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HADRONIC DECAYS OF EXCITED HEAVY QUARKONIA

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Abstract

We construct an effective Lagrangian for the hadronic decays of a heavy excited s-wave-spin-one quarkonium Ψ' into a lower s-wave-spin-one state Ψ. We show that reasonable fits to the measured invariant mass spectra in the charmonium and bottomonium systems can be obtained within this framework. The mass dependence of the various terms in the Lagrangian is discussed on the basis of a quark model.

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1 Introduction

Bound state systems consisting of a heavy quark and a heavy antiquark are fairly well described in terms of wave function models, where the potential among the two heavy quarks is determined from fitting a phenomenological ansatz to the data [1]. In this way many static properties of these systems may be understood even quantitatively.

As far as decays of heavy quarkonia are concerned there has been also some theoretical progress recently [2, 3]. For inclusive decays in which the two heavy quarks annihilate an effective theory approach has been developed allowing for a systematic treatment of these processes. This method puts the description of this type of decays on a model independent basis.

The theoretical framework of exclusive hadronic decays is in general still quite model dependent. However, for a large class of decay modes of excited quarkonia Ψ' we may use chiral symmetry arguments to construct an effective Hamiltonian for decays such as $\Psi' \to \Psi\pi$ or $\Psi' \to \Psi\pi\pi$ where π is a member of the Goldstone boson octet. This ansatz has been discussed already in the literature [4], however, without giving a systematic derivative expansion in the spirit of chiral perturbation theory.

Such an expansion may be performed starting from the infinite mass limit for the heavy quarkonium[‡]. The momentum of the heavy quarkonium scales with the heavy quark masses and hence it may not be used as an expansion parameter. In order to define a systematic derivative expansion we decompose the momentum $p(p')$ of the heavy quarkonium in the initial (final) state as $p'(p) = (m_Q + m_{\overline{Q}})v + k'(k)$, where k' and k are small residual momenta of the order of the binding energy of the heavy quarkonium, and m_Q and $m_{\overline{Q}}$ are the masses of the heavy quark and antiquark respectively. Note that unlike in Heavy Quark Effective Theory where systems with a single heavy quark are considered we cannot use the static limit for quarkonia states. Rather one has to employ the non-relativistic limit and the expansion in k/m_Q and $k'/m_{\overline{Q}}$ becomes an expansion in $|\vec{v}_{rel}|/c$, where $\vec{v}_{rel}$ is the relative (three)velocity of the two heavy quarks.

The velocity v is the velocity of the initial state quarkonium and hence the momenta of the final state heavy quarks will be of the order of the mass difference of the initial and final state quarkonia, which we shall assume to be small compared to the masses m_Q and $m_{\overline{Q}}$.

In the spirit of chiral perturbation theory one may now formulate a derivative expansion, which after Fourier transformation becomes an expansion in the pion momenta and the residual momenta k' and k. In the next section we shall use chiral symmetry to write down the leading as well as the chiral symmetry breaking terms in such an expansion. In section 3 we show that one may obtain a good

[‡]The idea we are proposing here is along the lines as in [5], where this method is applied to ϕ decays.

fit to the data, total rates as well as invariant mass spectra of the two pions in the decay $\Psi' \to \Psi\pi\pi$. We shall compare the charmonia and the bottomonia and discuss the dependence of the parameters in the effective Lagrangian on the bottom and charm mass in section 4. Conclusions are given in section 5.

2 Effective Lagrangian for Heavy Quarkonia Decays

We construct an effective Lagrangian for the decays of a heavy excited s-wave-spin-one quarkonium Ψ' into a lower s-wave-spin-one state Ψ. Let A_μ be the field of the initial excited s-wave 1^- state and B_μ the one of the final 1^- state. We shall consider here only quarkonia where $m_Q = m_{\overline{Q}}$, hence we look at charmonium or bottomonium systems.

The momenta of the heavy quarkonia are split into a large piece which scales with m_Q and a small residual part k which depends only weakly on the heavy quark mass

$$\begin{aligned} p' &= 2m_Q v + k' \text{ for } A_\mu \\ p &= 2m_Q v + k \text{ for } B_\mu \end{aligned}$$

In coordinate space this is achieved by a phase redefinition of the fields

$$\begin{aligned} A_\mu(x) &= \exp(-i2m_Q v\cdot x) A_\mu^{(v)}(x) \\ B_\mu(x) &= \exp(-i2m_Q v\cdot x) B_\mu^{(v)}(x) \end{aligned} \tag{1}$$

such that the derivative acting on the fields with superscript (v) yields now k (or k') which is small compared to the heavy mass. The fields $A^{(v)}$ and $B^{(v)}$ obey the equations of motion corresponding to static fields and are transverse with respect to the velocity vector v

$$iv\cdot\partial A_\mu^{(v)}(x) = 0, \quad v^\mu A_\mu^{(v)}(x) = 0 \tag{2}$$

$$iv\cdot\partial B_\mu^{(v)}(x) = 0, \quad v^\mu B_\mu^{(v)}(x) = 0 \tag{3}$$

We are interested in an effective Lagrangian for the hadronic decays of the type $A \to B\pi$ or $A \to B\pi\pi$, and we construct this Lagrangian using chiral symmetry. Higher order terms may be included by chiral perturbation theory, i.e. by a systematic expansion in the derivatives (these correspond to pion momenta or residual momenta for quarkonia) and in the light quark masses. The heavy quarkonia are singlets under the chiral symmetry, and hence the decays $A \to B\pi$ are forbidden in the chiral limit.

