag{116}$$

where $\alpha_s(\mu)$ is the onle-loop expression (43) for the running coupling coupling constant of QCD.

Finally, for radiative rare decays we have

$$R_{rare} = \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* C_7(m_b) \frac{e}{16\pi^2} m_b \sigma_{\mu\nu}(1+\gamma_5) s F^{\mu\nu} \tag{117}$$

where $C_7(m_b)$ is again a coefficient obtained from running between M_W and m_b. Its value is $C_7(m_b) \sim 0.3$, the corresponding analytical expression may be found in [49].

The inclusive decay rate for a heavy hadron H containing the quark Q may be related to a forward matrix element by

$$\begin{aligned}\Gamma \propto \sum_X (2\pi)^4 \delta^4(P_B - P_X) |\langle X|\mathcal{H}_{eff}|H(v)\rangle|^2 &= \int d^4x \, \langle H(v)|\mathcal{H}_{eff}(x)\mathcal{H}_{eff}^\dagger(0)|H(v)\rangle \quad (118)\\ &= 2 \text{ Im} \int d^4x \, \langle H(v)|T\{\mathcal{H}_{eff}(x)\mathcal{H}_{eff}^\dagger(0)\}|H(v)\rangle.\end{aligned}$$

where $|X\rangle$ is the final state, which is summed over to obtain the inclusive rate.

The matrix element appearing in (118) contains a large scale, namely the mass of the heavy quark. The first step towards a $1/m_Q$ expansion is to make this large scale explicit. This may be done by a phase redefinition. This leads to

$$\Gamma \propto 2 \text{ Im} \int d^4x \, e^{-im_Q vx} \, \langle H(v)|T\{\widetilde{\mathcal{H}}_{eff}(x)\widetilde{\mathcal{H}}_{eff}^\dagger(0)\}|H(v)\rangle \tag{119}$$

where

$$\widetilde{\mathcal{H}}_{eff} = \bar{Q}_v R \qquad Q_v = e^{-im_Q vx} Q \tag{120}$$

This relation exhibits the similarity between the cross-section calculation in deep inelastic scattering and the present approach to total rates. In deep inelastic scattering there appears a large scale, which is the momentum transfer to the leptons, while here the mass of the heavy quark appears as a large scale.

The next step is to perform an operator product expansion of the product of the two Hamiltonians. After the phase redefinition the remaining matrix element does not involve large momenta of the order of the heavy quark mass any more and hence a short-distance expansion becomes useful, if the mass m_Q is large compared to the scale $\bar{\Lambda}$ determining the matrix element. The next step is thus to perform an operator-product expansion, which has the general form

$$\int d^4x \, e^{im_Q vx} \, \langle H(v)|T\{\widetilde{\mathcal{H}}_{eff}(x)\widetilde{\mathcal{H}}_{eff}^\dagger(0)\}|H(v)\rangle = \sum_{n=0}^{\infty} \left(\frac{1}{2m_Q}\right)^n \hat{C}_{n+3}(\mu) \langle H(v)|\mathcal{O}_{n+3}|H(v)\rangle_\mu,$$

where $\mathcal{O}_n$ are operators of dimension n, with their matrix elements renormalized at scale μ, and $\hat{C}_n$ are the corresponding Wilson coefficients. These coefficients encode the short distance physics related to the heavy quark mass scale and may be calculated in perturbation theory. All long distance contributions connected to the hadronic scale $\bar{\Lambda}$ are contained in the matrix elements of the operators $\mathcal{O}_{n+3}$.

Still the matrix elements of $\mathcal{O}_{n+3}$ are not independent of the heavy quark mass scale, but this mass dependence may be expanded in powers of $1/m_Q$ by means of heavy quark effective theory. This is achived by expanding the heavy quark fields appearing in the operators $\mathcal{O}_n$ using (18) (or, equivalently, (22)) as well as the states by including the corrections to the Lagrangian given in (19) (or (23)) as time-ordered products. In this way the mass dependence of the total decay rate may be accessed completely within an expansion in $1/m_Q$.

