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CP violation and the CKM angle γ from angular distributions of untagged B_s decays governed by $\bar{b} \to \bar{c}u\bar{s}$

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Abstract

We demonstrate that time-dependent studies of angular distributions for B_s decays caused by $\bar{b} \to \bar{c}u\bar{s}$ quark-level transitions extract *cleanly* and *model-independently* the CKM angle γ. This CKM angle could be cleanly determined from *untagged* B_s decays alone, if the lifetime difference between the B_s mass eigenstates B_s^L and B_s^H is sizable. The time-dependences for the relevant *tagged* and *untagged* observables are given both in a general notation and in terms of linear polarization states and should exhibit large CP-violating effects. These observables may furthermore provide insights into the hadronization dynamics of the corresponding exclusive B_s decays thereby allowing tests of the factorization hypothesis.

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1 Introduction

Some time ago it was pointed out in [1] that a clean measurement of the angle γ of the usual "non-squashed" unitarity triangle [2] of the Cabibbo–Kobayashi–Maskawa matrix (CKM matrix) [3] is possible by studying the time dependence of the color-allowed decays $\overset{(-)}{B_s} \to D_s^{\pm} K^{\mp}$. A similar analysis of the color-suppressed modes $\overset{(-)}{B_s} \to \overset{(-)}{D^0}\, \phi$ provides in principle also clean information about γ [4]. Because current detectors have difficulties in observing the soft photon in $D_s^* \to D_s\gamma$ decays, Aleksan, Le Yaouanc, Oliver, Pene and Raynal employed several plausible assumptions to show that the CKM angle γ can still be extracted from partially reconstructed B_s modes where that soft photon could be missed [5].

Unfortunately in all these strategies *tagging*, i.e. the distinction between initially present B_s and $\overline{B_s}$ mesons, is essential. Moreover one has to resolve the rapid $B_s - \overline{B_s}$ oscillations, which may arise from the expected large mass difference $\Delta m \equiv m_H - m_L > 0$ between the mass eigenstates B_s^H ("heavy") and B_s^L ("light") [6]. This is a formidable experimental task. In a recent paper [7] these methods have been re-considered in light of the expected perceptible lifetime difference [8] between B_s^H and B_s^L. There it has been shown that the rapid Δmt–oscillations cancel in *untagged* data samples. Whereas the extraction of γ from untagged $\overset{(-)}{B_s} \to D_s^{\pm} K^{\mp}$ requires some mild additional theoretical input, it does not require any theory beyond the validity of the CKM model from untagged $\overset{(-)}{B_s} \to \overset{(-)}{D^0}\, \phi$ decays [7].

In a recent publication [9] we have investigated quasi two body modes $B_s \to X_1\, X_2$ into admixtures of different CP eigenstates where both X_1 and X_2 carry spin and continue to decay through CP-conserving interactions. The time-dependent angular distributions for the *untagged* decays $B_s \to D_s^{*+}\, D_s^{*-}$ and $B_s \to J/\psi\, \phi$ determine the Wolfenstein parameter η [10]. If one uses $|V_{ub}|/|V_{cb}|$ as an additional input, the CKM angle γ can be fixed. That input allows, however, also the determination of η (or γ) from the mixing-induced CP asymmetry of $B_d \to J/\psi\, K_S$ measuring $\sin 2\beta$ (β is another angle of the unitarity triangle [2]). Comparing these two results for η (or γ) obtained from B_s and B_d modes, respectively, an interesting test whether the $B_s - \overline{B_s}$ and $B_d - \overline{B_d}$ mixing phases are described by the Standard Model or receive additional contributions from "New Physics" can be preformed. Another application of the formalism developed in [9] is the point that a determination of γ is possible by using the $SU(2)$ isospin symmetry of strong interactions to relate *untagged* data samples of $B_s \to K^{*+}\, K^{*-}$ and $B_s \to K^{*0}\, \overline{K^{*0}}$.

Having all these results in mind it is quite natural to ask what can be learned from time-dependent *untagged* measurements of the angular distributions for $\overset{(-)}{B_s} \to D_s^{*\pm}\, K^{*\mp}$,

$D_{s1}(2536)^{\pm}K^{*\mp}$, $D_s^{**\pm}K^{*\mp}$ and $\overset{(-)}{B_s}\to\overset{(-)}{D^{*0}}\ \phi$, $\overset{(-)}{D_1}\ (2420)^0\phi$, $\overset{(-)}{D^{**0}}\ \phi$ or – more generally – from B_s modes governed by $\bar{b}\to\bar{c}u\bar{s}$ quark-level transitions. Since the photon(s) in the strong or electromagnetic decays of D_s^* and D^{*0} are more difficult to detect than charged particles for generic detectors, we listed also higher resonances because of their significant all-charged final states, such as $D_{s1}(2536)^+\to D^{*+}K^0, D^{*+}\to\pi^+D^0$ or $D_1(2420)^0\to D^{*+}\pi^-, D^{*+}\to\pi^+D^0$. The $K^{*\mp}$ in the above B_s-decays can be substituted by either a strange resonance or a collection of strange resonances with common spin and parity quantum numbers.