In the chiral limit only decays to an even number of pions are allowed, and the leading term in the derivative expansion obeying chiral symmetry consists of three terms

$$\mathcal{L}_0 = g A_\mu^{(v)} B^{(v)\mu *}\ \mathrm{Tr}[(\partial_\nu U)(\partial^\nu U)^\dagger] + g_1 A_\mu^{(v)} B^{(v)\mu *}\ \mathrm{Tr}[(v\cdot\partial U)(v\cdot\partial U)^\dagger]$$
$$+ g_2 A_\mu^{(v)} B_\nu^{(v)*}\ \mathrm{Tr}[(\partial^\mu U)(\partial^\nu U)^\dagger + (\partial^\mu U)^\dagger(\partial^\nu U)] + \text{ h.c.} \tag{4}$$

where U is a unitary 3×3 matrix that contains the Goldstone fields

$$U = \exp\left(i\Phi/F_0\right) \qquad \Phi = \sqrt{2}\begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \overline{K^0} & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix} \tag{5}$$

and F_0 is the pion decay constant[§] in the chiral limit. In previous considerations the second term in (4) has been often omitted, but since a consistent definition of the derivative expansion forces us to introduce the vector v, the second term is a perfectly allowed one. We shall show in the next section that the magnitude of the coupling g_1 is not small compared to g and that a reasonable fit to the invariant mass spectra of the pions may be obtained already from (4).

The quark mass matrix breaks chiral symmetry, and the leading terms of this kind are

$$\mathcal{L}_{S.B.} = g_3 A_\mu^{(v)} B^{(v)\mu *}\ \mathrm{Tr}[\mathcal{M}(U + U^\dagger - 2)]$$
$$+ i g' \epsilon^{\mu\nu\alpha\beta}\left[v_\mu A_\nu^{(v)} \partial_\alpha B_\beta^{(v)*} - (\partial_\mu A_\nu^{(v)}) v_\alpha B_\beta^{(v)*}\right]\ \mathrm{Tr}[\mathcal{M}(U - U^\dagger)]$$
$$+ \text{ h.c.} \tag{6}$$

where

$$\mathcal{M} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} \tag{7}$$

The two operators proportional to the ϵ tensor are relevant for decays involving an odd number of pions. The relative sign of the these terms is fixed by reparameterization invariance [6].

The Lagrangian in (4) has been considered also by Casalbuoni *et al.* [7], as well as the part proportional to g' in (6) [8]. In the latter case the authors include in addition the mixing and direct coupling to the singlet η', whereas we concentrate on the contributions from the octet according to [9].

3 Fit to the Data

3.1 $\Psi' \to \Psi\pi\pi$

Starting from the Lagrangians given in (4) and (6) we fit the experimental data on the decays $\psi(2S) \to J/\psi\pi^+\pi^-$ in the charmonium system, and $\Upsilon(aS) \to$

§ We use the convention in which the pion decay constant is $F_0 \simeq 93$ MeV.

$\Upsilon(bS)\pi^+\pi^-$ in the bottomonium system, where (a,b) takes the values $(2,1), (3,2)$ and $(3,1)$. The amplitude has the form

$$\mathcal{A}(\Psi' \to \Psi\pi^+\pi^-) = -\frac{4}{F_0^2}\left\{ \left[\frac{g}{2}(m_{\pi\pi}^2 - 2M_\pi^2) + g_1(v\cdot p_{\pi^+})(v\cdot p_{\pi^-}) + g_3 M_\pi^2\right]\epsilon_\Psi^* \cdot \epsilon_{\Psi'} \right.$$
$$\left. + g_2\left[p_{\pi^+\mu}p_{\pi^-\nu} + p_{\pi^+\nu}p_{\pi^-\mu}\right]\epsilon_\Psi^{*\,\mu}\epsilon_{\Psi'}{}^{\nu}\right\} \tag{8}$$

where $m_{\pi\pi}^2 = (p_{\pi^+} + p_{\pi^-})^2$ and ϵ_Ψ, $\epsilon_{\Psi'}$ are the polarization vectors of the heavy spin-one quarkonia . In the rest frame of the decaying particle Ψ' we have $v_\mu = (1,\vec{0})$ and the amplitude (8) reduces to the non-relativistic expression given by Brown and Cahn [10] a long time ago. In this frame the heavy final particle Ψ moves non-relativistically. If we neglect the motion of the final state heavy quarkonium, the amplitude proportional to g_2 describes the d-wave part of the dipion system, which is found to be suppressed experimentally to a few percent in the charmonium decay [11] as well as in the bottomonium decays [12, 13, 14]. This observation is supported in addition by some qualitative arguments that we will give in Sec.4. Thus we set $g_2 = 0$.

