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A Determination of the CKM-angle α using Mixing-induced CP Violation in the Decays $B_d \to \pi^+\pi^-$ and $B_d \to K^0\bar{K}^0$ *

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Abstract

We present a method of determining the CKM-angle α by performing simultaneous measurements of the mixing-induced CP asymmetries of the decays $B_d \to \pi^+\pi^-$ and $B_d \to K^0\bar{K}^0$. The accuracy of our approach is limited by $SU(3)$-breaking effects originating from $\bar{b} \to \bar{d}s\bar{s}$ QCD-penguin diagrams. Using plausible power-counting arguments we show that these uncertainties are expected to be of the same order as those arising through electroweak penguins in the standard Gronau-London-method in which α is extracted by means of isospin relations among $B \to \pi\pi$ decay amplitudes. Therefore our approach, which does not involve the experimentally difficult mode $B_d \to \pi^0\pi^0$ and is essentially unaffected by electroweak penguins, may be an interesting alternative to determine α.

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CP-violating asymmetries arising in nonleptonic B-decays (see e.g. refs. [1]-[8]) will play a central role in the determination of the angles α, β and γ in the unitarity triangle [9, 10] at future experimental B-physics projects. Unfortunately, these asymmetries are in general not related to the CKM-angles in a clean way, but suffer from uncertainties originating from so-called penguins. These contributions preclude in particlular a clean determination of the CKM-angle α by measuring the mixing-induced CP-violating asymmetry $\mathcal{A}_{\rm CP}^{\rm mix\text{-}ind}(B_d \to \pi^+\pi^-)$. In a pioneering paper [11], Gronau and London have presented a method to eliminate the uncertainty in this determination of α that is related to *QCD-penguins*. It uses isospin relations among $B_d \to \pi^+\pi^-$, $B_d \to \pi^0\pi^0$ and $B^\pm \to \pi^\pm\pi^0$ decay amplitudes and requires besides a time-dependent study of $B_d \to \pi^+\pi^-$ yielding $\mathcal{A}_{\rm CP}^{\rm mix\text{-}ind}(B_d \to \pi^+\pi^-)$ a measurement of the corresponding branching ratios.

However, there are not only QCD- but also *electroweak* penguin operators. Although one would expect naïvely that electroweak penguins should only play a minor role in nonleptonic B-decays, there are certain transitions that are affected significantly by these operators which become important in the presence of a heavy top-quark. This interesting feature has first been pointed out in refs. [12]-[14] and has been confirmed later by the authors of refs. [15]-[17]. As has been stressed first by Deshpande and He [18], the influence of electroweak penguins on the extraction of α by using the standard Gronau-London-method [11] could also be sizable. A more elaborate analysis [19] shows, however, that this impact is expected to be rather small, at most a few per cent.

In a recent paper [20] we have presented strategies for the experimental determination of electroweak penguin contributions to nonleptonic B-decays. These strategies allow in particular to control the electroweak penguin uncertainty affecting the extraction of the CKM-angle α in the Gronau-London-method [11]. Although this method of determining α is very clean from the theoretical point of view, it requires the measurement of the decay $B_d \to \pi^0\pi^0$ which is rather difficult. The very recent analysis by Kramer and Palmer [21] indicates a branching ratio $\mathrm{BR}(B_d \to \pi^0\pi^0) \lesssim \mathcal{O}(10^{-6})$. Therefore, it is important to search for other methods that allow a clean determination of α. Such methods are also needed for overconstraining the shape of the unitarity triangle.

Motivated by this experimental situation, Dunietz [22] has suggested an alternative way of extracting α that is based on the $SU(3)$ flavour symmetry of strong interactions [23]-[27] and uses time-dependent measurements of the modes $B_d \to \pi^+\pi^-$ and $B_s \to K^+K^-$. However in view of the large B_s^0–$\bar{B}_s^0$–mixing, the time-dependent analysis of the transition $B_s \to K^+K^-$ with the expected branching ratio at the $\mathcal{O}(10^{-5})$ level may be difficult as well.