While our note focusses on quasi two body modes where each body has a well-defined spin and parity, a complementary report discusses the effects when several resonances contribute to the final state [11]. In the former case the final states cannot be classified by their CP eigenvalues as in [9]. However, they can instead be classified by their parities. To this end linear polarization states [12] are particularly useful. As we will demonstrate in the present paper, the *untagged* angular distributions for such B_s decays may inform us in a clean way about γ, if the lifetime difference between B_s^H and B_s^L is in fact sizable. In particular we do not need any theoretical input to extract this quantity from the *untagged* data samples which exhibit in addition interesting CP-violating effects. Furthermore essentially the whole hadronization dynamics can be extracted from these angular correlations. Since, for example, the $\overset{(-)}{B_s}\to D_s^{*\pm}\ K^{*\mp}$, $D_{s1}(2536)^{\pm}K^{*\mp}$, $D_s^{**\pm}K^{*\mp}$ modes are color-allowed whereas the $\overset{(-)}{B_s}\to\overset{(-)}{D^{*0}}\ \phi$, $\overset{(-)}{D_1}\ (2420)^0\phi$, $\overset{(-)}{D^{**0}}\ \phi$ channels are color-suppressed, the factorization hypothesis [13, 14], which has some justification within the $1/N_{\mathrm{C}}$–expansion [15], should work quite well in the former case and should be very questionable in the latter case [16]. Therefore we expect significant non-factorizable contributions to the angular distributions for the $\overset{(-)}{B_s}\to\overset{(-)}{D^{*0}}\ \phi$, $\overset{(-)}{D_1}\ (2420)^0\phi$, $\overset{(-)}{D^{**0}}\ \phi$ decays. The explicit angular distributions for some of these decays will be given in a separate publication [17]. There also appropriate weighting functions are given allowing an efficient extraction of the corresponding observables from experimental data with the help of a *moment analysis* (see [18, 19]).

Our paper is organized as follows: The time-dependences of the observables of the angular distributions are calculated in Section 2 in terms of a general notation that allows an easy comparison with the results presented in [9]. In Section 3 these time-dependences are given in terms of linear polarization states which provide a useful tool to calculate the explicit angular distributions for final state configurations having definite parities. There we demonstrate explicitly that the observables of the *untagged* angular distributions for the $\bar{b}\to\bar{c}u\bar{s}$ (and $\bar{b}\to\bar{u}c\bar{s}$) decays suffice to extract the CKM angle γ. The issue of CP violation in untagged data samples is discussed in Section 4 and the main results of our paper are summarized briefly in Section 5.

2 Calculation of the general time-evolutions

In the case of the decays considered in this paper, the transition amplitudes for the quasi two body modes $\overline{B_s} \to X_1\,X_2$ and $B_s \to X_1\,X_2$ can be expressed as hadronic matrix elements of low energy effective Hamiltonians having the following structures:

$$\begin{aligned} H_{\text{eff}}\,(\overline{B_s} \to X_1\,X_2) &= \frac{G_{\mathrm{F}}}{\sqrt{2}}\overline{v}\left[C_1(\mu)\overline{O}_1 + C_2(\mu)\overline{O}_2\right] &(1)\\ H_{\text{eff}}\,(B_s \to X_1\,X_2) &= \frac{G_{\mathrm{F}}}{\sqrt{2}}v^*\left[C_1(\mu)O_1^\dagger + C_2(\mu)O_2^\dagger\right], &(2)\end{aligned}$$

where $\overline{v}$ and v denote appropriate CKM factors, $\overline{O}_k$ and O_k ($k \in \{1,2\}$) are four-quark operators ("current-current" operators in our case) and $C_1(\mu)$ and $C_2(\mu)$ are the Wilson coefficient functions of these operators. They can be calculated perturbatively and contain the whole short distance dynamics. As usual $\mu = \mathcal{O}(m_b)$ is a renormalization scale. To be definite, for $X_1\,X_2 \in \{D_s^{*+}\,K^{*-},\ D_{s1}(2536)^+K^{*-},\ D_s^{**+}K^{*-},\ D^{*0}\,\phi,\ D_1(2420)^0\phi,\ D^{**0}\phi\}$ we have

$$\begin{aligned} \overline{O}_1 &= (\bar{s}_\alpha u_\beta)_{\mathrm{V-A}}\,(\bar{c}_\beta b_\alpha)_{\mathrm{V-A}}\\ \overline{O}_2 &= (\bar{s}_\alpha u_\alpha)_{\mathrm{V-A}}\,(\bar{c}_\beta b_\beta)_{\mathrm{V-A}}\end{aligned} \qquad (3)$$

$$\begin{aligned} O_1 &= (\bar{s}_\alpha c_\beta)_{\mathrm{V-A}}\,(\bar{u}_\beta b_\alpha)_{\mathrm{V-A}}\\ O_2 &= (\bar{s}_\alpha c_\alpha)_{\mathrm{V-A}}\,(\bar{u}_\beta b_\beta)_{\mathrm{V-A}}\,,\end{aligned} \qquad (4)$$

where the greek indices denote $SU(3)_{\mathrm{C}}$ color indices, and the CKM factors are given by

$$\begin{aligned} \overline{v} &= V_{us}^* V_{cb}\\ v &= V_{cs}^* V_{ub}.\end{aligned} \qquad (5)$$

Nowadays the Wilson coefficients $C_1(\mu)$ and $C_2(\mu)$ are available beyond the leading logarithmic approximation [20, 21]. A nice review of such next-to-leading order calculations has been given recently in [22], and we refer the reader to that publication for the details of such calculations.

