Virtual Top Effects on Low-Mass Higgs Interactions at Next-to-Leading Order in QCD

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Abstract

We present the next-to-leading-order QCD corrections of $\mathcal{O}(\alpha_s^2 G_F M_t^2)$ to the low-M_H effective $\ell^+\ell^- H$, $W^+W^- H$, and ZZH interaction Lagrangians in the high-M_t limit. In the on-shell scheme formulated with G_F, the $\mathcal{O}(\alpha_s^2 G_F M_t^2)$ corrections support the $\mathcal{O}(\alpha_s G_F M_t^2)$ ones and further increase the screening of the $\mathcal{O}(G_F M_t^2)$ terms. The coefficients of $(\alpha_s/\pi)^2$ range from -6.847 to -16.201, being in line with the value -14.594 recently found for $\Delta\rho$. All four QCD expansions converge considerably more rapidly, if they are written with $\mu_t = m_t(\mu_t)$, where $m_t(\mu)$ is the $\overline{\mathrm{MS}}$ mass, rather than the pole mass, M_t.

PACS numbers: 12.15.-y, 12.38.Bx, 14.65.Ha, 14.80.Bn

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Now that the existence of the top quark has been established [1], the Higgs boson is the last missing link in the Standard Model (SM). The discovery of this particle and the study of its properties are among the most urgent goals of present and future high-energy colliding-beam experiments. The Higgs boson is currently being searched for with the CERN Large Electron-Positron Collider (LEP1) and the SLAC Linear Collider (SLC) via Bjorken's process [2], $e^+e^- \to Z \to f\bar{f}H$. At the present time, the failure of this search allows one to rule out the mass range $M_H \leq 64.3$ GeV at the 95% confidence level [3]. The hunt for the Higgs boson will be continued with LEP2 via the Higgs-strahlung mechanism [4], $e^+e^- \to ZH \to f\bar{f}H$. In next-generation e^+e^- linear supercolliders (NLC), also $e^+e^- \to \bar{\nu}_e\nu_e H$ via W^+W^- fusion and, to a lesser extent, $e^+e^- \to e^+e^-H$ via ZZ fusion will provide copious sources of Higgs bosons.

Once a novel scalar particle is discovered, it will be crucial to decide if it is the very Higgs boson of the SM or if it lives in some more extended Higgs sector. For that purpose, precise knowledge of the SM predictions will be mandatory, *i.e.*, quantum corrections must be taken into account. The status of the radiative corrections to the production and decay processes of the SM Higgs boson has recently been reviewed [5]. Since the top quark is by far the heaviest known elementary particle, with a pole mass of $M_t = (180 \pm 12)$ GeV [1], the leading high-M_t terms, of $\mathcal{O}(G_F M_t^2)$, are particularly important, and it is desirable to gain control over their QCD corrections. During the last year, a number of papers have appeared in which the two-loop $\mathcal{O}(\alpha_s G_F M_t^2)$ corrections to various Higgs-boson production and decay processes are presented. The list of these processes includes $H \to f\bar{f}$, with $f \neq b$ [6] and $f = b$ [7, 8], $Z \to f\bar{f}H$ and $e^+e^- \to ZH$ [9], $e^+e^- \to \bar{\nu}_e\nu_e H$ via W^+W^- fusion [10], $gg \to H$ [10, 11], and more [10]. In this paper, we shall proceed one step beyond and tackle with three-loop $\mathcal{O}(\alpha_s^2 G_F M_t^2)$ corrections. To simplify matters, we shall work in the limit $M_H \ll M_t$ and concentrate on reactions with colourless external legs. Such reactions typically involve the $\ell^+\ell^-H$, W^+W^-H, and ZZH couplings together with the gauge couplings of the W and Z bosons to the leptons. Our primary task is thus to find the next-to-leading QCD corrections to the low-M_H effective $\ell^+\ell^-H$, W^+W^-H, and ZZH interaction Lagrangians.

