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EXCLUSIVE SEMILEPTONIC TAU DECAYS *

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Abstract

We review current issues in exclusive semileptonic tau decays. We present the formalism of structure functions, and then discuss predictions for final states with kaons, for decays into four pions and for radiative corrections to the decay into a single pion.

I. INTRODUCTION

The experimental and theoretical investigation of τ lepton decays has received continuous attention since the τ has been discovered nearly twenty years ago. Branching ratios and distributions of the dominant modes are now determined with a precision of a few permill, posing a serious challenge for a refined theoretical treatment. The purely leptonic decay rates can be evaluated from first principles, providing thus a convenient calibration. The decay rate $\Gamma(\tau \to \nu\pi)$ can be predicted using the measured decay rate of the pion as input. Uncertainties arise at the permille level and are induced through the model dependence of radiative corrections. CVC allows to relate vector current decays e.g. into two or four pions to the corresponding cross sections measured in electron positron collisions. However, already now this approach is limited by the error of the input data. In the case of the two pion mode, the τ decay rate with a relative error below one % is already now about a factor five more precise than the predictions based on $\sigma(e^+e^- \to \pi\pi)$. For the remaining modes, induced by the axial current or by Cabibbo suppressed channels, a variety of theoretical assumptions must be employed, based on symmetry relations between different amplitudes which can be derived from isospin or $SU(3)$, or on dynamical constraints deduced from chiral Lagrangians at small momentum transfer. To saturate the form factors in the region of large

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Q^2 by resonances with momentum independent coupling is a natural choice in the context of the vector dominance model (VDM) but perhaps the most problematic assumption, which has to be tested by measurements of differential distributions. A particular powerful tool is provided by the analyses of angular distributions. The relevant information is conveniently encoded in structure functions [1] which, in turn, allow to reconstruct the form factors. τ decays are therefore a unique tool to study hadron physics in the low momentum region in order to test a variety of theoretical approaches and to derive resonance parameters which are not accessible elsewhere. An improved description of the hadronic matrix elements allows, furthermore, to exploit all (or at least most) channels for an analysis of the τ polarization P_τ in Z decays. Decays into a pion have been employed for this purpose from the very beginning; as a consequence of the improved understanding of the relevant distributions two and three pion modes are now equally important for the analysis. Since all multi-pion modes have equal (and maximal) analyzing power once the decay matrix element is known [2] this is clearly a worthwhile aim. Another reason to improve the theoretical description of τ production and decay is the search for new physics. Subtle deviations from the Standard Model like the influence of a charged Higgs boson on the question of universality could be amplified by the masses of the fermion involved in the reaction [3]. Last not least: only a steady improvement of the theoretical understanding allows to develop the Monte Carlo program TAUOLA [4], which is used to simulate τ decays in practically all e^+e^- experiments at LEP, SLC and CESR.

In this contribution a variety of topics and recent developments will be treated. In section 2 the formalism for the relation between form factors and structure functions and the transition to angular distributions will be reviewed [1] and the basic assumptions inherent in the theoretical models used to predict multimeson amplitudes will be presented [5]. In section 3 predictions for final states involving one or several kaons will be discussed, with emphasis on a recent update [6] of the theoretical predictions. Four pion modes will be discussed in section 4, which is based on ref. [7]. Radiative corrections [8] to $\tau \to \nu_\tau \pi(\gamma)$ will be discussed in the last, fifth section of this review.

II. FORM FACTORS, STRUCTURE FUNCTIONS AND $\Gamma(\tau \to \nu_\tau 3\pi)$

The matrix element for the τ decay into a multimeson final state can be written in the following form

$$\mathcal{M}(\tau \to \nu \text{ had}) = \frac{G_F}{\sqrt{2}} \bar{u}(l') \gamma_\mu (1-\gamma_5) u(l)\, J^\mu \tag{1}$$

The hadronic current J^μ is built up from the momenta of the mesons. For the three meson state it is characterized by four independent form factors. A convenient choice reads as follows [1]

$$\begin{aligned} J^\mu = & \left(g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2} \right) \left[(q_1 - q_3)_\nu F_1 + (q_2 - q_3)_\nu F_2 \right] \\ & + i\, \epsilon^{\mu\alpha\beta\gamma} q_{1\,\alpha} q_{2\,\beta} q_{3\,\gamma} F_3 + Q^\mu F_4 \end{aligned} \tag{2}$$