Applying a similar notation as in [9], we obtain the following transition amplitudes for decays of B_s and $\overline{B_s}$ mesons into a configuration f of the quasi two body state $X_1\,X_2$, where f is a label that defines the relative polarizations of the two hadrons X_1 and X_2:

$$\begin{aligned} \overline{A}_f &\equiv \langle (X_1\,X_2)_f | H_{\text{eff}}\,(\overline{B_s} \to X_1\,X_2) | \overline{B_s}\rangle = \frac{G_{\mathrm{F}}}{\sqrt{2}}\,\overline{v}\,\overline{M}_f &(6)\\ A_f &\equiv \langle (X_1\,X_2)_f | H_{\text{eff}}\,(B_s \to X_1\,X_2) | B_s\rangle = \eta_{\mathrm{P}}^f\, e^{i\phi_{\mathrm{CP}}(B_s)}\,\frac{G_{\mathrm{F}}}{\sqrt{2}}\,v^*\,M_f &(7)\end{aligned}$$

with

$$\begin{aligned} \overline{M}_f &\equiv \langle (X_1\,X_2)_f | C_1(\mu)\overline{O}_1 + C_2(\mu)\overline{O}_2 | \overline{B_s}\rangle &(8)\\ M_f &\equiv \langle (X_1\,X_2)_f^{\mathrm{C}} | C_1(\mu)O_1 + C_2(\mu)O_2 | \overline{B_s}\rangle. &(9)\end{aligned}$$

In order to evaluate (7) we have performed the CP transformations

$$\begin{aligned}
&\langle (X_1\, X_2)_f | C_1(\mu) O_1^\dagger + C_2(\mu) O_2^\dagger | B_s \rangle \\
&\quad = \langle (X_1\, X_2)_f | (\mathcal{CP})^\dagger (\mathcal{CP}) \left[C_1(\mu) O_1^\dagger + C_2(\mu) O_2^\dagger \right] (\mathcal{CP})^\dagger (\mathcal{CP}) | B_s \rangle \\
&\quad = \eta_{\rm P}^f \, e^{i\phi_{\rm CP}(B_s)} \, \langle (X_1\, X_2)_f^{\rm C} | C_1(\mu) O_1 + C_2(\mu) O_2 | \overline{B_s} \rangle
\end{aligned} \tag{10}$$

by taking into account the relations

$$(\mathcal{CP}) O_k^\dagger (\mathcal{CP})^\dagger = O_k \tag{11}$$

and

$$(\mathcal{CP}) | B_s \rangle = e^{i\phi_{\rm CP}(B_s)} \, | \overline{B_s} \rangle \tag{12}$$

$$(\mathcal{CP}) | (X_1\, X_2)_f \rangle = \eta_{\rm P}^f \, | (X_1\, X_2)_f^{\rm C} \rangle . \tag{13}$$

Here $\phi_{\rm CP}(B_s)$ parametrizes the applied CP phase convention and $\eta_{\rm P}^f \in \{-1, +1\}$ denotes the parity eigenvalues of the configurations f of $X_1\, X_2$. In terms of linear polarization amplitudes [12] (see also [23]) we have $\eta_{\rm P}^0 = \eta_{\rm P}^{||} = +1$ and $\eta_{\rm P}^{\perp} = -1$ for $X_1\, X_2 \in \{D_s^{*+} K^{*-}, D^{*0}\phi\}$. In contrast, for $X_1\, X_2 \in \{D_{s1}(2536)^+ K^{*-}, D_1(2420)^0 \phi\}$ we have $\eta_{\rm P}^0 = \eta_{\rm P}^{||} = -1$ and $\eta_{\rm P}^{\perp} = +1$.

Let us now consider the $\overline{B_s}$ and B_s decays into the charge-conjugate quasi two body states $(X_1\, X_2)^{\rm C}$. In the case relevant for the present paper corresponding to $X_1\, X_2 \in \{D_s^{*+} K^{*-},\ D_{s1}(2536)^+ K^{*-},\ D_s^{**+} K^{*-},\ D^{*0}\,\phi, D_1(2420)^0\phi,\ D^{**0}\phi\}$ we have $(X_1\, X_2)^{\rm C} \in \{D_s^{*-} K^{*+},\ D_{s1}(2536)^- K^{*+},\ D_s^{**-} K^{*+},\ \overline{D}^{*0}\,\phi,\ \overline{D}_1(2420)^0\phi,\ \overline{D}^{**0}\phi\}$, respectively. If the charge-conjugate states are present in a configuration f with parity eigenvalue $\eta_{\rm P}^f$, a similar calculation as sketched above yields

$$\begin{aligned}
\overline{A}_f^{\rm C} &\equiv \langle (X_1\, X_2)_f^{\rm C} | H_{\rm eff}(\overline{B_s} \to (X_1\, X_2)^{\rm C}) | \overline{B_s} \rangle = \frac{G_{\rm F}}{\sqrt{2}}\, v\, M_f &(14)\\
A_f^{\rm C} &\equiv \langle (X_1\, X_2)_f^{\rm C} | H_{\rm eff}(B_s \to (X_1\, X_2)^{\rm C}) | B_s \rangle = \eta_{\rm P}^f \, e^{i\phi_{\rm CP}(B_s)} \frac{G_{\rm F}}{\sqrt{2}}\, \overline{v}^*\, \overline{M}_f . &(15)
\end{aligned}$$

