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Structure Functions in Semihadronic Tau Decays

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We review a variety of topics related to hadronic structure functions in exclusive semihadronic tau decays. We introduce the concept of structure functions and summarize the most important concepts. We then calculate the decay $\tau \to 3\pi\nu_\tau$ for very small hadronic invariant mass to one loop in Chiral Perturbation Theory. New interesting features emerge with respect to the known results at tree level, in particular for the structure functions w_D and w_E. We discuss the prospects for experimental verification of our predictions. Finally, we discuss various issues at higher Q^2, related to hadronic resonance physics. Here we consider 2π, πK, 3π, $K\pi K$ and $\pi K\pi$ hadronic final states.

1. Overview

In this note we will present a variety of important topics related to differential distributions in semihadronic τ decays. First, in Sec. 2, we will introduce the concept of structure functions as a convenient tool to extract the information encoded in angular and invariant mass distributions. Then in Sec. 3, we will calculate the decay $\tau \to 3\pi\nu_\tau$ to one loop in Chiral Perturbation Theory. Finally, in Sec. 4, we will discuss various issues at higher Q^2 (resonance physics), considering 2π, πK, 3π, $K\pi K$ and $\pi K\pi$ final states.

2. Hadronic Structure Functions

Consider two and three meson decay modes of the τ,

$$\tau \to \left\{ \begin{array}{c} h_1(p_1)h_2(p_2) \\ h_1(p_1)h_2(p_2)h_3(p_3) \end{array} \right\} + \nu_\tau \tag{1}$$

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The amplitudes for these decays can be written as

$$\mathcal{M} = V_{\rm CKM}\frac{G_F}{\sqrt{2}} L_\mu H^\mu \tag{2}$$

where

$$L_\mu = \overline{u}_\nu \gamma_\mu \gamma_- u_\tau \tag{3}$$

and

$$H^\mu = \left\{ \begin{array}{c} \langle h_1 h_2 | V^\mu - A^\mu | 0\rangle \\ \langle h_1 h_2 h_3 | V^\mu - A^\mu | 0\rangle \end{array} \right. \tag{4}$$

The amplitudes can be parametrized by introducing form factors. In the general two meson case, there are two such form factors,

$$\begin{aligned} \langle h_1 h_2 | V^\mu - A^\mu | 0\rangle \quad &= \quad \underbrace{F_V(Q^2) T^{\mu\nu}(p_1 - p_2)_\nu}_{J^P=1^-} \\ &+ \quad \underbrace{F_S(Q^2) Q^\mu}_{J^P=O+} \end{aligned} \tag{5}$$

In the general three meson case, there are four of them,

$\langle h_1 h_2 h_3 | V^\mu - A^\mu | 0\rangle$

$$
\begin{aligned}
&= \underbrace{[(p_1-p_3)_\nu F_1 + (p_2-p_3)_\nu F_2]T^{\mu\nu}}_{J^P=1^+} \\
&+ \underbrace{i\epsilon^{\mu\alpha\beta\gamma} p_{1\alpha} p_{2\beta} p_{3\gamma} F_3}_{J^P=1^-} \\
&+ \underbrace{Q^\mu F_4}_{J^P=0^-} \qquad (6)
\end{aligned}
$$

where the F_i depend on three invariants, e.g. $F_i = F_i(Q^2, s_1, s_2)$. We have used

$$
\begin{aligned}
Q^\mu &:= \begin{cases} p_1^\mu + p_2^\mu \\ p_1^\mu + p_2^\mu + p_3^\mu \end{cases} \\
s_1 &:= (p_2+p_3)^2 \quad \text{and cyclic} \\
T^{\mu\nu} &:= g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2} \qquad (7)
\end{aligned}
$$

In specific cases, there are various simplifications. If the two mesons are two pions, $h_1 h_2 = \pi^- \pi^0$, then the vector current is conserved and the scalar form factor vanishes, $F_S \equiv 0$ for $m_u = m_d$. In the three pion case, $h_1 h_2 h_3 = \pi^-\pi^-\pi^+$ or $\pi^0\pi^0\pi^-$, Bose symmetry relates F_2 to F_1, via $F_2(Q^2, s_1, s_2) = F_1(Q^2, s_2, s_1)$. G parity conservation requires $F_3 \equiv 0$ for $m_u = m_d$, and PCAC requires $F_4 \equiv 0$ for $m_u = m_d = 0$.

