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TWO-LOOP CORRECTIONS TO FERMION PAIR PRODUCTION VERTICES†

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Abstract

Recent results on the entirely analytic calculation of second order corrections to massive quark pair production vertices induced by light quark flavours are reviewed. Based on the method presented in this talk the second order effects of the insertion of any massless one-loop vacuum polarization into the gluon line of the first order diagrams can be determined. The behaviour of the corrections at threshold for vector and axial-vector current induced massive quark production is discussed.

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1 Introduction

Within recent years a lot of work has been invested to determine two-loop radiative corrections to physical processes relevant for present and future collider experiments. The aim of this difficult work is twofold. On the one hand, one is looking for potentially large corrections and on the other one likes to diminish the size of the uncertainty of the corresponding one-loop calculations. Especially in the framework of electroweak processes involving hadrons these aspects are of importance, because the scale of the QCD coupling in the one-loop corrections can only be fixed by an explicit two-loop calculation.

Of special interest is the process of hadron production in e^+e^--collisions. Whereas the $\mathcal{O}(\alpha_s)$ corrections to this process have been calculated quite a long time ago [1, 2] for all mass and energy assignments, the complete $\mathcal{O}(\alpha_s^2)$ corrections have only been determined as a high energy expansion up to terms of $\mathcal{O}(M^4/s^2)$ [3], where $\sqrt{s}$ denotes the c.m. energy and M the mass of the produced quarks. As far as $b\bar{b}$-production at the Z peak is concerned this is definitely a good approximation. However, in view of forthcoming e^+e^--collision experiments (τ-charm-, B-factory, NLC), where quark pairs will be produced close to the threshold, the knowledge of the entire $\mathcal{O}(\alpha_s^2)$ corrections for all values of M^2/s is mandatory.

In our group two strategies are pursued to reach this aim. The first one consists of finding a numerical approximation by use of the Padé method. For that one takes information from the high energy regime, the threshold region and at momentum transfer zero and constructs an interpolating function based on conformal mapping of the physical momentum range onto the unit circle in the complex plane. The second strategy consists of the analytic calculation of the $\mathcal{O}(\alpha_s^2)$ corrections. Of course this seems to be an impossible task at the present stage. Especially certain classes of gluonic corrections (those which do not exist in the corresponding QED calculation) represent a great challenge. On the other hand, the second order corrections, which arise from the insertion of the vacuum polarization due to massless quarks into the gluon line of the first order diagrams (belonging to the so-called "non-singlet" corrections, see Fig. 1) are accessible. In particular, the corrections due to massless quarks are sufficient to carry out the BLM procedure [4], which sets the scale of the strong coupling in the first order corrections and, if one believes in the "naive non-abelianization" hypothesis [5], even might provide good estimates for the complete second order corrections.

In this talk I will report on recent developments of the analytical approach. I will concentrate on the calculation and the structure of the light fermionic non-singlet second order corrections. Because the calculation is most easily performed by using dispersion integration techniques, the results are derived in on-shell renormalized QED. The transition to the $\overline{\text{MS}}$ scheme and to QCD is indicated afterwards. The presentation is kept general and can be applied to fermion anti-fermion pair production vertices induced by any current. In particular, I will discuss the threshold behaviour and the associated scales of quark pair production via the vector and the axial-vector current. I will not talk about massive quark pair production mediated by singlet (or "double triangle") diagrams. For a presentation of the Padé method the interested reader is referred to the talk by M. Steinhauser [6] and the references given therein.

2 Calculation of the Light Fermionic Second Order Corrections

The non-singlet light fermionic second order corrections to the fermion anti-fermion pair production rate

$$R_{F\bar{F}} = r^{(0)} + \left(\frac{\alpha}{\pi}\right) r^{(1)} + \left(\frac{\alpha}{\pi}\right)^2 r^{(2)} + \dots, \tag{1}$$

correspond to the sum of all possible cuts of the diagrams depicted in Fig. 1.* In the frame of on-shell renormalized QED the basic idea to calculate those diagrams is to write the one-loop corrections to

*The reverse situation with primary production of light fermions and secondary radiation of massive ones has been treated in an earlier work [7] and shall not be discussed here. Like the singlet contribution they need not to be taken into account for the BLM procedure as they do not contribute to the running of the coupling.