Using these results and the well-known formalism describing $B_s - \overline{B_s}$ mixing [7, 24], we obtain the following expressions for initially, i.e. at $t = 0$, present B_s and $\overline{B_s}$ mesons:

$$\begin{aligned}
A_{\tilde f}^*(t)\, A_f(t) = \frac{G_{\rm F}^2}{2} |v|^2 \eta_{\rm P}^{\tilde f} \eta_{\rm P}^f M_{\tilde f}^* M_f \\
\times \left[|g_+(t)|^2 + \eta_{\rm P}^{\tilde f} \lambda_{\tilde f}^* g_+(t)\, g_-^*(t) + \eta_{\rm P}^f \lambda_f\, g_+^*(t)\, g_-(t) + \eta_{\rm P}^{\tilde f} \eta_{\rm P}^f \lambda_{\tilde f}^* \lambda_f\, |g_-(t)|^2 \right]
\end{aligned} \tag{16}$$

$$\begin{aligned}
\overline{A}_{\tilde f}^*(t)\, \overline{A}_f(t) = \frac{G_{\rm F}^2}{2} |v|^2 \eta_{\rm P}^{\tilde f} \eta_{\rm P}^f M_{\tilde f}^* M_f \\
\times \left[|g_-(t)|^2 + \eta_{\rm P}^{\tilde f} \lambda_{\tilde f}^* g_+^*(t)\, g_-(t) + \eta_{\rm P}^f \lambda_f\, g_+(t)\, g_-^*(t) + \eta_{\rm P}^{\tilde f} \eta_{\rm P}^f \lambda_{\tilde f}^* \lambda_f\, |g_+(t)|^2 \right]
\end{aligned} \tag{17}$$

$$A_{\tilde f}^{\mathrm{C}*}(t)\, A_f^{\mathrm{C}}(t) = \frac{G_{\mathrm{F}}^2}{2}\, |v|^2\, M_{\tilde f}^*\, M_f \tag{18}$$
$$\times \left[|g_-(t)|^2 + \eta_{\mathrm{P}}^{\tilde f}\, \lambda_{\tilde f}^{\mathrm{C}*}\, g_+^*(t)\, g_-(t) + \eta_{\mathrm{P}}^f\, \lambda_f^{\mathrm{C}}\, g_+(t)\, g_-^*(t) + \eta_{\mathrm{P}}^{\tilde f}\, \eta_{\mathrm{P}}^f\, \lambda_{\tilde f}^{\mathrm{C}*}\, \lambda_f^{\mathrm{C}}\, |g_+(t)|^2\right]$$

$$\overline{A}_{\tilde f}^{\mathrm{C}*}(t)\, \overline{A}_f^{\mathrm{C}}(t) = \frac{G_{\mathrm{F}}^2}{2}\, |v|^2\, M_{\tilde f}^*\, M_f \tag{19}$$
$$\times \left[|g_+(t)|^2 + \eta_{\mathrm{P}}^{\tilde f}\, \lambda_{\tilde f}^{\mathrm{C}*}\, g_+(t)\, g_-^*(t) + \eta_{\mathrm{P}}^f\, \lambda_f^{\mathrm{C}}\, g_+^*(t)\, g_-(t) + \eta_{\mathrm{P}}^{\tilde f}\, \eta_{\mathrm{P}}^f\, \lambda_{\tilde f}^{\mathrm{C}*}\, \lambda_f^{\mathrm{C}}\, |g_-(t)|^2\right],$$

where

$$|g_\pm(t)|^2 = \frac{1}{4}\left[e^{-\Gamma_L t} + e^{-\Gamma_H t} \pm 2e^{-\Gamma t}\cos(\Delta m t)\right] \tag{20}$$
$$g_+(t) g_-^*(t) = \frac{1}{4}\left[e^{-\Gamma_L t} - e^{-\Gamma_H t} - 2ie^{-\Gamma t}\sin(\Delta m t)\right] \tag{21}$$

with $\Gamma \equiv (\Gamma_L + \Gamma_H)/2$. The observable λ_f is defined through

$$\lambda_f \equiv -\,\eta_{\mathrm{P}}^f\, e^{-i\Theta_{M_{12}}^{(s)}}\, \frac{\overline{A}_f}{A_f} \tag{22}$$

with

$$\Theta_{M_{12}}^{(s)} = \pi + 2\arg(V_{ts}^* V_{tb}) - \phi_{\mathrm{CP}}(B_s) \tag{23}$$

denoting the phase of the off-diagonal element of the $B_s - \overline{B_s}$ mass matrix. Combining (22) with (23) and (6) and (7), we observe explicitly that the convention dependent phases $\phi_{\mathrm{CP}}(B_s)$ cancel (as they have to!) and arrive at

$$\lambda_f = \exp\left(-2\, i \arg\{V_{ts}^* V_{tb}\}\right)\, \frac{\overline{v}}{v^*}\, \frac{\overline{M}_f}{M_f}. \tag{24}$$

Correspondingly we have introduced

$$\lambda_f^{\mathrm{C}} \equiv -\frac{1}{\eta_{\mathrm{P}}^f\, e^{-i\Theta_{M_{12}}^{(s)}}}\, \frac{A_f^{\mathrm{C}}}{\overline{A}_f^{\mathrm{C}}} = \left[\exp\left(-2\, i \arg\{V_{ts}^* V_{tb}\}\right)\, \frac{\overline{v}}{v^*}\right]^* \frac{\overline{M}_f}{M_f}. \tag{25}$$

Note that $\lambda_{\tilde f}$ and $\lambda_{\tilde f}^{\mathrm{C}}$ can be obtained easily from (24) and (25) by replacing f with $\tilde f$.

