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The Scalar Contribution to $\tau \to K\pi\nu_\tau$

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Abstract

We consider the scalar form factor in $\tau \to K\pi\nu_\tau$ decays. It receives contributions both from the scalar resonance $K_0^*(1430)$ and from the scalar projection of off-shell vector resonances. We construct a model for the hadronic current which includes the vector resonances $K^*(892)$ and $K^*(1410)$ and the scalar resonance $K_0^*(1430)$. The parameters of the model are fixed by matching to the $O(p^4)$ predictions of chiral perturbation theory. Suitable angular correlations of the $K\pi$ system allow for a model independent separation of the vector and scalar form factor. Numerical results for the relevant structure functions are presented.

I. INTRODUCTION

Semileptonic decays of the τ lepton into $K^-\pi^0\nu_\tau$ and $\overline{K^0}\pi^-\nu_\tau$ final states provide a powerful probe of the strange sector of the weak charged current. Recently, the CLEO and LEP experiments have measured the branching fractions into $K\pi$ final states with fairly good precision [1–3]. Additional and more detailed information on the underlying strong interaction physics (the meson dominance model and resonance parameters, chiral perturbation theory, SU(3) flavor symmetry and isospin symmetry) can be extracted by an analysis of the hadronic system, *e.g.* by analyzing the invariant mass distribution and in particular angular distributions of the hadronic system as described below.

In this paper we will discuss the scalar form factor in these τ decay modes. The hadronic matrix elements can be parametrized in terms of two form factors, the vector form factor $F(Q^2)$ and the scalar form factor $F_S(Q^2)$. Both decay modes are expected to be dominated by the $K^*(892)$ vector resonance ($J^P = 1^-$), and in fact in our previous paper [4] we neglected the scalar form factor. A scalar resonance ($J^P = 0^+$), however, can enhance the scalar form factor. A good candidate is the $K_0^*(1430)$, and indeed there might be experimental evidence for an enhancement in the $K\pi$ invariant mass spectrum around $\sqrt{Q^2} = 1400\,\mathrm{MeV}$ [5]. The situation, however, is more complicated in two ways. Firstly, in this energy range, there is also a vector resonance $K^*(1410)$. Secondly, off-shell the $K^*(892)$ can have a scalar component, which contributes to the scalar form factor. Vector and scalar contributions can be disentangled experimentally in a model independent way by studying suitable angular distributions of the $K\pi$ system [6]. The relevant information in encoded in four structure functions. Three of these structure functions (W_B, W_{SA}, W_{SF}) can be measured even if the τ rest frame cannot be reconstructed. Of particular importance are the structure functions W_{SA} and W_{SF}, which arise from a nonvanishing spin-zero part of the hadronic matrix element. W_{SA} is essentially the spin-zero spectral function whereas W_{SF} arises from the interference of the spin-one part and the spin-zero part of the hadronic matrix element. We will show that a measurement of these structure functions allows to establish the $K_0^*(1430)$ contribution in a model independent way and might also yield interesting information about the off-shell behaviour of the K^* propagator.

The paper is organized as follows: In Sec. 2, we discuss the general structure of the matrix element in terms of the vector and the scalar form factors. We discuss predictions from chiral perturbation theory for the low energy limit of the form factors and derive a model for the form factors based on a meson dominance model. The parameters of the model are then fixed by matching it to the chiral perturbation theory limit. In Sec. 3, we define the structure functions, which determine the angular distributions and the decay rate. Numerical results and our conclusions are finally presented in Sec. 4 and 5, respectively.

II. THE HADRONIC MATRIX ELEMENTS

The matrix element $\mathcal{M}$ for the τ–decay into a kaon and a pion ($m_K = 0.493$ GeV, $m_\pi = 0.139$ GeV)

$$\tau(l,s) \rightarrow \nu(l',s') + K(q_1, m_K) + \pi(q_2, m_\pi) \tag{1}$$

can be written as a product of a leptonic (M_μ) and a hadronic vector current (J^μ) as

$$\mathcal{M} = \sin\theta_c \frac{G}{\sqrt{2}} M_\mu J^\mu \tag{2}$$

In Eq. (2), G denotes the Fermi-coupling constant and θ_c is the Cabibbo angle. The leptonic and hadronic currents are given by

$$M_\mu = \bar{u}(l', s')\gamma_\mu(g_V - g_A\gamma_5)u(l, s) \tag{3}$$

and

$$J^\mu = \langle K(q_1)\pi(q_2)|V^\mu(0)|0\rangle \tag{4}$$

In Eq. (3) s denotes the polarization 4-vector of the τ lepton satisfying

$$l_\mu s^\mu = 0 \qquad s_\mu s^\mu = -P^2 \tag{5}$$

P denotes the polarization of the τ in the laboratory frame. In the case of Z decays into τ pairs and after averaging over the τ directions the polarization is given by $P = \frac{-2\, v_\tau a_\tau}{v_\tau^2 + a_\tau^2}$ with $v_\tau = -1 + 4\sin^2\theta_W$ and $a_\tau = -1$; for lower energies effectively $P = 0$.

The most general ansatz for the hadronic vector current in Eq. (4) is characterized by two form factors $F^{K\pi}(Q^2)$ and $F_S^{K\pi}(Q^2)$

$$J^\mu = F^{K\pi}(Q^2)\, \left(g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2}\right)\, (q_1 - q_2)_\nu \;+\; F_S^{K\pi}(Q^2)\, Q^\mu \tag{6}$$

where

$$Q^\mu = q_1^\mu + q_2^\mu \tag{7}$$

The vector form factor $F^{K\pi}(Q^2)$ corresponds to the $J^P = 1^-$ component of the strange weak charged current, and the scalar form factor $F_S^{K\pi}(Q^2)$ to the $J^P = 0^+$ component. In the limit of $SU(3)$ flavor symmetry, $m_K^2 = m_\pi^2$, the vector current is conserved, and $F_S(Q^2)$ vanishes.