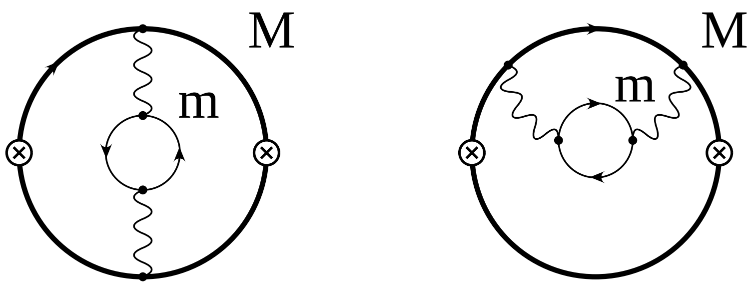

Figure 1: Non-singlet diagrams describing primary production of massive fermion pairs (with fermion mass M) with additional real or virtual radiation of a light fermion anti-fermion pair (with fermion mass m).

the photon propagator in terms of a dispersion relation

$$\begin{aligned} & \frac{-i}{p^2+i\epsilon}\left(g^{\mu\sigma}-\xi\frac{p^\mu\,p^\sigma}{p^2}\right)\Big(-\,\Pi_{\sigma\rho}(m^2,p^2)\Big)\frac{-i}{p^2+i\epsilon}\left(g^{\rho\nu}-\xi\frac{p^\rho\,p^\nu}{p^2}\right) \\ &= \frac{1}{3}\left(\frac{\alpha}{\pi}\right)\int\limits_{4\,m^2}^{\infty}\frac{\mathrm{d}\lambda^2}{\lambda^2}\left[\frac{-\,i\,g^{\mu\nu}}{p^2-\lambda^2+i\epsilon}\right]R_{f\bar{f}}(\lambda^2)\,+\dots\,, \end{aligned} \tag{2}$$

where $R_{f\bar{f}}$ is the Born cross-section for $f\bar{f}$ production in e^+e^--collisions normalized to the point cross-section. It is evident that eq. (2) represents a convolution of a massive vector boson propagator with the function $R_{f\bar{f}}$. The longitudinal and explicitly gauge dependent terms indicated by the dots are irrelevant due to current conservation. Thus the light fermionic second order corrections to $R_{F\bar{F}}$ can be written as a one-dimensional integral

$$r_f^{(2)}(M^2,s)=\frac{1}{3}\int\limits_{4\,m^2}^{\infty}\frac{\mathrm{d}\lambda^2}{\lambda^2}\,r^{(1)}(M^2,s,\lambda^2)\,R_{f\bar{f}}(m^2,\lambda^2)\,, \tag{3}$$

where $r^{(1)}(M^2,s,\lambda^2)$ represents the $\mathcal{O}(\alpha)$ corrections to the $F\bar{F}$ productions rate from the real and virtual radiation of a vector boson with mass λ at c.m. energy $\sqrt{s}$. For the special case of interest, $m\to 0$, the evaluation of the integral (3) can be simplified enormously by subtracting and adding $R_{f\bar{f}}$ at its asymptotic high energy value $R_\infty^f\equiv R_{f\bar{f}}(\infty)$:

$$\begin{aligned} r_f^{(2)}(M^2,s) &= \frac{1}{3}\int\limits_{4\,m^2}^{\infty}\frac{\mathrm{d}\lambda^2}{\lambda^2}\,r^{(1)}(M^2,s,\lambda^2)\,\Big[R_{f\bar{f}}(m^2,\lambda^2)-R_\infty^f\Big] \\ &\quad +\frac{1}{3}\,R_\infty^f\int\limits_{4\,m^2}^{\infty}\frac{\mathrm{d}\lambda^2}{\lambda^2}\,r^{(1)}(M^2,s,\lambda^2)\,. \end{aligned} \tag{4}$$