The remaining coupling constants g, g_1 and g_3 can be fitted to the total decay rate and the invariant mass spectra of the two pions. The data for the (normalized) differential rates for $\psi(2S) \to J/\psi\pi^+\pi^-$ and $\Upsilon(2S) \to \Upsilon(1S)\pi^+\pi^-$ are taken from Fig.7 in [12], for the $\Upsilon(3S)$ decays we consider the data from [14]. Our results are listed in Table 1 and the fits are shown in Fig.1 and Fig.2 . It is worth discussing briefly the results decay by decay.

- $\psi(2S) \to J/\psi\pi^+\pi^-$: We obtain already a good fit working in the chiral limit, i.e. by ignoring g_3. The ratio g_1/g is then given by

$$\left(\frac{g_1}{g}\right)_{c\bar{c}} = -0.35 \pm 0.03 \tag{9}$$

where the error is given by the fitting program, including the experimental uncertainties.

Varying g_3 in the range $g_3 = 0\ldots 2.1$ and fitting again g and g_1 gives an approximate linear relation among the triple (g, g_1, g_3),

$$(g, g_1, g_3) = \left(0.30 + \frac{1}{20}x, -0.11 - \frac{3}{10}x, x\right), \qquad x = 0\ldots 2.1 \tag{10}$$

where χ^2_{min} is found at $g_3 = 1.55$. The fit for χ^2_{min} is shown in the upper plot of Fig.1 .

- $\Upsilon(2S) \to \Upsilon(1S)\pi^+\pi^-$: A good fit is again obtained by putting $g_3 = 0$. Moreover, allowing for all three parameters to be free, this fit is included in the set of solutions for which $\chi^2 \leq (\chi^2_{min} + 0.01)$ holds. However, χ^2 as a function of the couplings g_1 and g_3 is very flat around χ^2_{min} and thus the errors of g_1 and g_3 in the all-parameter-fit are an order of magnitude larger than the values themselves. Therefore we do not indicate this fit here. In the case of $g_3 = 0$, g is of the same order as in the corresponding charmonium decay, whereas the ratio g_1/g differs roughly by a factor of 2,

$$\left(\frac{g_1}{g}\right)_{b\bar{b}} = -0.19 \pm 0.04 \tag{11}$$

The fit corresponding to $g_3 = 0$ is shown in the lower plot of Fig.1.

- $\Upsilon(3S) \to \Upsilon(2S)\pi^+\pi^-$: In this decay, the experimental data have rather large errors. A reasonable fit is obtained by using the current algebra amplitude, i.e. by putting $g_1 = g_3 = 0$. If we allow in addition for a variation of g_1 the χ^2 decreases a little bit, however, the error of g_1 is of the same order as the value itself. Altough g is larger by a factor of $\sim (5-6)$ in comparison to the $\Upsilon(2S)$ decay, the ratios g_1/g are close to each other (within a large error)

$$\left(\frac{g_1}{g}\right)_{b\bar{b}} = -0.23 \pm 0.22 \tag{12}$$

In the upper plot of Fig.2 we show the fit corresponding to $g_1 \neq 0$.

- $\Upsilon(3S) \to \Upsilon(1S)\pi^+\pi^-$: Here we are confronted with two aspects which are absent in the other decays. Firstly the phase space is considerably larger than in all the other decays discussed so far. Since chiral perturbation theory is valid only for momenta small compared to the scale $\Lambda_{\chi SB} = 4\pi F_0 \simeq 1.2$ GeV, the fits obtained with the amplitude (8) should be considered with some caution. Secondly the experimental data indicate that the dipion invariant mass distribution is double peaked [13, 14], whereas all the other decays have a single peak only.

The three parameter fit is indeed rather poor, however, it implies at least that there is a double peak (shown in the lower plot of Fig.2 as the solid line). In order to improve the situation, we pick up the idea of [15] by taking $\pi^+\pi^-$ final state interactions into account. Working in the approach where both the heavy particles are at rest, the first line of the amplitude in (8) represents the dipion system in an s-wave. The amplitude (for $g_2 = 0$) can therefore be improved by introducing the Omnès function [16]

$$\begin{aligned} \mathcal{A}(\Psi' \to \Psi\pi^+\pi^-)|_{g_2=0} &\longrightarrow \Omega(m^2_{\pi\pi})\mathcal{A}(\Psi' \to \Psi\pi^+\pi^-)|_{g_2=0} \\ \Omega(s) &= \exp\left(\frac{s}{\pi}\int_{4M_\pi^2}^{\infty} ds' \frac{\delta_0^0(s')}{s'(s'-s)}\right) \end{aligned} \tag{13}$$