Recently, the $\mathcal{O}(\alpha_s^2 G_F M_t^2)$ correction to $\Delta\rho$ has been calculated and found to be quite sizeable [12], being right at the edge of affecting ongoing precision tests of the standard electroweak theory. For $N_c = 3$ and $n_f = 6$, the QCD expansion of $\Delta\rho$ reads [12, 13]

$$\Delta\rho = 3X_t \left[1 - 2.859\,912\,a(1 + a\beta_0 L) - a^2(25.311\,305 - 1.786\,213\,n_f)\right], \tag{1}$$

where $X_t = (G_F M_t^2/8\pi^2\sqrt{2})$, $a = \alpha_s(\mu)/\pi$, $L = \ln(\mu^2/M_t^2)$, $\beta_0 = 11/4 - n_f/6$ is the one-loop coefficient of the QCD beta function, G_F is Fermi's constant, and μ is the QCD renormalization scale. It is of great theoretical interest to find out whether the occurrence of significant $\mathcal{O}(\alpha_s^2 G_F M_t^2)$ corrections is specific to $\Delta\rho$ or whether this is a common feature among the electroweak parameters with a quadratic M_t dependence at one loop. In the latter case, there must be some underlying principle which is responsible for this phenomenon. Our analysis will put us into a position where we can investigate this issue for four independent quantities.

We shall work in the electroweak on-shell renormalization scheme, with G_F as a basic parameter [14]. We shall take the colour gauge group to be SU(3), so that $N_c = C_A = 3$, $C_F = 4/3$, and $T_F = 1/2$. We shall explicitly include five massless quark flavours plus the massive top quark in our calculation, so that we have $n_f = 6$ active quark flavours altogether. We shall evaluate the strong coupling constant, $\alpha_s(\mu)$, at next-to-leading order in the modified minimal-subtraction ($\overline{\mathrm{MS}}$) scheme [15]. The W-, Z-, and Higgs-boson self-energies $\Pi_{WW}(q^2)$, $\Pi_{ZZ}(q^2)$, and $\Pi_{HH}(q^2)$ will be the basic ingredients of our analysis. In the case of $\Pi_{HH}(q^2)$, we shall actually need the first derivative $\Pi'_{HH}(q^2)$ for the Higgs-boson wave-function renormalization. Since we wish to extract the leading high-M_t terms, we may put $q^2 = 0$. While the $\mathcal{O}(\alpha_s^2 G_F M_t^2)$ results for $\Pi_{WW}(0)$ and $\Pi_{ZZ}(0)$ are now well established [12], $\Pi'_{HH}(0)$ requires a separate analysis, which we shall carry out here. Our calculation will proceed along the lines of Ref. [12]. We shall present our main results in this letter. The technical details and a variety of applications are reported elsewhere [16].

The Feynman diagrams pertinent to $\Pi_{HH}(q^2)$ in $\mathcal{O}(\alpha_s^2 G_F M_t^2)$ come in twenty different topologies. Typical specimen are depicted in Fig. 1. Using dimensional regularization, with $n = 4 - 2\epsilon$ space-time dimensions and a 't Hooft mass μ, and adopting from Ref. [17] the QCD coupling and mass counterterms in the $\overline{\mathrm{MS}}$ scheme, we find

$$\begin{aligned}\Pi'_{HH}(0) = {}& \frac{3G_F m_t^2(\mu)}{8\pi^2\sqrt{2}}\left\{\frac{2}{\epsilon} + 2l - \frac{4}{3} + a\left(-\frac{2}{\epsilon^2} + \frac{5}{3\epsilon} + 2l^2 - \frac{10}{3}l - \frac{37}{18}\right)\right. \\ & + a^2\left[\frac{5}{2\epsilon^3} - \frac{79}{12\epsilon^2} - \frac{1}{3\epsilon}\left(\zeta(3) - \frac{311}{36}\right) + \frac{5}{2}l^3 - \frac{7}{2}l^2 - l\left(\zeta(3) + \frac{1073}{72}\right)\right. \\ & \left.\left. - \frac{16}{3}\,\mathrm{Li}_4\left(\frac{1}{2}\right) + \frac{11}{3}\zeta(4) + \frac{37}{9}\zeta(3) + \frac{4}{3}\zeta(2)\ln^2 2 - \frac{2}{9}\ln^4 2 + \frac{17}{54}\right]\right\}, \end{aligned} \tag{2}$$

