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Theory of Top Quark Production and Decay†

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Abstract

Direct and indirect information on the top quark mass and its decay modes is reviewed. The theory of top production in hadron- and electron-positron-colliders is presented.

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Introduction

An extensive search for top quarks has been performed at electron-positron and hadron colliders for more than a decade. First evidence for top quark production in proton-antiproton collisions has been announced by the CDF collaboration in the spring of 1994. After collecting more luminosity subsequently both the CDF and the D0 experiments presented the definite analysis [1] which demonstrated not only the existence of the anticipated quark but at the same time also provided a kinematic determination of the top quark mass around 180 GeV and a production cross section consistent with the QCD predictions. The mass value is in perfect agreement with the indirect mass determinations based on precision measurements [2-7] of the electroweak parameters in e^+e^-annihilation and in lepton-nucleon scattering. Exploiting the quadratic top mass dependence of radiative corrections an indirect mass measurement of 180 GeV with a present uncertainty of roughly 20 GeV has been achieved.

The top quark completes the fermionic spectrum of the Standard Model. All its properties are uniquely fixed after the mass has been determined. However, as a consequence of its large mass and decay rate it will behave markedly different compared to the remaining five lighter quarks.

It is not just the obvious aim for completion which raises the interest in the top quark. With its mass comparable to the scale of the electroweak symmetry breaking it is plausible that top quark studies could provide an important clue for the understanding of fermion mass generation and the pattern of Yukawa couplings. In fact, it has been suggested that a top quark condensate could even be responsible for the mechanism of spontaneous symmetry breaking [8].

These lectures will be mainly concerned with top quark phenomenology within the framework of the Standard Model (SM). The precise understanding of its production and decay constitutes the basis of any search for deviations or physics beyond the SM.

The properties of the top quark will be covered in chapter 1. Direct and indirect determinations of its decay rates, decay distributions including QCD and electroweak corrections and decay modes predicted in supersymmetric extensions will be discussed. Top quark production at hadron colliders will be the subject of chapter 2. The production cross section and momentum distributions are important ingredients in any of the present analysis. An alternative reaction, namely top quark production through W-b-fusion allows to determine the W-b-$\bar{t}$ coupling and thus indirectly the top quark decay rate.

Perspectives for top studies at a future e^+e^-collider will be presented in chapter 3. An accurate determination of the top quark mass and its width to better than 1 GeV with a relative accuracy of about 10% seems feasible, and the electroweak couplings of the top quark could be precisely measured with the help of polarized beams. Of particular interest is the interplay between the large top quark decay rate and the binding through the QCD potential which will be also covered in chapter 3.

Chapter 1

The Profile of the Top Quark

Hadron collider experiments at the TEVATRON have firmly established the existence of the top quark and already provide a fairly accurate determination of its mass. The couplings of the top quark to the gauge bosons are uniquely fixed by the SM. Thus all its properties — its production cross section and its decay rate and distributions — can be predicted unambiguously.

The study of real top quarks at high energy colliders, in particular the observation of a peak in the invariant mass of its decay products, is certainly the most impressive proof of existence. Nevertheless, the indirect evidence for a top quark and the determination of its mass is not only of historical interest. The arguments which anticipated the existence of the top quark and its mass around 180 GeV illustrate the rigid structure of the SM, its selfconsistency and beauty. They will be presented in section 1.1.

These theoretical arguments have inspired the experimental searches. The upper limit on the top mass around 200 GeV deduced already relatively early from electroweak precision studies has provided encouragement that energies of present colliders were suited to complete this enterprise. The agreement between the most recent indirect mass determinations both through radiative corrections and through direct observation strengthens the present belief into the quantum field aspect of the theory. It furthermore justifies the corresponding line of reasoning concerning the search for the last remaining ingredient of the SM, the Higgs boson. Section 1.1 of this first chapter will, with this motivation in mind, be devoted to a discussion of the indirect information on the top quark, its existence and its mass. Top decays, including aspects of radiative corrections, polarisation effects and decays induced by physics beyond the SM will be covered in section 1.2.