The lowest-order term of the operator product expansion are the dimension-3 operators. Due to Lorentz invariance and parity there are only two combinations which may appear, namely $\bar{Q}_v \not{v} Q_v$ or $\bar{Q}_v Q_v$. Note that the Q_v operators differ from the full QCD operators only by a phase redefinition, and hence $\bar{Q}_v \not{v} Q_v = \bar{Q} \not{v} Q$ and $\bar{Q}_v Q_v = \bar{Q} Q$. The first combination is proportional to the Q-number current $\bar{Q}\gamma_\mu Q$, which is normalized even in full QCD, while the second differs from the first one only by terms of order $1/m_Q^2$

$$\bar{Q}_v Q_v = v_\mu \bar{Q}_v \gamma_\mu Q_v + \frac{1}{2m_Q^2} \bar{h}_v \left[(iD)^2 - (ivD)^2 + \frac{i}{2}\sigma_{\mu\nu} G^{\mu\nu} \right] h_v + \mathcal{O}(1/m_Q^3). \tag{121}$$

where $G_{\mu\nu}$ is the gluon field strength.

Thus the matrix elements of the dimension-3 contribution is known to be normalized; in the standard normalization of the states this implies

$$\langle H(v)|\mathcal{O}_3|H(v)\rangle = \langle H(v)|\bar{Q}_v \not{v} H_v|B(v)\rangle = 2m_H \tag{122}$$

where m_H is the mass of the heavy hadron. To lowest order in the heavy mass expansion we may furthermore replace $m_B = m_Q$ and hence we may evaluate the leading term in the $1/m_q$ expansion without any hadronic uncertainty. Generically the dimension-3 contribution yields the free quark decay rate. This has been previously used as a model for inclusive decays, but now it turns out to be the first term in a systematic $1/m_Q$ expansion of total rates.

A dimension-four operators contains an additional covariant derivative, and thus one has matrix elements of the type

$$\langle H(v)|\mathcal{O}_4|H(v)\rangle \propto \langle H(v)|\bar{Q}_v \Gamma D_\mu Q_v|H(v)\rangle = \mathcal{A}_\Gamma v_\mu \tag{123}$$

Since the equations of motion apply for this tree level matrix element, one finds that the constant $\mathcal{A}_\Gamma$ has to vanish, and thus there are no dimension-four contributions. This statement is completely equaivalent to Lukes theorem [31], since we are considering a forward matrix element, i.e. a matrix element at zero recoil [35].

The first non-trivial non-perturbative contribution comes from dimension-5 operators and are of order $1/m_Q^2$. For mesonic decays there are only the two parameters λ_1 and λ_2 given in (27) and (28), which correspond to matrix elements of the subleading terms of the Lagrangian. They parametrize the non-perturbative input in the order $1/m_Q^2$. For Λ_Q-type baryons the parameter λ_2 vanishes due to heavy quark spin symmetry, while the kinetic energy parameter λ_1 is non-zero as well. In the framework of the $1/m_Q$ expansion this leads to a difference in lifetimes between mesons and baryons.

4.2 Calculation of Total Decay Rates

In this subsection we shall collect the results for the total rates including the first non-trivial non-perturbative correction.

Inserting R_{sl} as given in (114) one obtains for the total inclusive semileptonic decay rate $B \to X_c \ell \nu$

$$\Gamma(B \to X_c \ell \nu) = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \left[\left(1 + \frac{\lambda_1}{2m_c^2}\right) f_1\left(\frac{m_c}{m_b}\right) - \frac{9\lambda_2}{2m_c^2} f_2\left(\frac{m_c}{m_b}\right) \right], \tag{124}$$

where the two f_j are phase-space functions

$$\begin{aligned} f_1(x) &= 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \log x, \\ f_2(x) &= 1 - \frac{8}{3}x^2 - 8x^4 + 8x^6 + \frac{5}{3}x^8 + 8x^4 \log x. \end{aligned} \tag{125}$$

The result for $B \to X_u \ell \nu_\ell$ is obtained from (124) as the limit $m_c \to 0$ and the replacement $V_{cb} \to V_{ub}$

$$\Gamma(B \to X_u \ell \nu) = \frac{G_F^2 m_b^5}{192\pi^3} |V_{ub}|^2 \left[1 + \frac{\lambda_1 - 9\lambda_2}{2m_b^2}\right]. \tag{126}$$