where $m_t(\mu)$ is the top-quark $\overline{\mathrm{MS}}$ mass, $l = \ln[\mu^2/m_t^2(\mu)]$, Li_4 is the quadrilogarithm, and ζ is Riemann's zeta function. In Eq. (2), we have omitted terms containing $\gamma_E - \ln(4\pi)$, where γ_E is Euler's constant. These may be retrieved by substituting $\mu^2 \to 4\pi e^{-\gamma_E}\mu^2$. We observe that, up to an overall minus sign, the ultraviolet divergences in Eq. (2) precisely match those of the corresponding expression for $\Pi_{WW}(0)/M_W^2$ in Ref. [12]. In the following, we shall employ M_t instead of $m_t(\mu)$, since M_t directly corresponds to the parameter which is being extracted from experiment [1]. The two-loop relation between M_t and $m_t(M_t)$ may be found in Ref. [17], and the μ evolution of $m_t(\mu)$ is determined by the respective renormalization-group (RG) equation.

The QCD corrections to the $\ell^+\ell^- H$ Yukawa coupling originate in the renormalizations of the Higgs-boson wave function and vacuum expectation value. For $M_H \ll M_t$, they may be accommodated in the $\ell^+\ell^- H$ interaction Lagrangian by writing [18]

$$\mathcal{L}_{\ell\ell H} = -2^{1/4} G_F^{1/2} m_\ell \bar{\ell}\ell H(1 + \delta_u), \tag{3}$$

where

$$\delta_u = -\frac{1}{2}\left[\frac{\Pi_{WW}(0)}{M_W^2} + \Pi'_{HH}(0)\right] \tag{4}$$

is manifestly finite, gauge independent, and RG invariant. Here, the subscript u is to remind us that this term appears as a universal building block in the radiative corrections to all production and decay processes of the Higgs boson. Combining Eq. (2) with the corresponding expression for $\Pi_{WW}(0)/M_W^2$ in Ref. [12] and eliminating $m_t(\mu)$ in favour of M_t, we obtain

$$\delta_u = \frac{7}{2}X_t\left[1 - 1.797\,105\,a(1 + a\beta_0 L) - a^2(28.649\,053 - 2.074\,701\,n_f)\right]. \tag{5}$$

Equation (5) reproduces the well-known $\mathcal{O}(G_F M_t^2)$ [18, 19] and $\mathcal{O}(\alpha_s G_F M_t^2)$ [6] terms.

Next, we shall derive the $\mathcal{O}(\alpha_s^2 G_F M_t^2)$ correction to the low-M_H effective W^+W^-H interaction Lagrangian. In contrast to the $\ell^+\ell^-H$ case, we are now faced with the task of computing genuine three-point amplitudes at three loops, which, at first sight, appears to be enormously hard. In fact, we are not aware of any three-loop calculation of a three-point function in the literature. Fortunately, in the limit that we are interested in, this problem may be reduced to one involving just three-loop two-point diagrams by means of a low-energy theorem, whose lowest-order version has been introduced in Refs. [4, 20]. Generally speaking, this theorem relates the amplitudes of two processes which differ by the insertion of an external Higgs-boson line carrying zero four-momentum. It allows us to compute a loop amplitude with an external Higgs boson which is light compared to the virtual particles by differentiating the respective amplitude without that Higgs boson with respect to the virtual-particle masses. In Refs. [7, 21], it has been shown how the applicability of this theorem may be extended beyond the leading order. Proceeding along the lines of Refs. [9, 10], we obtain

$$\mathcal{L}_{W^+W^-H} = 2^{5/4}G_F^{1/2}M_W^2 W_\mu^+ W^{-\mu} H(1 + \delta_{WWH}), \tag{6}$$

with

$$\delta_{WWH} = \delta_u + \left[1 - \frac{(m_t^0)^2\partial}{\partial(m_t^0)^2}\right]\frac{\Pi_{WW}(0)}{M_W^2}, \tag{7}$$

