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TOP QUARK PAIR PRODUCTION IN THE THRESHOLD REGION*

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Recent results on production and decays of polarized top quarks are reviewed. Top quark pair production in e^+e^- annihilation is considered near energy threshold. For longitudinally polarized electrons the produced top quarks and antiquarks are highly polarized. Dynamical effects originating from strong interactions and Higgs boson exchange in the $t-\bar{t}$ system can be calculated using the Green function method. Energy-angular distributions of leptons in semileptonic decays are sensitive to the polarization of the decaying top quark and to the Lorentz structure of the weak charged current.

1 Introduction

As the heaviest fermion of the Standard Model the top quark is an exciting new window on very high mass scale physics. There is no doubt that precise studies of top quark production and decays will provide us with new information about the mechanism of electroweak symmetry breaking. The analysis of polarized top quarks and their decays has recently attracted considerable attention[1,2,3]. For non-relativistic top quarks the polarization studies are free from hadronization ambiguities. This is due to the short lifetime of the top quark which is shorter than the formation time of top mesons and toponium resonances. Therefore top decays interrupt the process of hadronization at an early stage and practically eliminate associated non-perturbative effects.

The most efficient and flexible reaction producing polarized top quarks is pair production in e^+e^- annihilation with longitudinally polarized electron beams. For $e^+e^- \to t\bar{t}$ in the threshold region one can study decays of polarized top quarks under particularly convenient conditions: large event rates, well-identified rest frame of the top quark, and large degree of polarization. At the same time, thanks to the spectacular success of the polarization program at SLC [4], the longitudinal polarization of the electron beam will be an obvious option for a future linear collider.

In the present article some recent results[5,6,7] are presented for polarized top quark pair production in e^+e^- annihilation near production threshold. The Green function method [8,9,10,11] has been extended to the case of polarized t and $\bar{t}$. It has been shown that for the longitudinally polarized electron beam longitudinally polarized top quarks can be produced. The transverse and normal components of the top quark polarization have been calculated including effects of $S-P$ interference and rescattering corrections. Effects of Higgs particle exchange have been also studied. Semileptonic decays of polarized top quarks are briefly discussed. The cleanest spin analysis for the top quarks can be obtained from their semileptonic decay channels[2,12].

2 Pair production near threshold

The top quark is a short–lived particle. For the top mass m_t in the range 160–190 GeV its width Γ_t increases with m_t from 1 to 2 GeV. Thus Γ_t by far exceeds the hyperfine splitting for toponia and open top hadrons, the hadronization scale of about 200 MeV, and even the energy splitting between $1S$ and $2S$ $t\bar{t}$ resonances. Toponium resonances including the $1S$ state overlap each other. As a consequence the cross section for $t\bar{t}$ pair production near energy threshold has a rather simple and smooth shape. It can be shown[8,9] that for non-relativistic t and $\bar{t}$ the dominant contribution to the amplitude is given by the sum of the ladder diagrams corresponding to chromostatic interactions between top and antitop. The other contributions are suppressed by factors of order β^2 where β denotes the velocity of the top quark in the center-of-mass frame. The ladder diagram with n exchanges of gluons gives the contribution of order $(\alpha_s/\beta)^n$ where α_s is the strong coupling constant. In the threshold region $\beta \sim \alpha_s$ and all the contributions are of the same order. The sum of the ladder diagrams can be expressed through the Green function of the $t-\bar{t}$ system. The effects of the top quark width are incorporated through the complex energy $E + i\Gamma_t$, where $E = \sqrt{s} - 2m_t$ is the non-relativistic energy of the system. The idea [8,9] to use the Green function instead of summing over overlapping resonances has been also applied in numerical calculations of differential cross

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sections. Independent approaches have been developed for solving Schrödinger equation in position space [11] and Lippmann-Schwinger equation in momentum space [10,13]. The results of these two methods agree very well. One of the most important future applications[14] will be the determination of m_t and α_s. It has been pointed out[11] that the momentum dependence of the width for $t-\bar{t}$ system which arises at order α_s^2 may be important in quantitative studies. However, it has been conjectured[13] that order α_s^2 rescattering corrections to the width nearly cancel the negative contributions originating from phase space suppression. The remainder can be interpreted as due to time dilatation for t and $\bar{t}$ in the center-of-mass frame. Its effect on the total cross section is quite small and can be neglected. Recently an elegant proof of this conjecture has been found[15].