As it has been discussed above, the leading non-perturbative corrections in (124) and (126) are parametrized by λ_1 and λ_2. Estimates for these parameters have been discussed in section 2; in order to estimate the total effect of the non-perturbative effects we shall insert a range of values $-0.3 > \lambda_1 > -0.6$ GeV^2; from this we obtain

$$\frac{\lambda_1 - 9\lambda_2}{2m_b^2} \sim -(3 \cdots 4)\% \tag{127}$$

This means that the non-perturbative contributions are small, in particular compared to the perturbative ones, which have been calculated some time ago [50, 51]. For the decay $B \to X_u \ell \bar{\nu}_\ell$ the lowest order QCD corrections are given by

$$\Gamma(B \to X_u \ell \bar{\nu}_\ell) = \frac{G_F^2 m_b^5}{192\pi^3} |V_{ub}|^2 \left[1 + \frac{2\alpha}{3\pi}\left(\frac{25}{4} - \pi^2\right)\right] = 0.85 |V_{ub}|^2 \Gamma_b, \tag{128}$$

and thus the typical size of QCD radiative corrections is of the order of ten to twenty percent.

Similarly, one obtains the result for non-leptonic decays as

$$\Gamma(B \to X_c) = 3\frac{G_F^2 m_b^5}{192\pi^3} \left\{ A_1 f_1 \left(\frac{m_c}{m_b}\right) \left[1 + \frac{1}{2m_b^2}(\lambda_1 - 9\lambda_2)\right] - 48 A_2 f_3 \left(\frac{m_c}{m_b}\right) \frac{1}{2m_b^2} \lambda_2 \right\} \tag{129}$$

where the coefficients A_i are given by combination of the Wilson coefficents $C_\pm(m_b)$ (116)

$$A_1 = \frac{1}{3}[C_-^2(m_b) + 2C_+^2(m_b)], \qquad A_2 = \frac{1}{6}[C_+^2(m_b) - C_-^2(m_b)], \tag{130}$$

and $f_3(x) = (1 - x^2)^3$ is another phase space function. Again the non-perturbative corrections turn out to be small, in the region of a few percent compared to the leading term, and the perturbative corrections turn out to be much larger than this.

Finally, for the rare decay $B \to X_s \gamma$ one may as well calculate the non-perturbative contribution in terms of λ_1 and λ_2. One obtains

$$\Gamma(B \to X_s \gamma) = \frac{\alpha G_F^2}{16\pi^4} m_b^5 |V_{ts} V_{td}^*|^2 |C_7(m_b)|^2 \left[1 + \frac{1}{2m_b^2}(\lambda_1 - 9\lambda_2)\right], \tag{131}$$

and the relative size of the non-perturbative corrections is the same as in the $B \to X_u \ell \bar{\nu}_\ell$ decays.

Typically the non-pertubative corrections are much smaller than the radiative corrections. The only exception is the endpoint region of lepton energy spectra which receives both large perturbative as well as non-perturbative corrections. However, this is only a small region in phase space and the corections to the total rates remain moderate.

4.3 Lifetimes of Heavy Hadrons

The subject of heavy hadron lifetimes is strongly related to non-leptonic processes, which have been considered already some time ago [52]-[55]; however the application of the $1/m_Q$ expansion

———— FIG 1 ————-

Figure 1: Diagrams for non-leptonic decays of B mesons. The upper diagram corresponds to the leading term in the $1/m_Q$ expansion, the middle one to weak anihilation, and the lower one to Pauli interference. Diagrams taken from [56].

has turned many assumptions into quantitative arguments. As outlined in the last section the $1/m_Q$ expansion allows to calculate total rates, even for non-leptonic processes, and hence a QCD-based calculation of lifetimes becomes possible. A recent review of this subject is given in [56].

Studying the formulae obtained in the $1/m_Q$ expansion up to order $1/m_Q^2$ one finds that lifetime differences between B mesons do not occur up to this level of the expansion; in other words, any difference between the $B^\pm$ and the B^0 or the B_s lifetimes are induced by effects of the order $1/m_b^3$. These effects are due to dimension six operators of the four-quark form

$$\mathcal{O}_6 = (\bar{b}\Gamma q)(\bar{q}\Gamma b). \tag{132}$$

In fig.1 the relevant diagramms are shown. The two lower ones yield dimension six operators of the from (132); comparing to the terminology used in phenomenological models the middle diagram is called weak annihilation (WA) and the lower one Pauli interference (PI).

