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On the Corrections to Dashen's Theorem

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Abstract

The electromagnetic corrections to the masses of the pseudoscalar mesons π and K are considered. We calculate in chiral perturbation theory the contributions which arise from resonances within a photon loop at order $O(e^2 m_q)$. Within this approach we find rather moderate deviations to Dashen's theorem.

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1 Introduction

Dashen's theorem [1] states that the squared mass differences between the charged pseudoscalar mesons $\pi^\pm, K^\pm$ and their corresponding neutral partners π^0, K^0 are equal in the chiral limit, i.e., $\Delta M_K^2 - \Delta M_\pi^2 = 0$, where $\Delta M_P^2 = M_{P^\pm}^2 - M_{P^0}^2$. In recent years several groups have calculated the electromagnetic corrections to this relation from non-vanishing quark masses. The different conclusions are either that the violation is large [2, 3] or that it *may be* large [4, 5, 6].
The electromagnetic mass difference of the pions ΔM_π^2 has been determined in the chiral limit using current algebra by Das et al. [7]. Ecker et al. [8] have repeated the calculation in the framework of chiral perturbation theory (χPT) [9] by resonance exchange within a photon loop. The occurring divergences from these loops are absorbed by introducing an electromagnetic counterterm (with a coupling constant $\hat{C}$) in the chiral lagrangian. They find that the contribution from the loops is numerically very close to the experimental mass difference, and thus conclude that the finite part of $\hat{C}$ is almost zero.
In [2] the authors have calculated the Compton scattering of the pseudoscalar mesons including the resonances and determined from this amplitude the mass differences at order $O(e^2 m_q)$. They concluded first of all by using three low-energy relations that the one-loop result is finite, i.e., there is no need of a counterterm lagrangian at order $O(e^2 m_q)$ in order to renormalize the contributions from the resonances, secondly they found a strong violation of Dashen's theorem. We are in disagreement with both of these results.
In this article we proceed in an analogous manner to [8] for the case $m_q \neq 0$. We calculate in χPT the contributions of order $O(e^2 m_q)$ to the masses of the Goldstone bosons due to resonances. The divergences are absorbed in the corresponding electromagnetic counterterm lagrangian, associated with the couplings $\hat{K}_i$, where $i = 1, \ldots, 14$. The most general form of this lagrangian has been given in [5, 6, 10]. We find again that the contribution from the loops reproduces the measured mass difference ΔM_π^2 very well, and therefore we consider the finite parts of the $\hat{K}_i$ to be small. Using this assumption also for the calculation of ΔM_K^2, we may finally read off the corrections to Dashen's theorem from one-loop resonance exchange. The (scale dependent) result shows that the resonances lead to rather moderate deviations.
The article is organized as follows. In section 2 we present the ingredients

from χPT and the resonances needed for the calculation. In section 3 we give the contributions to the masses and to Dashen's theorem and renormalize the counterterm lagrangian. The numerical results and a short conclusion are given in section 4.

2 The Lagrangians at lowest and next-to-leading Order

The chiral lagrangian can be expanded in derivatives of the Goldstone fields and in the masses of the three light quarks. The power counting is established in the following way: The Goldstone fields are of order $O(p^0)$, a derivative ∂_μ, the vector and axial vector currents v_μ, a_μ count as quantities of $O(p)$ and the scalar (incorporating the masses) and pseudoscalar currents s, p are of order $O(p^2)$. The effective lagrangian starts at $O(p^2)$, denoted by $\mathcal{L}_2$. It is the non-linear σ-model lagrangian coupled to external fields, respects chiral symmetry $SU(3)_R \times SU(3)_L$, and is invariant under P and C transformations [9],

$$
\begin{aligned}
\mathcal{L}_2 &= \frac{F_0^2}{4}\langle d^\mu U^\dagger d_\mu U + \chi U^\dagger + \chi^\dagger U\rangle \\
d_\mu U &= \partial_\mu U - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu) \\
v_\mu &= Q A_\mu + \cdots \\
Q &= \frac{e}{3}\,\mathrm{diag}\,(2,-1,-1) \\
\chi &= 2B_0(s+ip) \\
s &= \mathrm{diag}\,(m_u, m_d, m_s) \\
F_\pi &= F_0\,[1 + O(m_q)] \\
B_0 &= -\frac{1}{F_0^2}\langle 0|\bar{u}u|0\rangle\,[1 + O(m_q)] \quad .
\end{aligned} \tag{1}
$$