So we have seen the the hadronic matrix element for two (three) mesons in the final state is characterized by two (four) complex functions of one (three) hadronic invariants. For experimental analyses, it turns out to be very useful to trade the two (four) complex functions for four (sixteen) real valued "structure functions" W_X, which are defined from the hadronic tensor $H^{\mu\nu}$ in the hadronic rest frame [1],

$$
H^{\mu\nu} := H^\mu H^{\nu\star} \qquad (8)
$$

For the precise definitions, the reader is referred to [1], the main points are summarized in Tab. 1. The differential decay rate is given by [1]:

$$
d\Gamma = \frac{G_F^2}{4M_\tau} |V_{\mathrm{CKM}}|^2 \sum_X L_X W_X \;\; dPS^{(4)} \qquad (9)
$$

Note that W_A, W_B and W_{SA} alone determine $d\Gamma/dQ^2$,

$$
\frac{d\Gamma(\tau \to h_1 h_2 h_3 \nu_\tau}{dQ^2} \propto \frac{(M_\tau^2 - Q^2)^2}{Q^4}
$$

Table 1
The structure functions

	$H^\mu \longrightarrow$		
$H^{\nu\star}\downarrow$	$J^P = 1^+$	$J^P = 1^-$	$J = 0$
$J^P = 1^+$	$\mathbf{W_A}$ $W_C W_D W_E$		
$J^P = 1^-$	$W_F W_G$ $W_H W_I$	$\mathbf{W_B}$	
$J = 0$	$W_{SB} W_{SC}$ $W_{SD} W_{SG}$	$W_{SF} W_{SG}$	$\mathbf{W_{SA}}$
		$\underbrace{\hphantom{xxxxxxxx}}_{h_1 h_2}$	
	$\underbrace{\hphantom{xxxxxxxxxxxxxxxx}}_{h_1 h_2 h_3}$		

$$
\times \quad \int ds_1\, ds_2 \left\{ (1 + \frac{Q^2}{M_\tau^2}) \frac{W_A + W_B}{6} + \frac{W_{SA}}{2} \right\}. \qquad (10)
$$

(Almost) all structure functions can be determined and disentangled from each other by studying angular correlations of the hadronic system, for details see [1].

3. Low Energy Expansion of the Decay $\tau \to 3\pi\nu_\tau$

1. Tau decays into three pions involve hadronic invariant masses $\sqrt{Q^2}$ from threshold $3M_\pi$ up to M_τ. Therefore theoretical predictions are difficult, and one has to resort to phenomenological models. However, for $\sqrt{Q^2}$ below about 600 MeV, chiral perturbation theory (CHPT) [2] allows to calculate the hadronic matrix elements in a systematic expansion in external momenta and quark masses.

This is interesting for two reasons. First, this calculation allows to understand the complicated three pion dynamics systematically in a small corner of phase space, which can be used to test and improve phenomenological models. Second, the small Q^2 region could become accessible experimentally in future at b and τ-charm factories.

Here we want to emphasize a few new and interesting features arising from the one loop calculation in CHPT. We will explain their origin in detail, and claim that they should be seen by the new high statistics experiments.