The four quark operators appearing at order $1/m_Q^3$ are usually estimated applying the vacuum insertion assumption, althought this procedure has been criticized recently [57]. It amounts to replace

$$\langle B|(\bar{b}\Gamma q)(\bar{q}\Gamma b)|B\rangle \to \langle B|(\bar{b}\Gamma q)|0\rangle\langle 0|(\bar{q}\Gamma b)|B\rangle$$

and hence the parameter entering the estimates of the lifetime differences is the decay constant f_B.

The WA piece has been considered in [53, 58] and has been found to be small, of the order of one percent. The dominant contribution comes from the PI diagram. For the lifetime of the B^- one obtains [56]

$$\Gamma(B^-) = \Gamma_{1/m_Q^2}(B) + \Delta\Gamma_{PI}(B^-) \tag{133}$$

where the PI contribution is

$$\Delta\Gamma_{PI}(B^-) = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2 \cdot 24\pi^2 \frac{f_B^2}{M_B^2}[C_+^2(m_b) - C_-^2(m_b) + \frac{1}{N_C}(C_+^2(m_b) + C_-^2(m_b))] \tag{134}$$

where $C_\pm$ have been given in (116). QCD radiative corrections change the sign of the PI contribution; the constructive interference at the scale M_W is turned into a destructive one at the scale m_b prolonging the lifetime of the B^- relative to the B_0. Using HQET the running of the coefficients below m_b has also been calculated; it has been found that the destructive interference is amplified by the running below m_b, since

$$\begin{aligned}\Delta\Gamma_{PI}(B^-) = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2 \cdot 24\pi^2 \frac{f_B^2}{M_B^2}\kappa^{-4} \quad &[(C_+^2(m_b) - C_-^2(m_b))\kappa^{9/2} + \frac{C_+^2(m_b) + C_-^2(m_b)}{3} \\ &-\frac{1}{9}(\kappa^{9/2} - 1)(C_+^2(m_b) - C_-^2(m_b))],\end{aligned} \tag{135}$$

where

$$\kappa = [\frac{\alpha_S(\mu_{had}^2)}{\alpha_S(m_b^2)}]^{1/b}, \text{ with } b = 11 - \frac{2}{3}n_F \tag{136}$$

and μ_{had} is some small hadronic scale, which may be defined e.g. by $\alpha_s(\mu_{had}) = 1$. Putting in numbers the result is [56]

$$\frac{\tau(B^-)}{\tau(B_d)} = 1 + 0.05 \cdot \frac{f_B^2}{(200 \text{ MeV})^2} \tag{137}$$

Hence the $1/m_Q$ expansion (in combination with vacuum insertion) predicts a slightly longer lifetime for the charged B meson compared to the neutral one. However, this is based on the vacuum insertion assumption which has been reconsidered recently in [57]; it is claimed that possible non-factorizable contributions may be enhanced by large prefactors thereby invalidating (137).

However, the estimate (137) is compatible with data; the latest compilation yields [59]

$$\frac{\tau(B^+)}{\tau(B^0)} = 1.019 \pm 0.048 \tag{138}$$

Applying the $1/m_Q$ expansion for the lifetimes also to the D system one finds large corrections. From the experimental side it is knwon that the lifetime differences are very large and that the naive parton model expectation fails by a large margin. This may be taken as an indication that the c quark mass is indeed not large enough to justify a $1/m_Q$ expansion, at least for the non-leptonic processes.