The brackets $\langle\cdots\rangle$ denote the trace in flavour space and U is a unitary 3×3 matrix that incorporates the fields of the eight pseudoscalar mesons,

$$
U = \exp\left(\frac{i\Phi}{F_0}\right)
$$

$$\Phi = \sqrt{2}\begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \overline{K^0} & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix} . \tag{2}$$

Note that the photon field A_μ is incorporated in the vector current v_μ. The corresponding kinetic term has to be added to $\mathcal{L}_2$,

$$\mathcal{L}^\gamma_{kin} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\left(\partial_\mu A^\mu\right)^2 , \tag{3}$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and the gauge fixing parameter chosen to be $\lambda = 1$. In order to maintain the usual chiral counting in $\mathcal{L}^\gamma_{kin}$, it is convenient to count the photon field as a quantity of order $O(p^0)$, and the electromagnetic coupling e of $O(p)$ [5].

The lowest order couplings of the pseudoscalar mesons to the resonances are linear in the resonance fields and start at order $O(p^2)$ [8, 11]. For the description of the fields we use the antisymmetric tensor notation for the vector and axialvector mesons, e.g., the vector octet has the form

$$V_{\mu\nu} = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0_{\mu\nu} + \frac{1}{\sqrt{6}}\omega_{8\,\mu\nu} & \rho^+_{\mu\nu} & K^{*+}_{\mu\nu} \\ \rho^-_{\mu\nu} & -\frac{1}{\sqrt{2}}\rho^0_{\mu\nu} + \frac{1}{\sqrt{6}}\omega_{8\,\mu\nu} & K^{*0}_{\mu\nu} \\ K^{*-}_{\mu\nu} & \overline{K^{*0}}_{\mu\nu} & -\frac{2}{\sqrt{6}}\omega_{8\,\mu\nu} \end{pmatrix} . \tag{4}$$

This method is discussed in detail in [8], we restrict ourselves on the formulae needed for the calculations in the following section. The relevant interaction lagrangian contains the octet fields only,

$$\begin{aligned} \mathcal{L}^V_2 &= \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f^{\mu\nu}_+\rangle + \frac{iG_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle \\ \mathcal{L}^A_2 &= \frac{F_A}{2\sqrt{2}}\langle A_{\mu\nu} f^{\mu\nu}_-\rangle \\ f^{\mu\nu}_\pm &= uF^{\mu\nu}_L u^\dagger \pm u^\dagger F^{\mu\nu}_R u \\ F^{\mu\nu}_{R,L} &= \partial^\mu(v^\nu \pm a^\nu) - \partial^\nu(v^\mu \pm a^\mu) - i[v^\mu \pm a^\mu, v^\nu \pm a^\nu] \\ u^\mu &= iu^\dagger d^\mu U u^\dagger = u^{\dagger\,\mu} \\ U &= u^2 \quad . \end{aligned} \tag{5}$$

The coupling constants are real and are not restricted by chiral symmetry [11], numerical estimates are given in [8]. In the kinetic lagrangian a covariant derivative acts on the vector and axialvector mesons,

$$\begin{aligned}
\mathcal{L}_{kin}^{R} &= -\frac{1}{2}\langle \nabla^\mu R_{\mu\nu} \nabla_\sigma R^{\sigma\nu} - \frac{1}{2} M_R^2 R_{\mu\nu} R^{\mu\nu} \rangle \qquad\qquad R = V, A \\
\nabla^\mu R_{\mu\nu} &= \partial^\mu R_{\mu\nu} + [\Gamma^\mu, R_{\mu\nu}] \\
\Gamma^\mu &= \frac{1}{2}\left\{ u^\dagger [\partial^\mu - i(v^\mu + a^\mu)] u + u[\partial^\mu - i(v^\mu - a^\mu)] u^\dagger \right\} \quad ,
\end{aligned} \tag{6}$$

where M_R is the corresponding mass in the chiral limit. Finally we collect all the different terms together into one lagrangian,

$$\mathcal{L}_2^{eff} = \mathcal{L}_2 + \mathcal{L}_2^R + \mathcal{L}_{kin}^\gamma + \mathcal{L}_{kin}^R \quad . \tag{7}$$