In the hadronic current in Eq. (6), the coefficient $-Q^\mu Q^\nu/Q^2$, which multiplies the vector form factor, seems to lead to a divergence in the limit $Q^2 \to 0$. This seeming singularity must be canceled by a corresponding $1/Q^2$ divergence in the scalar form factor. Therefore any physical admissible model must fulfill the condition

$$\lim_{Q^2\to 0} Q^2 F_S^{K\pi}(Q^2) = (m_K^2 - m_\pi^2) F^{K\pi}(0) \tag{8}$$

In the isospin symmetry limit, the form factor F for the two decay modes $\tau \to K^-\pi^0\nu_\tau$ and $\tau \to \overline{K^0}\pi^-\nu_\tau$ are related by a simple Clebsch-Gordan factor

$$F^{K^-\pi^0} = \frac{1}{\sqrt{2}} F^{\overline{K^0}\pi^-}; \qquad F_S^{K^-\pi^0} = \frac{1}{\sqrt{2}} F_S^{\overline{K^0}\pi^-} \tag{9}$$

We will specify the form factor $F^{\overline{K^0}\pi^-}$ and $F_S^{\overline{K^0}\pi^-}$ in the remaining part of this section.

Let us add two brief comments regarding the closely related decay modes $\tau \to 2\pi\nu_\tau$ and $\tau \to 2K\nu_\tau$. First, the corresponding scalar form factors in these decay modes vanish in the isospin symmetry limit, because in this limit the vector current is exactly conserved (CVC). However, CVC could be violated due to isospin violation. Such a violation could be tested by measuring the structure functions W_{SA} and W_{SF} (see below). In particular the structure function W_{SF} is expected to provide the most sensitive test of CVC due to the interference of a possible (small) spin-zero part with the (large) vector part. Any nonvanishing contribution to W_{SA} or W_{SF} would be a clear signal of CVC violation. Second, note that the scalar sector in these tau decays is also sensitive to possible new physics effects such as effects from charged Higgs exchange and possible CP violation [7]. These latter effects could in principle also be present in the $K\pi$ decay mode. However, we assume that these effects are much smaller than the contributions discussed in this paper and we will neglect them in the following.

A. Chiral perturbation theory

In this subsection we discuss the dynamical constraints on the form factors which can be deduced from chiral Lagrangians. For very small momentum transfers (small compared with the masses of the hadronic resonances), chiral perturbation theory provides standard model predictions for the hadronic matrix elements [8]. The form factors for $K^+ \to \pi^0 e^+ \nu_e$ and $K^0 \to \pi^- e^+ \nu_e$, which have been calculated in chiral perturbation theory to one loop in [9], are related by analytic continuation to those relevant for $\tau \to K\pi\nu_\tau$ (see also [10]). Expanding the results in a Taylor series around $Q^2 = 0$, one obtains

$$F^{\overline{K^0}\pi^-}(Q^2) = f(0)\big[1 + \frac{1}{6}\langle r^2\rangle_V^{K\pi} Q^2 + \cdots\big]$$

$$F_S^{\overline{K^0}\pi^-}(Q^2) = \frac{m_K^2 - m_\pi^2}{Q^2} f_S(0)\big[1 + \frac{1}{6}\langle r^2\rangle_S^{K\pi} Q^2 + \cdots\big] \tag{10}$$

At leading order (tree level), chiral perturbation theory predicts $f(0) = f_S(0) = 1$ and $\langle r^2\rangle_V^{K\pi} = \langle r^2\rangle_S^{K\pi} = 0$. At next-to-leading order, both $f(0)$ and $f_S(0)$ differ slightly from unity. Numerically, $f(0) = f_S(0) = 0.978$ in the isospin symmetry limit [9,10]. For our purposes, however, this small deviation from unity may be neglected, because we neglect other effects of the same order of magnitude anyway (Note that isospin breakage, for example, yields $f^{K^-\pi^0}(0) = 0.988$ and $f^{\overline{K^0}\pi^-}(0) = 0.977$). Therefore we will assume

$$f(0) = f_S(0) = 1 \tag{11}$$

At one-loop order, the chiral perturbation theory predictions for the vector and scalar radii as determined in [9] are numerically given by

$$\langle r^2\rangle_V^{K\pi} = 0.37\,\mathrm{fm}^2 = 9.55\,\mathrm{GeV}^{-2} \tag{12}$$

$$\langle r^2\rangle_S^{K\pi} = 0.20\,\mathrm{fm}^2 = 5.14\,\mathrm{GeV}^{-2} \tag{13}$$

Of course, chiral perturbation theory fulfills the condition in Eq. (8).

B. Meson Dominance Model

The form factors in the region of large Q^2 are now discussed in the framework of a resonance exchange model. We assume that the hadronic current for $\tau \to K\pi\nu_\tau$ is dominated by three interfering resonances, the vector mesons $K^*(892)$ and $K^*(1410) \equiv K^{*\prime}$ (with $\Gamma_{K^*} = 0.05$ GeV, $\Gamma_{K^{*\prime}} = 0.227$ GeV) and the scalar resonance $K_0^*(1430)$ (with $\Gamma_{K_0^*} = 0.287$ GeV). Assuming constant meson couplings and choosing a certain form for the meson propagators (see the discussion after (23)), this gives the hadronic current

$$\begin{aligned} J^{\overline{K^0}\pi^-\,\mu}(q_1,q_2) = \frac{c_V}{1+\beta_{K^*}} \Bigg[&\left(g^{\mu\nu} - \frac{Q^\mu Q^\nu}{m_{K^*}^2}\right) \mathrm{BW}_{K^*}(Q^2)\ (q_1-q_2)_\nu \\ &+ \beta_{K^*} \left(g^{\mu\nu} - \frac{Q^\mu Q^\nu}{m_{K^{*\prime}}^2}\right) \mathrm{BW}_{K^{*\prime}}(Q^2)\ (q_1-q_2)_\nu \Bigg] \\ &+ \frac{m_K^2 - m_\pi^2}{m_{K_0^*}^2}\, c_S\, \mathrm{BW}_{K_0^*}(Q^2)\ Q^\mu \end{aligned} \tag{14}$$

Here, β_{K^*} describes the relative contribution of the $K^{*\prime}$ as compared to the dominant K^*. c_V determines the overall strength of the vector resonances and contains the relevant meson coupling constants. Similarly, c_S determines the strength of the scalar resonance channel. In the $SU(3)$ flavour symmetry limit ($m_K = m_\pi$), this scalar contribution vanishes. Therefore we have factored out a coefficient proportional to the symmetry breaking, such that c_S is a number of order one.