Real or imaginary parts of bilinear combinations of decay amplitudes like those given in (16)-(19) govern the angular distributions for the decay products of X_1 and X_2. In this paper we are focussing on *untagged* angular distributions, where one does not distinguish between initially present B_s and $\overline{B_s}$ mesons. The corresponding observables for $\overset{(-)}{B_s} \to X_1\, X_2$ and $\overset{(-)}{B_s} \to (X_1\, X_2)^{\mathrm{C}}$ are related to real or imaginary parts of

$$\left[A_{\tilde f}^*(t)\, A_f(t)\right] \equiv \overline{A}_{\tilde f}^*(t)\, \overline{A}_f(t) + A_{\tilde f}^*(t)\, A_f(t) = \frac{G_{\mathrm{F}}^2}{4}\, |v|^2\, \eta_{\mathrm{P}}^{\tilde f}\, \eta_{\mathrm{P}}^f\, M_{\tilde f}^*\, M_f \tag{26}$$
$$\times \left[\left(1 + \eta_{\mathrm{P}}^{\tilde f}\, \eta_{\mathrm{P}}^f \lambda_{\tilde f}^* \lambda_f\right)\left(e^{-\Gamma_L t} + e^{-\Gamma_H t}\right) + \left(\eta_{\mathrm{P}}^{\tilde f}\, \lambda_{\tilde f}^* + \eta_{\mathrm{P}}^f\, \lambda_f\right)\left(e^{-\Gamma_L t} - e^{-\Gamma_H t}\right)\right]$$

and

$$\left[A_{\tilde{f}}^{\mathrm{C}*}(t)\,A_f^{\mathrm{C}}(t)\right] \equiv \overline{A}_{\tilde{f}}^{\mathrm{C}*}(t)\,\overline{A}_f^{\mathrm{C}}(t) + A_{\tilde{f}}^{\mathrm{C}*}(t)\,A_f^{\mathrm{C}}(t) = \frac{G_{\mathrm{F}}^2}{4}\,|v|^2\,M_{\tilde{f}}^*\,M_f \qquad (27)$$
$$\times\left[\left(1+\eta_{\mathrm{P}}^{\tilde{f}}\,\eta_{\mathrm{P}}^{f}\lambda_{\tilde{f}}^{\mathrm{C}*}\,\lambda_f^{\mathrm{C}}\right)\left(e^{-\Gamma_L t}+e^{-\Gamma_H t}\right)+\left(\eta_{\mathrm{P}}^{\tilde{f}}\,\lambda_{\tilde{f}}^{\mathrm{C}*}+\eta_{\mathrm{P}}^{f}\,\lambda_f^{\mathrm{C}}\right)\left(e^{-\Gamma_L t}-e^{-\Gamma_H t}\right)\right],$$

respectively. In order to evaluate these equations we have combined (16)-(19) with the explicit time-dependences of (20) and (21). At present such *untagged* studies are obviously much more efficient from an experimental point of view than tagged analyses. In the distant future it will be feasible to collect also *tagged* data samples of B_s decays and to resolve the rapid oscillatory Δmt–terms. The corresponding tagged observables are given in (16)-(19).

Let us after these general considerations become more specific in the following section. There we give the time-evolutions in terms of linear polarization states and demonstrate that the *untagged* observables evolving as real or imaginary parts of (26) and (27) suffice to extract the CKM angle γ.

3 The extraction of the CKM angle γ

Since it is convenient to give the angular distributions in terms of the linear polarization states $f \in \{\,0\,,\,\|\,,\,\perp\,\}$ (see [12, 23]), let us summarize the corresponding time-dependences in this section. The linear polarization states are characterized by the parity eigenvalues η_{P}^f. If we introduce the quantity

$$R_f \equiv |R_f|e^{i\rho_f} \equiv \frac{|\overline{v}|}{|v|}\frac{\overline{M}_f}{M_f}, \qquad (28)$$

where ρ_f is a CP-conserving strong phase originating from strong final state interaction processes, we have in our specific case $X_1\,X_2 \in \{D_s^{*+}K^{*-},\ D_{s1}(2536)^+K^{*-},\ D_s^{**+}K^{*-},\ D^{*0}\phi,\ D_1(2420)^0\phi,\ D^{**0}\phi\}$

$$\lambda_f \;=\; e^{-i\gamma}R_f \qquad (29)$$
$$\lambda_f^{\mathrm{C}} \;=\; e^{+i\gamma}R_f, \qquad (30)$$

where γ is the notoriously difficult to measure CKM angle of the unitarity triangle [2]. Using (5) and the Wolfenstein expansion [10] of the CKM matrix by neglecting terms of $\mathcal{O}(\lambda^2)$, where $\lambda = \sin\theta_{\mathrm{C}} = 0.22$ is related to the Cabibbo angle, we obtain

$$R_f = \frac{1}{R_b}\frac{\overline{M}_f}{M_f} \qquad (31)$$

with

$$R_b \equiv \frac{1}{\lambda}\frac{|V_{ub}|}{|V_{cb}|}. \tag{32}$$

The CKM factor R_b is constrained by present experimental data to lie within the range $R_b = 0.36 \pm 0.08$ [25, 26, 27].