The lifetimes of the two neutral B mesons are equal up to terms of order $1/m_b^3$, hence one expects

$$\bar{\tau}(B_d) = \bar{\tau}(B_s) \tag{139}$$

to a good accuracy. Here $\bar{\tau}$ denotes the average lifetime of the two mass eigenstates of the $B^0 - \bar{B}^0$ system. While the lifetimes in the B_d system are practically the same such that the difference may be neglected, it has been pointed out that this is not necessarily the case in the B_s system; the lifetime difference has been estimated in [60] to be

$$\frac{\Delta\Gamma(B_s)}{\bar{\Gamma}(B_s)} = \frac{\Gamma(B_{s,short}) - \Gamma(B_{s,long})}{\bar{\Gamma}(B_s)} = 0.18 \cdot \frac{f_{B_s}^2}{(200 \text{ MeV})^2} \tag{140}$$

Hence this difference may be as large twenty percent and thus it cannot be neglected any more e.g. in an analysis of B_s-$\bar{B}_s$ mixing.

Finally we shall consider the b baryon lifetimes. Here we expect the differences between the meson and the baryon lifetimes to be of order $1/m^2$, since the matrix elements of the kinetic energy operator as well as of the chromomagnetic moment operator are different between baryons and mesons; in particular, due to spin symmetry the matrix element of the chromomagnetic moment operator vanishes for Λ-type baryons, since the light degrees of freedom are in a spin-0 state. Probably only the Λ-type baryons will be stable against electromagnetic and strong decays, and hence their lifetime will be determined by the weak decay of the heavy quark.

Theoretical estimates of the Λ_b lifetime have been attempted in [56, 57] and yield the expectation that

$$\frac{\tau(\Lambda_b)}{\tau(B_d)} \gtrsim 0.9. \tag{141}$$

This has to be confronted with the recent data [59]

$$\frac{\tau(\Lambda_b)}{\tau(B_d)} = 0.80 \pm 0.05. \tag{142}$$

Thus the lifetime is slightly below the expectation, although this is not yet a significant deviation from the prediction. In particular, older data indicated that this lifetime ratio could have been as low as 0.7; such a low value would clearly indicate a theoretical problem in the $1/m_Q$ expansion of the lifetimes, but the new data are in better agreement with the theoretical expectations.

4.4 Lepton Energy Spectra

The method of the operator-product expansion may also be used to obtain the non-perturbative corrections to the charged lepton energy spectrum [8]. In this case the operator product expansion is applied not to the full effective Hamiltonian, but rather only to the hadronic currents. The rate is written as a product of the hadronic and leptonic tensor

$$d\Gamma = \frac{G_F^2}{4m_B}|V_{Qq}|^2 W_{\mu\nu}\Lambda^{\mu\nu} d(PS), \tag{143}$$

where $d(PS)$ is the phase-space differential. The short-distance expansion is then performed for the two currents appearing in the hadronic tensor. Redefining the phase of the heavy-quark fields as in (11) one finds that the momentum transfer variable relevant for the short-distance expansion is $m_Q v - q$, where q is the momentum transfer to the leptons.

The structure of the expansion for the spectrum is identical to the one of the total rate. The contribution of the dimension-3 operators yields the free-quark decay spectrum, there are no contributions from dimension-4 operators, and the $1/m_b^2$ corrections are parametrized in terms of λ_1 and λ_2. Calculating the spectrum for $B \to X_c \ell \nu$ yields [9]-[12]

$$\begin{aligned}\frac{d\Gamma}{dy} = \frac{G_F^2 \mid V_{cb}\mid^2 m_b^5}{192\pi^3}\Theta(1-y-\rho)y^2 \Big[&\Big\{3(1-\rho)(1-R^2) - 2y(1-R^3)\Big\} \\ &+ \frac{\lambda_1}{[m_b(1-y)]^2}(3R^2 - 4R^3) - \frac{\lambda_1}{m_b^2(1-y)}(R^2 - 2R^3) \\ &- \frac{3\lambda_2}{m_b^2(1-y)}(2R + 3R^2 - 5R^3) + \frac{\lambda_1}{3m_b^2}[5y - 2(3-\rho)R^2 + 4R^3] \\ &+ \frac{\lambda_2}{m_b^2}[(6+5y) - 12R - (9-5\rho)R^2 + 10R^3]\Big] + \mathcal{O}\left[(\Lambda/[m_b(1-y)])^3\right]\end{aligned} \tag{144}$$

where we have defined

$$\rho = \left(\frac{m_c}{m_b}\right)^2 \quad R = \frac{\rho}{1-y} \tag{145}$$

and

$$y = 2E_\ell/m_b \tag{146}$$

is the rescaled energy of the charged lepton.