The Breit-Wigner factors are defined by

$$\mathrm{BW}_X[Q^2] = \frac{M_X^2}{[M_X^2 - Q^2 - i\sqrt{Q^2}\Gamma_X(Q^2)]}\,, \tag{15}$$

with energy dependent widths $\Gamma_X(Q^2)$

$$\begin{aligned} \Gamma_X(Q^2) &= \Gamma_X \frac{M_X^2}{Q^2}\left(\frac{p}{p_X}\right)^{2n+1} \\ p &= \frac{1}{2\sqrt{Q^2}}\sqrt{[Q^2 - (M_K^2+M_\pi^2)^2]\,[Q^2 - (M_K^2 - M_\pi^2)]} \\ p_X &= \frac{1}{2M_X}\sqrt{[M_X^2 - (M_K^2+M_\pi^2)^2]\,[M_X^2 - (M_K^2 - M_\pi^2)]} \end{aligned} \tag{16}$$

where $n = 1$ for the K^* and the $K^{*\prime}$ (p-wave phase space) and $n = 0$ for the K_0^* (s-wave phase space) [11].

Comparing Eq. (14) with Eq. (6) yields

$$F^{\overline{K^0}\pi^-}(Q^2) = \frac{c_V}{1+\beta_{K^*}}\left[\mathrm{BW}_{K^*}(Q^2) + \beta_{K^*}\mathrm{BW}_{K^{*\prime}}(Q^2)\right] \tag{17}$$

$$F_S^{\overline{K^0}\pi^-}(Q^2) = \frac{m_K^2 - m_\pi^2}{Q^2}\frac{c_V}{1+\beta_{K^*}}\left[\frac{m_{K^*}^2 - Q^2}{m_{K^*}^2}\mathrm{BW}_{K^*}(Q^2) + \beta_{K^*}\frac{m_{K^{*\prime}}^2 - Q^2}{m_{K^{*\prime}}^2}\mathrm{BW}_{K^{*\prime}}(Q^2)\right]$$

$$+\frac{m_K^2-m_\pi^2}{m_{K_0^*}^2}\, c_S\, \mathrm{BW}_{K_0^*}(Q^2) \tag{18}$$

$$\approx (m_K^2-m_\pi^2)\Big[\frac{c_V}{Q^2}+\frac{c_S}{m_{K_0^*}^2}\mathrm{BW}_{K_0^*}(Q^2)\Big]$$

where the approximation in the last line

$$\frac{m_{K^*}^2-Q^2}{m_{K^*}^2}\mathrm{BW}_{K^*}(Q^2)\approx\frac{m_{K^*}^2-Q^2}{m_{K^*}^2}\frac{m_{K^*}^2}{m_{K^*}^2-Q^2}=1 \tag{19}$$

(and similarly for $K^*\to K^{*'}$) becomes exact in the narrow width limit Γ_{K^*}, $\Gamma_{K^{*'}}\to 0$, or below threshold, $Q^2<(m_K+m_\pi)^2$. Note that this limit implies that the off-shell contribution of the vector resonances to the scalar form factor is effectively non-resonant. The Breit-Wigner enhancement is canceled by the off-shellness requirement. Thus the K_0^* will be dominant in F_S unless c_S is unnaturally small.

C. Matching

We now match the meson dominance model to chiral perturbation theory . Expanding the form factor predictions from the meson dominance model in Eqs. (17,18) around $Q^2=0$ yields

$$F^{\overline{K^0}\pi^-}(Q^2)=c_V\left[1+\frac{1}{1+\beta_{K^*}}\left(\frac{1}{m_{K^*}^2}+\frac{\beta_{K^*}}{m_{K^{*'}}^2}\right)Q^2+\cdots\right] \tag{20}$$

$$F_S^{\overline{K^0}\pi^-}(Q^2)=\frac{m_K^2-m_\pi^2}{Q^2}\left[c_V+\frac{c_S}{m_{K_0^*}^2}Q^2+\cdots\right] \tag{21}$$

We now compare this to the constraints from chiral perturbation theory in Eq. (10). Matching the leading order terms yields

$$c_V=1 \tag{22}$$

Note that with this choice, we fulfill the condition in Eq. (8) and we simultaneously reproduce the correct leading terms for the vector and scalar form factors. This relies on our choice for the meson propagators in Eq. (14). Due to the fact that the mesons are objects with structure, the form of their propagators becomes model dependent in the off-shell region [12]. In Eq. (14), we used a K^* propagator proportional to

$$\left(g^{\mu\nu}-\frac{Q^\mu Q^\nu}{m_{K^*}^2}\right) \tag{23}$$

and not, as sometimes suggested, proportional to

$$\left(g^{\mu\nu}-\frac{Q^\mu Q^\nu}{Q^2}\right) \tag{24}$$

instead. The latter form would have led to a vanishing contribution of the vector resonances to the scalar form factor, and we would not be able to fulfill the condition in Eq. (8), which is necessary to insure that the current is finite in the limit $Q^2 \to 0$. Note that the scalar resonance does not enter at this point, because its contribution starts at higher order in $\mathcal{O}(Q^2)$. With the form of Eq. (23), however, we simultaneously reproduce the correct low energy limit for the vector and the scalar form factor in a natural way. This gives some credibility to our choice for the off-shell propagator and the vector meson dominance model.

Matching the next-to-leading terms requires

$$\frac{1}{1+\beta_{K^*}}\left(\frac{1}{m_{K^*}^2}+\frac{\beta_{K^*}}{m_{K^{*\prime}}^2}\right) = \frac{\langle r^2\rangle_V^{K\pi}}{6} \tag{25}$$

$$\frac{c_S}{m_{K_0^*}^2} = \frac{\langle r^2\rangle_S^{K\pi}}{6} \tag{26}$$

The second equation determines c_S to

$$c_S = \frac{m_{K_0^*}^2}{6}\langle r^2\rangle_S^{K\pi} = 1.7 \tag{27}$$

We would like to emphasize that the exact numerical value for c_S should not be taken too seriously, in view of the large extrapolation from $Q^2 = 0$ to $Q^2 = m_{K_0^*}^2$.

Eq. (25) could be used to determine β_{K^*} (resulting in $\beta_{K^*} \approx -0.3$). Note, however, that a small uncertainty in $\langle r^2\rangle_V^{K\pi}$ translates into a large uncertainty in β_{K^*}. Therefore we will instead use the value for $\beta_{K^*} = -0.135$, which has been determined in [4] from a fit to the branching ratio[1] of $\tau \to K\pi\nu_\tau$. With this value, the left hand and the right hand sides of the first equation in Eq. (25) are $1.37\,\mathrm{GeV}^{-2}$ and $1.59\,\mathrm{GeV}^{-2}$, respectively. In view of the uncertainty of $\langle r^2\rangle_V^{K\pi}$, mainly from higher orders in chiral perturbation theory, this 15% mismatch is certainly acceptable.