The one-loop electromagnetic mass shifts of the pseudoscalar mesons calculated with this lagrangian (see section 3) contain divergences that can be absorbed in a counterterm lagrangian. In its general form, this lagrangian has one term of order $O(e^2)$ and 14 terms of $O(e^2p^2)$ [5, 6, 10],

$$\begin{aligned}
\mathcal{L}_2^C &= \hat{C}\langle QUQU^\dagger \rangle \\
\mathcal{L}_4^C &= \hat{K}_1 F_0^2 \langle d^\mu U^\dagger d_\mu U \rangle \langle Q^2 \rangle + \hat{K}_2 F_0^2 \langle d^\mu U^\dagger d_\mu U \rangle \langle QUQU^\dagger \rangle \\
&+ \hat{K}_3 F_0^2 \left(\langle d^\mu U^\dagger Q U \rangle \langle d_\mu U^\dagger Q U \rangle + \langle d^\mu U Q U^\dagger \rangle \langle d_\mu U Q U^\dagger \rangle \right) \\
&+ \hat{K}_4 F_0^2 \langle d^\mu U^\dagger Q U \rangle \langle d_\mu U Q U^\dagger \rangle \\
&+ \hat{K}_5 F_0^2 \langle \left(d^\mu U^\dagger d_\mu U + d^\mu U d_\mu U^\dagger \right) Q^2 \rangle \\
&+ \hat{K}_6 F_0^2 \langle d^\mu U^\dagger d_\mu U Q U^\dagger Q U + d^\mu U d_\mu U^\dagger Q U Q U^\dagger \rangle \\
&+ \hat{K}_7 F_0^2 \langle \chi U^\dagger + \chi^\dagger U \rangle \langle Q^2 \rangle + \hat{K}_8 F_0^2 \langle \chi U^\dagger + \chi^\dagger U \rangle \langle QUQU^\dagger \rangle \\
&+ \hat{K}_9 F_0^2 \langle (\chi U^\dagger + \chi^\dagger U + U^\dagger \chi + U \chi^\dagger) Q^2 \rangle \\
&+ \hat{K}_{10} F_0^2 \langle (\chi^\dagger U + U^\dagger \chi) Q U^\dagger Q U + (\chi U^\dagger + U \chi^\dagger) Q U Q U^\dagger \rangle \\
&+ \hat{K}_{11} F_0^2 \langle (\chi^\dagger U - U^\dagger \chi) Q U^\dagger Q U + (\chi U^\dagger - U \chi^\dagger) Q U Q U^\dagger \rangle \\
&+ \hat{K}_{12} F_0^2 \langle d^\mu U^\dagger \left[c_\mu^R Q, Q \right] U + d^\mu U \left[c_\mu^L Q, Q \right] U^\dagger \rangle \\
&+ \hat{K}_{13} F_0^2 \langle c^{R\,\mu} Q U c_\mu^L Q U^\dagger \rangle + \hat{K}_{14} F_0^2 \langle c^{R\,\mu} Q c_\mu^R Q + c^{L\,\mu} c_\mu^L Q \rangle \\
&+ O(p^4, e^4) \quad .
\end{aligned} \tag{8}$$

with $c_\mu^{R,L} Q = -i\,[v_\mu \pm a_\mu, Q]$. The three last terms contribute only to matrix elements with external fields, we are therefore left with 12 relevant counterterms. Note that we have omitted terms which come either from the purely

strong or the purely electromagnetic sector in $\mathcal{L}_4^C$.
At this point it is worthwhile to discuss the connection of the present formalism to the usual χPT without resonances where the Goldstone bosons and the (virtual) photons are the only interacting particles. For this purpose we consider the electromagnetic mass of the charged pion. In χPT the lagrangian has the form up to and including $O(e^2p^2)$