	g	g_1	g_3	$\chi^2/d.f.$
$\psi(2S) \to J/\psi\pi^+\pi^-$	0.30 ± 0.01	-0.11 ± 0.01	0^*	25.3/24
	0.38 ± 0.03	-0.59 ± 0.19	1.55 ± 0.58	20.1/23
$\Upsilon(2S) \to \Upsilon(1S)\pi^+\pi^-$	0.25 ± 0.01	-0.05 ± 0.01	0^*	25.3/19
$\Upsilon(3S) \to \Upsilon(2S)\pi^+\pi^-$	1.19 ± 0.04	0^*	0^*	26.2/21
	1.53 ± 0.33	-0.36 ± 0.34	0^*	25.0/20
$\Upsilon(3S) \to \Upsilon(1S)\pi^+\pi^-$	0.026 ± 0.001	-0.087 ± 0.003	0.54 ± 0.02	87.6/29

Table 1: Values of the coupling constants obtained from fits to the data as indicated in the text. The 0^* means that this parameter is ignored in this fit.

where $\delta_0^0(s)$ (denoted as δ from here on) is the energy dependent phase shift of the isospin $I = 0$ s-wave $\pi\pi$ scattering amplitude. We decompose δ in a low energy phase shift δ_{LE} ("low" with respect to the integration range of Ω) and in a part coming from a (sharp) scalar resonance, δ_R. For the low energy tail we use the parametrization of Schenk [17], which reproduces the measured data up to $s^{1/2} = 0.9$ GeV,

$$\begin{aligned}
\delta &= \delta_{LE} + \delta_R \quad , \\
\tan\delta_{LE}(s) &= \left(1 - \frac{4M_\pi^2}{s}\right)^{1/2} \frac{4M_\pi^2 - s_0}{s - s_0} \\
&\quad \times \left[a_0 - \tilde{b}_0\left(1 - \frac{s}{4M_\pi^2}\right) + c_0\left(1 - \frac{s}{4M_\pi^2}\right)^2\right] \quad , \\
\tilde{b}_0 &= b_0 + a_0\frac{4M_\pi^2}{4M_\pi^2 - s_0} + a_0^3 \quad , \\
\delta_R(s) &= \pi\theta(s - M_R^2)
\end{aligned} \tag{14}$$

with the numerical values

$$a_0 = 0.2, \qquad b_0 = 0.24, \qquad c_0 = 0, \qquad s_0^{1/2} = 0.865\ GeV \tag{15}$$

and the resonance mass M_R is left as a free parameter. We repeat the fit procedure and get the following result,

$$\Upsilon(3S) \to \Upsilon(1S)\pi^+\pi^- :$$

g	g_1	g_3	M_R [GeV]	$\chi^2/d.f.$
0.014 ± 0.001	-0.040 ± 0.002	0.21 ± 0.01	1.17 ± 0.07	$26.6/28$

(16)

The fit is in good agreement with the data, see lower plot of Fig.2 (dashed line). The parameters g_1 and g_3 are larger than g, which has decreased (compared to the $\Upsilon(2S)$ decay) by a factor of $\sim (10-15)$. The resonance mass M_R is around 1 GeV, as it is expected from experimental data on δ [18] and from the corresponding scalar resonances $f_0(980)$ and $f_0(1300)$ (or a mixture of them) listed by the PDG [19].

In summary we find that the data on the decays $\Psi' \to \Psi\pi\pi$ can be fitted in most cases with two parameters g and g_1 to a good accuracy. In the charmonium system we find the ratio $g_1/g = 0.35$, whereas for bottomonia we have $g_1/g \simeq 0.2$. The observation, that the data can be fitted accurately with operators allowed in the chiral limit, has first been made by Novikov and Shifman [20]. An exception to this rule is found in the transition $\Upsilon(3S) \to \Upsilon(1S)\pi^+\pi^-$, where the amplitude (8) does not give a good description. The situation can be improved by taking $\pi\pi$ rescattering into account using the parametrization of [17] and a scalar resonance, the mass of which is preferably around 1 GeV.

3.2 $\Psi' \to \Psi\pi$

In the case of the decay amplitudes of $A \to B\pi$ where $\pi = \pi^0, \eta$ experimental values exist in the charmonium system, upper limits are known for the $\Upsilon(2S)$ decays and for the transition $\Upsilon(3S) \to \Upsilon(1S)\eta$ [19]. For the $\Psi'(2S)$ decays we consider the coupling g' in the Lagrangian (6) with respect to the same decay channel in the different systems,

$$\left|\frac{g'_{b\bar{b}}}{g'_{c\bar{c}}}\right|_{\pi} = \frac{M_{\Upsilon(2S)}}{M_{\psi(2S)}} \left[\frac{\Gamma\left(\Upsilon(2S) \to \Upsilon(1S)\pi\right)}{\Gamma\left(\psi(2S) \to J/\psi\pi\right)}\right]^{1/2} \left(\frac{|\vec{p_\psi}|}{|\vec{p_\Upsilon}|}\right)^{3/2}$$