III. DECAY RATE, ANGULAR DISTRIBUTIONS AND STRUCTURE FUNCTIONS

In this section, we introduce the formalism of the structure functions which allow for a model independent separation of the vector and scalar contribution to the $K\pi$ mode.

[1]In [4], we used the value $\mathcal{B}(K\pi\nu_\tau) = 1.36 \pm 0.08\%$ from [13]. This yielded $\beta_{K^*} = -0.135$, very close to the corresponding strength of the ρ' contribution to the ρ in $\tau \to \pi\pi\nu_\tau$ [14], as is expected in the limit of $SU(3)$ flavor symmetry. In the meanwhile, a new preprint by CLEO has appeared [1], with new data on the $\tau \to \bar{K}^0\pi^-\nu_\tau$. Adding to this mode the CLEO 94 data [2] on $\tau \to K^-\pi^0\nu_\tau$ yields $\mathcal{B}(K\pi\nu_\tau) = 1.21 \pm 0.14\%$. However, additionally taking the isospin constraint into account, (the $2:1$ ratio between the two modes), which is not well satisfied by the data, CLEO obtains $\mathcal{B}(K\pi\nu_\tau) = 1.11 \pm 0.12\%$. Note that this fit value is several standard deviations lower than the old world average. It leads to a value of β_{K^*} of about $\beta_{K^*} = -0.05 \pm 0.05$, compatible with zero. In the rest of this paper we will use $\beta_{K^*} = -0.135$ as standard value, but we also compare with $\beta_{K^*} = 0$.

Interesting additional informations, such as the relative sign of spin-zero and spin-one part in the hadronic matrix element, will also become possible through the measurement of the structure functions. The formalism is largely based on [6]. We will, however, specify the results in [6] for the tau decay mode into $K\pi\nu_\tau$.

The differential decay rate is obtained from

$$d\Gamma(\tau \to \nu_\tau K\pi) = \frac{1}{2m_\tau}\frac{G^2}{2}\sin^2\theta_c\ \{L_{\mu\nu}H^{\mu\nu}\}\ d\mathrm{PS}^{(3)} \tag{28}$$

where $L_{\mu\nu} = M_\mu(M_\nu)^\dagger$ and $H^{\mu\nu} = J^\mu(J^\nu)^\dagger$ with M_μ and J^μ defined in Eqs. (3,4).

In the following, the $\tau \to K\pi\nu_\tau$ decay mode is analyzed in the hadronic rest frame $\vec{q}_1 + \vec{q}_2 = 0$. After integration over the unobserved neutrino direction, the phase space is parametrized in the K-π-rest frame by

$$d\mathrm{PS}^{(3)} = \frac{1}{(4\pi)^3}\frac{(m_\tau^2 - Q^2)}{m_\tau^2}\,|\vec{q}_1|\,\frac{dQ^2}{\sqrt{Q^2}}\,\frac{d\cos\beta}{2}\,\frac{d\cos\theta}{2} \tag{29}$$

The angle β in Eq. (29) denotes the angle between the direction of the kaon ($\hat{q}_1 = \vec{q}_1/|\vec{q}_1|$) and the direction of the laboratory $\vec{n}_L$ viewed from the hadronic rest frame

$$\cos\beta = \vec{n}_L \cdot \hat{q}_1 \tag{30}$$

$\vec{n}_L$ is obtained from $\vec{n}_L = -\vec{n}_Q$, where $\vec{n}_Q$ denotes the direction of the K-π–system in the laboratory[2].

The angle θ ($0 \le \theta \le \pi$) in Eq. (29) is the angle between the direction of flight of the τ in the laboratory frame (= direction of the τ polarization) and the direction of the hadrons as seen in the τ rest frame. Note that the cosine of the angle θ is observable even in experiments, where the τ direction cannot be measured experimentally. This is because $\cos\theta$ can be calculated [15,16,6] from the energy E_h of the hadronic system with respect to the laboratory frame

$$\cos\theta = \frac{(2xm_\tau^2 - m_\tau^2 - Q^2)}{(m_\tau^2 - Q^2)\sqrt{1 - 4m_\tau^2/s}} \tag{31}$$

with

$$x = 2\frac{E_h}{\sqrt{s}} \qquad s = 4E_{beam}^2 \tag{32}$$

Of particular importance for the subsequent discussion is ψ, the angle between the direction of the laboratory and the τ as seen from the hadronic rest frame. Again the cosine of this angle can be calculated from the hadronic energy E_h [15,16,6]. One has:

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[2]Since the τ direction $\vec{n}_\tau$ as seen from the hadronic rest frame cannot in general be determined in the present e^+e^-–experiments, it is not possible to study the angular distribution of $\hat{q}_1$ with respect to $\vec{n}_\tau$.

$$\cos\psi = \frac{x(m_\tau^2 + Q^2) - 2Q^2}{(m_\tau^2 - Q^2)\sqrt{x^2 - 4Q^2/s}} \tag{33}$$

We are now in the position to evaluate the lepton and hadron tensors $L_{\mu\nu}$ and $H^{\mu\nu}$ in Eq. (28).

The angular (β) and E_h (through θ and ψ) dependence of the matrix element $L_{\mu\nu}H^{\mu\nu}$ can be disentangled by introducing suitable linear combinations of density matrix elements of the hadronic-system[3]:

$$L_{\mu\nu}H^{\mu\nu} = (g_V^2 + g_A^2)(m_\tau^2 - Q^2)\,(\,\bar{L}_B W_B + \bar{L}_{SA}W_{SA} + \bar{L}_{SF}W_{SF} + \bar{L}_{SG}W_{SG}\,) \tag{34}$$