F_1 and F_2 determine the spin one component of the axial current induced amplitude, F_4 the spin zero part which is obviously given by the matrix element of the divergence of the axial current. The vector current induced amplitude is responsible for the formfactor F_3. All form factors may contribute in the general three meson case. The three pion case, however, allows for significant simplifications. G-parity implies $F_3 = 0$, Bose symmetry relates F_1 and F_2 and PCAC leads to $F_4 = 0$. Instead of 16 independent real functions which characterize the general hadronic tensor $H^{\mu\nu} = J^\mu J^{*\nu}$ one thus deals with four functions of Q^2, s_1, s_2 only. A convenient basis has been found in ref. [1] which is related to the form factors in a straightforward way. In the three pion rest frame, and with the x, y coordinates alligned with the $\vec{q}_1, \vec{q}_2$ plane (see Fig. 1), these four "structure functions" are given by

$$
\begin{aligned}
W_A = \;& (x_1^2 + x_3^2)\,|F_1|^2 + (x_2^2 + x_3^2)\,|F_2|^2 \\
& + 2(x_1 x_2 - x_3^2)\,\mathrm{Re}\,(F_1 F_2^*) \\
W_C = \;& (x_1^2 - x_3^2)\,|F_1|^2 + (x_2^2 - x_3^2)\,|F_2|^2 \\
& + 2(x_1 x_2 + x_3^2)\,\mathrm{Re}\,(F_1 F_2^*) \\
W_D = \;& 2\left[x_1 x_3\,|F_1|^2 - x_2 x_3\,|F_2|^2\right. \\
& \left. + x_3(x_2 - x_1)\,\mathrm{Re}\,(F_1 F_2^*)\right] \\
W_E = \;& -2x_3(x_1 + x_2)\,\mathrm{Im}\,(F_1 F_2^*)
\end{aligned}
\tag{3}
$$

The variables x_i are defined by $x_1 = q_1^x - q_3^x$, $x_2 = q_2^x - q_3^x$, $x_3 = q_1^y = -q_2^y$, where q_i^x (q_i^y) denotes the x (y) component of the momentum of meson i in the hadronic rest frame. W_A governs the rate and the distributions in the Dalitz plot, the remaining functions determine the angular distribution. If the τ is unpolarized and its rest frame is reconstructed, the angular distribution of the three pions with respect to the τ momentum (as seen from the 3 pion rest frame) is given by

$$
\begin{aligned}
\frac{dN}{d\cos\beta\, d\gamma} \sim\;& \left[\left(1 - m_\tau^2/Q^2\right)(1 - \cos^2\beta) + 2m_\tau^2/Q^2\right]\, W_A \\
& - \left(1 - m_\tau^2/Q^2\right)\sin^2\beta\,\left(\cos 2\gamma\, W_C - \sin 2\gamma\, W_D\right) \\
& + 2\,\cos\beta\, W_E
\end{aligned}
\tag{4}
$$

with the definition of the angles indicated in Fig. 1. In ref. [1] it has been demonstrated that these structure functions can also be measured in present experiments despite the fact that the neutrino momentum is missing and hence the analysis has to be performed without knowledge of the τ rest frame. Predictions for the structure functions have been made in ref. [1] (see also Fig. 2) based on the model of ref. [5], and in fact first comparisons between theory and experiment have been performed in ref. [9] In this model (earlier variants based on a similar approach can be found in ref. [10])

the normalization of the form factor is determined in the chiral limit:

$$
F_1 = F_2 = \frac{2\sqrt{2}}{3 f_\pi}\cos\theta_c \tag{5}
$$

For large Q^2, s_1 and s_2 these formfactors are modulated by Breit Wigner resonances in the 3π and 2π channel:

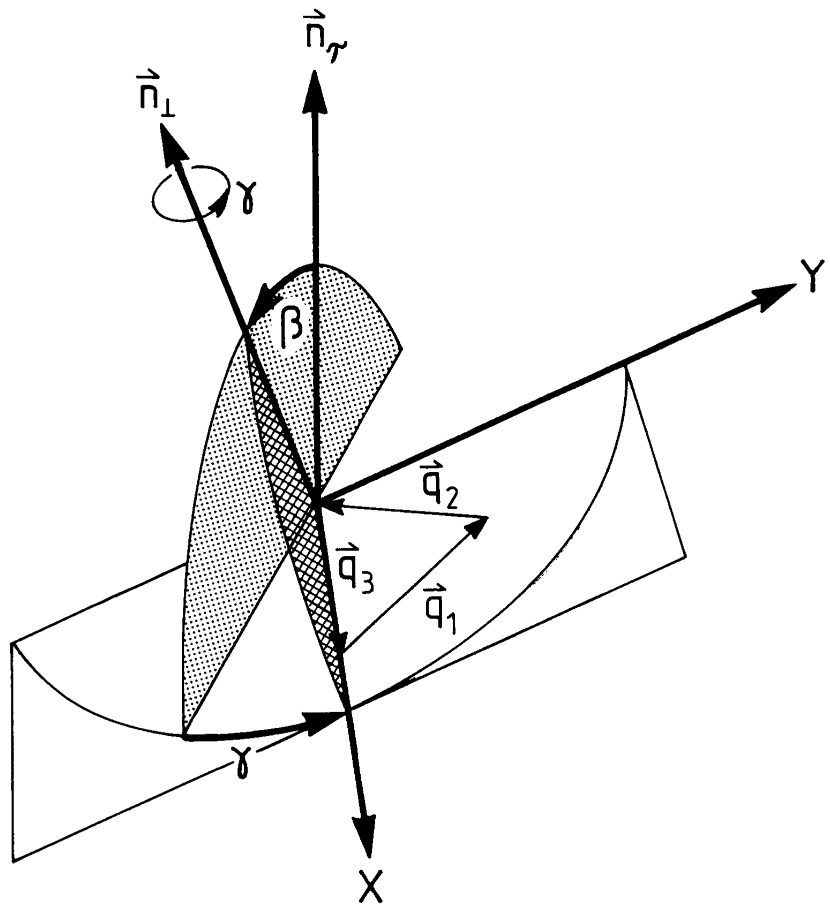

FIG. 1. Definition of the angles β and γ.

$$F_1(Q^2, s_2) = \frac{2\sqrt{2}}{3f_\pi} \cos\theta_c BW_{A_1}(Q^2)\, BW_\rho(s_2) \tag{6}$$

$$F_2(Q^2, s_1) = \frac{2\sqrt{2}}{3f_\pi} \cos\theta_c BW_{A_1}(Q^2)\, BW_\rho(s_1) \tag{7}$$

More detailled studies, testing the magnitude of amplitudes induced by F_3 and F_4, are possible. Since these affect the angular distributions through interference terms, they should be well accessible in measurements, already with the statistics of ongoing experiments.

III. FINAL STATES WITH KAONS [6,11]

A rich variety of final states has become accessible recently with improved particle identification: modes with one or two kaons have been analyzed and even the three kaon final state is kinematically accessible in τ decays. Considering the multitude of charge assignments, even the bookkeeping for the various reactions becomes tedious. The ansatz for the $K\pi$ channel is still straightforward. Dominated by K^* and its radial excitation $K^{*\prime}$, the form factor is constructed in close similarity to the one of the two pion mode, which proceeds through the ρ and ρ' mesons, of course modified by proper $SU(3)$ factors. In fact, the