$$|\vec{p_\Psi}| = \frac{1}{2M_{\Psi'}}\left\{\left[M^2_{\Psi'} - (M_\Psi + M_\pi)^2\right]\left[M^2_{\Psi'} - (M_\Psi - M_\pi)^2\right]\right\}^{1/2} \qquad (17)$$

where the stronger limit is found in the η decay channel,

$$\left|\frac{g'_{b\bar{b}}}{g'_{c\bar{c}}}\right|_{\pi^0} < 3.08\,, \qquad \left|\frac{g'_{b\bar{b}}}{g'_{c\bar{c}}}\right|_{\eta} < 0.57 \qquad (18)$$

For $\Upsilon(3S) \to \Upsilon(1S)\eta$ the phase space is much larger than in the other decays, i.e. the η is by no means soft. The present upper limits for bottomonia in the η decay channel are similar to each other

$$\frac{\Gamma(\Upsilon(3S) \to \Upsilon(1S)\eta)}{\Gamma_{total}} < 2.2 \times 10^{-3} \,, \qquad \frac{\Gamma(\Upsilon(2S) \to \Upsilon(1S)\eta)}{\Gamma_{total}} < 2 \times 10^{-3} \tag{19}$$

However, equal branching ratios lead to completely different g',

$$\left|g'_{\Upsilon(3S)}\right| \simeq \frac{\left|g'_{\Upsilon(2S)}\right|}{14} \qquad (\textit{equal branching ratios}) \tag{20}$$

mainly due to the difference in phase space.

4 Discussion of the Results

In this section we shall discuss our results, in particular we shall try to give some arguments concerning the mass dependence of the coefficients g, g_1 and g' (neglecting contributions driven by the pion mass, i.e. $g_3 = 0$). We achieve an at least qualitative understanding by representing the coefficients in the Lagrangian by matrix elements of heavy quark operators. Hence we consider a model in which the heavy quarks and the light pseudoscalar mesons are the relevant degrees of freedom; in this respect this resembles the chiral quark model of Georgi and Manohar [21]. Note that we omit in the following discussion the decay $\Upsilon(3S) \to \Upsilon(1S)\pi^+\pi^-$ which is somewhat beyond the scope of our method due to the large phase space for the pions.

First of all, this model easily explains the fact that the pion-d-wave contribution is suppressed, since this at least requires a dimension-6-operator for the heavy quarks, which would be contained in the operator

$$\mathcal{L}_{d-wave} = \frac{\gamma_2}{\Lambda^3}(\bar{Q}_v\gamma_\mu Q_v)(\bar{Q}_v\gamma_\nu Q_v)\mathrm{Tr}\ \left[(\partial^\mu U)(\partial^\nu U^\dagger) + (\partial^\nu U)(\partial^\mu U^\dagger)\right] \tag{21}$$

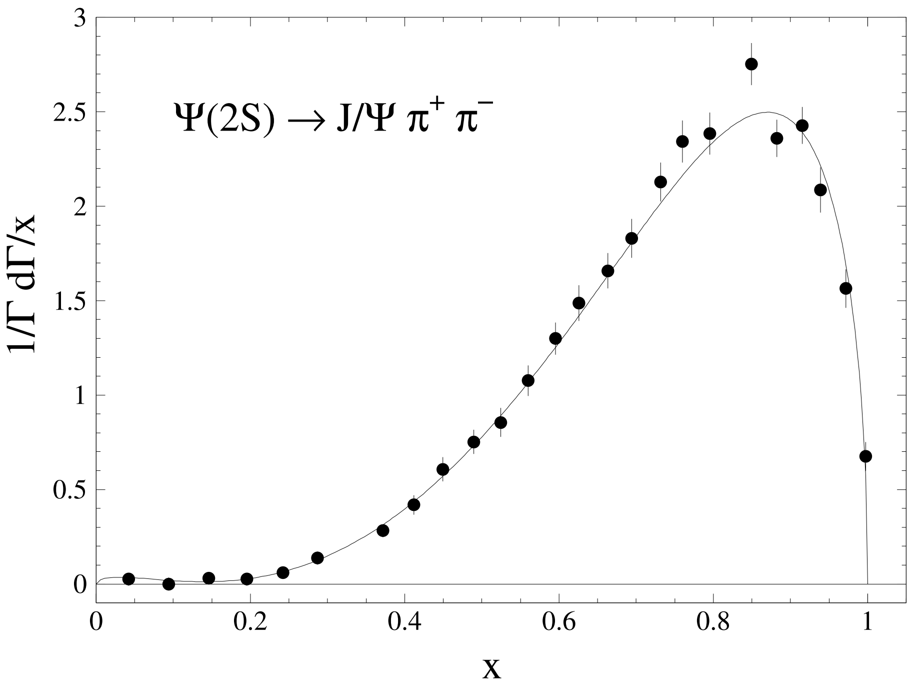
while the leading contribution to the pion-s-wave is proportional to a dimension-3-operator for the heavy quarks,

$$\mathcal{L}_{s-wave} \propto (\bar{Q}_v Q_v) \tag{22}$$

Here Q_v are the operators of heavy quarks which have been rescaled with a phase according to

$$Q_v = \exp(imvx)Q$$

and the scale Λ is at least of the order of the chiral symmetry breaking scale $\Lambda_{\chi SB}$ or maybe even the heavy quark mass; in the latter case the d-wave contribution to this decay would be more strongly suppressed in the Υ decays as compared to the J/ψ transitions.