Because $R_{f\bar{f}}$ reaches its asymptotic value already for λ^2 much smaller than the hard scales M^2 and s we can replace $r^{(1)}(M^2,s,\lambda^2)$ in the first integrand on the r.h.s. of eq. (4) by $r^{(1)}(M^2,s,0)$, the one-loop contribution to $R_{F\bar{F}}$ in eq. (1). The Bloch-Nordsieck theorem [8] assures that the corresponding limit exists. The first integral can then be trivially expressed in terms of the moment

$$R_0^f\;\equiv\;\int\limits_0^1\mathrm{d}x\,\Big[\,R_{f\bar{f}}\Big(\frac{4\,m^2}{x}\Big)-R_\infty^f\,\Big]\,. \tag{5}$$

The evaluation of the second integral constitutes the main effort. The final result for the second order corrections due to one light fermion f can be written in the form ($x=f$)

$$r_x^{(2)}\;=\;-\frac{1}{3}\Big[\,R_\infty^x\,\ln\frac{m^2}{s}-R_0^x\,\Big]\,r^{(1)}+R_\infty^x\,\delta^{(2)}\,, \tag{6}$$

where $\delta^{(2)}$ is a complicated function of the ratio M^2/s involving Tri- (Li_3), Di- (Li_2) and usual logarithms [9, 10]. For the light fermionic corrections we are interested in the moments read

$$R_\infty^f = 1\,, \qquad R_0^f = \ln 4 - \frac{5}{3}\,. \tag{7}$$

The result in the on-shell scheme can now be easily transferred to the $\overline{\mathrm{MS}}$ scheme by expressing the fine structure constant α in terms of the $\overline{\mathrm{MS}}$ coupling

$$\alpha = \alpha_{\overline{\mathrm{MS}}}(\mu^2)\left(1 + \frac{\alpha_{\overline{\mathrm{MS}}}(\mu^2)}{\pi}\,\frac{1}{3}\,R_\infty^f\,\ln\frac{m^2}{\mu^2}\right) + \mathcal{O}(\alpha^3_{\overline{\mathrm{MS}}})\,, \tag{8}$$

which effectively replaces $\ln(m^2/s)$ in (6) by $\ln(\mu^2/s)$. At this point I would like to emphasize that the moments can also be extracted directly from the vacuum polarization function due to a massless fermion anti-fermion pair ($x = f$),

$$\Pi^{x,\overline{\mathrm{MS}}}_{massless}(q^2) \;=\; -\frac{\alpha_{\overline{\mathrm{MS}}}(\mu^2)}{3\,\pi}\left[R_\infty^x\,\ln\frac{-q^2}{4\,\mu^2} + R_0^x\right]. \tag{9}$$

To arrive at the corresponding light fermionic second order corrections in the frame of QCD we have to multiply the corresponding SU(3) group theoretical factors $N_c = 3$, $T = 1/2$ and $C_F = 4/3$. In particular the moments for light quarks in QCD read $R_n^{f,\mathrm{QCD}} = T\,R_n^f$ ($n = \infty, 0$).

3 The Method of Moments

The method described above can be applied to determine the corrections from the insertion of any massless vacuum polarization into the gluon line, as long as the absorptive part of the vacuum polarization function approaches a constant in the high energy limit. This is equivalent to the occurrence of at most one single power of the logarithm $\ln(-q^2/\mu^2)$ in the renormalized vacuum polarization function in $D = 4$ dimensions and is certainly true for any one-loop vacuum polarization function. Thus we can easily determine for example the second order corrections from the insertion of a vacuum polarization due to massless coloured scalars into the gluon line. According to eq. (9) the scalar moments read

$$R_\infty^{s,\mathrm{QCD}} = T\left(\frac{1}{4}\right), \qquad R_0^{s,\mathrm{QCD}} = T\left(\frac{1}{4}\ln 4 - \frac{2}{3}\right). \tag{10}$$