In this letter we would like to propose a different method of extracting α. In order to eliminate the penguin contributions, we use time-dependent measurements of the modes $B_d \to \pi^+\pi^-$ and $B_d \to K^0\bar{K}^0$ yielding the corresponding mixing-induced CP-violating asymmetries and employ the $SU(3)$ flavour symmetry of strong interactions [23]-[27] to derive relations among the corresponding decay amplitudes. The transition $B_d \to K^0\bar{K}^0$ is – in contrast to $B_s \to K^+K^-$ – a pure penguin-induced mode with a branching ratio $\mathcal{O}(10^{-6})$ [21, 28]. Yet because of smaller B_d^0–$\bar{B}_d^0$–mixing, time-dependent studies of this channel may probably be easier for experimentalists than those of the decay $B_s \to K^+K^-$. As we will see in a moment, our approach is essentially unaffected by electroweak penguins.

In the previous literature it has been claimed by several authors that the Standard Model predicts *vanishing* CP-violating asymmetries for decays such as $B_d \to K_S K_S$ or $B_d \to K^0\bar{K}^0$ (the CP asymmetries of both channels are equal) because of the cancellation of weak decay- and mixing-phases (see e.g. refs. [3, 7, 8]). This result is however only correct, if the $\bar{b} \to \bar{d}$ QCD-penguin amplitudes are dominated by internal top-quark exchanges. As has been pointed out in refs. [28, 29], QCD-penguins with internal up- and charm-quarks may generally also play a significant role and in the case of $B_d \to K^0\bar{K}^0$ could lead to rather large CP asymmetries of $\mathcal{O}(10-50)\%$ [28]. Unfortunately, these asymmetries suffer from large hadronic uncertainties and are therefore not related to CKM-angles in a clean way. Nevertheless, $\mathcal{A}_{\text{CP}}^{\text{mix-ind}}(B_d \to K^0\bar{K}^0)$ may be combined with additional inputs to determine α in a clean way as we will demonstrate in this letter.

In our discussion it is convenient to use the description of $B \to PP$ decays given by Gronau, Hernández, London and Rosner in refs. [19] and [30]-[35]. Using the same notation as these authors, the $B_d^0 \to \pi^+\pi^-$ and $B_d^0 \to K^0\bar{K}^0$ decay amplitudes take the form

$$\begin{aligned} A(B_d^0 \to \pi^+\pi^-) &= -\left[(T+E)+(P+PA)+c_u P_{\text{EW}}^{\text{C}}\right] \\ A(B_d^0 \to K^0\bar{K}^0) &= \left[(P+PA+P_3)+c_s P_{\text{EW}}^{\text{C}}\right], \end{aligned} \tag{1}$$

where T and E describe $\bar{b} \to \bar{u}u\bar{d}$ colour-allowed tree-level and exchange amplitudes, respectively, P denotes $\bar{b} \to \bar{d}$ QCD-penguins, PA is related to QCD-penguin annihilation diagrams and P_{EW}^{C} to colour-suppressed $\bar{b} \to \bar{d}$ electroweak penguins. The term P_3 describes $SU(3)$-breaking effects that are introduced through the creation of a $s\bar{s}$ pair in the $\bar{b} \to \bar{d}$ QCD-penguin diagrams [35]. If we follow the plausible arguments of Gronau et al. outlined in [19, 35], we expect the following hierarchy of the different topologies present in (1):

$$\begin{aligned} 1 &: |T| \\ \mathcal{O}(\bar{\lambda}) &: |P| \\ \mathcal{O}(\bar{\lambda}^2) &: |E|, \quad |P_3| \\ \mathcal{O}(\bar{\lambda}^3) &: |PA|, \quad \left|P_{\text{EW}}^{\text{C}}\right|. \end{aligned} \tag{2}$$

Note that the parameter $\bar{\lambda} = \mathcal{O}(0.2)$ appearing in these relations is not related to the usual Wolfenstein parameter λ. It has been introduced by Gronau et al. just to keep track of the expected orders of magnitudes. We have named this quantity $\bar{\lambda}$ in order not to confuse it with Wolfenstein's λ.