1.1 Indirect information

1.1.1 Indirect evidence for the top quark

Several experimental results already prior to its discovery did provide strong evidence that the fermion spectrum of the Standard Model

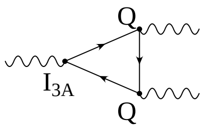
$$\begin{bmatrix} \nu_e \\ e^- \end{bmatrix}_L \; e_R^- \qquad \begin{bmatrix} \nu_\mu \\ \mu^- \end{bmatrix}_L \; \mu_R^- \qquad \begin{bmatrix} \nu_\tau \\ \tau^- \end{bmatrix}_L \; \tau_R^-$$

$$\begin{bmatrix} u \\ d \end{bmatrix}_L \; \begin{matrix} u_R \\ d_R \end{matrix} \qquad \begin{bmatrix} c \\ s \end{bmatrix}_L \; \begin{matrix} c_R \\ s_R \end{matrix} \qquad \begin{bmatrix} t \\ b \end{bmatrix}_L \; \begin{matrix} t_R \\ b_R \end{matrix}$$

does include the top quark, imprinting the same multiplet structure on the third family as the first two families. The evidence is based on theoretical selfconsistency (absence of anomalies), the absence of flavour changing neutral currents (FCNC) and measurements of the weak isospin of the b quark which has been proved to be non-zero, $I_3 = -1/2$, thus demanding an $I_3 = +1/2$ partner in this isospin multiplet.

Absence of triangle anomalies

A compelling argument for the existence of top quarks follows from a theoretical consistency requirement. The renormalizability of the Standard Model demands the absence of triangle anomalies. Triangular fermion loops built-up by an axialvector charge $I_{3A} = -I_{3L}$ combined with two electric vector charges Q would spoil the renormalizability of the gauge theory. Since the anomalies do not depend on the masses of the fermions circulating in the loops, it is sufficient to demand that the sum



$$\sim \sum_L I_{3A} Q^2 = -\sum_L I_3 \left[I_3 + \frac{1}{2}Y\right]^2$$
$$\sim \sum_L Y \sim \sum_L Q$$

of all contributions be zero. Such a requirement can be translated into a condition on the electric charges of all the left-handed fermions

$$\sum_L Q = 0. \tag{1.1}$$

This condition is met in a complete standard family in which the electric charges of the lepton plus those of all color components of the up and down quarks add up to zero,

$$\sum_L Q = -1 + 3 \times \left[\left(+\frac{2}{3}\right) + \left(-\frac{1}{3}\right)\right] = 0.$$

If the top quark were absent from the third family, the condition would be violated and the Standard Model would be theoretically inconsistent.

Absence of FCNC decays

Mixing between quarks which belong to different isospin multiplets

$$\begin{bmatrix} c \\ s' \end{bmatrix}_L \quad b'_L \qquad\qquad \begin{aligned} s'_L &= s_L \cos\vartheta' + b_L \sin\vartheta' \\ b'_L &= -s_L \sin\vartheta' + b_L \cos\vartheta' \end{aligned}$$

generates non-diagonal neutral current couplings, i.e. the breaking of the GIM mechanism

$$\begin{aligned} < I_3 > &= +\frac{1}{2}(\bar{c}_L, c_L) - \frac{1}{2}\left(\bar{s}'_L, s'_L\right) \\ &= \frac{1}{2}(\bar{c}_L, c_L) - -\frac{1}{2}(\bar{s}_L, s_L)\cos^2\vartheta' - \frac{1}{2}\left(\bar{b}_L, b_L\right)\sin^2\vartheta' \\ &\quad -\frac{1}{2}\sin\vartheta' \cos\vartheta' \left((\bar{s}_L, b_L) + \left(\bar{b}_L, s_L\right)\right). \end{aligned}$$

The non-diagonal current induces flavor-changing neutral lepton pair decays $b \rightarrow s + l^+ l^-$ which have been estimated to be a substantial fraction of all semileptonic B meson decays. The relative strenth of neutral versus charged current induced rate is essentially given by

$$\frac{\Gamma_{\rm NC}}{\Gamma_{\rm CC}} \sim \frac{1}{2}\left(\frac{M_W^2}{M_Z^2}\right)^2 \frac{(v_b^2 + a_b^2)(v_e^2 + a_e^2)}{(1+1)(1+1)} \sim 0.06. \tag{1.2}$$

Taking the proper momentum dependence of the matrix element and the phase space into account one finds [9]

$$\frac{\mathrm{BR}\,(B \rightarrow l^+ l^- X)}{\mathrm{BR}\,(B \rightarrow l^+ \nu_l X)} \geq 0.12. \tag{1.3}$$

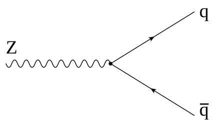
This ratio is four orders of magnitude larger than a bound set by the UA1 Collaboration [10, 11]

$$\frac{\mathrm{BR}\,(B \rightarrow \mu^+ \mu^- X)}{\mathrm{BR}\,(B \rightarrow \mu \nu_\mu X)} < \frac{5.0 \times 10^{-5}}{0.103 \pm 0.005}. \tag{1.4}$$

so that the working hypothesis of an isosinglet b quark is clearly ruled out experimentally also by this method.