This expression is somewhat complicated, but it simplifies for the decay $B \to X_u \ell \nu$ since then the mass of the quark in the final state may be neglected. One finds

$$\frac{d\Gamma}{dy} = \frac{G_F^2 \mid V_{ub}\mid^2 m_b^5}{192\pi^3}\Bigg[\left(2y^2(3-2y) + \frac{10y^2}{3}\frac{\lambda_1}{m_b^2} + 2y(6+5y)\frac{\lambda_2}{m_b^2}\right)\Theta(1-y)$$

———- FIG 2 ————–

Figure 2: The electron spectrum for free quark $b \to c$ decay (dashed line), free quark $b \to u$ decay (grey line), and $B \to X_c e \bar{\nu}_e$ decay including $1/m_b^2$ corrections (solid line) with $\lambda_1 = -0.5$ GeV^2 and $\lambda_2 = 0.12$ GeV^2. The figure is from [11].

$$-\frac{\lambda_1 + 33\lambda_2}{3m_b^2}\delta(1-y) - \frac{\lambda_1}{3m_b^2}\delta'(1-y)\Bigg] \tag{147}$$

Figure 2 shows the distributions for inclusive semileptonic decays of B mesons. The spectrum close to the endpoint, where the lepton energy becomes maximal, exhibits a sharp spike as $y \to y_{max}$. In this region we have we have

$$\frac{d\Gamma}{dy} \propto \Theta(1-y-\rho)\left[2 + \frac{\lambda_1}{(m_Q(1-y))^2}\left(\frac{\rho}{1-\rho}\right)^2\left\{3 - 4\left(\frac{\rho}{1-\rho}\right)\right\}\right], \tag{148}$$

which behaves like δ-functions and its derivatives as $\rho \to 0$, which can be seen in (147). This behaviour indicates a breakdown of the operator product expansion close to the endpoint, since for the spectra the expansion parameter is not $1/m_Q$, but rather $1/(m_Q - qv)$, which becomes $1/(m_Q[1-y])$ after the integration over the neutrino momentum. In order to obtain a description of the endpoint region, one has to perform some resummation of the operator product expansion.

4.5 Resummation in the Endpoint Region

Very close to the endpoint of the inclusive semileptonic decay spectra only a few resonances contribute. In this resonance region one may not expect to have a good description of the spectrum using an approach based on parton-hadron duality; here a sum over a few resonances will be appropriate.

In the variable y the size of this resonance region is however of the order of $(\bar{\Lambda}/m_Q)^2$ and thus small. In a larger region of the order $\bar{\Lambda}/m_Q$, which we shall call the endpoint region, many resonances contribute and one may hope to describe the spectrum in this region using parton-hadron duality.

It has been argued in [61] that the δ-function-like singularities appearing in (147) may be reinterpreted as the expansion of a non-perturbative function describing the spectrum in the endpoint region. Keeping only the singular terms of (147) we write

$$\frac{1}{\Gamma_b}\frac{d\Gamma}{dy} = 2y^2(3-2y)S(y), \tag{149}$$

where

$$S(y) = \Theta(1-y) + \sum_{n=0}^{\infty} a_n \delta^{(n)}(1-y) \tag{150}$$

is a non-perturbative function given in terms of the moments a_n of the spectrum, taken over the endpoint region. These moments themselves have an expansion in $1/m_Q$ such that $a_n \sim 1/m_Q^{n+1}$, and we shall consider only the leading term in the expansion of the moments, corresponding to the most singular contribution to the endpoint region.

Comparing (147) with (149) and (150) one obtains that

$$a_0 = \int dy(S(y) - \Theta(1-y)) = \mathcal{O}(1/m_Q^2) \tag{151}$$

$$a_1 = \int y(S(y) - \Theta(1-y)) = -\frac{\lambda_1}{3m_Q^2} \tag{152}$$

where the integral extends over the endpoint region.

