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Tau Polarimetry with Multi-Meson States †

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Abstract

It is demonstrated that the analyzing power of multi-meson final states in semileptonic τ decays with respect to the τ spin is equal and maximal for all decay modes.

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The determination of the polarization of τ leptons produced in Z decays has lead to an important determination of the τ coupling to the Z boson, rivaling those from the forward-backward asymmetry of leptons or from the left-right asymmetry measured with longitudinally polarized beams. It is well known that the decay mode into a single pion leads to optimal analyzing power which expresses itself in an angular distribution of pions from decays of polarized τ 's of the form

$$dN \propto (1 + f \cos\theta) \tag{1}$$

where $f = 1$. The angle between τ spin and pion momentum is denoted by θ. Decays into the ρ or a_1 meson or higher excitations exhibit reduced analyzing power, e.g. $f = (M^2 - 2Q^2)/(M^2 + 2Q^2)$ for a spin 1 final state of mass Q [1]. In [2, 3] it has been argued that significant analyzing power can be recovered by exploiting information encoded in the momenta of the (pseudoscalar) mesons which are the actual decay products and are observed in the experiment. Detailed models have been used for the two- and three-meson channels to identify various angular distributions which enhance the sensitivity.

In this brief comment we would like to demonstrate explicitly that maximal sensitivity can be recovered for *any* multi-meson final state, once the dynamics of the decay matrix element is known. Ingredients are the knowledge of all meson momenta and information about the τ rest frame. The latter is equivalent to reconstruction of the actual direction of flight of the τ and can be achieved in e^+e^- experiments with the help of vertex detectors [4].

The argument is based on the observation that the squared matrix element for semileptonic τ can always be written (in the τ restframe) in the form

$$\mathcal{M} \propto 1 - \vec{h}\vec{s}. \tag{2}$$

The τ-spin direction is denoted by $\vec{s}$ and the polarimeter vector $\vec{h}$ is a function of all meson momenta. It has length $\vec{h} = 1$ and its direction therefore gives the (negative) spin direction of the original τ with unit probability. This holds true for meson final states only — the spin information is strongly diluted for leptonic τ decays as a consequence of averaging over the electron spin and neutrino momenta corresponding to a reduction of the length of $\vec{h}$. It is, however, retained in the direction of the $\bar{\nu}_e$. For top decays, the e^+ direction preserves the full analyzing power [5].

The aim of this short comment is to demonstrate that the norm of $\vec{h}$ is in fact equal to one for all semileptonic decays.

The matrix element for the semileptonic decay $\tau \to \nu_\tau + X$ can be written in the form

$$\mathcal{M} = \frac{G}{\sqrt{2}}\bar{u}(N)\gamma^\mu(1-\gamma_5)u(P)J_\mu, \tag{3}$$

where $J_\mu \equiv \langle X|V_\mu - A_\mu|0\rangle$ denotes the matrix element of the $V - A$ current relevant for the specific final state X. The vector J_μ depends in general on the momenta of all hadrons. The squared matrix element for the decay of a τ with spin s and mass M then reads

$$|\mathcal{M}|^2 = G^2(\omega + H_\mu s^\mu), \tag{4}$$

with

$$\omega = P_\mu(\Pi_\mu + \Pi^5_\mu) \qquad\qquad H_\mu = \frac{1}{M}(M^2 g_\mu{}^\nu - P_\mu P^\nu)(\Pi_\nu + \Pi^5_\nu) \tag{5}$$

and

$$\Pi_\mu = 2\left[(J^* \cdot N)J_\mu + (J \cdot N)J^*_\mu - (J^* \cdot J)N_\mu\right], \qquad \Pi^5_\mu = 2\,\mathrm{Im}\,\epsilon_\mu{}^{\nu\rho\sigma} J^*_\nu J_\rho N_\sigma. \tag{6}$$

This formula was derived in [2] and constitutes the basis for the simulation of spin effects in TAUOLA [6].

In the τ-rest frame, the function ω coincides with the time component of the four vector $\Pi_\mu + \Pi^5_\mu$ (multiplied with M) and the vector $\vec{H}$ with its space component (multiplied with M). The assertion that $|\vec{h}| = |\vec{H}/\omega| = 1$ is therefore equivalent to the statement that $\Pi_\mu + \Pi^5_\mu$ is null-vector. A simple calculation demonstrates that

$$\Pi^5_\mu \Pi^\mu = 0, \qquad\qquad \Pi^5_\mu \Pi^{5\mu} = -\Pi_\mu \Pi^\mu \tag{7}$$

and hence

$$(\Pi_\mu + \Pi^5_\mu)(\Pi^\mu + \Pi^{5\mu}) = 0 \tag{8}$$

which proves our assertion and gives, at the same time, a prescription of how to construct the direction of the spin on an event-by-event basis. Obviously, in order to perform this analysis, all hadron momenta must be measured and the dependence of the current J on these momenta known — either from theoretical considerations or from fits to experimentally measured distributions.

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