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_2^Q + \mathcal{L}_4^Q \\ \mathcal{L}_2^Q &= \mathcal{L}_2 + C\langle QUQU^\dagger\rangle, \qquad\qquad \mathcal{L}_4^Q = \sum_{i=1}^{14} K_i O_i \end{aligned} \tag{9}$$

where C and K_i are low energy constants. They are independent of the Goldstone bosons masses and parameterize all the underlying physics (including resonances) of χPT. $\mathcal{L}_2$ is given in (1) and the operators O_i are identical to those in (8). Neglecting the contributions of the order $O(e^2m_q)$ for a moment, the pion mass is [8]

$$M_{\pi^\pm}^2 = \frac{2e^2}{F_0^2}C + O(e^2m_q) \tag{10}$$

entirely determined by the coupling constant C. In the resonance approach $M_{\pi^\pm}^2$ gets contributions from resonance-photon loops already at order $O(e^2)$ (see graphs (c) and (d) in Figure 1)

$$M_{\pi^\pm}^2 = M_{\pi^\pm}^2|_{loops} + \frac{2e^2}{F_0^2}\hat{C} + O(e^2m_q) \tag{11}$$

The loop term contains a divergent and a finite part and is completely determined by the resonance parameters. The divergences are absorbed by renormalizing the coupling constants $\hat{C}$ [8] (see section 3). The connection to χPT without resonances is then given by the relation [8]

$$\begin{aligned} C &= C^R(\mu) + \hat{C}(\mu) \\ C^R(\mu) &= \frac{F_0^2}{2e^2}M_{\pi^\pm}^2|_{loops\ (finite)} \end{aligned} \tag{12}$$

where $C^R(\mu)$ and $\hat{C}(\mu)$ are finite and the scale dependence cancels in the sum. Relation (12) says that the coupling constant C is split in a part from resonances (C^R) and another part from non-resonant physics ($\hat{C}$). This

ansatz of separating resonant and non-resonant contributions to the low-energy parameters has been originally made for the strong interaction sector at next to leading order [8]. In this case resonance exchange gives tree-level contributions and no renormalization is needed. In the electromagnetic case however, contributions arise from resonances with photons in loops and we renormalize the non-resonant part of the coupling constant, i.e. $\hat{C}$ at order $O(e^2)$.
In an analogous fashion the above procedure can be carried out up to the order $O(e^2p^2)$. The couplings K_i of $\mathcal{L}_4^Q$ are in general divergent, since they absorb the divergences of the one-loop functional generated by $\mathcal{L}_2^Q$ [5, 6, 10]. At a specific scale point the renormalized coupling constants $K_i^r(\mu)$ can be split in two parts

$$K_i^r(\mu_0) = K_i^R(\mu_0) + \hat{K}_i(\mu_0) \tag{13}$$

where the terms on the right-hand side are taken after renormalization of $\hat{K}_i$ (see section 3) and are thus finite. The choice of the scale point μ_0 is not a priori fixed. Like in the strong sector [8] we consider μ_0 in the range of the lowest lying resonances, i.e. in the range from 0.5 to 1.0 GeV.

In the strong sector it was found that the resonances saturate the low-energy parameters almost completely [8]. In additon the authors have found that the same conclusion holds for the electromagnetic coupling constant C leading to $\hat{C}(\mu) \approx 0$. Consequently we *assume* that the $K_i^r(\mu)$ are also saturated by resonance contributions, i.e. we put

$$\hat{K}_i(\mu) \approx 0 \quad . \tag{14}$$

As we will see in section 4 this assumption works well in the case of ΔM_π^2.

3 Corrections to Dashen's Theorem

Using the lagrangian given in (7) it is a straightforward process to calculate the mass shift between the charged pseudoscalar mesons $\pi^\pm, K^\pm$ and their corresponding neutral partner π^0, K^0 at the one-loop level. The relevant diagrams for the mass of the charged pion are shown in Fig.1. Graph (a) contains the off-shell pion form factor, (b) vanishes in dimensional regularization and (c) is called "modified seagull graph". Graph (d) contains an a_1-pole. The mass of the neutral pion does not get contributions from the