The non-perturbative function implements a resummation of the most singular terms contributing to the endpoint and, in the language of deep inelastic scattering, corresponds to the leading twist contribution. This resummation has been studied in QCD [63, 62] and the function $S(y)$ may be related to the distribution of the light cone component of the heavy quark residual momentum inside the heavy meson. The latter is a fundamental function for inclusive heavy-to-light transitions, which has been defined in [62]

$$f(k_+) = \frac{1}{2M_H}\langle H(v)|\, \bar{h}_v\, \delta(k_+ - iD_+)\, h_v\, |H(v)\rangle, \tag{153}$$

where $k_+ = k_0 + k_3$ is the positive light cone component of the residual momentum k. The relation between the two functions S and f is given by

$$S(y) = \frac{1}{m_Q} \int\limits_{-m_Q(1-y)}^{\bar{\Lambda}} dk_+ f(k_+) \tag{154}$$

from which we infer that the n^{th} moment of the endpoint region is given in terms of the matrix element $\langle H(v)|\bar{h}_v(iD_+)^n h_v|H(v)\rangle$.

The function f is a universal distribution function, which appears in all heavy-to-light inclusive decays; another example is the decay $B \to X_s\gamma$ [64, 62], where this function determines the photon-energy spectrum in a region of order $1/m_Q$ around the K^* peak.

In principle f has to be determined by other methods than the $1/m_Q$ expansion, e.g. from lattice calculations or from a model, or it has to be determined from experiment by measuring the photon spectrum in $B \to X_s\gamma$ or the lepton spectrum in $B \to X_u\ell\bar{\nu}$. In the context of the model ACCMM model [51] f has been calculated in [65].

Some of the properties of f are known. Its support is $-\infty < k_+ < \bar{\Lambda}$, it is normalized to unity, and its first moment vanishes. Its second moment is given by a_1, and its third moment has been estimated [62, 35]. A one-parameter model for f has been suggested in [63], which incorporates the known features of f

$$f(k_+) = \frac{32}{\pi^2\bar{\Lambda}}\,(1-x)^2 \exp\left\{ -\frac{4}{\pi}\,(1-x)^2 \right\} \Theta(1-x), \tag{155}$$

where $x = k_+/\bar{\Lambda}$, and the choice $\bar{\Lambda} = 570$ MeV yields reasonable values for the moments. In fig. 3 we show the spectrum for $B \to X_u\ell\nu_\ell$ using the ansatz (155).

Including the non-perturbative effects yields a reasonably behaved spectrum in the endpoint region and the δ-function-like singularities have disappeared. Furthermore, the spectrum now extends beyond the parton model endpoint; it is shifted from $E_\ell^{max} = m_Q/2$ to the physical endpoint $E_\ell^{max} = M_H/2$, since f is non-vanishing for positive values of $k_+ < \bar{\Lambda} = M_H - m_Q$.

———- FIG 3 —————

Figure 3: Charged-lepton spectrum in $B \to X_u \ell \bar{\nu}$ decays. The solid line is (149) with the ansatz (155), the dashed line shows the prediction of the free-quark decay model. The figure is from [63].

5 Conclusions

The development of the field of heavy quark physics has been indeed remarkable over the last few years, experimentally as well as theoretically. From the experimental side, the progress in the technology of detectors (e.g. silicon vertex detectors) opened the possibility to study b physics even at machines which originally were not designed for this kind of research. In this way also the high energy colliders (in particular LEP and TEVATRON) could contribute substantially in this area, since they allow to measure states (such as the B_s and the b flavoured baryons) which lie above the threshold of the $\Upsilon(4s)$-B-factories.

From the theoretical side the heavy quark limit and HQET brought an important success, since it provides a model independent and QCD based framework for the description of processes involving heavy quarks. The effective theory approach has originally been formulated for exclusive decays but in the past few years a heavy mass expansion has been set up also for inclusive transitions.

As far as exclusive heavy to heavy decays are concerned, the additional symmetries of the heavy mass limit restrict the number of non-perturbative functions in a model independent way; furthermore, heavy quark symmetries fix the absolute normalization of some of the transition amplitudes at the point of maximum momentum transfer. Phenomenologically this has improved our knowledge on the CKM matrix element V_{cb} dramatically; with the value $|V_{cb}| = (39.5 \pm 2.0) \times 10^{-3}$ the relative precision of this CKM matrix element is now about 5% and thus at a level of the precision with which the Cabbibo angle is known.