2. Sometimes it is useful to parametrize the hadronic matrix element using conventions which differ from the notation introduced in Sec. 2. One can also use three different form factors F, G and H, and introduce isospin indices:

$$\begin{aligned}
H_\mu &= \langle \pi^i(p_1)\pi^j(p_2)\pi^k(p_3)|A^l_\mu(0)|0\rangle \\
&= \delta^{ij}\delta^{kl}\Big[(p_1+p_2)_\mu G(s_1,s_2,s_3) \\
&+ (p_1-p_2)_\mu H(s_1,s_2,s_3) \\
&+ p_{3\,\mu} F(s_1,s_2,s_3)\Big] + \text{permutations} \qquad (11)
\end{aligned}$$

F, G and H are isospin and Lorentz scalars and can be written as functions of s_1, s_2 and s_3. This general matrix element can be projected on a physical matrix element in a trivial manner: isospin symmetry allows to relate the matrix element of the different decay channels. We have calculated them in the framework of CHPT to one loop. The complete analytic results and a very detailed analysis are given elsewhere [3]. Here we would like to discuss some interesting features of this decay as predicted by CHPT at the level of structure functions, showing how they derive from basic properties of the form factors.

We will consider only four structure functions W_X, the ones related only to the spin 1 part of the matrix element. They can be expressed as:

$$\begin{aligned}
W_A &= (x_1+x_2)^2|K|^2 \\
&+ \left[(x_1-x_2)^2+4x_3^2\right]|H|^2 \\
&+ 2(x_1^2-x_2^2)\mathrm{Re}(KH^*) \\
W_C &= W_A - 8x_3^2|H|^2 \\
W_D &= 4x_3\Big[(x_1-x_2)|H|^2 \\
&+ (x_1+x_2)\mathrm{Re}(KH^*)\Big] \\
W_E &= 4x_3(x_1+x_2)\mathrm{Im}(KH^*)\,, \qquad (12)
\end{aligned}$$

where x_i are simple kinematical functions (see Ref. [1] for details), and $K=(G-F)/3$. W_A governs the rate and the distributions in the Dalitz plot, while the remaining functions determine the angular distribution.

From CHPT to one loop we find:

$$\begin{aligned}
K &= \frac{i}{F_\pi}\Big\{\frac{2}{3}+\frac{1}{3F_\pi^2}\Big[s_3\bar{J}(s_3)-\frac{M_\pi^2}{2}\Big(\bar{J}(s_1) \\
&+ \bar{J}(s_2)\Big)\Big]+\frac{1}{288\pi^2F_\pi^2}\Big[4\bar{l}_1(s_3-2M_\pi^2) \\
&- 2\bar{l}_2(s_1+s_2-4M_\pi^2)+12M_\pi^2\bar{l}_4+2\bar{l}_6Q^2 \\
&+ s_1+s_2-6(s_3+M_\pi^2)\Big]+O(p^4)\Big\} \\
H &= \frac{i}{F_\pi}\Big\{\frac{1}{6F_\pi^2}\Big[\Big(5M_\pi^2-2s_1\Big)\bar{J}(s_1) \\
&- \Big(5M_\pi^2-2s_2\Big)\bar{J}(s_2)\Big] \\
&+ \frac{1}{96\pi^2F_\pi^2}\Big(\frac{5}{3}-2\bar{l}_2\Big)(s_1-s_2) \\
&+ O(p^4)\Big\}\,. \qquad (13)
\end{aligned}$$

where

$$\begin{aligned}
\bar{J}(s) &= \frac{1}{16\pi^2}\left(\sigma\ln\frac{\sigma-1}{\sigma+1}+2\right) \\
\sigma &= \sqrt{1-4M_\pi^2/s} \qquad (14)
\end{aligned}$$

The $\bar{l}_i$ are the renormalized coupling constants of CHPT at $O(p^4)$ (see [3] for numerical values).

As required by Bose symmetry K (H) is symmetric (antisymmetric) under the exchange of s_1 and s_2. Isospin symmetry requires that the form factors in the $2\pi^0\pi^-$ mode are equal to the isospin symmetric form factors (13) multiplied by a factor $\sqrt{2}$, whereas the form factors in the $2\pi^-\pi^+$ mode are given by:

$$\begin{aligned}
K^{(--+)} &= -\frac{1}{\sqrt{2}}\Big[K(s_1,s_3,s_2)+K(s_2,s_3,s_1) \\
&- H(s_1,s_3,s_2)-H(s_2,s_3,s_1)\Big] \\
H^{(--+)} &= \frac{1}{\sqrt{2}}\Big[3(K(s_1,s_3,s_2)-K(s_2,s_3,s_1)) \\
&+ H(s_1,s_3,s_2)-H(s_2,s_3,s_1)\Big]\,.
\end{aligned}$$