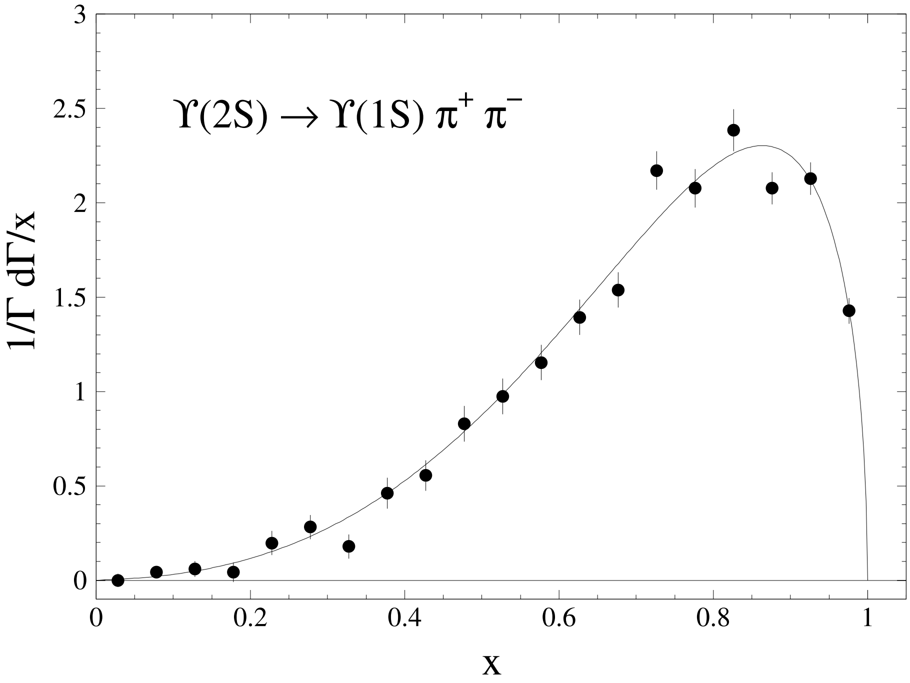


Figure 1: The curves show the fit as indicated in the text to the normalized invariant mass spectra of the two pions with $x = (m_{\pi\pi} - 2M_\pi)/(M_{2S} - M_{1S} - 2M_\pi)$ and $m_{\pi\pi} = (p_{\pi^+} + p_{\pi^-})$. The experimental data are taken from [12].