The Green function method has been generalized to the case of polarized top quark pair production in e^+e^- annihilation[5,3] and in $\gamma\gamma$ collisions[16]. In the non-relativistic approximation, neglecting contributions of order β^2 the calculation of polarized cross sections can be reduced to solving of the Lippmann-Schwinger equations for the S wave and for the P wave Green functions:

$$G\,(p,E) = G_0(p,E)\,\left[1+\int\frac{d^3q}{(2\pi)^3}\,V\,(\mathbf{p}-\mathbf{q})\;G(q,E)\right] \tag{1}$$

$$F\,(p,E) = G_0(p,E)\left[1+\int\frac{d^3q}{(2\pi)^3}\frac{\mathbf{p}\cdot\mathbf{q}}{p^2}V\,(\mathbf{p}-\mathbf{q})\,F(q,E)\right] \tag{2}$$

where $\mathbf{p}$ is top quark momentum, $G_0(p,E)$ denotes the free Green function for the $t-\bar{t}$ system

$$G_0(p,E) \;=\; = \;\left(E-\frac{p^2}{m_t}+\mathrm{i}\Gamma_t\right)^{-1} \tag{3}$$

and $V\,(\mathbf{p}-\mathbf{q})$ denotes the chromostatic potential in momentum space. In our numerical calculations a potential[13] was used given by the two-loop perturbative formula at large momenta and by Richardson's potential at intermediate and small ones. The S wave Green functions G governs the momentum distribution of the top quark

$$\frac{d\sigma}{dp}\sim\mathcal{D}_{S-S}(p,E) \;=\; p^2\,|\,G(p,E)\,|^2 \tag{4}$$

Interference between S and P partial waves[17] results in a non-trivial momentum and angular distribution which for e^+e^- annihilation and linearly polarized beams reads[5]:

$$\frac{d\sigma}{dp\,d\Omega}\sim\mathcal{D}_{S-S}(p,E)\;[1\,+\,2\,C_{FB}(\chi)\,\varphi_R(p,E)\,\cos\vartheta] \tag{5}$$

where ϑ is the angle between the electron and the top quark, and $\varphi_R(p,E)$ denotes the real part of the function

$$\varphi(p,E) \;=\; \frac{\left(1-\frac{4\alpha_s}{3\pi}\right)\,p\,F^*(p,E)}{\left(1-\frac{8\alpha_s}{3\pi}\right)m_t\,G^*(p,E)} \tag{6}$$

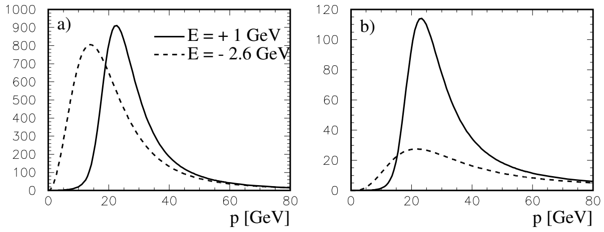

Figure 1: Top quark momentum – angular distribution functions: a) $\mathcal{D}_{S-S}(p,E)$ and b) $\mathcal{D}_{S-P}(p,E)$ for $m_t = 174$ GeV.

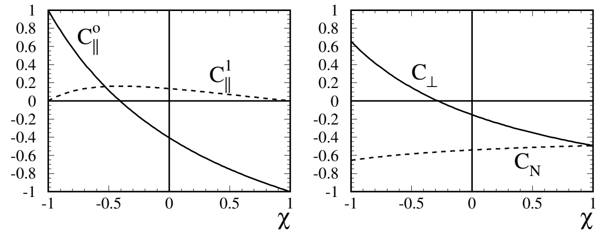

Figure 2: Coefficient functions: a) $C^0_\parallel(\chi)$ – solid line and $C^1_\parallel(\chi)$ – dashed line, b) $C_\perp(\chi)$ – solid line and $C_N(\chi)$ – dashed line.

C_{FB} depends on the electroweak charges of electron and top quark and the variable

$$\chi = \frac{P_{e^+}-P_{e^-}}{1-P_{e^+}P_{e^-}} \tag{7}$$

where P_{e^+} and P_{e^-} denote the linear polarizations of the beams. The functions $\mathcal{D}_{S-S}(p,E)$ and

$$\mathcal{D}_{S-P}(p,E) \;=\; \mathcal{D}_{S-S}(p,E)\,\varphi_R(p,E) \tag{8}$$

are shown in Fig.1 for two energies close to the threshold. To describe polarized top quark production in the threshold region it is convenient to define the components of its polarization vector with respect to the triplet of orthogonal unit vectors: $\hat{n}_\parallel$, $\hat{n}_\perp$ and $\hat{n}_{_N}$, where $\hat{n}_\parallel$ points in the direction of the e^- beam, $\hat{n}_{_N}\sim\vec{p}_{e^-}\times\vec{p}_t$ is normal to the production plane and $\hat{n}_\perp = \hat{n}_{_N}\times\hat{n}_\parallel$. Retaining only the terms up to $\mathcal{O}(\beta)$ one derives[5] the following expressions for the components of the polarization vector, as functions of E, p, ϑ and χ:

$$\mathcal{P}_\parallel(p,E,\vartheta,\chi) \;=\; C^0_\parallel(\chi)+C^1_\parallel(\chi)\,\varphi_R(p,E)\,\cos\vartheta \tag{9}$$

$$\mathcal{P}_\perp(p,E,\vartheta,\chi) \;=\; C_\perp(\chi)\,\varphi_R(p,E)\,\sin\vartheta \tag{10}$$

$$\mathcal{P}_N(p,E,\vartheta,\chi) \;=\; C_N(\chi)\,\varphi_I(p,E)\,\sin\vartheta \tag{11}$$

where $\varphi_R(p,E)$ and $\varphi_I(p,E)$ denote the real and imaginary parts of the function $\varphi(p,E)$ defined in eq.(6). The energy dependence of all the coefficient functions $C(\chi)$ is very weak and can be neglected. In Fig.2a the coefficient functions $C^0_\parallel(\chi)$ and $C^1_\parallel(\chi)$ are shown. It is evident that for maximal and minimal values of $\chi=\pm1$ the top quark is maximally polarized along the direction

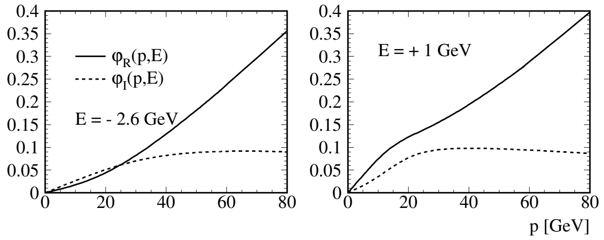

Figure 3: Momentum dependence of the functions $\varphi_R(p, E)$ (solid lines) and $\varphi_I(p, E)$ (dashed lines): a) E = -2.6 GeV, and b) E = 1 GeV.

of the incoming electron. The correction to the parallel polarization is quite small, so, to a good approximation $\mathcal{P}_\parallel$ and $C^0_\parallel(\chi)$ are equal. The shape of the latter function depends on the electroweak charges of the top quark. For example if we change the values of $Zt\bar{t}$ couplings rotating the (v_t, a_t) vector by $\pm 0.1rd$ then $C^0_\parallel(0.8) = -.91 \pm .02$, where the central value denotes the Standard Model result, $C^0_\parallel(0) = -.41^{+.10}_{-.09}$, and $C^0_\parallel(-0.8) = .58^{+.07}_{-.08}$. Thus, measuring the top quark parallel polarization as a function of χ one can measure v_t and a_t. This demonstrates that polarization studies close to threshold are very promising indeed. The other components of the top polarization can be also interesting and the corresponding coefficient functions are plotted in Fig.2b. Momentum dependence of the functions $\varphi_R(p, E)$ and $\varphi_I(p, E)$ is shown in Fig.3 for two energies in the threshold region.

3 Semileptonic decays of top quarks

The energy and angular distributions of the charged leptons and the neutrinos are sensitive to the polarization of the decaying heavy quark. Therefore they can be used in determination of this polarization. Compact analytic formulae have been recently found[18] for QCD corrections to the energy-angular distributions of the charged leptons and the neutrinos in semileptonic decays of the top quark. In the rest frame of the decaying top quark the double differential energy-angular distribution of the charged lepton is the product of the energy distribution and the angular distribution[19]. QCD corrections essentially do not spoil this factorization[20]. Thus the polarization analysing power of the charged lepton energy-angular distribution is maximal and hence far superior to other distributions discussed in the following. In particular for the neutrino energy-angular distribution factorization does not hold. It follows that the charged lepton angular distribution in $t \to Wb \to b\ell^+\nu$ decays is significantly more sensitive towards the polarization of t than the angular distributions of W and ν. The neutrino distributions, however, are more sensitive to deviations from $V - A$ and can be used in testing this basic assumption about the Lorentz structure of the charged weak current[12]. The charged lepton is likely to be the less energetic lepton because its energy spectrum is softer than that of the neutrino. For large values of m_t the angular distribution of the less energetic lepton is a more efficient analyser of top polarization than the angular distribution of neutrinos. For m_t in the range 150-200 GeV it is also better than the direction of W boson. For semileptonic t decays this observation is not very useful. However, for the hadronic $t \to Wb \to b\bar{u}d$ decays the angular distribution of the less energetic jet can be used[2].

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