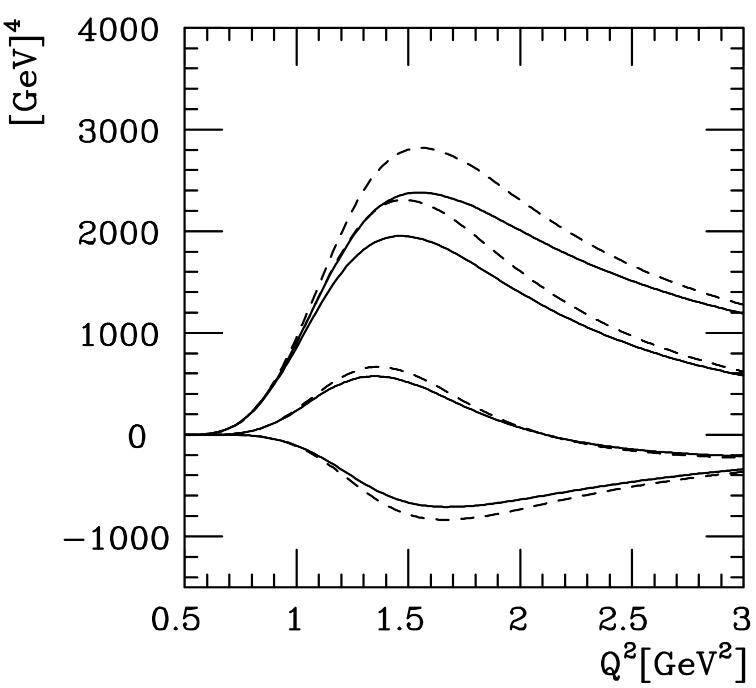

FIG. 2. Spin one hadronic structure functions W_A, W_C, W_D and W_E (from top to bottom) for $\tau \to \nu 3\pi$ as a function of Q^2. Results are shown for two sets of a_1 parameters: $m_{A_1} = 1.251$ GeV, $\Gamma_{a_1} = 0.599$ GeV (solid) and $m_{A_1} = 1.251$ GeV, $\Gamma_{a_1} = 0.550$ GeV (dashed)

relative coupling strength of K^* and $K^{*\prime}$ is identical to the one of ρ and ρ' in the two pion case.

The choice of a specific form for the $K\pi$ resonant amplitude is an important ingredient for the $K\pi\pi$ and $KK\pi$ channels [6] to be discussed now. The construction of the relevant form factors proceeds in close similarity to the approach discussed in section 3 for the three pion channel. For small momenta the form factors F_1 and F_2 are again derived from chiral Lagrangians. In addition, one finds in the same limit a non-vanishing F_3. For example,

$$F_3^{(K^-\pi^-K^+)}(q_i = 0) = \frac{-1}{2\sqrt{2}\pi^2 f_\pi^3} \tag{8}$$

and similarly for the other kaonic modes (except for $K^-\pi^0K^0$ and $\pi^0\pi^0K^-$ where F_3 vanishes in the chiral limit). This term is induced by the celebrated Wess-Zumino anomaly and has originally been discussed in the context of τ decays in ref. [12] Vector as well as axial current induced amplitudes are present leading to five additional non-vanishing structure functions (compared to the three pion mode) and to more complicated angular distributions [1]. The incorporation of resonances in the three meson channel and in various two meson channels proceeds essentially as in the three pion mode. A number of additional complications do however arise: The form factor $F_2\,(F_3)$ vanishes in the soft meson limit for $K^-\pi^0K^0\,(K^-\pi^0K^0,\ \pi^0\pi^0K^-)$. On the other hand it is evident that spin one resonances are present in all three subchannels $K^-K^0 \equiv \rho$, $K^-\pi^0 \equiv K^{-\star}$ and $K^0\pi^0 \equiv K^{0\star}$. Hence $F_2\,(F_3)$ must be different from zero for non-vanishing q_i. The resulting choice of amplitudes is treated in ref. [6] These relations provide important cross checks on the consistency of different measurements.

TABLE I. Predictions for the normalized decay widths $\Gamma(abc)/\Gamma_e$ and the branching ratios $\mathcal{B}(abc)$ for the various channels. The contribution from the vector current is listed in column 3 and available experimental data are listed in column 5.