3. Numerical results for the structure functions are given in Figs. 1–3. We plot s_1, s_2 averaged functions $w_X(Q^2) =$

$$= \begin{cases} \int ds_1\, ds_2 W_X(Q^2, s_1, s_2) \\ \quad \text{for} \quad X = A, C \\ \\ \int ds_1\, ds_2\, \text{sign}(s_1 - s_2)\, W_X(Q^2, s_1, s_2) \\ \quad \text{for} \quad X = D, E \end{cases}$$

where $Q^\mu = p_1^\mu + p_2^\mu + p_3^\mu$.

Corrections to the tree level results soon become very important. Because of this, and because of the obvious difficulty in accessing experimentally the region close to threshold, significant direct tests of these predictions in the low energy region will require very high statistics. The $O(p^4)$ chiral prediction for the branching ratio for $\left(\tau \to \nu_\tau + (3\pi)(Q^2 \le Q^2_{\max})\right)$ is of the order of 10^{-5} for $Q_{\max} = 600\,\text{MeV}$ and 10^{-4} for $Q_{\max} = 700\,\text{MeV}$ [3]. This shows that experimental tests in the low energy region are difficult but not hopeless. Moreover, some of the qualitative features of the structure functions, which the chiral expansion allows to understand in detail, will also have consequences at higher energies and so can be tested experimentally more easily.

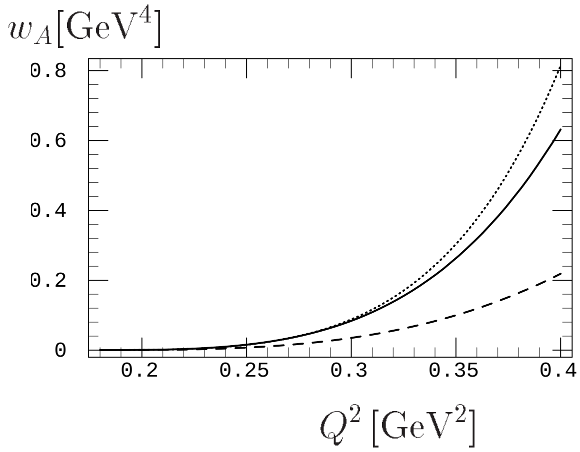
The first feature, which the chiral expansion predicts, is that $w_A^{(--+)} \approx w_C^{(--+)} \approx w_A^{(00-)} \approx w_C^{(00-)}$. In fact, the four plots can not be distinguished from each other within the resolution of Fig. 1. The equality of the structure functions for the two charge modes indicates that w_A and w_C are dominated by the [210] partition [4]. We have explicitly verified that this is the case. The near equality of w_A and w_C at low energies can be understood algebraically. In Eq. (12) we read that

$$W_A - W_C = 8x_3^2|H|^2\,. \tag{15}$$

H vanishes at tree level according to (13). Thus the difference starts as the *square* of a quantity of $O(p^2)$. Furthermore, H is kinematically suppressed at low energies, because it vanishes for $s_1 = s_2$. Therefore $w_A \approx w_C$ should be valid even at energies well above those where one would trust a one loop CHPT calculation.

In Fig. 1 we also compare to the prediction from the vector meson dominance model (VMD) in [5].

Figure 1. Integrated structure function $w_A(Q^2)$: CHPT prediction at tree level (dashed), one loop (solid) and from a vector meson dominance model (dotted). The four functions, w_A and w_C for both modes $2\pi^-\pi^+$ and $2\pi^0\pi^-$ all look identical within the resolution of this diagram.



Below $\sqrt{Q^2}$ of about 600 MeV, where we would trust CHPT, we find very good agreement between the VMD model and the one loop chiral prediction.