Partial width $\Gamma(Z \rightarrow b\bar{b})$ and forward-backward asymmetry of b quarks

The Z boson couples to quarks through vector and axial–vector charges with the well–known strength



$$= \sqrt{\frac{G_F m_Z^2}{2\sqrt{2}}}\gamma_\mu \left[v_q - a_q \gamma_5\right].$$

Depending on the isospin assignment of righthanded and lefthanded quark fields these charges are defined as

$$v_q = 2(I_q^{3L} + I_q^{3R}) - 4e_q \sin^2 \vartheta_W \quad (1.5)$$
$$a_q = 2(I_q^{3L} - I_q^{3R}) \quad (1.6)$$

For the present application the *Born approximation* in the massless limit provides an adequate representation of the partial Z decay rate

$$\Gamma_B(Z \to b\bar{b}) \approx \frac{G_F m_Z^3}{8\sqrt{2}\pi} \beta \left(v_b^2 + a_b^2\right). \quad (1.7)$$

In the Standard Model $2I_q^{3R} = 0$ and $2I_q^{3L} = \pm 1$ for up/down quarks respectively.

The ratio between the predictions in the context of a topless model and the SM amounts to

$$\frac{\Gamma^{\text{topless}}}{\Gamma^{\text{SM}}} = \frac{(4Q_b \sin^2 \theta_W)^2}{(1 + 4Q_b \sin^2 \theta_W)^2 + 1} \approx 1/13 \quad (1.8)$$

whereas theory and LEP experiments are well consistent

$$\frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to \text{had})} = \begin{cases} 0.2155 \pm 0.0004 & \text{theory} \quad [3] \\ 0.2219 \pm 0.0017 & \text{experiment} \quad [3] \end{cases} \quad (1.9)$$

ruling out the $I = 0$ assignement for the b-quark. The forward-backward asymmetry at the Z resonance

$$A^{\text{FB}} = \frac{3}{4} A_e A_b \quad (1.10)$$

with

$$A_f \equiv \frac{2 v_f a_f}{v_f^2 + a_f^2} \quad (1.11)$$

is sensitive toward the relative size of vector and axial b quark couplings. Up to a sign, the first of these factors, $A_e \approx 0.15$, can be interpreted as the degree of longitudinal Z polarisation, $P_Z = -A_e$, which is induced by the electron coupling even for unpolarized beams. For longitudinally polarized beams it can be replaced by unity. The second factor represents essentially the analyzing power of b quarks. With a predicted value of 0.93 it is close to its maximum in the SM. In fact, this high analysing power is the reason for the large sensitivity of A^{FB} toward $\sin^2 \theta_W$ [12]. For a fictitious topless model A_b is zero. The most recent experimental results from LEP and SLC are displayed in Fig. 1.1.

A remaining sign ambiguity is finally resolved by the interference between NC and electromagnetic amplitude. It leads to a forward backward asymmetry at low energies

$$A_{\text{FB}} = -\frac{3 G_F s}{16\sqrt{2}\pi\alpha} \frac{a_e a_b}{Q_e Q_b} \quad (1.12)$$