If we express the hadronic matrix elements M_f defined by (9) in the form

$$M_f = |M_f| e^{i\vartheta_f}, \tag{33}$$

where ϑ_f denotes a CP-conserving strong phase shift, the time-dependent *untagged* observables corresponding to the linear polarization states [12] are in the case of $\overset{(-)}{B_s} \to X_1\, X_2$ given by

$$\begin{aligned}\left[|A_0(t)|^2\right] &= \frac{G_{\mathrm{F}}^2}{4}|V_{ub}V_{cs}|^2|M_0|^2 \\ &\times \left[\left(1+|R_0|^2\right)\left(e^{-\Gamma_L t}+e^{-\Gamma_H t}\right)+2|R_0|\cos(\rho_0-\gamma)\left(e^{-\Gamma_L t}-e^{-\Gamma_H t}\right)\right]\end{aligned} \tag{34}$$

$$\begin{aligned}\left[|A_{\|}(t)|^2\right] &= \frac{G_{\mathrm{F}}^2}{4}|V_{ub}V_{cs}|^2|M_{\|}|^2 \\ &\times \left[\left(1+|R_{\|}|^2\right)\left(e^{-\Gamma_L t}+e^{-\Gamma_H t}\right)+2|R_{\|}|\cos(\rho_{\|}-\gamma)\left(e^{-\Gamma_L t}-e^{-\Gamma_H t}\right)\right]\end{aligned} \tag{35}$$

$$\begin{aligned}\left[|A_{\perp}(t)|^2\right] &= \frac{G_{\mathrm{F}}^2}{4}|V_{ub}V_{cs}|^2|M_{\perp}|^2 \\ &\times \left[\left(1+|R_{\perp}|^2\right)\left(e^{-\Gamma_L t}+e^{-\Gamma_H t}\right)-2|R_{\perp}|\cos(\rho_{\perp}-\gamma)\left(e^{-\Gamma_L t}-e^{-\Gamma_H t}\right)\right]\end{aligned} \tag{36}$$

$$\begin{aligned}\left[A_0^*(t)A_{\|}(t)\right] &= \frac{G_{\mathrm{F}}^2}{4}|V_{ub}V_{cs}|^2|M_0||M_{\|}|e^{i(\vartheta_{\|}-\vartheta_0)}\left[\left(1+|R_0||R_{\|}|e^{i(\rho_{\|}-\rho_0)}\right)\right. \\ &\times \left.\left(e^{-\Gamma_L t}+e^{-\Gamma_H t}\right)+\left(|R_0|e^{i(\gamma-\rho_0)}+|R_{\|}|e^{-i(\gamma-\rho_{\|})}\right)\left(e^{-\Gamma_L t}-e^{-\Gamma_H t}\right)\right]\end{aligned} \tag{37}$$

$$\begin{aligned}\left[A_{\|}^*(t)A_{\perp}(t)\right] &= -\frac{G_{\mathrm{F}}^2}{4}|V_{ub}V_{cs}|^2|M_{\|}||M_{\perp}|e^{i(\vartheta_{\perp}-\vartheta_{\|})}\left[\left(1-|R_{\|}||R_{\perp}|e^{i(\rho_{\perp}-\rho_{\|})}\right)\right. \\ &\times \left.\left(e^{-\Gamma_L t}+e^{-\Gamma_H t}\right)+\left(|R_{\|}|e^{i(\gamma-\rho_{\|})}-|R_{\perp}|e^{-i(\gamma-\rho_{\perp})}\right)\left(e^{-\Gamma_L t}-e^{-\Gamma_H t}\right)\right]\end{aligned} \tag{38}$$

$$\begin{aligned}\left[A_0^*(t)A_{\perp}(t)\right] &= -\frac{G_{\mathrm{F}}^2}{4}|V_{ub}V_{cs}|^2|M_0||M_{\perp}|e^{i(\vartheta_{\perp}-\vartheta_0)}\left[\left(1-|R_0||R_{\perp}|e^{i(\rho_{\perp}-\rho_0)}\right)\right. \\ &\times \left.\left(e^{-\Gamma_L t}+e^{-\Gamma_H t}\right)+\left(|R_0|e^{i(\gamma-\rho_0)}-|R_{\perp}|e^{-i(\gamma-\rho_{\perp})}\right)\left(e^{-\Gamma_L t}-e^{-\Gamma_H t}\right)\right].\end{aligned} \tag{39}$$

For the untagged decays into the charge conjugate two body states we obtain on the other hand the following expressions:

$$\begin{aligned}\left[|A_0^{\mathrm{C}}(t)|^2\right] &= \frac{G_{\mathrm{F}}^2}{4}|V_{ub}V_{cs}|^2|M_0|^2 \\ &\times\left[\left(1+|R_0|^2\right)\left(e^{-\Gamma_L t}+e^{-\Gamma_H t}\right)+2|R_0|\cos(\rho_0+\gamma)\left(e^{-\Gamma_L t}-e^{-\Gamma_H t}\right)\right]\end{aligned} \tag{40}$$

$$\begin{aligned}\left[|A_\parallel^{\mathrm{C}}(t)|^2\right] &= \frac{G_{\mathrm{F}}^2}{4}|V_{ub}V_{cs}|^2|M_\parallel|^2 \\ &\times\left[\left(1+|R_\parallel|^2\right)\left(e^{-\Gamma_L t}+e^{-\Gamma_H t}\right)+2|R_\parallel|\cos(\rho_\parallel+\gamma)\left(e^{-\Gamma_L t}-e^{-\Gamma_H t}\right)\right]\end{aligned} \tag{41}$$

$$\begin{aligned}\left[|A_\perp^{\mathrm{C}}(t)|^2\right] &= \frac{G_{\mathrm{F}}^2}{4}|V_{ub}V_{cs}|^2|M_\perp|^2 \\ &\times\left[\left(1+|R_\perp|^2\right)\left(e^{-\Gamma_L t}+e^{-\Gamma_H t}\right)-2|R_\perp|\cos(\rho_\perp+\gamma)\left(e^{-\Gamma_L t}-e^{-\Gamma_H t}\right)\right]\end{aligned} \tag{42}$$