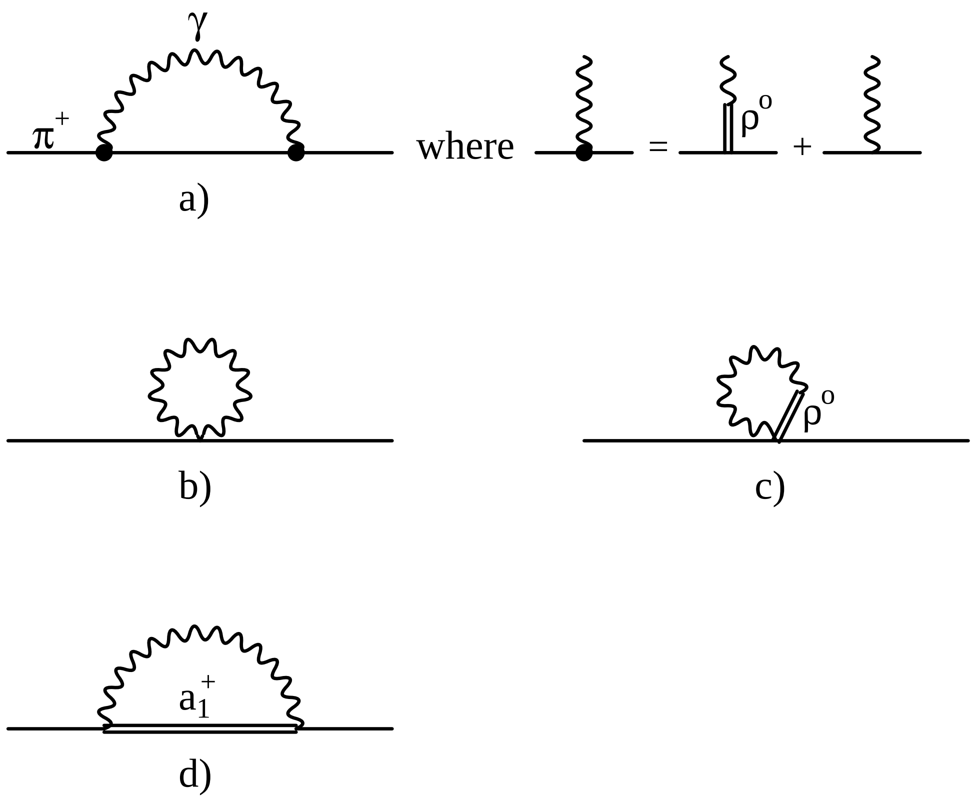

Figure 1: One-loop contributions to the electromagnetic mass shift of $\pi^{\pm}$.

loops.
If we take the resonances to be in the $SU(3)$ limit according to (6), i.e., all vector resonances have the same mass M_V and all axialvector resonances the mass M_A, we get the contributions listed below. For the graphs with the pion form factor,

$$\begin{aligned}
\Delta_{p.f.} M_\pi^2 &= -ie^2 \int \frac{d^4q}{(2\pi)^4} \frac{q^2 + 4\nu + 4M_\pi^2}{q^2(q^2+2\nu)} \\
&\quad -i\frac{8e^2 F_V G_V}{F_0^2} \int \frac{d^4q}{(2\pi)^4} \frac{q^2 M_\pi^2 - \nu^2}{q^2(q^2+2\nu)(M_V^2-q^2)}
\end{aligned} \tag{15}$$

$$-i\frac{4e^2F_V^2G_V^2}{F_0^4}\int\frac{d^4q}{(2\pi)^4}\frac{q^2[q^2M_\pi^2-\nu^2]}{q^2(q^2+2\nu)(M_V^2-q^2)^2} \quad ,$$

where $\nu = pq$ and p is the momentum of the pion. Using the relation $F_V G_V = F_0^2$ [11] we obtain

$$\Delta_{p.f.}M_\pi^2 = -ie^2M_V^4\int\frac{d^4q}{(2\pi)^4}\frac{2\nu+4M_\pi^2}{q^2(q^2+2\nu)(M_V^2-q^2)^2} \quad . \tag{16}$$

