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TOP QUARK PRODUCTION TO $\mathcal{O}(\alpha_s^2)$ † ‡

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Abstract

The cross-section for the production of $t\bar{t}$ pairs in e^+e^--annihilation via a virtual photon is determined up to order α_s^2. For the cm-energies of interest the effect of the top mass may not be considered as a small parameter and the full mass dependence must be included.

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Quark mass effects can often be considered as small perturbations in the analysis of the total cross sections for hadron production in e^+e^--annihilation. The approximation $m_q = 0$, supplemented by "mass corrections" of order m_q^2/s is adequate for most purposes and has lead to reliable predictions including terms of order α_s^3 [1]. The situation is drastically different for top quark production at a linear collider, where $2\,M_t$ and $E_{cm} = \sqrt{s}$ will be of comparable magnitude throughout. With this motivation in mind the QCD corrections to the vector current correlator have been calculated in [2, 3, 4] up to $\mathcal{O}(\alpha_s^2)$ including the full quark mass dependence. Since the corresponding results for the axial vector current are not yet available, the subsequent discussion is strictly applicable for the production through the virtual photon only. The results presented below should therefore not be considered as absolute predictions but rather as indicative for the magnitude and importance of the second order corrections. Our notation and conventions are based on [3]. The cross-section normalized to the point cross-section can be cast into the following form:

$$
\begin{aligned}
R &= \frac{\sigma(e^+e^- \to \gamma^* \to t\bar{t})}{\sigma_{point}} \\
&= Q_t^2 \Bigg[R^{(0)} + \left(\frac{\alpha_s(M_t^2)}{\pi}\right) C_F\, R^{(1)} \\
&\qquad + \left(\frac{\alpha_s(M_t^2)}{\pi}\right)^2 \left(C_F^2\, R_A^{(2)} + C_F\, C_A\, R_{NA}^{(2)} + C_F\, T\, n_l\, R_l^{(2)} + C_F\, T\, R_F^{(2)} \right) \Bigg] \\
&\quad + \sum_{q=u,d,s,c,b} Q_q^2 \left(\frac{\alpha_s(M_t^2)}{\pi}\right)^2 C_F\, T\, R_g^{(2)}\,. \qquad (1)
\end{aligned}
$$

M_t denotes the top quark pole mass and $\alpha_s(M_t^2)$ the $\overline{\mathrm{MS}}$-renormalized strong coupling constant at the scale M_t with $n_l = 5$ light flavours. The abelian and non-abelian parts $R_A^{(2)}$ and $R_{NA}^{(2)}$ are taken from [3], where the Padé approximation method has been employed, whereas $R_l^{(2)}$ and $R_F^{(2)}$ originate from massless and top quark loop insertions, respectively, into the gluon propagator and are given in closed analytical form in [2]. The last term, $R_g^{(2)}$ originates from gluon splitting into $t\bar{t}$ and has been calculated in [5]. To fix our notation explicitely we note in passing that

$$
\begin{aligned}
R^{(0)} &= \frac{3}{2}\beta\,(3-\beta^2)\,, \\
R^{(1)} &= 3\Bigg\{ \frac{(3-\beta^2)\,(1+\beta^2)}{2} \left[2\,\mathrm{Li}_2(p) + \mathrm{Li}_2(p^2) + \ln p \left(2\,\ln(1-p) + \ln(1+p)\right) \right] \\
&\quad - \beta\,(3-\beta^2) \left(2\,\ln(1-p) + \ln(1+p)\right) \\
&\quad - \frac{(1-\beta)\,(33 - 39\,\beta - 17\,\beta^2 + 7\,\beta^3)}{16} \ln p + \frac{3\,\beta\,(5 - 3\,\beta^2)}{8} \Bigg\}\,, \qquad (2)
\end{aligned}
$$

where

$$
p = \frac{1-\beta}{1+\beta}\,, \qquad \beta = \sqrt{1 - 4\,\frac{M_t^2}{s}}\,. \qquad (3)
$$

$\sqrt{s}$ [GeV]	400	500	600	700
β	0.48	0.71	0.81	0.87
$R^{(0)}$	2.0084	2.6673	2.8513	2.9228
$C_F\,R^{(1)}$	17.6874	10.7599	7.7942	6.2463
$C_F^2\,R_A^{(2)}$	3.5(1)	-21.4(1)	-19.63(1)	-16.32(1)
$C_F\,C_A\,R_{NA}^{(2)}$	117.8(2)	31.9(1)	4.03(1)	-8.61(1)
$n_l C_F\,T\,R_l^{(2)}$	-32.4356	-6.8356	0.8496	4.1789
$C_F\,T\,R_F^{(2)}$	0.7044	1.0441	1.2375	1.3834
$\sum_q Q_q^2/Q_t^2\,C_F\,T\,R_g^{(2)}$	$1.2\;\;10^{-6}$	0.0005	0.0051	0.0209
$R^{(2)}$	89.5(3)	4.7(2)	-13.50(2)	-19.35(2)
$R^{(\alpha_s=0.115)}$	1.200	1.348	1.376	1.382
$R^{(\alpha_s=0.120)}$	1.213	1.354	1.380	1.385
$R^{(\alpha_s=0.125)}$	1.227	1.360	1.384	1.388

Table 1: Results for the different contributions to R for $M_t = 175$ GeV and the cm-energies $E_{cm} = 400, 500, 600, 700$ GeV. The total cross-section via photon exchange is given for the three different values of $\alpha_s(M_t^2)$ corresponding to $\alpha_s(M_Z^2) = 0.115, 0.120$ and 0.125.