$$\begin{aligned}\left[A_0^{\mathrm{C}*}(t)A_\parallel^{\mathrm{C}}(t)\right] &= \frac{G_{\mathrm{F}}^2}{4}|V_{ub}V_{cs}|^2|M_0||M_\parallel|e^{i(\vartheta_\parallel-\vartheta_0)}\left[\left(1+|R_0||R_\parallel|e^{i(\rho_\parallel-\rho_0)}\right)\right. \\ &\times\left.\left(e^{-\Gamma_L t}+e^{-\Gamma_H t}\right)+\left(|R_0|e^{-i(\gamma+\rho_0)}+|R_\parallel|e^{i(\gamma+\rho_\parallel)}\right)\left(e^{-\Gamma_L t}-e^{-\Gamma_H t}\right)\right]\end{aligned} \tag{43}$$

$$\begin{aligned}\left[A_\parallel^{\mathrm{C}*}(t)A_\perp^{\mathrm{C}}(t)\right] &= \frac{G_{\mathrm{F}}^2}{4}|V_{ub}V_{cs}|^2|M_\parallel||M_\perp|e^{i(\vartheta_\perp-\vartheta_\parallel)}\left[\left(1-|R_\parallel||R_\perp|e^{i(\rho_\perp-\rho_\parallel)}\right)\right. \\ &\times\left.\left(e^{-\Gamma_L t}+e^{-\Gamma_H t}\right)+\left(|R_\parallel|e^{-i(\gamma+\rho_\parallel)}-|R_\perp|e^{i(\gamma+\rho_\perp)}\right)\left(e^{-\Gamma_L t}-e^{-\Gamma_H t}\right)\right]\end{aligned} \tag{44}$$

$$\begin{aligned}\left[A_0^{\mathrm{C}*}(t)A_\perp^{\mathrm{C}}(t)\right] &= \frac{G_{\mathrm{F}}^2}{4}|V_{ub}V_{cs}|^2|M_0||M_\perp|e^{i(\vartheta_\perp-\vartheta_0)}\left[\left(1-|R_0||R_\perp|e^{i(\rho_\perp-\rho_0)}\right)\right. \\ &\times\left.\left(e^{-\Gamma_L t}+e^{-\Gamma_H t}\right)+\left(|R_0|e^{-i(\gamma+\rho_0)}-|R_\perp|e^{i(\gamma+\rho_\perp)}\right)\left(e^{-\Gamma_L t}-e^{-\Gamma_H t}\right)\right].\end{aligned} \tag{45}$$

Combining these equations appropriately – each of them represents a certain measurement – a determination of γ and of the strong phase shifts is possible without using any additional input. This can be seen as follows:

Let us consider the untagged observables corresponding to (34), (35) and to the real part of (37). From these rates the *ratios* of the coefficients of $e^{-\Gamma_L t}-e^{-\Gamma_H t}$ and of $e^{-\Gamma_L t}+e^{-\Gamma_H t}$ can be determined. The overall normalizations of these rates cancel in the ratios which are given by

$$u_f \equiv \frac{2|R_f|\cos(\rho_f-\gamma)}{1+|R_f|^2} \qquad (f\in\{0,\,\parallel\}) \tag{46}$$

and

$$u_{0,\|} \equiv \frac{|R_0|\cos(\vartheta_\| - \vartheta_0 - \rho_0 + \gamma) + |R_\||\cos(\vartheta_\| - \vartheta_0 + \rho_\| - \gamma)}{\cos(\vartheta_\| - \vartheta_0) + |R_0||R_\||\cos(\vartheta_\| - \vartheta_0 + \rho_\| - \rho_0)}, \tag{47}$$

respectively, and depend thus only on $|R_0|$, ρ_0, $|R_\||$, $\rho_\|$, $\vartheta_\| - \vartheta_0$ and on the CKM angle γ. Using in addition the observables of the untagged B_s decays into the charge conjugate final states that are related to (40), (41) and to the real part of (43), we can determine similar ratios of the coefficients of $e^{-\Gamma_L t} - e^{-\Gamma_H t}$ and $e^{-\Gamma_L t} + e^{-\Gamma_H t}$. These charge conjugate ratios, which are given by

$$u_f^{\mathrm{C}} \equiv \frac{2|R_f|\cos(\rho_f + \gamma)}{1 + |R_f|^2} \qquad (f \in \{0, \|\}) \tag{48}$$

and

$$u_{0,\|}^{\mathrm{C}} \equiv \frac{|R_0|\cos(\vartheta_\| - \vartheta_0 - \rho_0 - \gamma) + |R_\||\cos(\vartheta_\| - \vartheta_0 + \rho_\| + \gamma)}{\cos(\vartheta_\| - \vartheta_0) + |R_0||R_\||\cos(\vartheta_\| - \vartheta_0 + \rho_\| - \rho_0)}, \tag{49}$$

respectively, depend on the same six "unknowns" as (46) and (47) determined from (34), (35) and (37). We have therefore six observables at our disposal to determine the six "unknowns" $|R_0|$, ρ_0, $|R_\||$, $\rho_\|$, $\vartheta_\| - \vartheta_0$, γ. In particular we are in a position to extract the CKM angle γ. Using furthermore the observables we have not considered so far, certain discrete ambiguities are resolved and also $|R_\perp|$, $\rho_\perp$, $\vartheta_\perp - \vartheta_0$ can be determined. Note that the overall normalizations of the rates corresponding to (34)-(45) inform us about $|V_{ub}V_{cs}| \cdot |M_f|$, where $f \in \{0, \|, \perp\}$.