Consequently, if we neglect the terms of $\mathcal{O}(\bar{\lambda}^3)$, we obtain

$$\begin{aligned} A(B_d^0 \to \pi^+\pi^-) &= -[(T+E)+P] \\ A(B_d^0 \to K^0\bar{K}^0) &= P + P_3. \end{aligned} \tag{3}$$

Within this approximation, terms of $\mathcal{O}(\bar{\lambda}^4)$, i.e. $SU(3)$-breaking corrections to the PA and $P_{\rm EW}^{\rm C}$ amplitudes, which have not been written explicitly in (1), have also to be neglected.

Rotating the $\bar{B}_d^0 \to \pi^+\pi^-$ and $\bar{B}_d^0 \to K^0\bar{K}^0$ amplitudes by the phase factor $e^{-2i\beta}$, we find

$$\begin{aligned} e^{-2i\beta}A(\bar{B}_d^0 \to \pi^+\pi^-) &= -\left[e^{2i\alpha}(T+E) + e^{-2i\beta}\bar{P}\right] \\ e^{-2i\beta}A(\bar{B}_d^0 \to K^0\bar{K}^0) &= e^{-2i\beta}\left(\bar{P}+\bar{P}_3\right), \end{aligned} \tag{4}$$

where we have used the relation

$$e^{-2i\beta}(\bar{T}+\bar{E}) = e^{-2i(\beta+\gamma)}(T+E) = e^{2i\alpha}(T+E). \tag{5}$$

Using (3) and (4) it is an easy exercise to eliminate P and $\bar{P}$ and to derive the following relations:

$$A(B_d^0 \to K^0\bar{K}^0) + (T+E) - P_3 + A(B_d^0 \to \pi^+\pi^-) = 0 \tag{6}$$

$$e^{-2i\beta}A(\bar{B}_d^0 \to K^0\bar{K}^0) + e^{2i\alpha}(T+E) - e^{-2i\beta}\bar{P}_3 + e^{-2i\beta}A(\bar{B}_d^0 \to \pi^+\pi^-) = 0, \tag{7}$$

which have been represented graphically in the complex plane in Fig. 1. If the $\bar{b} \to \bar{d}$ QCD-penguins were dominated by internal top-quark exchanges, we would have $e^{-2i\beta}\bar{P}_3 = P_3$. However, as has been shown in refs. [28, 29], QCD-penguins with internal up- and charm-quarks are expected to lead to sizable corrections to this relation.

The angles ψ and ϕ appearing in Fig. 1 can be determined *directly* by measuring the mixing-induced CP asymmetries of the decays $B_d \to K^0\bar{K}^0$ and $B_d \to \pi^+\pi^-$, respectively, which are given by [20]

$$\mathcal{A}_{\rm CP}^{\text{mix-ind}}(B_d \to K^0\bar{K}^0) = -\frac{2|A(\bar{B}_d^0 \to K^0\bar{K}^0)||A(B_d^0 \to K^0\bar{K}^0)|}{|A(\bar{B}_d^0 \to K^0\bar{K}^0)|^2 + |A(B_d^0 \to K^0\bar{K}^0)|^2}\sin\psi \tag{8}$$

$$\mathcal{A}_{\rm CP}^{\text{mix-ind}}(B_d \to \pi^+\pi^-) = -\frac{2|A(\bar{B}_d^0 \to \pi^+\pi^-)||A(B_d^0 \to \pi^+\pi^-)|}{|A(\bar{B}_d^0 \to \pi^+\pi^-)|^2 + |A(B_d^0 \to \pi^+\pi^-)|^2}\sin\phi \tag{9}$$

and enter the formulae for the corresponding time-dependent CP asymmetries in the following way:

$$a_{\rm CP}(t) \equiv \frac{\Gamma(B^0_d(t)\to f)-\Gamma(\bar{B}^0_d(t)\to f)}{\Gamma(B^0_d(t)\to f)+\Gamma(\bar{B}^0_d(t)\to f)} = \mathcal{A}^{\rm dir}_{\rm CP}(B_d\to f)\cos(\Delta M_d t)+\mathcal{A}^{\text{mix-ind}}_{\rm CP}(B_d\to f)\sin(\Delta M_d t). \tag{10}$$