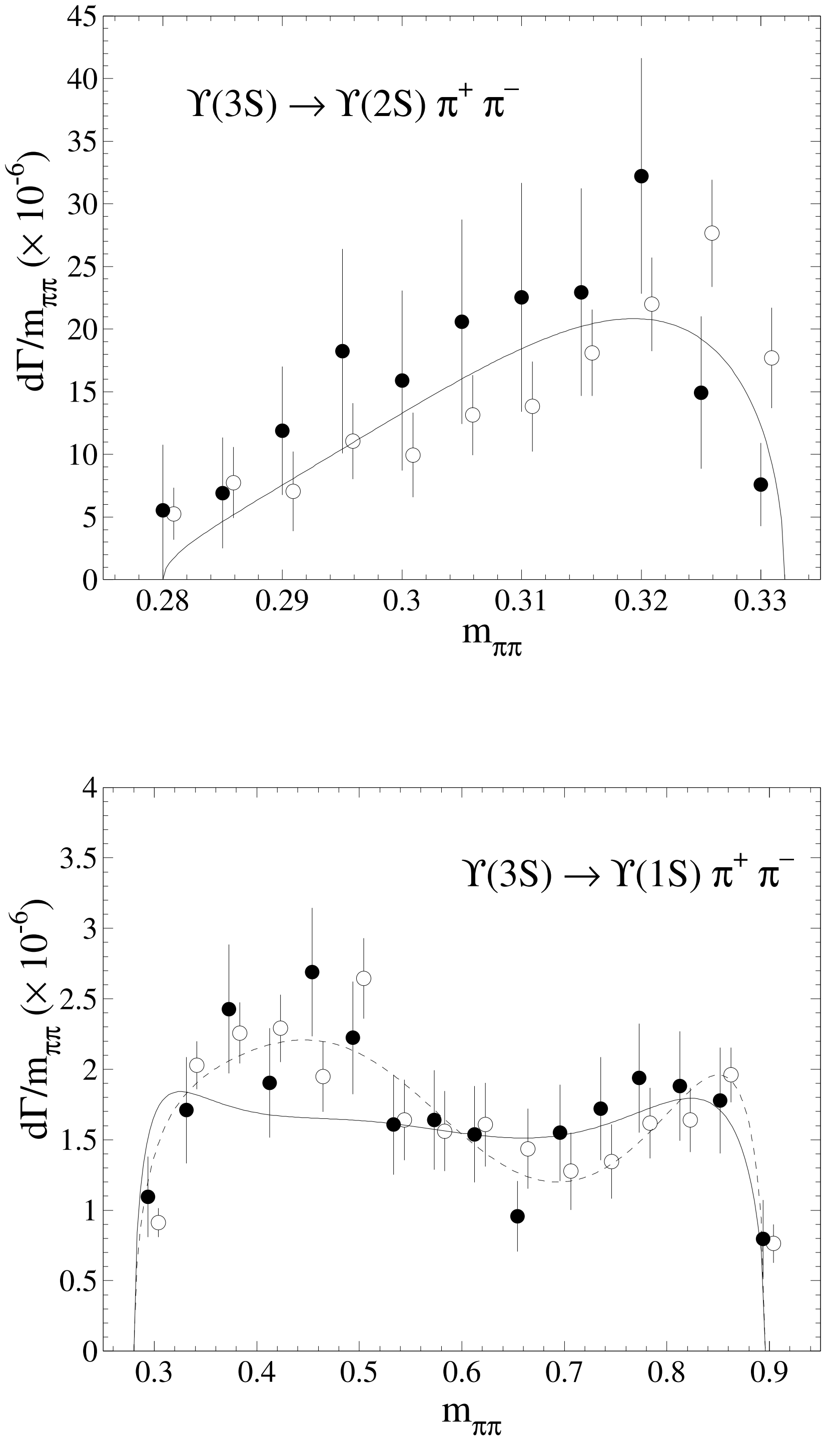


Figure 2: The dots represent the data from the exclusive deacys, the circles mark the inclusive distributions. The latter are slightly shifted to the right for clarity. The straight lines show the fit to the invariant mass spectra of the two pions, the dashed curve in the case of $\Upsilon(3S) \to \Upsilon(1S)\pi^+\pi^-$ represents the fit with $\pi^+\pi^-$ final state interactions taken into account (see text). The experimental data are taken from [14].

We have argued that the d-wave contribution is suppressed and hence we have only two operators left in the chiral limit. Both operators contribute to the s-wave of the decays $\Psi' \to \Psi\pi^+\pi^-$. Employing the model discussed above, we have the corresponding two operators on the quark level

$$\begin{aligned}\mathcal{L}_{quark} &= \gamma(\bar{Q}_v Q_v)\mathrm{Tr}\ \left[(\partial^\mu U)(\partial_\mu U^\dagger)\right] \\ &\quad + \gamma_1(\bar{Q}_v Q_v)\mathrm{Tr}\ \left[(v\cdot\partial U)(v\cdot\partial U^\dagger)\right]\end{aligned} \tag{23}$$

The coupling constants g and g_1 are related to the couplings in γ and γ_1 the quark Lagrangian $\mathcal{L}_{quark}$ through the relations

$$g\langle\Psi|A^{(v)\mu}B^{(v)*}_\mu|\Psi'\rangle = g\epsilon^*_\Psi\cdot\epsilon_{\Psi'} = \gamma\langle\Psi|\bar{Q}_v Q_v|\Psi'\rangle \tag{24}$$

$$g_1\langle\Psi|A^{(v)\mu}B^{(v)*}_\mu|\Psi'\rangle = g_1\epsilon^*_\Psi\cdot\epsilon_{\Psi'} = \gamma_1\langle\Psi|\bar{Q}_v Q_v|\Psi'\rangle \tag{25}$$

From the combined chiral and heavy mass expansion one would expect that both coupling constants γ and γ_1 do not depend on the heavy quark mass. As far as the mass dependence of g and g_1 is concerned, one has to take into account the mass dependence of the matrix element $\langle\Psi|\bar{Q}_v Q_v|\Psi'\rangle$. For the operators itself there exists a heavy mass limit, while the mass dependence of the states may not be accessed within the framework of the $1/m_Q$ expansion [3]. To this end we have to rely on simple dimensionality arguments. Assuming that the states are normalized to unity we may write the matrix elements as

$$\langle\Psi|\bar{Q}_v Q_v|\Psi'\rangle \propto E^3 \tag{26}$$

where E is a scale related to the binding of the heavy quarkonia. Here we may consider two extreme cases. The first one is the purely coulombic scenario, in which the parameter E is the heavy mass times the fine structure constant

$$E = \alpha_s(m_Q)m_Q \tag{27}$$

and therefore one would expect a strong scaling with the heavy mass. This is, however, not supported by the energy differences in the known quarkonia, where e.g. the level spacing $M_{2S} - M_{1S}$ is practically the same in the J/ψ system and the Υ system. On the other hand, the leptonic decay widths are determined by the value of the wave function at the origin and for this case one has a significant mass dependence.