The modified seagull graph gives

$$\Delta_{s.g.}M_\pi^2 = i\frac{e^2F_V^2}{F_0^2}(3-\epsilon)\int\frac{d^4q}{(2\pi)^4}\frac{1}{M_V^2-q^2} \tag{17}$$

with $\epsilon = 4-d$, and finally for the a_1-pole graph, where unlike [2] we get an additional second term,

$$\begin{aligned}\Delta_{a_1}M_\pi^2 &= -i\frac{e^2F_A^2}{F_0^2}(3-\epsilon)\int\frac{d^4q}{(2\pi)^4}\frac{1}{M_A^2-q^2} \\ &- i\frac{e^2F_A^2}{F_0^2}\int\frac{d^4q}{(2\pi)^4}\frac{q^2\left[M_\pi^2+(3-\epsilon)\nu\right]+(2-\epsilon)\nu^2}{q^2\left[M_A^2-(q+p)^2\right]} \quad .\end{aligned} \tag{18}$$

We now add the contribution from $\mathcal{L}_2^C$ and $\mathcal{L}_4^C$ to the mass shift [5, 6] and evaluate the integrals,

$$\begin{aligned}\Delta M_\pi^2 &= -\frac{3e^2}{F_0^2 16\pi^2}\left[F_V^2M_V^2\left(\ln\frac{M_V^2}{\mu^2}+\frac{2}{3}\right)-F_A^2M_A^2\left(\ln\frac{M_A^2}{\mu^2}+\frac{2}{3}\right)\right] \\ &\quad -\frac{e^2F_A^2}{F_0^2 16\pi^2}M_\pi^2\left[2+\frac{3}{2}\ln\frac{M_A^2}{\mu^2}+I_1\left(\frac{M_\pi^2}{M_A^2}\right)\right] \\ &\quad +\frac{2e^2}{16\pi^2}M_\pi^2\left[\frac{7}{2}-\frac{3}{2}\ln\frac{M_\pi^2}{M_V^2}+I_2\left(\frac{M_\pi^2}{M_V^2}\right)\right] \\ &\quad +\frac{2e^2\hat{C}}{F_0^2}-\frac{6e^2}{F_0^2}(F_V^2M_V^2-F_A^2M_A^2)\lambda \\ &\quad +8e^2M_K^2\hat{K}_8+2e^2M_\pi^2\hat{R}_\pi-\frac{3e^2F_A^2}{F_0^2}M_\pi^2\lambda\end{aligned} \tag{19}$$

with

$$I_1(z) = \int_0^1 x\ln[x-x(1-x)z]\,dx$$

$$
\begin{aligned}
I_2(z) &= \int_0^1 (1+x)\left\{\ln[x+(1-x)^2 z] - \frac{x}{x+(1-x)^2 z}\right\} dx \\
\hat{R}_\pi &= -2\hat{K}_3 + \hat{K}_4 + 2\hat{K}_8 + 4\hat{K}_{10} + 4\hat{K}_{11} \qquad (20)
\end{aligned}
$$

The divergences of the resonance-photon loops show up as poles in $d = 4$ dimensions. They are collected in the terms proportional to λ

$$
\lambda = \frac{\mu^{d-4}}{16\pi^2}\left\{\frac{1}{d-4} - \frac{1}{2}[\ln 4\pi + \Gamma'(1) + 1]\right\} \quad . \qquad (21)
$$

The occurring divergences are now canceled by renormalizing the contributions from non-resonant physics, i.e. the coupling constants $\hat{C}$ and $\hat{K}_i$. The divergence of the order $O(e^2)$ (fourth line of equation (19)) is absorbed by putting

$$
\hat{C} = \hat{C}(\mu) + 3(F_V^2 M_V^2 - F_A^2 M_A^2)\lambda \qquad (22)
$$

and that of the order $O(e^2 m_q)$ (fifth line of equation (19)) by the relation

$$
\hat{R}_\pi = \hat{R}_\pi(\mu) + \frac{3F_A^2}{2F_0^2}\lambda \quad . \qquad (23)
$$

Using the second Weinberg sum rule [12]

$$
F_V^2 M_V^2 - F_A^2 M_A^2 = 0 \quad , \qquad (24)
$$

the divergence in (22) cancels, but the divergence in (23) does not. Even if we used an extension of this sum rule to order $O(m_q)$ [13],

$$
F_\rho^2 M_\rho^2 - F_{a_1}^2 M_{a_1}^2 \simeq F_\pi^2 M_\pi^2 \qquad (25)
$$

and assumed $F_A = F_0$ [11], the divergence would not cancel, on the contrary, it would become larger.

We finally get the result

$$
\begin{aligned}
\Delta M_\pi^2 = &-\frac{3e^2}{F_0^2 16\pi^2} F_V^2 M_V^2 \ln\frac{M_V^2}{M_A^2} \\
&-\frac{e^2 F_A^2}{F_0^2 16\pi^2} M_\pi^2 \left[2 + \frac{3}{2}\ln\frac{M_A^2}{\mu^2} + I_1\left(\frac{M_\pi^2}{M_A^2}\right)\right] \\
&+\frac{2e^2}{16\pi^2} M_\pi^2 \left[\frac{7}{2} - \frac{3}{2}\ln\frac{M_\pi^2}{M_V^2} + I_2\left(\frac{M_\pi^2}{M_V^2}\right)\right] \qquad (26) \\
&+\frac{2e^2\hat{C}}{F_0^2} + 8e^2 M_K^2 \hat{K}_8 + 2e^2 M_\pi^2 \hat{R}_\pi(\mu) \quad ,
\end{aligned}
$$

where we used (24) to simplify the first term. In the chiral limit ΔM_π^2 reduces to the expression given in [8].
The mass difference for the kaons is determined in an analogous way, in the contribution from the loops we merely have to replace M_π^2 by M_K^2. Finally the formula for the corrections to Dashen's theorem may be read off,

$$\begin{aligned}
\Delta M_K^2 - \Delta M_\pi^2 \;&=\; -\frac{e^2 F_A^2}{F_0^2 16\pi^2}\left\{ M_K^2\left[2+\frac{3}{2}\ln\frac{M_A^2}{\mu^2}+I_1\left(\frac{M_K^2}{M_A^2}\right)\right]\right. \\
&\qquad\qquad \left. -M_\pi^2\left[2+\frac{3}{2}\ln\frac{M_A^2}{\mu^2}+I_1\left(\frac{M_\pi^2}{M_A^2}\right)\right]\right\} \\
&\quad +\frac{2e^2}{16\pi^2}\left\{ M_K^2\left[\frac{7}{2}-\frac{3}{2}\ln\frac{M_K^2}{M_V^2}+I_2\left(\frac{M_K^2}{M_V^2}\right)\right]\right. \\
&\qquad\qquad \left. -M_\pi^2\left[\frac{7}{2}-\frac{3}{2}\ln\frac{M_\pi^2}{M_V^2}+I_2\left(\frac{M_\pi^2}{M_V^2}\right)\right]\right\} \qquad (27) \\
&\quad -2e^2 M_K^2\left[\frac{2}{3}\hat{S}_K(\mu)+4\hat{K}_8\right]+2e^2M_\pi^2\left[\frac{2}{3}\hat{S}_\pi-\hat{R}_\pi(\mu)\right] \quad ,
\end{aligned}$$

where $\hat{S}_{\pi,K}$ represent the contributions from the counterterm lagrangian to ΔM_K^2,

$$\begin{aligned}
\hat{S}_\pi &= 3\hat{K}_8+\hat{K}_9+\hat{K}_{10} \\
\hat{S}_K &= \hat{K}_5+\hat{K}_6-6\hat{K}_8-6\hat{K}_{10}-6\hat{K}_{11} \\
\hat{S}_K &= \hat{S}_K(\mu)+\frac{3F_A^2}{2F_0^2}\lambda \quad . \qquad (28)
\end{aligned}$$

4 Numerical Results and Conclusion

We put F_0 equal to the physical pion decay constant, $F_\pi = 92.4$ MeV and the masses of the mesons to $M_\pi = 135$ MeV, $M_K = 495$ MeV. We take $F_V = 154$ MeV [8] and $M_V = M_\rho = 770$ MeV. To eliminate the parameters of the axialvector resonances we use Weinberg's sum rules [12],

$$F_V^2 - F_A^2 = F_0^2 \qquad\qquad F_V^2 M_V^2 - F_A^2 M_A^2 = 0 \quad . \qquad (29)$$

The contributions from the counterterm lagrangian are not known so far. In [8] it was found that the experimental mass difference ΔM_π^2 at order $O(e^2)$

is well reproduced by the resonance-photon loops and therefore the authors conclude that the contributions from non-resonant physics are small, i.e. $\hat{C} \approx 0$. In analogy we *assume* for the numerical evaluation the dominance of the resonant contributions at order $O(e^2 m_q)$, i.e. we put $\hat{K}_i(\mu) \approx 0$.
Putting the numbers in (26) we get for the contribution from the loops to ΔM_π^2 at the scale points $\mu = (0.5, 0.77, 1)$ GeV (see Fig.2a)

$$\Delta M_\pi^2|_{loops} = 2M_\pi \times (\ 5.0\ ,\ 5.1\ ,\ 5.1\ )\ \text{MeV} \quad . \tag{30}$$

which is in nice agreement with the experimental value $\Delta M_\pi^2|_{exp.} = 2M_\pi \times 4.6$ MeV [14]. Using resonance saturation in the Kaon system as well, we obtain for the corrections to Dashen's theorem (again at the scale points $\mu = (0.5, 0.77, 1)$ GeV)

$$\Delta M_K^2 - \Delta M_\pi^2 = (\ -0.13\ ,\ 0.17\ ,\ 0.36\ ) \times 10^{-3}\ (\text{GeV})^2 \quad , \tag{31}$$

which are smaller than the values found in the literature,

$$\Delta M_K^2 - \Delta M_\pi^2 = \left\{ \begin{array}{ccc} 1.23 & & [2] \\ 1.3 \pm 0.4 & \times 10^{-3}\ (\text{GeV})^2 & [3] \\ 0.55 \pm 0.25 & & [4] \end{array} \right. \tag{32}$$

Of course, in order to get a scale independent result, the counterterms are not allowed to vanish completely.

In [2] the authors calculated the Compton scattering of the Goldstone bosons within the same model that we have used in the present article and determined the corrections to Dashen's theorem by closing the photon line. Their calculation is finite (without counterterms) and gives a considerably large value for $\Delta M_K^2 - \Delta M_\pi^2$. The difference to our result may be identified in (18), where we have found an additional (singular) term that gives a large negative and scale dependent contribution. The two results are compared in Fig.2b. Note that in [2] the physical masses for the resonances are used in the calculation of ΔM_K^2, whereas we work in the $SU(3)$ limit throughout.
The other calculations are not strongly connected to our approach, for a discussion of the value given in [4] we refer to [5].
We therefore conclude that taking into account the resonances at the one-loop level and working strictly in the $SU(3)$ limit for the resonances leads to moderate rather than large corrections to Dashen's theorem. Possibly strong violations must come from higher loop corrections or from non-resonant physics.

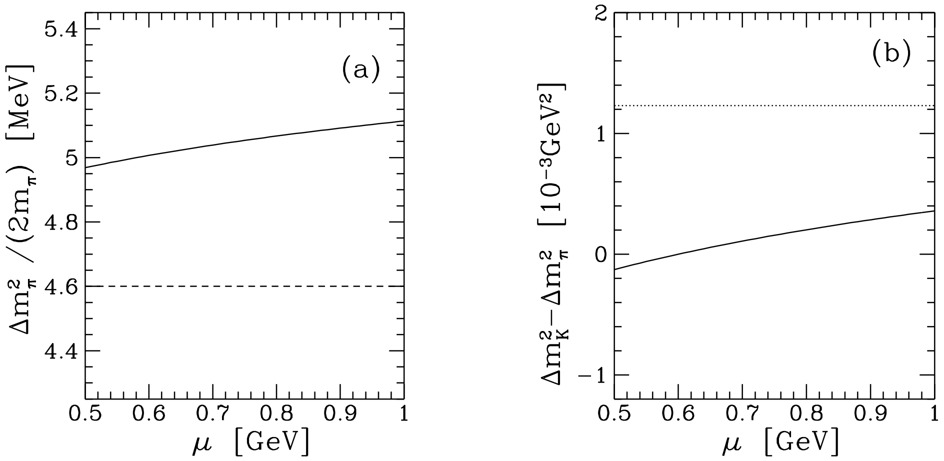

Figure 2: The solid lines show our results, the dashed and dotted curves represent in a) the experimental value [14], in b) the result of [2], respectively.

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