where m_t^0 is the bare top-quark mass. In Ref. [12], $\Pi_{WW}(0)$ is expressed in terms of $m_t(\mu)$. Thus, we have to undo the top-quark mass renormalization [17] before we can apply Eq. (7). Then, after evaluating the right-hand side of Eq. (7), we introduce M_t and so obtain

$$\delta_{WWH} = -\frac{5}{2}X_t\left[1 - 2.284\,053\,a(1 + a\beta_0 L) - a^2(20.612\,157 - 1.632\,629\,n_f)\right]. \tag{8}$$

We recover the well-known $\mathcal{O}(G_F M_t^2)$ [19, 22] and $\mathcal{O}(\alpha_s G_F M_t^2)$ [10] terms.

The derivation of the $\mathcal{O}(\alpha_s^2 G_F M_t^2)$ correction to the low-M_H effective ZZH interaction Lagrangian proceeds in close analogy to the W^+W^-H case, and we merely list the result:

$$\mathcal{L}_{ZZH} = 2^{1/4}G_F^{1/2}M_Z^2 Z_\mu Z^\mu H(1 + \delta_{ZZH}), \tag{9}$$

where

$$\delta_{ZZH} = -\frac{5}{2}X_t\left[1 - 4.684\,053\,a(1 + a\beta_0 L) - a^2(18.410\,658 - 1.927\,313\,n_f)\right]. \qquad (10)$$

Equation (10) contains the well-known $\mathcal{O}(G_F M_t^2)$ [19, 23] and $\mathcal{O}(\alpha_s G_F M_t^2)$ [9] terms. The analytic versions of Eqs. (5), (8), and (10) for N_c arbitrary and in terms of fundamental functions and one master integral, which may be solved numerically with high precision [12], are included in Ref. [16].

We have presented the three-loop $\mathcal{O}(\alpha_s^2 G_F M_t^2)$ corrections to the effective Lagrangians for the interactions of light Higgs bosons with pairs of charged leptons, W bosons, and Z bosons in the SM. As a corollary, we note that $\Gamma(H \to \ell^+\ell^-)$, $\Gamma(H \to W^+W^-)$, and $\Gamma(H \to ZZ)$ receive the correction factors $(1 + \delta_u)^2$, $(1 + \delta_{WWH})^2$, and $(1 + \delta_{ZZH})^2$, respectively. At first sight, the conditions $M_H > 2M_V$, with $V = W, Z$, which are necessary for the decays $H \to W^+W^-$ and $H \to ZZ$ to happen, seem to conflict with our assumption that $M_H \ll M_t$. However, as may be already gleaned from the corresponding one-loop analyses [22, 23], the actual expansion parameter is $r = (M_H^2/4M_t^2)$, which is still small against unity if $M_H \gtrsim 2M_V$. *E.g.*, at one loop, the term linear in r to be included within the square brackets of Eq. (10) has the coefficient $+4/25$. Moreover, these results may be used to refine the theoretical predictions for a variety of four- and five-point production and decay processes of light Higgs bosons at present and future e^+e^- colliders. This is relegated to our comprehensive report [16].