4. In Fig. 2 and 3 we plot $w_D(Q^2)$ and $w_E(Q^2)$. These are much smaller than w_A, and correspondingly more difficult to be measured. A measurement of them in the low energy region can only be considered at a b or τ-charm factory.

The behaviour of these structure functions is rather interesting. The one loop predictions from CHPT differ strongly for the two charge modes. The origin of this sign difference can again be understood in detail: neglecting $|H|^2$ that as discussed above is tiny near threshold, W_D and W_E are the real and imaginary part of the same function:

$$4x_3(x_1 + x_2)KH^* \tag{16}$$

The leading contribution to this function comes from the interference of the tree level part of K and the one loop part of H. At tree level K ex-

Figure 2. Integrated structure function $w_D(Q^2)$. CHPT at one loop for $2\pi^-\pi^+$ (solid) and for $2\pi^0\pi^-$ (dashed-dotted) and from the VMD model in [5] (dotted, identical prediction for both charge modes)

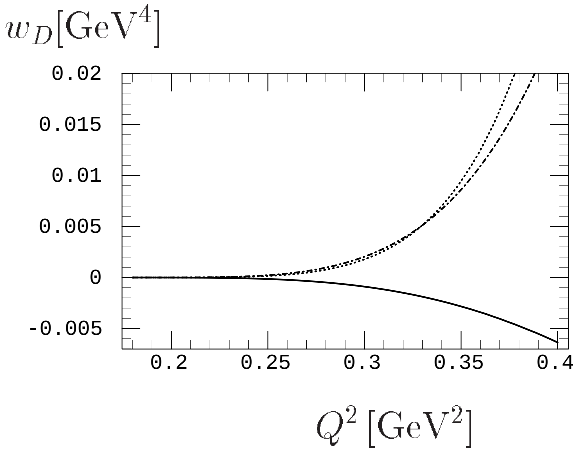
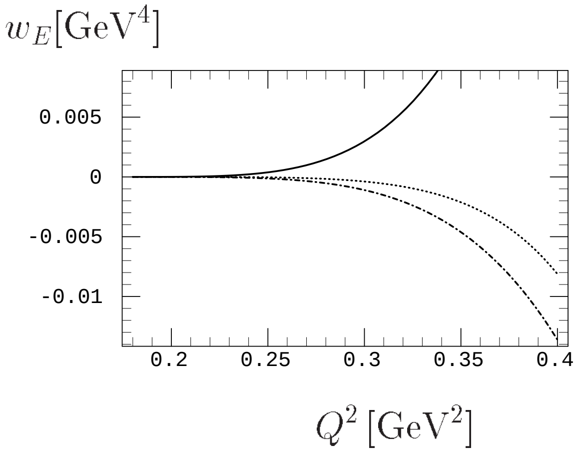


Figure 3. Integrated structure function $w_E(Q^2)$. CHPT at one loop for $2\pi^-\pi^+$ (solid) and for $2\pi^0\pi^-$ (dashed-dotted) and from the VMD model in [5] (dotted, identical prediction for both charge modes)



hibits a simple sign difference for the two charge modes:

$$
\begin{aligned}
K^{(--+)} &= \frac{i\sqrt{2}}{F_\pi}(-2/3 + O(p^2)) \\
K^{(00-)} &= \frac{i\sqrt{2}}{F_\pi}(2/3 + O(p^2)). \qquad (17)
\end{aligned}
$$

Since H does not change sign between the two charge modes the same sign difference as in K at tree level is reproduced in the two structure functions W_D and W_E. In our numerical calculation the sign difference remains up to $\sqrt{Q^2} \simeq 0.6$ GeV, where large corrections to this calculation are already expected.