channel (abc)	$\left(\frac{\Gamma(abc)}{\Gamma_e}\right)$	$\left(\frac{\Gamma(abc)}{\Gamma_e}\right)_V$	$\mathcal{B}(abc)$ [%]	$\mathcal{B}(abc)^{(expt.)}$ [%]
$K^-\pi^-K^+$	.011	.0045	.20	$(.20 \pm .07)$
$K^0\pi^-\overline{K^0}$	.011	.0045	.20	
$K_S\pi^-K_S$	.0027	.0008	.048	$(.021 \pm .006)$
$K_S\pi^-K_L$	.0058	.0029	.10	
$K^-\pi^0K^0$	.0090	.0032	.16	$(.12 \pm .04)$
$\eta\pi^-\pi^0$	.0108	.0108	.19	$(.170 \pm .028)$
$\pi^0\pi^0K^-$	.0080	.0007	.14	$(.09 \pm .03)$
$K^-\pi^-\pi^+$	.043	.0043	.77	$(.40 \pm .09)$
$\pi^-\overline{K^0}\pi^0$	.054	.0058	.96	$(.41 \pm .07)$

Depending on the details of the experimental setup, one may either observe K^0 and $\bar{K}^0$ (if the kaons interact in the detector) or K_S and K_L, if only their decay products are observed. The rates for $K_L\pi^-K_L$ and $K_S\pi^-K_S$ are identical, the rate for $K_S\pi^-K_L$ is about a factor 2.1 higher. Note that the first relation is a strict consequence of CP symmetry, the second one depends on the dynamics of the decay amplitude. Table 1 displays the theoretical predictions for the vector and axial vector induced rate separately and compares them with the experimental results presented at this conference [13] [using $B(\tau \to \nu\nu_e e) = 17.8\%$]. Experiment and theory are in satisfactory agreement.

IV. τ DECAYS INTO FOUR PIONS, CVC AND e^+e^- ANNIHILATION.

In principle the total and differential decay rate into four pions can be related to measurements of the cross section for e^+e^- annihilation into four pions.

$$\begin{aligned} d\Gamma(\tau \to \nu\pi^-3\pi^0) &\sim d\sigma(e^+e^- \to 2\pi^-2\pi^+) \\ d\Gamma(\tau \to \nu 2\pi^-\pi^+\pi^0) &\sim d\sigma(e^+e^- \to 2\pi^-2\pi^+) \\ &\quad + 2\, d\sigma(e^+e^- \to \pi^+\pi^-2\pi^0) \end{aligned} \tag{9}$$

However, already now the τ decays rate is comparable in precision to the e^+e^- data. In particular the data for $e^+e^- \to \pi^+\pi^-2\pi^0$ exhibit significant discrepancies between different experiments (see Fig. 3) which are not apparent from the theoretical predictions based on simply averaging the different sets of data [14]. The model developed in ref. [7] prefers the experimental results with the smaller cross section. Precise data from τ decays can help to resolve the problem. The model for the four pion channel was originally formulated in refs. [15,4] and has been recently improved and expanded in ref. [7]. The amplitude is again normalized to the predictions in the chiral limit, supplemented by appropriate resonances

