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RADIATION OF LIGHT FERMIONS IN HEAVY FERMION PRODUCTION

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Abstract

Recent analytic calculations on the rate for the production of a pair of massive fermions in e^+e^- annihilation plus real or virtual radiation of a pair of massless fermions are discussed. The contributions for real and virtual radiation are displayed separately. The asymptotic behaviour close to threshold is given in a compact form and an application to the angular distribution of massive quarks close to threshold is presented.

1. Introduction

The hadron production cross section belongs to the best measured quantities in e^+e^- collision experiments. Measurements exist in low and intermediate energy ranges as well as around the Z peak. A precise perturbative determination of the heavy quark production cross section $\sigma(e^+e^- \to \gamma^* \to Q\bar{Q})$, $(Q = c, b, t)$ is therefore a crucial input to present and future interpretation of experimental data. Especially future experiments at the τ–charm or the B factory and the intended NLC require calculations valid for the whole energy range above threshold. The $\mathcal{O}(\alpha_s)$ calculation including the full mass dependence has been performed a long time ago [1], whereas a complete second order computation is still unknown. Asymptotic expansions [2] of the second order contribution are available in the limit of vanishing fermion masses or including mass corrections through an expansion in m^2/s up to order m^4/s^2. To third order the corresponding expansion is known up to order m^2/s. These expansions, however, are inadequate near threshold.

This talk reports on recent analytic calculations of the complete $\mathcal{O}(\alpha_s^2)$ fermionic (n_f) contributions to the cross section valid for the full physical energy range. This corresponds to the evaluation of all possible cuts of the diagrams depicted in Fig. 1. The resulting expressions describe real radiation of a secondary fermion pair $(q\bar{q})$ off one of the primary fermions $(Q\bar{Q})$ and the corresponding virtual effects. *2. Calculation* The $\mathcal{O}(\alpha^2)$ QED calculation of the ratio $R_{Q\bar{Q}q\bar{q}} = \sigma(e^+e^- \to \gamma^* \to Q\bar{Q}q\bar{q})/\sigma_{pt}$, where $\sigma_{pt} = 4\pi\alpha^2/(3s)$, can be reduced to the following phase space integral

$$R_{Q\bar{Q}q\bar{q}} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right) \int\limits_{4m_q^2}^{(\sqrt{s}-2m_q)^2} \frac{\mathrm{d}\lambda^2}{\lambda^2} R_{Q\bar{Q}\gamma}(\lambda^2) R_{q\bar{q}}\,, \tag{1}$$

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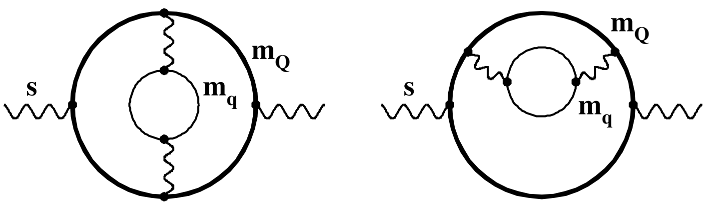

Figure 1: *Characteristic Feynman diagrams describing the production of a pair of massive and a (real or virtual) pair of massless fermions.*

where $R_{Q\bar{Q}\gamma}$ and $R_{q\bar{q}}$ denote the the normalized cross section of $Q\bar{Q}$ production with additional radiation of the photon with mass λ and of $q\bar{q}$ production respectively. The corresponding virtual contribution reads

$$R_{Q\bar{Q}}^{q\bar{q},\,virt} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)\int\limits_{4m_q^2}^{\infty}\frac{\mathrm{d}\lambda^2}{\lambda^2}R_{Q\bar{Q}}^{\gamma,\,virt}(\lambda^2)R_{q\bar{q}}\,, \tag{2}$$

where $R_{Q\bar{Q}}^{\gamma,\,virt}$ describes the $\mathcal{O}(\alpha)$ virtual corrections to the $Q\bar{Q}$ production cross section due to exchange of a massive photon with mass λ. For the electromagnetic Dirac (F_1) and Pauli (F_2) form factors relations in close analogy to (2) exist. The evaluation of eqs. (1) and (2) for $m_Q = 0$ is fairly simple and was performed in [3, 4]. The case of interest, $m_Q, s \gg m_q$, leads to much more involved calculations [5]. The solutions have the following form

$$R_{Q\bar{Q}q\bar{q}} = \left(\frac{\alpha}{\pi}\right)^2\left[f_R^{(2)}\ln^2\frac{m_q^2}{s} + f_R^{(1)}\ln\frac{m_q^2}{s} + f_R^{(0)}\right]\,,$$
$$R_{Q\bar{Q}}^{q\bar{q},\,virt} = \left(\frac{\alpha_s}{\pi}\right)^2\left[f_V^{(2)}\ln^2\frac{m_q^2}{s} + f_V^{(1)}\ln\frac{m_q^2}{s} + f_V^{(0)}\right]\,.$$