Note that CHPT predicts $w_D^{(--+)} < 0$ and $w_E^{(--+)} > 0$, which should be correct near threshold. However, experimental data are available for $Q^2 \geq 0.8$ GeV^2 [6], which indicate $w_D^{(--+)} > 0$ and $w_E^{(--+)} < 0$. Thus, unless the higher orders in the chiral expansion completely change the CHPT result, somewhere between threshold and $Q^2 \sim 0.8$ GeV^2 there must be a zero for both structure functions in the $2\pi^-\pi^+$ mode. This feature is absent in all the phenomenological models in the literature of which we are aware. We can not predict from the chiral expansion, whether these zeros will occur at very low energies or only a little below $Q^2 = (M_\rho + M_\pi)^2 \approx 0.8$ GeV^2. We suggest a careful experimental examination of w_D and w_E from $Q^2 = 0.8$ GeV^2 downwards.

5. Let us summarize our main results. First, we predict that w_A and w_C are very similar at threshold. This has already been noted in phenomenological models, but we have shown that $w_A = w_C$ is actually a low energy theorem of QCD, which receives corrections only at next-to-next-to leading order. So the symmetry structure of QCD requires $w_A \approx w_C$ well beyond the very low energy region.

Second, near threshold we find a sign difference between the two charge modes for w_D and w_E. Taking into account also the available experimental data in $2\pi^-\pi^+$ channel we conclude that in this channel both structure functions should have a zero somewhere between threshold and $Q^2 \sim 0.8$ GeV^2. This feature is absent in any of the phenomenological models of which we are aware, and its experimental verification would be the first evidence for the presence of the [300] partition state [4] in tau decays into three pions.

With the same method we can calculate also the scalar part of the matrix element close to threshold. It turns out that, compared to the one loop prediction from CHPT, this scalar part is underestimated, if present at all, in phenomenological models. A detailed analysis of this and other topics, such as the application of CHPT to decays into two pions, is given elsewhere [3].

4. Issues at Higher Q^2: Resonances

1. $\pi^-\pi^0$ final states. There are some discrepancies between experimental data and predictions using the "chirally normalized vector meson dominance", or "CN-VMD" model for $\pi K\pi$ final states [7,8]. Prompted by this, we have started to systematically *test* the assumptions of the CN-VMD model, beginning with the simplest mode, i.e. the two pion final state.

A completely general ansatz for the relevant form factor $F_V(Q^2)$ is

$$\begin{aligned} F_V(Q^2) &= \frac{f_\rho(Q^2) g_{\rho\pi\pi}(Q^2)}{M_\rho^2} \mathrm{BW}_\rho(Q^2) \\ &+ \frac{f_{\rho'}(Q^2) g_{\rho'\pi\pi}(Q^2)}{M_{\rho'}^2} \mathrm{BW}_{\rho'}(Q^2) \\ &+ \cdots \end{aligned} \tag{18}$$

where the $\cdots$ may indicate both higher resonances (ρ'', ...) and possible non-resonant contributions. We use Breit-Wigner resonance factors normalized to $\mathrm{BW}_X(0) = 1$,

$$\mathrm{BW}_X(Q^2) = \frac{M_X^2}{M_X^2 - Q^2 - i\sqrt{Q^2}\,\Gamma_X(Q^2)} \tag{19}$$

The on-shell values of the various couplings can (approximately) be determined from experimental data,

$$\begin{aligned} |g_{\rho\pi\pi}(M_\rho^2)| &= 6.05 \\ |g_{\rho'\pi\pi}(M_{\rho'}^2)| &= 1.39 \\ |f_\rho(M_\rho^2)| &= 0.117\,\mathrm{GeV}^2 \\ |f_{\rho'}(M_{\rho'})| &= 0.18\,\mathrm{GeV}^2 \end{aligned} \tag{20}$$

If we *assume* that the meson couplings are approximately constant from $Q^2 = 0$ up to the relevant resonance masses, we find

$$\begin{aligned} F_V(Q^2) &= \underbrace{\frac{f_\rho(M_\rho^2) g_{\rho\pi\pi}(M_\rho^2)}{M_\rho^2}}_{1.20} \mathrm{BW}_\rho(Q^2) \\ &+ \underbrace{\frac{f_{\rho'}(M_{\rho'}^2) g_{\rho'\pi\pi}(M_{\rho'}^2)}{M_{\rho'}^2}}_{-0.12} \mathrm{BW}_{\rho'}(Q^2) \\ &+ \cdots \end{aligned} \tag{21}$$