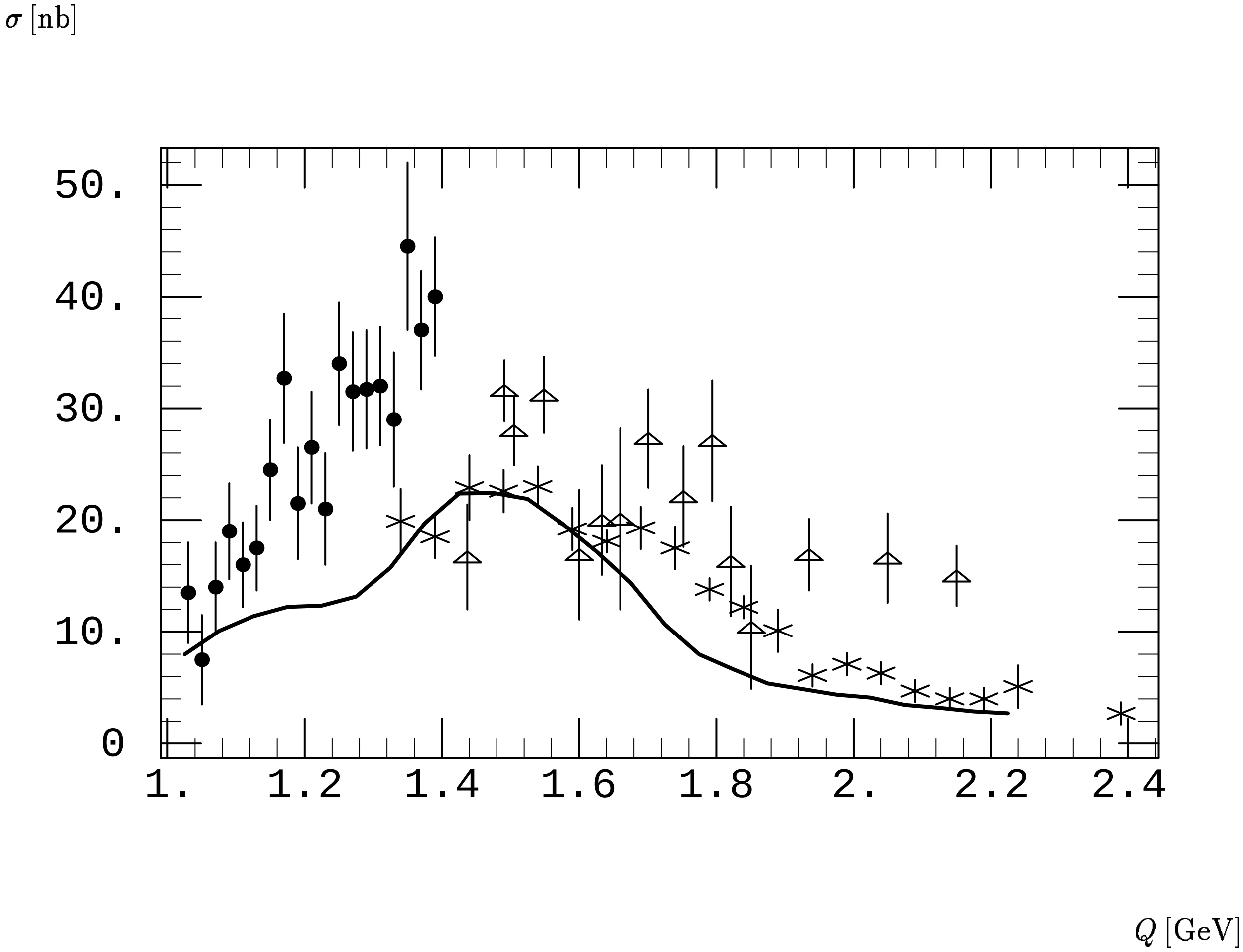

FIG. 3. Cross section for $e^+e^- \to \pi^+\pi^-2\pi^0$: Data from Nowosibirsk (dots with error bars) [18], from Frascati (triangles) [19] and from Orsay (crosses) [20], and the prediction from the model in ref. [7] (solid line)

in the two pion ($=\rho$) and three pion ($=a_1$) channel. In addition the anomalous $\pi\omega(\to 3\pi)$ mode is added, with a $\rho\omega\pi$ coupling strength derived from experimental information on $\omega \to \pi\gamma$ and VDM [16]. The $\omega\pi$ contribution to the 4π is expected to proceed via a vector current. However, a violation of G-parity would allow the $\omega\pi$ system to be in an axial vector state, which could be revealed by an analyses of the angular distribution in the $\omega\pi$ mode as introduced in ref. [17].

Predictions for the differential distributions of the model are in reasonable agreement with the experimental results.

V. RADIATIVE CORRECTIONS TO $\tau \to \nu\pi(\gamma)$

Neglecting radiative corrections, the decay rate of the τ lepton into a pion $\Gamma(\tau \to \pi\nu)$ can be unambiguously predicted from the pion decay rate since both quantities are proportional to the pion decay constant. However, virtual as well as real photon emision are sensitive to