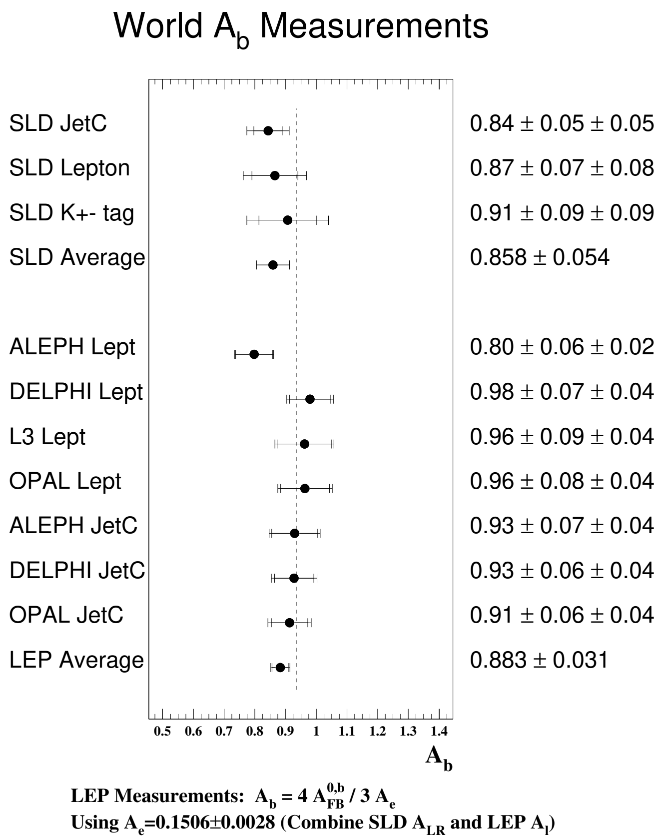


Figure 1.1: Experimental results for the asymmetry parameter A_b [13].

which has been studied in particular at PEP, PETRA, and most recently with highest precision at Tristan at a cm energy of 58 GeV [14]. Using the data available shortly after the turnon of LEP and combining $\Gamma_{b\bar{b}}$, A_b, and $A_{\rm FB}$

$$\begin{array}{lcl} \{I_3^L(b)\}_{exp} = -0.490^{+0.015}_{-0.012} & \rightarrow & I_3^L(b) = -1/2 \\ \{I_3^R(b)\}_{exp} = -0.028 \pm 0.056 & \rightarrow & I_3^R(b) = 0 \end{array}$$

has been obtained already some time ago [15]. As shown in Fig. 1.2 all measurements are nicely consistent with the predictions of the SM[1]. The isospin assignment of the Standard Model is thus well confirmed.

Figure 1.2: The weak isospins $I_3^L(b)$ and $I_3^R(b)$ of the left– and right–handed b quark components, extracted from the data on $\Gamma(Z \rightarrow b\bar{b})$ and $A_{FB}(b)$ at LEP, and PETRA/PEP and TRISTAN, Ref.[15].

[1]For a discussion of the most recent results for R_b, however, see [3].

1.1.2 Mass limits and indirect mass determinations

Theoretical constraints

Present theoretical analyses of the Standard Model are based almost exclusively on perturbation theory. If this method is assumed to apply also to the top-quark sector, in particular when linked to the Higgs sector, the top mass must be bounded as the strength of the Yukawa-Higgs-top coupling is determined by this parameter. The following consistency conditions must be met:

Perturbative Yukawa coupling $g_Y\ (ttH)$

Defining the Yukawa coupling in the Standard Model through

$$\mathcal{L}_Y = g_Y \left(\frac{v+H}{\sqrt{2}}\right)(\bar{t}_L t_R + \text{h.c.}) \tag{1.13}$$

the coupling constant g_Y is related to the top mass by

$$g_Y\ (ttH) = m_t \sqrt{2\sqrt{2}G_F}. \tag{1.14}$$

Demanding the effective expansion parameter $g_Y^2/4\pi$ to be smaller than 1, the top mass is bounded to

$$m_t < \sqrt{\frac{\sqrt{2}\pi}{G_F}} \approx 620\text{GeV}. \tag{1.15}$$

For a top mass of 180 GeV the coupling $g_Y^2/4\pi \approx 0.085$ is comfortably small so that perturbation theory can safely be applied in this region.

Unitarity bound

At asymptotic energies the amplitude of the zeroth partial wave for elastic $t\bar{t}$ scattering in the color singlet same-helicity channel [16]

$$\begin{aligned} a_0\,(t\bar{t} \rightarrow t\bar{t}) &= -\frac{3g_Y^2}{8\pi} \\ &= -\frac{3G_F m_t^2}{4\sqrt{2}\pi} \end{aligned} \tag{1.16}$$

grows quadratically with the top mass. Unitarity however demands this real amplitude to be bounded by $|\text{Re}\, a_0| \leq 1/2$. This condition translates to

$$m_t < \sqrt{\frac{2\sqrt{2}\pi}{3G_F}} \approx 500\,\text{GeV}. \tag{1.17}$$

The bound improves by taking into account the running of the Yukawa coupling [17]. These arguments are equally applicable for any additional species of chiral fermions with mass induced via spontaneous symmetry breaking.