Without considering the dynamics of the binding mechanism in detail, one cannot say much about the mass dependence of the matrix elements of the quark operators. For definiteness we consider the scenario in which E is independent of the mass of the heavy quark; consequently we would expect that g and g_1 are independent of the heavy quark mass, since the parameters γ and γ_1 in the quark level Lagrangian $\mathcal{L}_{quark}$ are mass independent. At least for the coupling g this is supported by data, since $g_{b\bar{b}}/g_{c\bar{c}}$ is practically unity for $\psi(2S) \to J/\psi\pi^+\pi^-$ and $\Upsilon(2S) \to \Upsilon(1S)\pi^+\pi^-$.

However, this scenario also implies that g_1/g should not scale with the heavy mass. From our fits we find that the coupling g_1 is larger in the J/ψ system by a factor between two or three compared to the $\Upsilon(2S)$ decay, while g_1/g for the different excited Υ states is (within the errors) constant, see eqs. (11,12).

The only consistent interpretation of this factor between g_1/g in the charm and bottom systems (given the scenario we are considering) is that the difference is induced by the QCD running of the corresponding coupling

$$\frac{g_{1\,b\bar{b}}}{g_{1\,c\bar{c}}} = \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^\kappa, \qquad \kappa \simeq 2-3 \tag{28}$$

Consequently the second term of our effective Lagrangian must be governed at the level of QCD by an operator with a sufficiently large anomalous dimension. However, if our assumption that the matrix elements of the quark operators are mass independent is not valid, the mass dependence of g_1/g can be also due to the mass dependence of these matrix elements.

Let us finally also consider the chiral symmetry breaking contribution relevant for the decays $\psi(2S) \to J/\psi\pi$ and $\Upsilon(2S) \to \Upsilon(1S)\pi$. Interpreting this transition again in terms of a dimension-3 operator for the heavy quarks one writes

$$\mathcal{L}'_{quark} = \gamma'(\bar{Q}_v\gamma_5 Q_v)\mathrm{Tr}\ \left[(U - U^\dagger)\right] \tag{29}$$

The leading term in the heavy mass expansion will vanish for this operator, since

$$\bar{Q}_v\gamma_5 Q_v = \mathcal{O}(1/m_Q)$$

and hence one expects the coefficients g' to scale as

$$\frac{g'_{b\bar{b}}}{g'_{c\bar{c}}} = \frac{m_c}{m_b} \simeq 0.3 \tag{30}$$

assuming again that the mass dependence of the matrix elements is weak.

5 Conclusions

We have formulated an effective theory approach for the hadronic decays of heavy quarkonia based on the chiral limit for the light degrees of freedom and on the $1/m_Q$ expansion for the heavy ones. The leading term in both the inverse of the heavy quark mass and the inverse of the chiral symmetry breaking scale consists of three terms multiplied by coupling constants which we have fitted from experiment. The fit to the invariant mass spectrum of the two pions in the decays $\psi(2S) \to J/\psi\pi^+\pi^-$ and $\Upsilon(2S) \to \Upsilon(1S)\pi^+\pi^-$ shown in Fig.1, as well as

$\Upsilon(3S) \to \Upsilon(2S)\pi^+\pi^-$ depicted in the upper plot of Fig.2, is quite satisfactory. In addition we have fitted the decay $\Upsilon(3S) \to \Upsilon(1S)\pi^+\pi^-$, where however, unlike in the other cases, the fit is rather poor (solid line in the lower plot of Fig.2). The situation can be improved by taking $\pi\pi$ final state interactions into account, see the dashed line in the lower plot of Fig.2.

We also have discussed the mass dependence of the various terms in the Lagrangian which unfortunately may not be considered entirely in the effective theory. We model the leading terms of the effective Lagrangian by switching to a description in which the relevant degrees of freedom are pions and heavy quarks. This model resembles the chiral quark model as proposed by Georgi and Manohar [21]. In this picture the coupling constants of the effective Lagrangian for the quarkonia and the pions may be reexpressed in terms of the ones of the underlying quark Lagrangian and certain matrix elements of heavy quark operators between heavy quarkonia states. The mass dependence of the matrix elements may not be evaluated from chiral and heavy quark symmetry and thus requires additional input. Using a specific assumption on the mass dependence of these matrix elements we arrive at definite predictions for the ratios $g_{b\bar{b}}/g_{c\bar{c}}$ and $g'_{b\bar{b}}/g'_{c\bar{c}}$. The predictions are consistent with what is found experimentally, although for $g'_{b\bar{b}}/g'_{c\bar{c}}$ only an experimental bound exists and more data is needed to confirm or falsify our values.

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