The decomposition of the lepton- and hadron-tensor in Eq. (34) has the advantage that the coefficients $L_{B,SA,SF,SG}$ contain all $\beta, E_h(\theta,\psi)$ and τ–polarization dependence (see below), whereas the hadronic structure functions $W_{B,SA,SF,SG}$ depend only on Q^2 and the form factors F and F_S of the hadronic current. The angular coefficients $\bar{L}_{B,SA,SF,SG}$ in Eq. (34) are given by [6]:

$$\begin{aligned}
\bar{L}_B &= 2/3\,K_1 \;+\; K_2 \;-\; 2/3\,\overline{K}_1\,(3\cos^2\beta - 1)/2 \\
\bar{L}_{SA} &= K_2 \\
\bar{L}_{SF} &= -\overline{K}_2\,\cos\beta \\
\bar{L}_{SG} &= 0
\end{aligned} \tag{35}$$

with

$$\begin{aligned}
K_1 &= 1 - \gamma_{VA}P\cos\theta - (m_\tau^2/Q^2)\,(1 + \gamma_{VA}P\cos\theta) \\
K_2 &= (m_\tau^2/Q^2)\,(1 + \gamma_{VA}P\cos\theta) \\
\overline{K}_1 &= K_1\,(3\cos^2\psi - 1)/2 - 3/2\,K_4\,\sin 2\psi \\
\overline{K}_2 &= K_2\,\cos\psi \;+\; K_4\,\sin\psi \\
K_4 &= \sqrt{m_\tau^2/Q^2}\;\gamma_{VA}\,P\sin\theta
\end{aligned} \tag{36}$$

where

$$\gamma_{VA} = \frac{2g_V g_A}{g_V^2 + g_A^2} \tag{37}$$

In the standard model $\gamma_{VA} = 1$.
For $P = 0$, the case relevant in the low energy region $\sqrt{S} \approx 10$ GeV, Eq. (35) simplifies to

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[3]The most general decomposition of $L_{\mu\nu}H^{\mu\nu}$ (for two body decays) in terms of density matrix elements (or structure functions) W_X of the hadronic system has two additional terms $\bar{L}_A W_A + \bar{L}_E W_E$ [6]. However, W_A and W_E vanish in the case of tau decays into two pseudoscalar mesons. Nonvanishing W_A and W_E arise for example in decay modes with a vector and a pseudoscalar [17].)

$$
\begin{aligned}
\bar{L}_B &= \frac{1}{3}\left(2+\frac{m_\tau^2}{Q^2}\right) - \frac{1}{6}\left(1-\frac{m_\tau^2}{Q^2}\right)\left(3\cos^2\psi - 1\right)\left(3\cos^2\beta - 1\right) \\
\bar{L}_{SA} &= \frac{m_\tau^2}{Q^2} \\
\bar{L}_{SF} &= -\frac{m_\tau^2}{Q^2}\cos\psi\,\cos\beta
\end{aligned}
\tag{38}
$$

Note that the angular coefficient $\bar{L}_{SG}$ vanishes, if the hadronic rest frame is experimentally not known and only the distribution in β is considered. One may alternatively conceive of an experiment where the τ direction can be determined. This would allow to analyze the distribution of the hadronic system in term of two kinematical angles [6] and in particular, $\bar{L}_{SG}$ would not vanish. The relevant angular coefficients can be found in appendix B of [6].

The hadronic structure functions $W_{B,SA,SF,SG}$ are related to the form factors in Eq. (6). The dependence can be obtained from Eq. (34) in [6], with the replacements $x_4 \to 2\,\vec{q}_1$, $F_3 \to -iF$, $F_4 \to F_S$. One has:

$$
\begin{aligned}
W_B &= 4(\vec{q}_1)^2\,|F|^2 \\
W_{SA} &= Q^2\,|F_S|^2 \\
W_{SF} &= 4\sqrt{Q^2}|\vec{q}_1|\,\mathrm{Re}\,[FF_S^*] \\
W_{SG} &= -4\sqrt{Q^2}|\vec{q}_1|\,\mathrm{Im}\,[FF_S^*]
\end{aligned}
\tag{39}
$$

where $|\vec{q}_1| = q_1^z$ is the momentum of the kaon in the rest frame of the hadronic system.

$$
q_1^z = \frac{1}{2\sqrt{Q^2}}\left([Q^2-m_1^2-m_2^2]^2 - 4m_1^2m_2^2\right)^{1/2} \qquad E_1^2 = (q_1^z)^2 + m_1^2 \tag{40}
$$

The hadronic structure functions $W_{B,SA,SF,SG}$ are linearly related to the density matrix elements of the K-π system:

$$
\begin{array}{lclcl}
W_B &=& \tilde{H}^{00} &=& H^{33} \\
W_{SA} &=& \tilde{H}^{ss} &=& H^{00} \\
W_{SF} &=& \tilde{H}^{s0}+\tilde{H}^{0s} &=& H^{03}+H^{30} \\
W_{SG} &=& -i(\tilde{H}^{s0}-\tilde{H}^{0s}) &=& -i(H^{03}-H^{30})
\end{array}
\tag{41}
$$

where

$$
\tilde{H}^{\sigma\sigma'} = \epsilon_\mu(\sigma)H^{\mu\nu}\epsilon_\nu^*(\sigma') \tag{42}
$$

and

$$
\epsilon_\mu(s) = (1;0,0,0) \qquad\qquad \epsilon_\mu(0) = (0;0,0,1) \tag{43}
$$

are the polarization vectors for a hadronic system in a spin one ($\epsilon(0)$) or spin zero ($\epsilon(s)$) state with the z–direction aligned with $\hat{q}_1$. The r.h.s of Eq. (41) refers to the space-space components $H^{mn} = H_{mn}$ $(m,n = 0,1,2,3)$ of $H^{\mu\nu}$.

With Eqs. (28-43) the most general angular distribution of the $K\pi$ final state in the decay of a polarized τ is presented. Integrating over the angles β and θ we obtain the formula for the differential decay rate $d\Gamma/dQ^2$:

$$\frac{d\Gamma}{dQ^2} = \frac{G^2 \sin^2\theta_c}{2^8\pi^3}(g_V^2 + g_A^2)\frac{(m_\tau^2 - Q^2)^2}{m_\tau Q^{3/2}}\ |q_1^z|\ \frac{2Q^2 + m_\tau^2}{3m_\tau^2}\left\{W_B + \frac{3m_\tau^2}{2Q^2+m_\tau^2}W_{SA}\right\} \tag{44}$$