Obviously the major goal of this approach is the extraction of the CKM angle γ. However, also the the quantities $|R_f|$ and the strong phase shifts ρ_f, ϑ_f are of interest, since they allow insights into the hadronization dynamics of the corresponding four-quark operators.

4 CP violation

There are many CP-violating observables that can be constructed from tagged time-dependent measurements. Some of them survive even when only untagged data samples are used. The most striking *untagged* CP-violating observable is

$$\begin{aligned}\mathrm{Im}\left\{[A_f^*(t)\,A_\perp(t)]\right\} + \mathrm{Im}\left\{[A_f^{\mathrm{C}*}(t)\,A_\perp^{\mathrm{C}}(t)]\right\} = -\frac{G_{\mathrm{F}}^2}{2}|V_{ub}V_{cs}|^2|M_f||M_\perp| \\ \times\{|R_f|\cos(\rho_f + \vartheta_f - \vartheta_\perp) + |R_\perp|\cos(\rho_\perp + \vartheta_\perp - \vartheta_f)\}\left(e^{-\Gamma_L t} - e^{-\Gamma_H t}\right)\sin\gamma,\end{aligned} \tag{50}$$

where $f \in \{0, \|\}$. Note that the plus sign on the l.h.s. of that equation is due to the fact that the parity eigenvalues of the final state configurations f and $\perp$ arising in the "mixed" combinations are different. The CP observable (50) is proportional to $\sin\gamma$ and occurs even when all strong phase shifts vanish. This CP-violating effect can be potentially very large as can be seen by employing the factorization assumption which implies vanishing strong phase shifts.

In contrast, to observe CP violation in the *untagged* interference term involving final state configurations with equal parity eigenvalues requires non-vanishing strong phase shifts as can be seen from the corresponding CP-violating observable

$$\begin{aligned}\mathrm{Re}\,\left\{[A_0^*(t)\,A_{\|}(t)]\right\} - \mathrm{Re}\,\left\{[A_0^{\mathrm{C}*}(t)\,A_{\|}^{\mathrm{C}}(t)]\right\} &= \frac{G_{\mathrm{F}}^2}{2}|V_{ub}V_{cs}|^2|M_0||M_{\|}| \\ \times\left\{|R_0|\sin(\rho_0+\vartheta_0-\vartheta_{\|}) + |R_{\|}|\sin(\rho_{\|}+\vartheta_{\|}-\vartheta_0)\right\}&\left(e^{-\Gamma_L t}-e^{-\Gamma_H t}\right)\sin\gamma. \quad (51)\end{aligned}$$

The last category of CP-violating effects in *untagged* data samples is related to

$$\begin{aligned}&\left[|A_f(t)|^2\right] - \left[|A_f^{\mathrm{C}}(t)|^2\right] \\ &\quad = \eta_{\mathrm{P}}^f G_{\mathrm{F}}^2|V_{ub}V_{cs}|^2|M_f|^2|R_f|\sin\rho_f\left(e^{-\Gamma_L t}-e^{-\Gamma_H t}\right)\sin\gamma \qquad (52)\end{aligned}$$

with $f\in\{0\,,\,\|\,,\,\perp\}$ and requires also non-vanishing strong phase shifts. This last category is the only one that has been considered so far in the literature [7].

5 Summary

We have calculated the time-dependences of the observables of angular distributions for B_s decays caused by $\bar{b}\to\bar{c}u\bar{s}$ quark-level transitions both in a general notation and in terms of linear polarization states. Examples for exclusive modes belonging to this decay category are the color-allowed and color-suppressed channels $\overset{(-)}{B_s}\to D_s^{*\pm}\,K^{*\mp}$, $D_{s1}(2536)^{\pm}K^{*\mp}$, $D_s^{**\pm}K^{*\mp}$ and $\overset{(-)}{B_s}\to\overset{(-)}{D^{*0}}\,\phi$, $\overset{(-)}{D_1}\,(2420)^0\phi$, $\overset{(-)}{D^{**0}}\,\phi$, respectively. Since charged particles are easier to detect for generic detectors than the photon(s) in the strong or electromagnetic decays of D_s^* and D^{*0}, we have also listed higher resonances exhibiting significant all-charged final states. The information that is provided by the corresponding angular correlations allows – without any theoretical input – the extraction both of the notoriously difficult to measure CKM angle γ and of the whole hadronization dynamics of these decays thereby allowing e.g. tests of the factorization hypothesis.

If the lifetime difference between the B_s mass eigenstates B_s^L and B_s^H is sizable, as is indicated by certain present theoretical analyses, even *untagged* B_s data samples suffice to accomplish this ambitious task. Interestingly, some of the many CP-violating observables that can be constructed from tagged measurements survive also in that *untagged* case and are potentially very large. One class of these untagged CP-violating observables is proportional to $\sin\gamma$ and arises even when all strong phase shifts vanish.

From an experimental point of view, untagged analyses of B_s-meson decays are obviously much more efficient than tagged studies. The feasibility of our *untagged* strategies for extracting γ in a clean way depends, however, crucially on a sizable lifetime difference of the B_s system. Even if this lifetime splitting should turn out to be too small for untagged analyses, once a non-vanishing lifetime difference has been established experimentally, the formalism presented in our paper must be used in the case of tagged measurements in order to extract γ correctly. Clearly time will tell and an exciting future may lie ahead of us.

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