Real and virtual corrections individually exhibit mass singularities in the limit $m_q \to 0$. The quadratic logarithms correspond to the soft photon singularities of the respective first order results. However, $f_R^{(2)} = -f_V^{(2)}$, and thus at least the leading logarithmic terms cancel if the sum of real and virtual contributions is considered. A linear logarithm in m_q^2/s remains in the sum and can be absorbed into a running $\overline{\mathrm{MS}}$ coupling $\alpha(\mu^2)$ at the scale $\mu^2 = s$. Due to the universality of the fermionic corrections in the second order we only have to replace one factor of α by $2/3\,\alpha_s$ to arrive at the corresponding QCD expressions. It is interesting that the functions $f_{R/V}^{(1/2)}$ can already be deduced without any effort from first order calculations. Only $f_R^{(0)}$ and $f_V^{(0)}$ require an actual second order evaluation and thus constitute the main effort. Their analytic results can completely be expressed in terms of Tri- (Li_3), Di- (Li_2) and usual logarithms. From the complete analytic results high energy expansions in terms of m_Q^2/s can be derived which are in agreement with prior asymptotic calculations [2]. Near threshold the results have to be expanded in terms of β, the relative velocity of the $Q\bar{Q}$ pair. Real radiation is suppressed by β^3 whereas the virtual contributions give a finite result at $\beta = 0$. Using the running coupling constant, defined by

$$\alpha(\mu^2) = \alpha\Big[1 - \frac{1}{3}\frac{\alpha}{\pi}\Big(\ln\frac{m_q^2}{\mu^2} + \frac{5}{3}\Big)\Big]\,,$$

the combination $F_1 + F_2$ which is relevant for the threshold cross section, assumes a particularly simple form

$$F_1 + F_2 = 1 + \frac{\alpha(s\beta^2)\pi}{4\beta} - 2\frac{\alpha(se^{3/4}/4)}{\pi} + \mathcal{O}(\beta)\,.$$

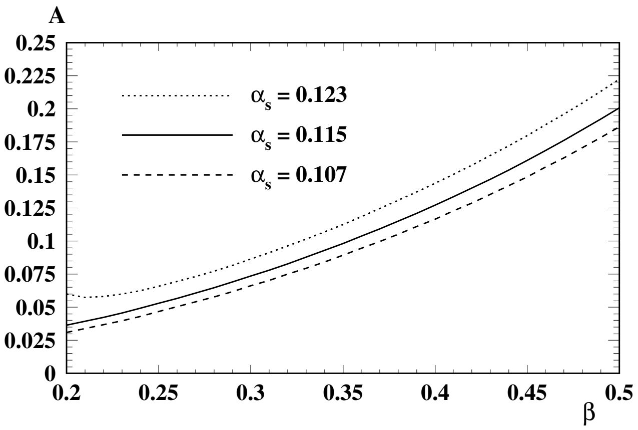

Figure 2: Sensitivity of the anisotropy A for $e^+e^- \to b\,\bar{b}$ to changes in $\alpha_{\overline{MS}}(M_Z^2)$.

The scale in the Coulomb exchange term $\propto 1/\beta$ is equal to the relative momentum of the $Q\bar{Q}$ pair, whereas in the transvers photon term the hard scale $\sim s/4$ has to be chosen. Up to second order in α the running coupling governing the Coulomb singularity is thus identical to the coupling in the potential. Similar conclusions can be drawn for each of the form factors individually. Employing the BLM procedure this scale setting prescription can be transfered to QCD. *3. Angular Distribution in* $e^+e^- \to Q\bar{Q}$ Because mesons with one heavy quark Q $(Q = b, c)$ follow the heavy quark direction we can use our results to predict the meson angular distribution near the $Q\bar{Q}$ production threshold‡. The quantity of interest is the "anisotropy" A in the cm angular distribution

$$\frac{\mathrm{dN}}{\mathrm{d}\cos^2\theta} \sim 1 + A\cos^2\theta\,,$$

which measures the D–wave contribution with the characteristic $\cos^2\theta$ behaviour relative to the S–wave. Due to the angular distributions specific for the exclusive pseudoscalar or vector meson channels the result can only be applied above the $B^*\bar{B}^*$ or $D^*\bar{D}^*$ threshold, where the $Q\bar{Q}$ distribution is saturated by the sum of the various channels ($B\bar{B}$, $B\bar{B}^*+cc.$, $B^*\bar{B}^*$, etc.). To determine the anisotropy only virtual corrections have to be taken into account. In the V-scheme this leads to the result [6]

$$A = \tilde{A}/(1-\tilde{A})\,,$$

$$\tilde{A} = \frac{\beta^2}{2}\,\frac{\left(1 - 4\,\frac{\alpha_V(m^2 e^{7/6})}{\pi}\right)}{\left(1 - \frac{16}{3}\,\frac{\alpha_V(m^2 e^{3/4})}{\pi}\right)}\,\frac{1-e^{-x_s}}{1-e^{-x_s'}}\,\frac{\alpha_V(4\,m^2\,\beta^2/e)}{\alpha_V(4\,m^2\,\beta^2)}\,,$$

where

$$x_s = \frac{4\,\pi}{3}\,\frac{\alpha_V(4\,m^2\beta^2)}{\beta}\,,\quad x_s' = \frac{4\,\pi}{3}\,\frac{\alpha_V(4\,m^2\beta^2/e)}{\beta}$$

and a resummation of the Coulomb terms has been performed. It is evident that in the anisotropy A four different scales are involved. In the factor arising from the Coulomb exchange the scale is smaller in the numerator (D-wave) than in the denominator (S-wave). This behaviour is consistent with qualitative considerations based on the relative distances relevant for S- and D-waves in the Coulomb part. In Fig. 2 the sensitivity of the anisotropy in the $b\bar{b}$ production angular distribution is demonstrated for variations of the value for $\alpha_{\overline{MS}}(M_Z^2)$.

‡ Of course, the results can also be applied to $t\bar{t}$ or $\tau^-\tau^+$ pair production near threshold.

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