We are also in a position to calculate the second order gluonic self-energy contribution to massive quark production, as illustrated in Fig. 2, by determining the corresponding moments of the gluonic and ghost contributions to the $\mathcal{O}(\alpha_s)$ gluon propagator corrections,

$$\begin{aligned} R_\infty^{g,\mathrm{QCD}} &= C_A\left(-\frac{5}{4} - \frac{3}{8}\xi\right), \\ R_0^{g,\mathrm{QCD}} &= C_A\left(\frac{31}{12} - \frac{3}{4}\xi + \frac{3}{16}\xi^2 + \left(-\frac{5}{4} - \frac{3}{8}\xi\right)\ln 4\right), \end{aligned} \tag{11}$$

The gauge parameter ξ is defined via the gluon propagator in lowest order

$$\frac{i}{p^2 + i\,\epsilon}\left(-\,g^{\mu\nu} + \xi\,\frac{p^\mu\,p^\nu}{p^2}\right). \tag{12}$$

Evidently the gluonic moments and $r_g^{(2),\mathrm{QCD}}$, defined in analogy to eq. (6), are not gauge invariant. However, for $\xi = 4$ the combination $\alpha_s^2\,R_\infty^{g,\mathrm{QCD}}/(3\pi)$ coincides with the complete gluonic $\mathcal{O}(\alpha_s^2)$ contribution to the QCD β-function, and thus $r_g^{(2),\mathrm{QCD}}|_{\xi=4}$ accounts for the leading logarithmic behaviour of the sum of all gluonic $\mathcal{O}(\alpha_s^2)$ diagrams in the high energy limit. It is not surprising that such a gauge can be found. However, it is a remarkable fact that for the same choice of ξ also the gluonic contributions to the perturbative QCD potential [11] can solely be expressed in terms of the moments (11). This point will be examined in more detail in the next section. Further application of the method of moments to corrections to $R_{F\bar{F}}$ beyond the second order in the strong coupling can be found in [9, 10].

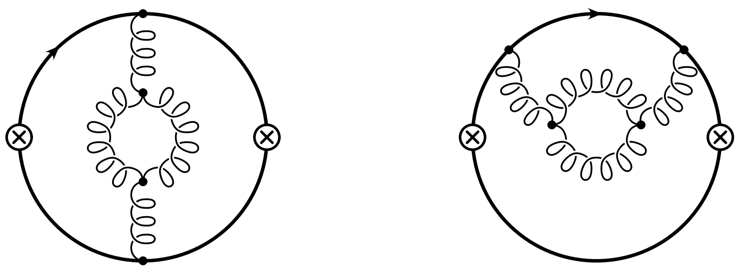

Figure 2: Gluonic self-energy contribution to heavy quark production. The ghost diagrams are not depicted explicitly.

4 The Threshold Behaviour

The examination of the threshold region is of special interest. Here, the contributions from the exchange of longitudinal and transversal polarized gluons can be clearly distinguished. The former gives rise to the instantaneous Coulomb interaction which cancels one power of $\beta = \sqrt{1-4\,M^2/s}$ from the phase space and leads to the well-known finiteness of the vector current cross-section at threshold. The latter is responsible for the relativistic corrections. Both contributions are governed by different momentum scales. From general considerations one can infer that the scale relevant for the instantaneous Coulomb interaction should be of the order of the relative momentum of the produced massive quarks, $\sqrt{s}\,\beta$, whereas the scale for the hard/transverse corrections should be of the order of the c.m. energy, $\sqrt{s}$. However, only an exact calculation of the non-singlet light fermionic second order corrections allows for an accurate determination of those scales via the BLM scale setting procedure.

Taking in account only the second order effects of n_f light quark species the threshold expansion of the massive quark pair production rate induced by the vector current reads