IV. NUMERICAL RESULTS

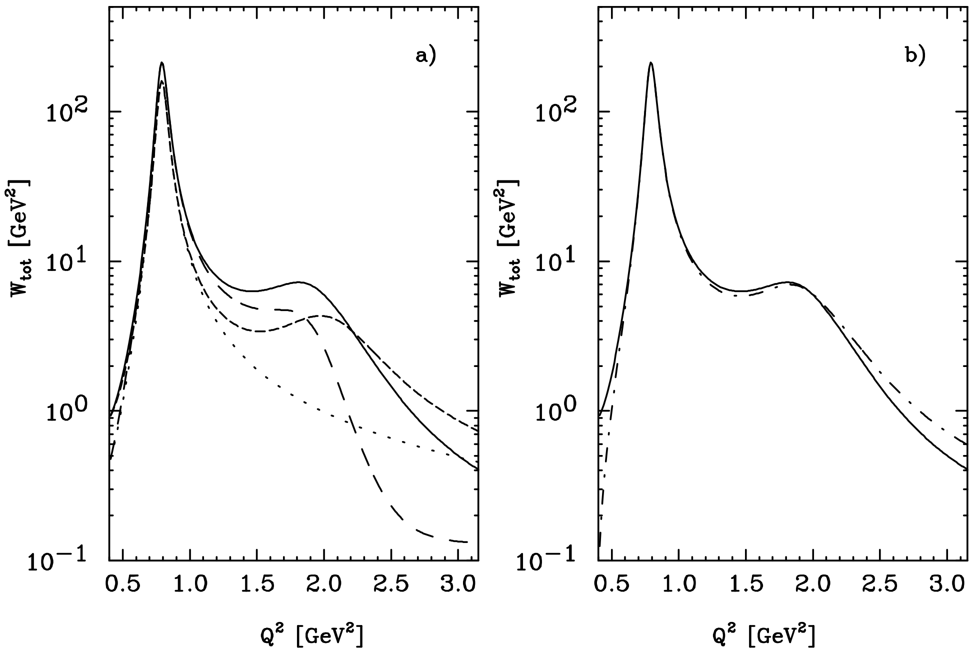

FIG. 1. a) Q^2 dependence of W_{tot} for $\beta_{K^*} = -0.135$, $c_S = 1.7$ (solid); $\beta_{K^*} = -0.135$, $c_S = 0$ (long dashed); $\beta_{K^*} = 0$, $c_S = 1.7$ (short dashed); $\beta_{K^*} = 0$, $c_S = 0$ (dotted). b) The Q^2 dependence of W_{tot} for $\beta_{K^*} = -0.135$, $c_S = 1.7$ is compared with the prediction from the purely transverse vector meson propagator in (24) (dotted-dashed)

The following numerical studies are shown for the full $K\pi$ final state, *i.e.* with a contribution of one third from $K^-\pi^0$ mode and two thirds from $\bar{K}^0\pi^-$.

In Fig. 1 we display $W_{tot}(Q^2)$, which is defined as

$$W_{tot}(Q^2) = W_B(Q^2) + \frac{3m_\tau^2}{2Q^2+m_\tau^2}W_{SA}(Q^2) \tag{45}$$

and which according to Eq. (44) determines the kaon-pion invariant mass spectrum $d\Gamma/dQ^2$.

The solid line gives the prediction from our model, including the K_0^* resonance (with $c_S = 1.7$) and the $K^{*\prime}$ resonance (with $\beta_{K^*} = -0.135$). We predict a clearly visible shoulder around $Q^2 \approx 2\,\mathrm{GeV}^2$. In Fig. 1a we compare with the results obtained without the K_0^*,

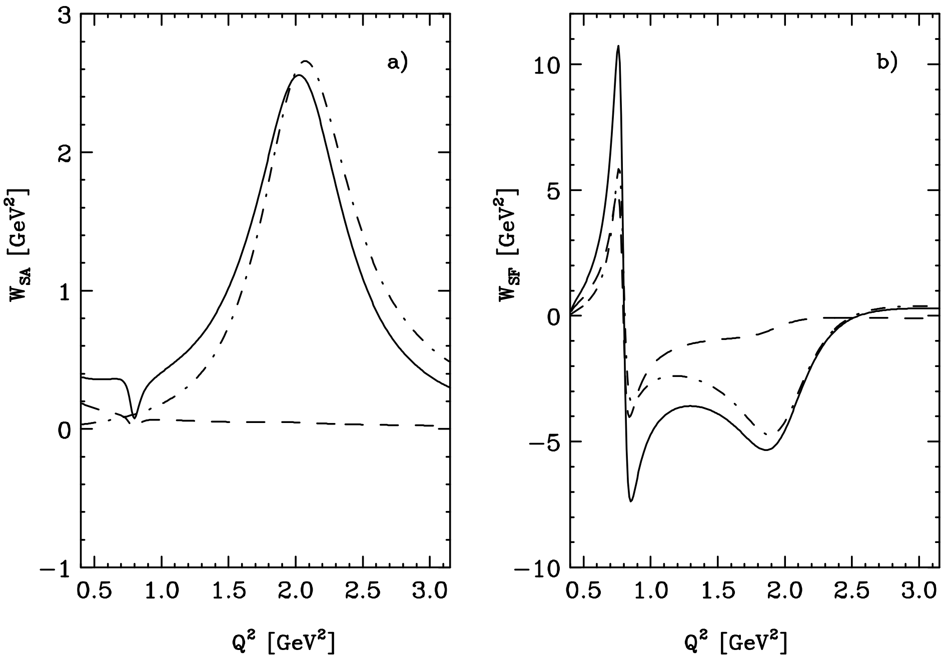

FIG. 2. Q^2 dependence of W_{SA} (a) and W_{SF} (b) for $c_S = 1.7$ (solid), $c_S = 0$ (long dashed), $c_S = 1.7$ but without the scalar off-shell K^* contributions (dotted-dashed). The parameter β_{K^*} is fixed to -0.135.

i.e. with $c_S = 0$, and without the $K^{*\prime}$, i.e. with $\beta_{K^*} = 0$. The two resonances lead to a sizable enhancement around $Q^2 \approx 2\,\mathrm{GeV}^2$. Within our model, we find that the $K^{*'}$ contributes mainly below its on-shell mass $Q^2 = 2\,\mathrm{GeV}^2$, because the negative interference with the tail of the $K^*(892)$ is negative above this value. Therefore an enhancement in the $Q^2 = 2.0 \cdots 2.5\,\mathrm{GeV}$ regime would give some indication of the presence of a K_0^* contribution. This conclusion, however, is model dependent because it depends on the relative phase of the $K^*(892)$ and the $K^{*'}$ (negative β_{K^*}). Predictions for the Q^2 dependence of W_{tot} without higher mass resonances $K^{*'}$ and K_0^* are shown as the dotted curve in Fig. 1a.