β is the velocity of one of the produced top quarks in the $t\bar{t}$ cm frame. The result for the different contributions $R_x^{(i)}$ are presented in Table 1 for various energies, adopting a top mass value of 175 GeV. The error indicated for $R_A^{(2)}$ and $R_{NA}^{(2)}$ originates from the possible choice of different Padé approximants. As $R_l^{(2)}$, $R_F^{(2)}$ and $R_g^{(2)}$ are known analytically no error is specified in this case. $R^{(2)}$ denotes the sum of all second order correction functions. To arrive at the prediction for R, the correction functions have to be multiplied by the quark charge and the corresponding power of the strong coupling constant evaluated at the scale M_t. For $\alpha_s(M_Z^2) = 0.115, 0.120$ and 0.125 this corresponds to $\alpha_s(M_t^2) = 0.105, 0.109$ and 0.113, respectively. The complete predictions for the three values of α_s are also presented in Table 1. In Fig. 1a R is plotted against $\sqrt{s}$ including successively higher orders in α_s. The dependence on the choice of $\alpha_s(M_Z^2)$ is shown in Fig. 1b.

References

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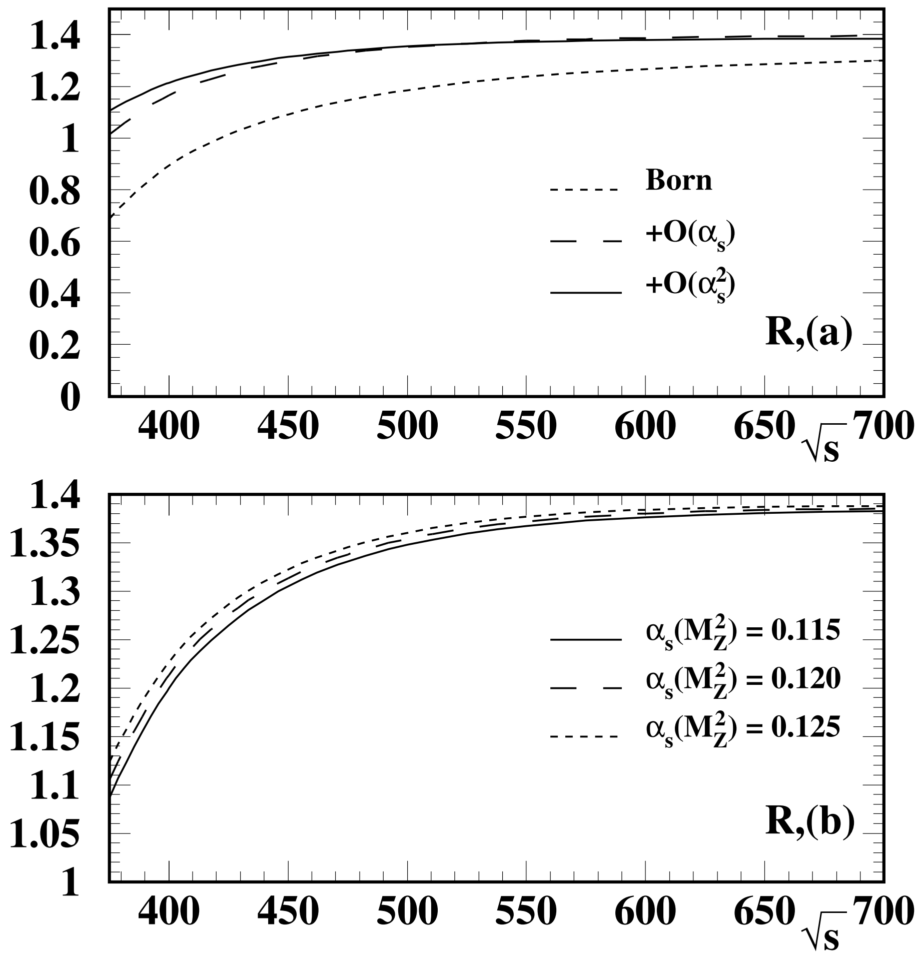

Figure 1: R as function of $\sqrt{s}$. In figure (b) the $\mathcal{O}(\alpha_s^2)$ prediction to R is plotted for different choices of $\alpha_s(M_Z^2)$.