$$\begin{aligned} R^V &= N_c\,C_F\,\frac{3}{4}\,\alpha_s(\mu^2)\,\pi\left[1+\frac{1}{3}\left(\frac{\alpha_s(\mu^2)}{\pi}\right)n_f\left[R_\infty^{f,\mathrm{QCD}}\ln\frac{s\,\beta^2}{4\,\mu^2}+R_0^{f,\mathrm{QCD}}\right]\right] \\ &\quad + N_c\,\frac{3}{2}\left\{1-4\,C_F\left(\frac{\alpha_s(\mu^2)}{\pi}\right)\left[1+\frac{1}{3}\left(\frac{\alpha_s(\mu^2)}{\pi}\right)n_f\left[R_\infty^{f,\mathrm{QCD}}\left(\ln\frac{s}{16\,\mu^2}+\frac{3}{4}\right)+R_0^{f,\mathrm{QCD}}\right]\right]\right\}\beta \\ &= N_c\,C_F\,\frac{3}{4}\,\alpha_s(\mu^2)\,\pi\left[1-n_f\,\Pi^{f,\overline{\mathrm{MS}}}_{massless}(-s\,\beta^2)\right] \\ &\quad + N_c\,\frac{3}{2}\left\{1-4\,C_F\left(\frac{\alpha_s(\mu^2)}{\pi}\right)\left[1-n_f\,\Pi^{f,\overline{\mathrm{MS}}}_{massless}(-\frac{s}{4}\,e^{3/4})\right]\right\}\beta \end{aligned} \tag{13}$$

including terms up to $\mathcal{O}(\beta)$. In the V-scheme, where the strong coupling is defined as the all order effective charge in the QCD potential

$$V_{\mathrm{QCD}}(Q^2) \;=\; -4\pi\,C_F\,\frac{\alpha_V(Q^2)}{Q^2}\,, \tag{14}$$

with [11, 13]

$$\begin{aligned} \alpha_V(Q^2) &= \alpha_s(\mu^2)\left\{1+\frac{\alpha_s(\mu^2)}{3\,\pi}\left[n_f\,T\left(\ln\frac{Q^2}{\mu^2}-\frac{5}{3}\right)+C_A\left(-\frac{11}{4}\ln\frac{Q^2}{\mu^2}+\frac{31}{12}\right)\right]\right\}+\mathcal{O}(\alpha_s^3) \\ &= \alpha_s(\mu^2)\left\{1-n_f\,\Pi^{f,\overline{\mathrm{MS}}}_{massless}(-Q^2)-\Pi^{g,\overline{\mathrm{MS}}}_{massless}(-Q^2)|_{\xi=4}\right\}+\mathcal{O}(\alpha_s^3)\,, \end{aligned} \tag{15}$$

the corresponding BLM scales can be directly read off the arguments of the vacuum polarization functions $\Pi^{f,\overline{\mathrm{MS}}}_{massless}$ on the r.h.s. of eq. (13). The reader should note that the second line of eq. (15) is a new observation [9] which has not been made in the original paper [11]. Obviously the leading second order threshold contribution to R^V from massless quarks is in one-to-one correspondence to the form of the QCD potential. This is in accordance with non-relativistic quantum mechanics. As a consequence the leading second order gluonic threshold contributions to R^V, which are proportional to the colour factor C_A (and represent purely non-abelian contributions) can be inferred direcly by adding $\Pi^{g,\overline{\rm MS}}_{massless}(s\,\beta^2)|_{\xi=4}$ on the r.h.s. of eq. (13). A similar conclusion is impossible for the hard corrections, because they represent effects which cannot be described by the instantaneous Coulomb potential.

Taking into account only the second order non-singlet effects of n_f light quarks flavours the axial-vector induced massive quark pair production rate at threshold can be cast into form

$$\begin{aligned} R^A &= N_c\,C_F\,\frac{1}{2}\,\alpha_s(\mu^2)\,\pi\left[1-\Pi^{x,\overline{\rm MS}}_{massless}(-s\,\beta^2\,e^{-2})\right]\beta^2 \\ &\quad + N_c\left\{1-2\,C_F\left(\frac{\alpha}{\pi}\right)\left[1-\Pi^{x,\overline{\rm MS}}_{massless}(-\frac{s}{4}\,e^{1/2})\right]\right\}\beta^3+\mathcal{O}(\beta^4)\,. \end{aligned} \tag{16}$$

The reader should note that the leading contribution to R^A is of $\mathcal{O}(\beta^2)$. In analogy to the vector current case the BLM scales in the V-scheme can be identified from eq. (16). The $\mathcal{O}(\beta^2)$ second order gluonic corrections $\propto C_A$ can be determined in complete analogy to the vector current case. A formal proof of this statement will be presented in later work.

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