The branching ratios corresponding to the various predictions in Fig. 1a are (the relative contribution from the scalar current is given in brackets):

$$\begin{array}{llll}
\beta_{K^*} = -0.135, & c_S = 1.7: & \mathcal{B}(K\pi) = 1.41\% & [4\%] \\
\beta_{K^*} = -0.135, & c_S = 0.0: & \mathcal{B}(K\pi) = 1.36\% & [0.6\%] \\
\beta_{K^*} = 0, & c_S = 1.7: & \mathcal{B}(K\pi) = 1.04\% & [5.7\%] \\
\beta_{K^*} = 0, & c_S = 0: & \mathcal{B}(K\pi) = 0.99\% & [0.7\%].
\end{array}$$

The value of c_S hardly affects the total decay rate. The effect of β_{K^*} is basically due to the change in the normalization of the $K^*(892)$ contribution in Eq. (17). The effect of the direct $K^*(1410)$ resonance contribution is phase space suppressed and similar in size to the effect of the K_0^* contribution via c_S.

In Fig. 1b, we explore the sensitivity of the invariant mass distribution to the scalar projection of the off-shell vector resonances K^* and $K^{*'}$. The prediction for W_{tot} from our model with the vector propagator in Eq. (23) is compared with the prediction based on the purely transverse vector propagator in Eq. (24). Sizable differences are found only very small ($Q^2 < 0.6$ GeV^2) or very large ($Q^2 > 2.4$ GeV^2) invariant mass, where the rate is extremely small. Note that the difference in W_{tot} in Fig. 1b is entirely due to the difference in W_{SA}.

The studies in Fig. 1 show that from a measurement of the $K\pi$ invariant mass spectrum alone, a scalar resonance contribution from the K_0^* or scalar off-shell effects from vector resonances can not be established reliably. We will show in the following that the formalism of the structure functions allows for a much more detailed and model independent investigation of these effects. Remember that a measurement of W_{SA} separates the scalar from the vector contribution to the rate, whereas W_{SF} and W_{SG} measure the real and imaginary part of the interference term.

We give the predictions for the structure function W_{SA} in Fig. 2a. One observes a clear peak from the K_0^* resonance around $2.1\,\mathrm{GeV}^2$, which is absent for $c_S = 0$. Thus an experimental confirmation of this enhancement would firmly establish the K_0^* contribution. Note that the value of the parameter β_{K^*} hardly affects W_{SA}, as is obvious from Eq. (18).

Additional information about the spin zero part of the matrix element (in particular the relative sign of the spin zero and the spin one part) can be extracted from the structure function W_{SF}, which is shown in Fig. 2b. This structure function changes sign at the K^* resonance and receives fairly large contributions due to the interference effects with the (large) spin one part of the matrix element. In contrast to W_{SA} this structure function receives sizable contributions even without the presence of the scalar resonance K_0^* (dashed curve). Furthermore, there is some dependence on the higher $K^{*'}$ contribution in W_{SF} (see below).

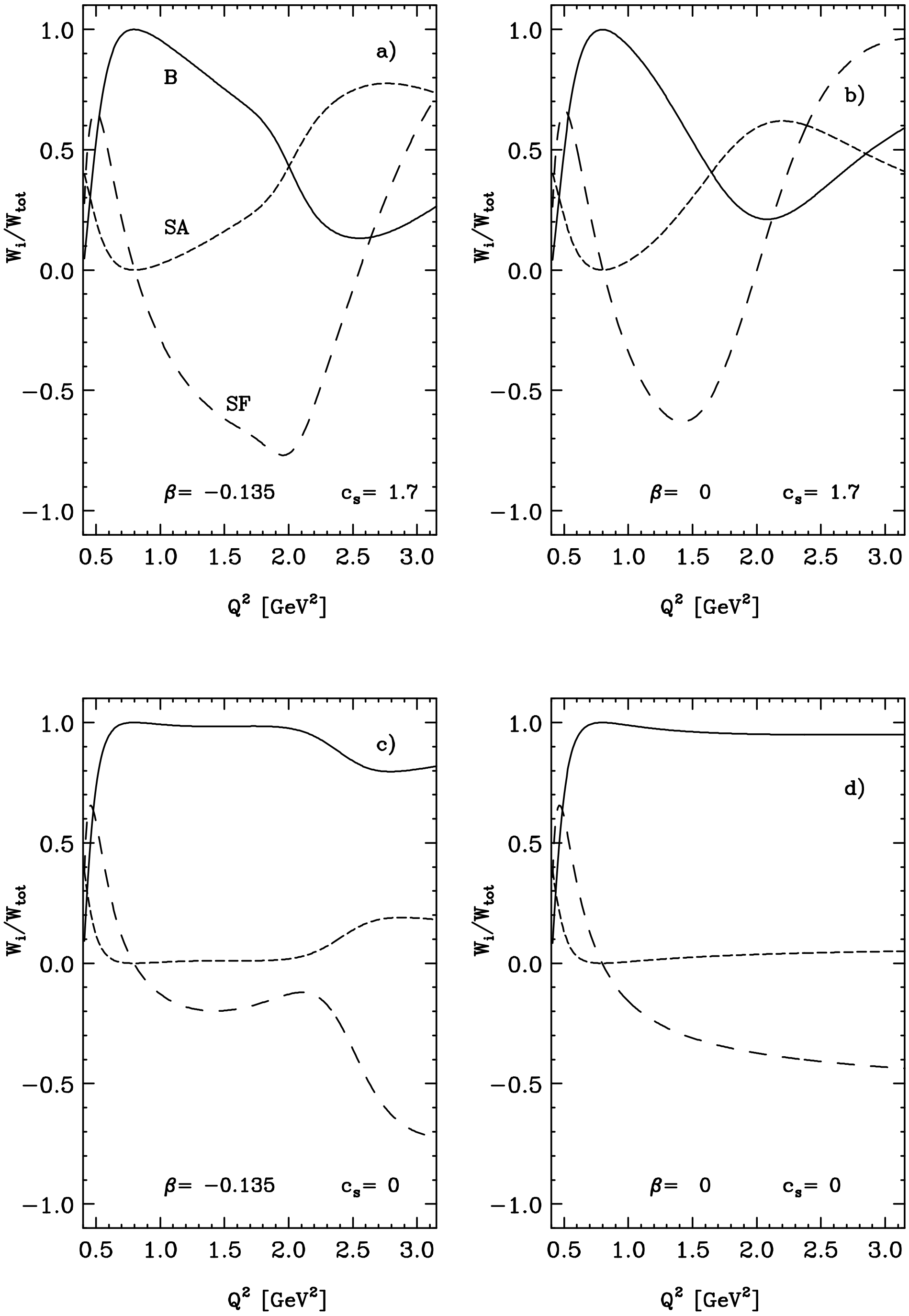

FIG. 3. Normalized structure functions W_B/W_{tot} (solid), W_{SA}/W_{tot} (short dashed), W_{SF}/W_{tot} (long dashed), for $c_S = 1.7$ (a,b) and $c_S = 0$ (c,d). The parameter β_{K^*} is -0.135 in (a,c) and 0 in (b,d).

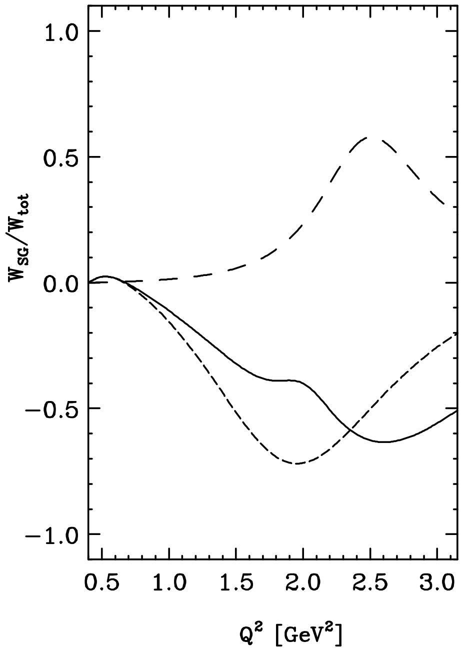

FIG. 4. Normalized structure function W_{SG}/W_{tot} as a function of Q^2 for $\beta_{K^\star} = -0.135$, $c_S = 1.7$ (solid); $\beta_{K^\star} = -0.135$, $c_S = 0$ (long dashed); $\beta_{K^\star} = 0$, $c_S = 1.7$ (short dashed).

In Figs. 2a and b, we also compare the results from the two different vector meson propagators in Eqs. (23,24). Regarding W_{SA}, we find in Fig. 2a that the interference of the K_0^* with the K^* off-shell scalar projection from the propagator in (23) shifts the K_0^* resonance peak to slightly smaller Q^2 values. This effect appears to be too small to be established experimentally. Another effect, however, of the scalar part of the K^* off-shell propagator is the dip close to the K^* resonance $Q^2 = 0.8\,\mathrm{GeV}^2$. An experimental confirmation of this dip would support the use of (23) for the K^* propagator. Of course, W_{SA} is very small in this mass region, and so only future high statistics experiment (b or tau-charm factories) could be sensitive enough to study this effect. As far as W_{SF} is concerned, there is some sensitivity to the choice of the off-shell propagator (compare the solid and the dotted-dashed curve). However, there is no qualitative difference, and in view of the uncertainty in the precise value of c_S, it does not seem to be possible to decide between the two forms of the K^* propagator from W_{SF}.

In Fig. 3, we display results for the three structure functions W_B, W_{SA} and W_{SF}, normalized to W_{tot}. We give results both with (Fig. 3a,b) and without (Fig. 3c,d) the K_0^* and with (a,c) and without (b,d) the $K^{*\prime}$ contributions. Comparing Figs. 3a,b and Fig. 3c,d we find that the scalar resonance leads to a strong enhancement of W_{SA}/W_{tot} above $Q^2 = 1.5\,\mathrm{GeV}^2$ as expected from Fig. 2a. The large negative value for the ratio of W_{SF}/W_{tot} in the range $Q^2 = 1 - 2$ GeV^2 is also an effect of the K_0^* scalar resonance.

Comparing the results in (a,c) and (b,d), we find that the value of $\beta_{K^\star}$ does not change the overall shapes of the W_i/W_{tot} very much. Note that the nonvanishing contribution to W_{SA}/W_{tot} and W_{SF}/W_{tot} in Fig. 3c,d is entirely due to the interference of the scalar contribution from the off-shell K^* with the vector part. The effect of the off-shell K^* in the results shown in Fig. 3a,b is small.

Finally, we present the Q^2 distribution for the ratio of W_{SG}/W_{tot} in Fig. 4. Remember that W_{SG} measures the imaginary part of the form factors (FF_S^*) and requires nontrivial phases of the amplitudes. These strong interaction phases are essential for an observation of possible CP violation effects in the difference of W_{SG} for the τ^+ and the τ^- decay [7]. Note however, that W_{SG} can only be measured in experiments where the τ direction in the laboratory frame can be determined as explained in Sec. III.

V. CONCLUSIONS

We have presented a meson dominance model for the the decay $\tau \to K\pi\nu_\tau$, which includes the vector resonances $K^*(892)$, $K^*(1410)$ and the scalar $K_0^*(1430)$. We have estimated the size of the K_0^* contribution by matching our model to the one loop prediction of chiral perturbation theory, which determines the amplitude at very small momentum transfers.

Based on our model, we have calculated the vector and the scalar form factors, which determine the hadronic matrix element. The scalar form factor receives contributions both from the K_0^* and from the off-shell spin-0 projection of the K^*.

We have presented predictions for the kaon-pion invariant mass spectrum. Although some indication for or against a sizable K_0^* contribution can be found in this spectrum, a clear disentanglement of the $K^*(1410)$ and the $K_0^*(1430)$ contributions is possible only by analyzing the hadronic structure functions. Both W_{SA}, the purely scalar structure function, and W_{SF}, which measures vector-scalar interference, are interesting in this respect. A measurement of $W_{SA}(Q^2)$ allows for a clear establishment of the K_0^* contribution, while $W_{SF}(Q^2)$ yields additional information about the scalar part, in particular its relative sign.

We also studied the issue of the off-shell K^* propagator. We used a propagator for massive vector mesons with a spin-0 component in the off-shell region, rather than a strictly transverse propagator. The vector meson dominance model can only produce the correct low energy limit if this scalar component is included in the vector meson propagator. Experimentally, this scalar component could best be studied by measuring $W_{SA}(Q^2)$ close to the $K^*(892)$ mass. An explicit experimental confirmation, however, is difficult and will require very high statistics.

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