Here we have used the experimental knowledge about the negative relative phase between the ρ and the ρ' contributions. Note that no reliable error estimates are possible here. For the value $f_{\rho'}(M_{\rho'}^2) g_{\rho'\pi\pi}(M_{\rho'}^2)/M_{\rho'}^2 = 0.12$ we get an error of ± 0.02 from the uncertainty in $M_{\rho'}$, but the uncertainty from $g_{\rho'\pi\pi}$ is unknown and might potentially be larger.

We will now compare these results with the CN-VMD model. If we dominate with the ρ only, we get

$$F_V(Q^2) = 1 \times \mathrm{BW}_\rho(Q^2) \tag{22}$$

Compared to (21), we find a 20% discrepancy in the strength of the ρ contribution. This has already been observed in [9]. If, however, we include the first two resonances, we get

$$\begin{aligned} F_V(Q^2) &= \underbrace{\frac{1}{1+\beta}}_{1.17} \times \mathrm{BW}_\rho(Q^2) \\ &+ \underbrace{\frac{\beta}{1+\beta}}_{-0.17} \times \mathrm{BW}_{\rho'}(Q^2) \end{aligned} \tag{23}$$

where we have taken β from the fit $N = 1$ in [1], and with three resonances, we get (fit $N = 2$ in [1])

$$\begin{aligned} F_V(Q^2) &= \underbrace{\frac{1}{1+\beta+\gamma}}_{1.16} \times \mathrm{BW}_\rho(Q^2) \\ &+ \underbrace{\frac{\beta}{1+\beta+\gamma}}_{-0.12} \times \mathrm{BW}_{\rho'}(Q^2) \end{aligned}$$

$$+ \quad \underbrace{\frac{\gamma}{1+\beta+\gamma}}_{-0.04} \times \mathrm{BW}_{\rho''}(Q^2) \qquad (24)$$

These numbers show that the CN-VMD model compares reasonably well with (21), if the first two or three ρ like states are included. This gives us some confidence that the CN-VMD model is indeed a good approximation. A more detailed study is in preparation [10].

2. $K\pi$ final states. There are two such final states, $\overline{K}^0\pi^-$ and $K^-\pi^0$, which are related by isospin. Due to $m_s > m_u, m_d$, both F_V and F_S may contribute [11]. Relevant resonances with the correct quantum numbers are:

$$\begin{array}{lll} J^P = 1^- \ (F_V): & K^*(892) \quad \text{and} \quad K^*(1410) \\ J^P = 0^+ \ (F_S): & K_0^*(1430) \end{array}$$

In [12], we have constructed a model with includes both the $K^*(1410)$ and the $K_0^*(1430)$.

The relative strength $\beta_{K^*} \approx -0.135$ of the $K^*(1410)$ in the vector channel was estimated from BR($K\pi$) (note that $\beta_{K^*} \approx \beta_\rho$, which is consistent with $SU(3)$ flavor symmetry expectations), and the strength of the $K_0^*(1430)$ by matching to $O(p^4)$ Chiral Perturbation Theory. We found that the $K^*(1410)$ and the $K_0^*(1430)$ contribute with comparable strength (both about 5% to the rate). Thus one has to disentangle them using angular distributions, i.e. by measuring W_B, W_{SA} and W_{SF}.

3. 3π final states. Some important tasks in these final states are to establish the presence of a scalar contribution and to measure its size, and to put upper limits on the ρ' contribution in two pion subresonances.

4. $K\pi K$ and $\pi K\pi$ final states. As opposed to the case of the 3π, here both the vector and the axial vector currents can contribute (i.e. both the a_1 and the ρ', or both the K_1 and the $K^{*\prime}$). Theoretical predictions for their ratio $V : A$ vary considerably [13,9,14,7,15]. Thus it should be measured, using angular distributions. Understanding these decay modes may turn out to be very useful for measurements of possible non standard model CP-violation [16]. This is because suitable interference terms between the vector and the axial vector contribution can be measured without polarized taus, and without knowledge of the τ rest frame; and because the two interfering amplitudes (vector and axial vector) may have comparable strengths.