Here, we would like to focus attention on an interesting theoretical point. In fact, our analysis allows us to recognize a certain universal pattern in the structure of the QCD perturbation series. In addition to $\Delta\rho$, we have now three more independent observables with quadratic M_t dependence at our disposal for which the QCD expansion is known up to next-to-leading order, namely δ_u, δ_{WWH}, and δ_{ZZH}. In the on-shell scheme of electroweak and QCD renormalization, these four electroweak parameters exhibit striking common properties. In fact, the leading- and next-to-leading-order QCD corrections act in the same direction and screen the $\mathcal{O}(G_F M_t^2)$ terms. Furthermore, the sets of α_s/π and $(\alpha_s/\pi)^2$ coefficients each lie in the same ball park. For the choice $\mu = M_t$, the coefficients of α_s/π range between -1.797 and -4.684, and those of $(\alpha_s/\pi)^2$ between -6.847 and -16.201. We would like to point out that the corresponding coefficients of the ratio μ_t^2/M_t^2, where $\mu_t = m_t(\mu_t)$, are -2.667 and -11.140 [16], *i.e.*, they lie right in the centres of these ranges. Therefore, it suggests itself that the use of M_t is the origin of these similarities. In fact, if we express the QCD expansions in terms of μ_t rather than M_t and choose $\mu = \mu_t$, then the coefficients of α_s/π and $(\alpha_s/\pi)^2$ nicely group themselves around zero; they range from -2.017 to 0.870 and from -3.970 to 1.344, respectively [16]. This indicates that the perturbation expansions converge more rapidly if we renormalize the top-quark mass according to the $\overline{\mathrm{MS}}$ scheme. Without going into details, we would like to mention that the study of renormalons [24] offers a possible theoretical explanation of this observation. Since the on-shell and $\overline{\mathrm{MS}}$ results coincide in lowest order, this does, of course, not imply that the QCD corrections are any smaller in the $\overline{\mathrm{MS}}$ scheme. It just means that, as a rule, the $\mathcal{O}(G_F M_t^2)$ terms with M_t replaced by the two-loop expression for μ_t [16] are likely to provide fair approximations for the full three-loop results. In all the cases considered here, the QCD corrections now appear to be well under control.

Finally, we would like to test Broadhurst's rule concerning the naïve non-abelianization of QCD [25]. Guided by the observation that the n_f-independent term of β_0 emerges from the coefficient of n_f by multiplication with $-33/2$, Broadhurst conjectured that this very relation between the n_f-independent term and the coefficient of n_f approximately holds for any observable at next-to-leading order in QCD. This rule is trivially satisfied for the L-dependent terms in Eqs. (1), (5), (8), and (10). As for the constant terms of $\mathcal{O}(\alpha_s^2 G_F M_t^2)$ in these expressions, multiplication of the coefficients of n_f with $-33/2$ yields -29.5, -34.2, -26.9, and -31.8, which has to be compared with the respective n_f-independent terms, -25.3, -28.6, -20.6, and -18.4. We observe that, in all these cases, Broadhurst's rule correctly predicts the sign and the order of magnitude of the n_f-independent terms, which are much harder to compute. Except for the last case, these predictions are, in fact, very close to the true values.

We would like to thank Bill Bardeen, David Broadhurst, Kostja Chetyrkin, and Michael Spira for very useful discussions. One of us (BAK) is indebted to the FNAL Theory Group for inviting him as a Guest Scientist. He is also grateful to the Phenomenology Department of the University of Wisconsin at Madison for the great hospitality extended to him during a visit when a major part of his work on this project was carried out.

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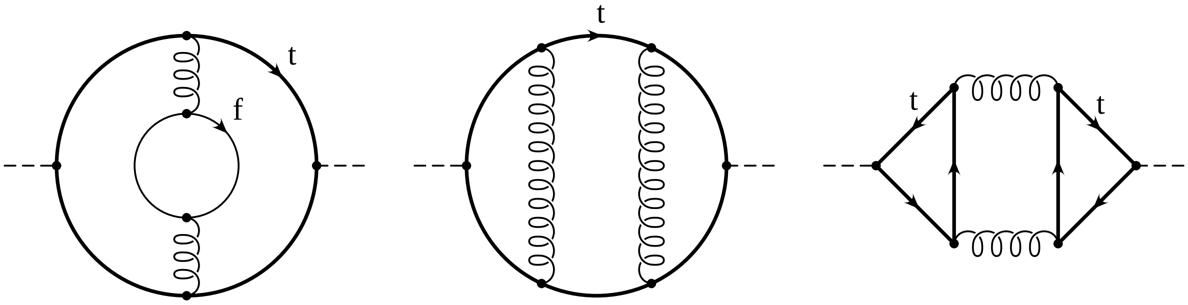

Figure 1: Typical Feynman diagrams pertinent to $\Pi_{HH}(q^2)$ in $\mathcal{O}(\alpha_s^2 G_F M_t^2)$. f stands for any quark.