In heavy to light decays heavy quark symmetries do not work as efficiently; in this case only the relative normalization of B decays versus the corresponding D decays may be obtained. From the experimental side there are first measurements of $B \to \pi \ell \nu$ and $B \to \rho \ell \nu$ from the CLEO collaboration and an extraction of the CKM matrix element V_{ub} from these processes is still to some extent model dependent. The latest value for this CKM matrix element is $V_{ub} = (3.3 \pm 0.2^{+0.3}_{-0.4} \pm 0.7) \times 10^{-3}$ where the last error is due to the model dependence.

HQET does not yet have much to say about exclusive non-leptonic decays; even for the decays $B \to D^{(*)} D_s^{(*)}$, which involves three heavy quarks, heavy quark symmetries are not sufficient to yield useful relations between the decay rates [66]. Of course, with additional assumptions such as factorization one can go ahead and relate the non-leptonic decays to the semileptonic ones; however, this is a very strong assumption and it is not clear in what sense factorization is an approximation. On the other side, the data on the non-leptonic B decays support factorization, and first attempts to understand this from QCD and HQET have been untertaken [67]; however, the problem of the exclusive non-leptonic decays still needs clarification and hopefully the heavy mass expansion will also be useful here.

The $1/m_Q$ expansion obtained from the OPE and HQET offers the unique possibility to calculate the transition rates for inclusive decays in a QCD based and model independent framework. The leading term of this expansion is always the free quark decay, and the first non-trivial corrections are in general given by the mean kinetic energy of the heavy quark inside the heavy hadron λ_1 and the matrix element λ_2 of the chromomagnetic moment operator.

The method also allows us to calculate differential distributions, such as the charged lepton energy spectrum in inclusive semileptonic decays of heavy hadrons. For this case, the expansion parameter is the inverse of the energy release $m_b - 2E_\ell$, where E_ℓ is the lepton energy. Close to the endpoint, the energy release is small and thus the expansion in its inverse powers becomes useless. In this kinematic region one may partially resume the $1/m_Q$ expansion, obtaining a result closely analogous to the leading twist term in deep inelastic scattering. Particularly in the endpoint region a non-perturbative function is needed which corresponds to the parton distributions parametrizing the deep inelastic scattering.

The leading term of the $1/m_Q$ expansion is the free quark decay, and the result for the semileptonic branching fraction in this approximation has been well known for some time. The first non-perturbative corrections turn out to be quite small and hence the main corrections are the perturbative QCD corrections, where in a recent calculation also the effects of finite charm quark mass have been taken into account [68, 69]. The radiative corrections lower the semileptonic branching fraction somewhat compared to the parton model.

There has been some discussion on the issue of the semileptonic branching fraction triggered by the fact that the data used to be as low as $Br(B \to X\ell\nu) \sim 10.5\%$ with a relative error of about ten percent. Such a low value for the semileptonic branching fraction in combination with tha charm counting in B decays would indicate some theoretical problem; however, the recent LEP data yield a value of $Br(B \to X\ell\nu) = (11.5 \pm 0.3)\%$ which is compatible with the theoretical expectations.

The expansion in powers of the inverse quark mass has become the standard tool in heavy quark physics and with the forthcomming experiments one may expect a strong improvement in our knowledge of the CKM sector of the SM, in particular a test of CP violation as it is encoded in the CKM matrix of the SM.

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Figure Captions

Figure 1 Diagrams for non-leptonic decays of B mesons. The upper diagram corresponds to the leading term in the $1/m_Q$ expansion, the middle one to weak anihilation, and the lower one to Pauli interference. Diagrams taken from [56].

Figure 2 The electron spectrum for free quark $b \to c$ decay (dashed line), free quark $b \to u$ decay (grey line), and $B \to X_c e \bar{\nu}_e$ decay including $1/m_b^2$ corrections (solid line) with $\lambda_1 = -0.5$ GeV^2 and $\lambda_2 = 0.12$ GeV^2. The figure is from [11].

Figure 3 Charged-lepton spectrum in $B \to X_u \ell \bar{\nu}$ decays. The solid line is (149) with the ansatz (155), the dashed line shows the prediction of the free-quark decay model. The figure is from [63].