Another interesting issue regarding the $\pi K\pi$ final states are the two axial resonances with strangeness, the $K_1(1270)$ and the $K_1(1400)$, which both can contribute here.

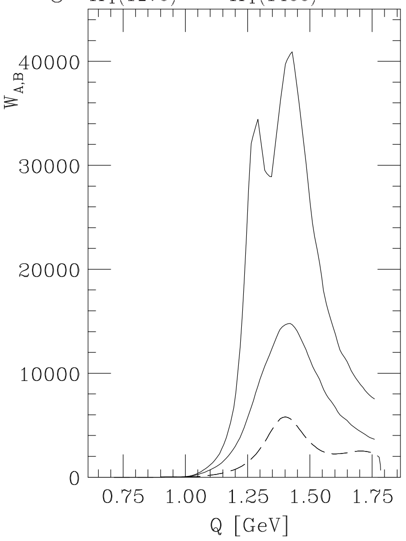
We believe that there are good reasons to suspect that the K_1 widths given in the particle data book [17] might be considerably too small [8]. These reasons are: (i) Otherwise the $BR(\pi K\pi)$ within the CN-VMD model turn out to be about a factor of $2\cdots3$ too large. Of course, the obvious thought is that this might be due to a failure of the CN-VMD model. However, due to the fact that most of the other predictions of the model for three meson final states agree reasonably well with data, and due to the various symmetry relations between the different three meson final states, we are actually unable to find any other natural explanation. This led us to suspect that indeed the true K_1 widths might be larger. (ii) Given that $\Gamma_{a_1} \approx 400\cdots600\,\mathrm{MeV}$, the K_1 widths in [17] appear unnaturally small. (iii) Experiments on which the numbers quoted in [17] are based, have made the same assumptions for the parametrization of background amplitudes, which in the past yielded very small values for Γ_{a_1}, and which subsequently were superseded by cleaner measurements of Γ_{a_1} using τ decays. Thus our conclusion is that one should *measure* the K_1 resonance parameters in $\tau \to \pi K\pi\nu_\tau$. In Fig. 4 we show that indeed w_B is very sensitive to the K_1 parameters. These issues will be discussed in more detail in a forthcoming publication [10].

5. Conclusions

Let us conclude by making some suggestions to experimentalists about what we believe to be interesting and feasible measurements.

First, let us consider decays with three pions. We would suggest to measure w_D and w_E for both charge modes, $2\pi^-\pi^+$ and $2\pi^0\pi^-$. These two structure functions might very well be different for the two modes. This would be the first evidence for the [300] partition class, which decays

Figure 4. Structure functions for $\tau \to K^- \pi^- \pi^+$: $w_A(Q^2)$ (solid) and $w_B(Q^2)$ (dashed). The larger solid curve with two clearly distinguishable peaks is obtained using the RPP-96 values for $\Gamma_{K_1(1270)}$ and $\Gamma_{K_1(1400)}$, the smaller solid curve using $\Gamma_{K_1(1270)} = \Gamma_{K_1(1400)} = 250\,\mathrm{MeV}$



differently into $2\pi^-\pi^+$ and $2\pi^0\pi^-$. Furthermore, we would like to urge to measure w_D and w_E for Q^2 less or nearly equal to $0.8\,\mathrm{GeV}^2$ for the $2\pi^-\pi^+$ charge mode. There is most probably a zero and a change of sign both in w_D and w_E in this mode, somewhere between $Q^2 = 9M_\pi^2 \cdots 0.8\,\mathrm{GeV}^2$.

We also suggest to measure the scalar contribution via w_{SB}, and to try to put limits on the ρ' contribution in the two-pion subresonances.

Secondly, regarding $\tau \to \pi K \pi \nu_\tau$, we suggest to measure the parameters of the K_1 resonances using this τ decay mode.

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