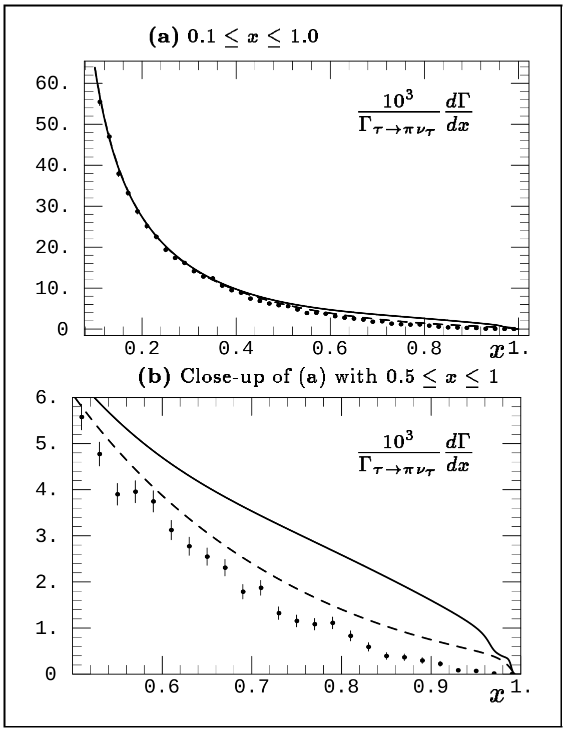

FIG. 4. Photon energy spectrum of the decay $\tau \to \pi\nu_\tau\gamma$: Total prediction (solid), internal bremsstrahlung alone (dashed) and the prediction from PHOTOS (dots with statistical error bars)

different regions of the respective form factors, and hence introduce a finite correction $\delta R_{\tau/\pi}$ in the ratio

$$R_{\tau/\pi} = \frac{\Gamma(\tau \to \nu_\tau \pi(\gamma))}{\Gamma(\pi \to \nu_\mu \mu(\gamma))}$$
$$= \frac{m_\tau^3}{2m_\pi m_\mu^2} \frac{(1 - m_\pi^2/m_\tau^2)^2}{(1 - m_\mu^2/m_\pi^2)^2} \left(1 + \delta R_{\tau/\pi}\right) \tag{10}$$

A detailled study of these form factors and their impact on $R_{\tau/\pi}$ has been performed in ref. [8]. The behaviour of the form factors at very small momentum transfers is fixed by low energy theorems like the axial anomaly and by experimental results obtained in radiative pion decay. In the region of intermediate momenta phenomenological parameterizations inspired by vector meson dominance are employed. The uncertainty in the final prediction is dominated by the ignorance of the form factor in this region. This ansatz is sufficient to predict the photon energy spectrum over the full kinematical region. As demonstrated in Fig. 4, the model prediction is in fair agreement with the prediction from PHOTOS [21], which is based on a semiclassical approximation. The virtual radiation extends over the full kinematical range with Q^2 even up to M_Z^2. For large momentum transfer the quark content of the pion plays the dominant role. Integrating over the quark-antiquark wave function leads to a short distance contribution proportional to f_π. Varying the model parameters and the cutoff μ_{cut}^2 which separates long and short distance contributions one arrives at a result for $\delta R_{\tau/\pi}$ and equally important at an estimate of the theoretical uncertainty of the prediction

$$\delta R_{\tau/\pi} = (0.16 \pm 0.14)\% \tag{11}$$

Similarly one obtains a result for the kaon mode

$$\delta R_{\tau/K} = (0.90 \pm 0.22)\% \tag{12}$$

The prediction for the pion mode should be contrasted with a recent estimate by Marciano and Sirlin [22], which in terms of $R_{\tau/\pi}$ reads

$$\delta R_{\tau/\pi} = (0.67 \pm 1.)\% \tag{13}$$

where the error reflects the authors' estimate of the missing long distance corrections which are not included in their work. From the work of ref. [8] it is evident that the τ decay rate to $\nu\pi(\gamma)$ and $\nu K(\gamma)$ can provide a unique test of lepton universality. Extracting the decay rate from $\mathcal{B}(\nu\pi(\gamma))$ and the τ lifetime, the equality of τ and muon coupling to the charged current can be tested with a precision up to one permille.

VI. SUMMARY

Exclusive semileptonic τ decays offer a unique tool to test the Standard Model in the low energy region. Resonance parameters and form factors can be determined with results complementary to those obtained in low energy e^+e^- machines. The τ polarization can be determined from all decay modes with optimal analyzing power. Isospin and $SU(3)$ symmetry can be investigated. Lepton universality can be tested with a precision of about one permille. Deviations from the Standard Model induced by loop effects or additional tree level contributions *e.g.* from a two Higgs doublet model can thus be explored in an interesting range of parameters.

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