Here, $\mathcal{A}^{\rm dir}_{\rm CP}(B_d\to f)$ describes direct CP violation and is given by

$$\mathcal{A}^{\rm dir}_{\rm CP}(B_d\to f)=\frac{|A(B^0_d\to f)|^2-|A(\bar{B}^0_d\to f)|^2}{|A(B^0_d\to f)|^2+|A(\bar{B}^0_d\to f)|^2}, \tag{11}$$

whereas ΔM_d denotes the mass splitting of the physical B^0_d–$\bar{B}^0_d$–mixing eigenstates. Note that eq. (10) is only valid in the case of B_d-decays into final CP-eigenstates $|f\rangle$ satisfying $(\mathcal{CP})|f\rangle=\pm|f\rangle$. This requirement is fulfilled by the B_d-modes considered in this letter. Let us note that we would have $\psi=0$ if we neglected the QCD-penguins with internal up- and charm-quark exchanges in the mode $B_d\to K^0\bar{K}^0$, and $\phi=2\alpha$ if we omitted the penguin contributions to the decay $B_d\to\pi^+\pi^-$.

The knowledge of ψ and ϕ together with the branching ratios for the decays $B^0_d\to K^0\bar{K}^0$ and $B^0_d\to\pi^+\pi^-$ and their CP-conjugates, respectively, specifies the dashed and solid triangles shown in Fig. 1. If we knew in addition the angle σ fixing the relative orientation of these two triangles, the angle α' in Fig. 1 could be determined. It is related to the CKM-angle α through

$$\alpha=\alpha'+\delta\alpha, \tag{12}$$

where

$$\delta\alpha=\mathcal{O}\left(\frac{1}{2}\frac{|P_3|+|\bar{P}_3|}{|T+E|}\right)=\mathcal{O}(\bar{\lambda}^2). \tag{13}$$

Note that $\delta\alpha$ is of the same order in $\bar{\lambda}$ as the uncertainty affecting the determination of α by using the $B\to\pi\pi$ approach proposed by Gronau and London [11, 19]. In contrast to eq. (13), the latter uncertainty is not related to $SU(3)$-breaking effects but originates from electroweak penguin operators.

While the angles ψ and ϕ are measured by the CP-violating asymmetries (8) and (9), respectively, the angle σ can only be determined in an *indirect* way. To this end, let us neglect the $SU(3)$-breaking effects described by the amplitudes P_3 and $\bar{P}_3$ which are both $\mathcal{O}(\bar{\lambda}^2)$. If we define the quantities

$$\begin{array}{rclrcl} A & \equiv & |A(B^0_d\to\pi^+\pi^-)|, & \bar{A} & \equiv & |A(\bar{B}^0_d\to\pi^+\pi^-)|,\\ B & \equiv & |A(B^0_d\to K^0\bar{K}^0)|, & \bar{B} & \equiv & |A(\bar{B}^0_d\to K^0\bar{K}^0)|, \end{array} \tag{14}$$

we obtain in this strict $SU(3)$-symmetric case the following equations from Fig. 1:

$$A^2+B^2-2AB\cos(\psi+\sigma) = |T+E|^2 \tag{15}$$

$$\bar{A}^2+\bar{B}^2-2\bar{A}\bar{B}\cos(\sigma+\phi) = |T+E|^2. \tag{16}$$

Combining (15) and (16) yields the equation

$$a\cos\sigma - b\sin\sigma = c, \tag{17}$$

where we have introduced the quantities a, b and c through

$$a \equiv \bar{A}\bar{B}\cos\phi - AB\cos\psi \tag{18}$$

$$b \equiv \bar{A}\bar{B}\sin\phi - AB\sin\psi \tag{19}$$

$$c \equiv \frac{1}{2}\left(\bar{A}^2 + \bar{B}^2 - A^2 - B^2\right). \tag{20}$$

Its solution can be written in the form

$$\tan\sigma = \frac{-bc \pm a\sqrt{a^2+b^2-c^2}}{ac \pm b\sqrt{a^2+b^2-c^2}} \tag{21}$$

and fixes $\tan\sigma$ up to a two-fold ambiguity corresponding to "+" and "−", respectively. Consequently, σ can be determined up to a four-fold ambiguity. Note that there would be no ambiguity in $\tan\sigma$ in the special cases $c = 0$, which corresponds to the limit of no direct CP violation in the decays $B_d \to \pi^+\pi^-$ and $B_d \to K^0\bar{K}^0$, and $a^2 + b^2 - c^2 = 0$. The angles ψ and ϕ determined by using (8) and (9), respectively, suffer also from two-fold ambiguities which are a characteristic feature of the determination of angles by using CP-violating asymmetries or amplitude relations. Taking into account additional information from other processes, it should be possible to exclude certain solutions and to resolve these ambiguities. In particular the future knowledge of the shape of the unitarity triangle obtained from loop induced transitions (see e.g. [36]) should be useful in this respect.

Using σ determined by means of eq. (21), both the angle α and the quantity $|T+E|$ can be extracted in the limit of vanishing $SU(3)$-breaking, i.e. $P_3 = \bar{P}_3 = 0$, as can be seen from Fig. 1. One could easily generalize the equations above by including the effect of P_3 and $\bar{P}_3$. This would modify σ and consequently α by corrections of $\mathcal{O}(\bar{\lambda}^2)$. Due to the lack of knowledge of the exact values of P_3 and $\bar{P}_3$ this generalization would not improve the accuracy of our method at present.

Consequently, combining all these considerations (see also eq. (13)), we expect the uncertainty in the determination of α in our approach to be of $\mathcal{O}(\bar{\lambda}^2)$. It should be stressed – as has already been done in refs. [19, 35] – that this estimate should not be taken too literally since $\bar{\lambda} = \mathcal{O}(0.2)$ is not a small number. Therefore, in practice the accuracy of our approach may well be of $\mathcal{O}(\bar{\lambda}^{2\pm1})$. In order to control it in a quantitative way, we have to deal with the $SU(3)$-breaking contributions P_3 and $\bar{P}_3$ which is beyond the scope of this letter. In this respect the $\mathcal{O}(\bar{\lambda}^2)$ electroweak penguin uncertainty affecting the determination of α in the Gronau-London-method [11] is in better shape as we have shown in ref. [20]. Performing measurements of the branching ratios of certain $B \to \pi K$ channels, which are expected to be of $\mathcal{O}(10^{-5})$, these electroweak penguin effects can be determined in principle.

In summary we have presented a determination of the CKM-angle α by using mixing-induced CP violation in the decays $B_d \to \pi^+\pi^-$ and $B_d \to K^0\bar{K}^0$. Interestingly enough, the accuracy of our method, which is limited by $SU(3)$-breaking effects related to the creation of $s\bar{s}$ pairs in $\bar{b} \to \bar{d}$ QCD-penguin processes, is expected to be of the same order in $\bar{\lambda}$, i.e. $\mathcal{O}(\bar{\lambda}^2)$, as the one arising from electroweak penguins in the original $B \to \pi\pi$ approach of Gronau and London. As we stated above, the electroweak penguin uncertainties in the latter method can be brought under control as demonstrated in ref. [20], whereas this is not the case of the $\mathcal{O}(\bar{\lambda}^2)$ $SU(3)$-breaking effects present in the method described here. Despite of this our method may be an interesting alternative to determine the CKM-angle α in a rather clean way. An advantage of our approach is the fact that it does not involve a measurement of the decay $B_d \to \pi^0\pi^0$ which is considered to be difficult. However, we need instead a time-dependent analysis of the pure penguin-induced mode $B_d \to K^0\bar{K}^0$. Experimentalists will find out which method can be performed easier in practice. It is needless to say that a comparison of α determinations by means of these two methods would give another test of the CKM picture of CP violation.

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Figure Caption

Fig. 1: A different strategy for determining the CKM-angle α.

Figure 1: