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Electroweak Fermion-loop Contributions to the Muon Anomalous Magnetic Moment

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Abstract

The two-loop electroweak corrections to the anomalous magnetic moment of the muon, generated by fermionic loops, are calculated. An interesting role of the top quark in the anomaly cancellation is observed. New corrections, including terms of order $G_\mu \alpha m_\mu^2 m_t^2 / M_W^2$, are computed and a class of diagrams previously thought to vanish are found to be important. The total fermionic correction is $-(23 \pm 3) \times 10^{-11}$ which decreases the electroweak effects on $g-2$, predicted from one-loop calculations, by 12%. We give an updated theoretical prediction for $g-2$ of the muon.

[1]The complete postscript file of this preprint, including figures, is available via anonymous ftp at ttpux2.physik.uni-karlsruhe.de (129.13.102.139) as /ttp95-19/ttp95-19.ps or via www at http://ttpux2.physik.uni-karlsruhe.de/cgi-bin/preprints/ Report-no: TTP95-19

The anomalous magnetic moment of the muon, $a_\mu \equiv (g_\mu - 2)/2$, provides a precision test of the standard model and potential window to "new physics" effects. The current experimental average [1]

$$a_\mu^{\rm exp} = 116592300(840) \times 10^{-11} \tag{1}$$

is in good accord with theory and constrains physics beyond the standard model such as [2, 3] supersymmetry, excited leptons, compositeness, etc. A new experiment [4] being prepared at Brookhaven National Laboratory is expected to reduce the uncertainty in $a_\mu^{\rm exp}$ to below $\pm 40 \times 10^{-11}$, more than a factor of 20 improvement. At that level, electroweak loop corrections become important and new physics at the multi-TeV scale is explored.

To fully exploit the anticipated experimental improvement, the standard model theoretical prediction for a_μ should be known with comparable precision. That requires the confluence of calculational effort involving very high order QED loops, hadronic loop contributions, and even two loop electroweak effects. Indeed, the contributions to a_μ are traditionally divided up into

$$a_\mu = a_\mu^{\rm QED} + a_\mu^{\rm Hadronic} + a_\mu^{\rm EW} \tag{2}$$

The QED loops have been computed to very high order [5]

$$\begin{aligned} a_\mu^{\rm QED} &= \frac{\alpha}{2\pi} + 0.76585738(6)\left(\frac{\alpha}{\pi}\right)^2 + 24.0454(4)\left(\frac{\alpha}{\pi}\right)^3 \\ &\quad +126.14(43)\left(\frac{\alpha}{\pi}\right)^4 + 930(170)\left(\frac{\alpha}{\pi}\right)^5 \end{aligned} \tag{3}$$

where in the calculation of the τ lepton loops we used $m_\tau = 1777$ MeV. Employing $\alpha^{-1} = 137.0359895(61)$ recommended by the PDG [1] gives

$$a_\mu^{\rm QED} = 116584708(5) \times 10^{-11} \tag{4}$$

The uncertainty could be further reduced by a factor of 2, if we chose to use α as determined from the electron $g_e - 2$, $\alpha^{-1} = 137.03599222(94)$; however, in either case it is well within the $\pm 40 \times 10^{-11}$ experimental goal.

A recent reexamination [6] of hadronic vacuum polarization at the $\mathcal{O}(\alpha/\pi)^2$ level, utilizing $e^+e^- \to$ hadrons data via a dispersion relation, gives

$$a_\mu^{\rm Hadronic}(\text{vac. pol.}) = 7023.5(152.6) \times 10^{-11} \tag{5}$$

Unfortunately, the uncertainty has not yet reached the hoped for level of precision. However, it is anticipated [7] that ongoing improvements in $e^+e^- \to$ hadrons data near the ρ meson resonance (which weighs heavily in (5)) and theoretical input in the higher energy region will significantly reduce the uncertainty. Nevertheless, the goal of going below $\pm 40 \times 10^{-11}$ remains a formidable challenge.

The result in (5) must be supplemented by higher order, $\mathcal{O}(\alpha/\pi)^3$, hadronic vacuum polarization effects [2, 3]

$$a_\mu^{\rm Hadronic}(\text{H.O. vac. pol.}) = -90(5) \times 10^{-11} \tag{6}$$

and the light by light hadronic amplitudes [8, 9]

$$a_\mu^{\rm Hadronic}(\text{light by light}) = 8(9) \times 10^{-11} \tag{7}$$

Altogether, one finds

$$a_\mu^{\rm Hadronic} = 6942(153) \times 10^{-11} \tag{8}$$

Now we come to the electroweak contributions to a_μ, the primary focus of this work. At the one loop level, they are well known [10, 11, 12, 13, 14]

$$\begin{aligned} a_\mu^{\rm EW}(1\,\text{loop}) &= \frac{5}{3}\frac{\mathrm{G}_\mu \mathrm{m}_\mu^2}{8\sqrt{2}\pi^2} \\ &\times \left[1 + \frac{1}{5}(1-4s_W^2)^2 + \mathcal{O}\left(\frac{m_\mu^2}{M^2}\right)\right] \\ &= 195 \times 10^{-11} \end{aligned} \tag{9}$$

where $G_\mu = 1.16639(1)\times 10^{-5}\ \mathrm{GeV}^{-2}$, $M = M_W$ or $M_{\rm Higgs}$, and the weak mixing angle is defined by $\sin^2\theta_W \equiv s_W^2 = 1 - M_W^2/M_Z^2$. We can safely neglect the $\mathcal{O}\left(m_\mu^2/M^2\right)$ terms in (9). Also, throughout this paper we neglect terms suppressed by the factor $1-4s_W^2$ whenever it simplifies the expressions without affecting accuracy.

The one loop estimate of electroweak effects is about 5 times the anticipated experimental accuracy. Naively, one would expect higher order electroweak contributions to be of relative $\mathcal{O}(\alpha/\pi)$ and hence insignificant. However, an interesting study by Kukhto, Kuraev, Schiller, and Silagadze [15] (KKSS) found that not to be the case. They showed that 2 loop electroweak contributions are quite large and must be included in any serious estimate of $a_\mu^{\rm EW}$ or confrontation with future experiments.

Including 2 loops and making the approximations mentioned above, $a_\mu^{\rm EW}$ becomes

$$a_\mu^{\rm EW} = \frac{5}{3}\frac{G_\mu m_\mu^2}{8\sqrt{2}\pi^2}\left(1 + C\frac{\alpha}{\pi}\right) \tag{10}$$

It is natural to separate the subset of the two loop electroweak contributions which contain a fermion loop

$$C = C^{\rm ferm} + C^{\rm bos} \tag{11}$$

Parts of both $C^{\rm ferm}$ and $C^{\rm bos}$ have been calculated by KKSS. Denoting the non calculated contributions by R_f (for fermionic loops) and by R_b (for the remaining diagrams) the KKSS results can be written as

$$\begin{aligned} C^{\rm ferm} &= -\frac{18}{5}\ln\left(\frac{M_Z^2}{m_\mu^2}\right) - \frac{9}{5}\ln\left(\frac{M_Z^2}{m_\tau^2}\right) + 1 + \frac{8}{15}\pi^2 + R_f \\ C^{\rm bos} &= -\frac{49}{15}\ln\left(\frac{M_Z^2}{m_\mu^2}\right) + R_b \end{aligned} \tag{12}$$

The known parts reduce the electroweak contribution $a_\mu^{\rm EW}$ by about 24% (-46×10^{-11}), a significant decrease. A full calculation of R_b is quite a daunting task because of the large number of diagrams. It has been estimated by KKSS to influence the correction factor C at the level of 10%. However, only a full 2 loop calculation will tell us if that is the case.

In the present paper we reexamine the fermionic loops contributing to the two loop electroweak corrections and calculate R_f. We find that a significant subset of diagrams has been neglected in previous studies. In particular we find that the large logs of the ratios of M_Z and lepton masses contributing to $C^{\rm ferm}$ are cancelled by the corresponding quark diagrams. We also obtain new relatively large non-logarithmic corrections of $\mathcal{O}(m_t^2/M_W^2)$ and $\mathcal{O}(1)$ terms.

In our calculation we chose the ratio of muon and vector boson masses as an expansion parameter in the calculation of diagrams contributing to $g-2$. Such asymptotic expansions have recently obtained firm theoretical foundation [16]. After the expansion we still have to perform two loop integrals, which however contain at most one mass scale. The calculation of such integrals is further facilitated by the integration by parts method [17] and the symbolic manipulation programs written in FORM [18]. In some cases we used packages SHELL2 [19] and MINCER [20] to check our results.

We use dimensional regularization with the dimension of space-time equal $D = 4-2\varepsilon$ and neglect terms containing γ_E and $\ln 4\pi\mu$ which accompany the poles $1/\varepsilon$ and vanish in the sum; this explains the appearance of logs of dimensionful quantities in the intermediate (divergent) results.

For the discussion of hadronic loops we use the following quark masses $m_u = m_d = 0.3$ GeV, $m_s = 0.5$ GeV, $m_c = 1.5$ GeV, $m_b = 4.5$ GeV, $m_t = 176$ GeV. We perform the computations in the 't Hooft-Feynman gauge. The basic two loop diagrams with fermion loop contributing to muon $g-2$ are shown in fig. 1. In addition we have to consider diagrams obtained by replacing vector boson propagators by corresponding Goldstone bosons as well as mirror counterparts of asymmetric diagrams.

Contributions of diagrams with a fermion loop connected to the muon line via two charged bosons are shown in fig. 1(a-c). Isospin $+1/2$ fermions are denoted by u and the isospin -1/2 fermions by d. We first consider the case when fermions in the loop belong to the first two generations. Here the masses of the fermions in the loop do not influence the result very much and we neglect them. The ratio of the neglected terms to the result is at most of the order of $(m_c^2/M_W^2)\ln(m_c^2/M_W^2) < 0.3\%$, with m_c denoting the mass of the charm quark ≈ 1.5 GeV. In ref. [21] it has been argued that the diagrams 1(a) and 1(b) vanish by virtue of Furry's theorem. We find that this is not true even after adding contributions of all fermions in a generation. Furry's theorem consists in the observation that contributions of diagrams with two different orientations of the fermion loop mutually cancel. This is not the case for diagrams 1(a) and 1(b) because for every fermion flavor interacting with the external photon there is only one possible orientation of the fermion line. Only those parts of the expressions which contain a single γ_5 cancel out after adding contributions from the up-type quark, down-type quark and from the lepton.

Adding contributions of quarks and leptons we obtain for a single light generation

(we put the relevant Kobayashi-Maskawa matrix elements equal 1)

$$\Delta C_{\text{light}}^{\text{ferm}} = \frac{2}{3s_W^2} \tag{13}$$

In the third generation we can neglect only masses of τ and of the b quark. For the τ lepton loop we obtain

$$\Delta C_{\tau}^{\text{ferm}} = \frac{1}{60s_W^2} \tag{14}$$

For the sum of all diagrams containing top and bottom quark loops we find

$$\begin{aligned} \Delta C_{\text{tb}}^{\text{ferm}} &= \frac{3}{5s_W^2}\left[\frac{m_t^2}{M_W^2}\left(-\frac{8}{3}-\frac{5}{4\varepsilon}+\frac{5}{2}\ln(m_t M_W)\right)\right. \\ &\quad \left. -\frac{1}{12}-\frac{1}{2}\ln\frac{m_t^2}{M_W^2}\right] \end{aligned} \tag{15}$$

We computed $\mathcal{O}(M_W^2/m_t^2)$ and $\mathcal{O}(M_W^4/m_t^4)$ corrections to this formula. Those terms turn out to have small coefficients which render them numerically insignificant. The singular terms m_t^2/ε will be canceled by renormalization of the W boson mass present in the one loop electroweak contributions to muon $g-2$.

We now consider the diagrams with a photon-photon-Z coupling induced by fermion loops shown in fig. 1(d). For electron or muon in the loop we obtain

$$\begin{aligned} \Delta C_{1d}^{\text{ferm}}(e) &= -\frac{9}{5}\left(\ln\frac{M_Z^2}{m_\mu^2}+\frac{5}{6}\right) \\ \Delta C_{1d}^{\text{ferm}}(\mu) &= -\frac{9}{5}\left(\ln\frac{M_Z^2}{m_\mu^2}-\frac{8}{27}\pi^2+\frac{11}{18}\right) \end{aligned} \tag{16}$$

in agreement with [15]. In that reference one also finds a formula for the τ lepton which can be generalized for all fermions sufficiently heavier than the muon and lighter than the Z boson

$$\Delta C_{1d}^{\text{ferm}}(f) = \frac{18}{5} I_{3f} Q_f^2 \left(\ln\frac{M_Z^2}{m_f^2}-2\right) \tag{17}$$

In practice this formula can be used for all quark loops except for the top quark, for which we find

$$\Delta C_{1d}^{\text{ferm}}(t) = \frac{M_Z^2}{m_t^2}\left(\frac{2}{3}+\frac{2}{5}\ln\frac{m_t^2}{M_Z^2}\right) \tag{18}$$

In ref. [15] the total fermionic two loop effect on muon $g-2$ was estimated by summing only electron, muon, and τ contributions to diagram 1(d). It has been concluded that the source of large corrections are logarithms of ratios of these light fermion masses to the mass of the Z boson. Such treatment is incomplete and misleading. We can see from the formula (17) that the sum over *all* fermions (see discussion in [3]) in the first two generations leads to the cancellation of M_Z dependent logarithms, due to the no-anomaly condition $\sum_f I_{3f} Q_f^2 = 0$. This pattern no longer holds for the third generation; here, due to the large mass of the top quark, its contribution is suppressed by a factor M_Z^2/m_t^2. This leads to the appearance of the logarithm of Z boson mass in the sum of all contributions to 1(d)

$$\begin{aligned}\Delta C_{1d}^{\text{ferm}} &= -\frac{18}{5}\ln\frac{(m_u m_c M_Z)^{4/3}}{(m_d m_s m_b)^{1/3} m_\mu^2 m_\tau} \\ &\quad -5 + \frac{8}{15}\pi^2 + \frac{M_Z^2}{m_t^2}\left(\frac{2}{3} + \frac{2}{5}\ln\frac{m_t^2}{M_Z^2}\right)\end{aligned} \tag{19}$$

The first line of (19) gives the dominant contributions of the diagram 1(d) from all fermions. We note that the mass of the top quark is absent in this part reflecting the suppression of top loop discussed above. The second line summarizes the corrections to the dominant effect from the electron, muon, and from the top quark. Numerically we obtain $\Delta a_{1d} = -14.4 \times 10^{-11}$ in contrast with the value given in [21, 15] $\Delta a_{1d(e,\mu,\tau)} \approx -25.6 \times 10^{-11}$. This reduction of the correction is caused by the cancellation of the M_Z dependent logarithms. We stress that the M_Z which is still present in the main part of (19) is caused by the large mass of the top quark and suppression of its contribution.

To summarize this part we note that the large numerical value of the sum of diagrams 1(d) is generated by large mass splittings among the fermions. This is the main difference between our result and the result of [15]: imagine a model in which all fermions had equal masses, then the sum of the three leptonic contributions discussed in [15] would be equal $3 \times \Delta a_{1d(\mu)}$, whereas we find that the total correction due to 1(d) would vanish. We believe to have found a qualitatively new type of the top quark effect: it is namely the extremely large mass of the top quark which determines the shape of the main part of the formula (19), although the numerical value of m_t is completely irrelevant. What is important is that $M_Z^2/m_t^2 \ll 1$.

We now discuss the remaining diagrams, neglected in [15]. In the diagram 1(e) we have to distinguish two cases, again treating top quark separately. For the light fermions we find, in contrast to the diagram 1(d), that the logarithmic factors are suppressed by extra powers of m_f^2/M_Z^2; we retain good accuracy by taking massless fermions. We find

$$\Delta C_{1e}^{\text{ferm}}(\text{light } f) = -\frac{I_{f3}^2 - 2s_W^2 I_{f3} Q_f + 2s_W^4 Q_f^2}{15 s_W^2 c_W^2} \tag{20}$$

This contribution becomes sizable after adding the top quark effect. For all fermions together we find

$$\begin{aligned}\Delta C_{1e}^{\text{ferm}} = -\frac{3}{5 s_W^2 c_W^2}\Bigg[&\frac{2}{3} - \frac{4}{3}s_W^2 + \frac{16}{9}s_W^4 \\ &-\frac{m_t^2}{M_Z^2}\left(\frac{17}{24} + \frac{5}{8\varepsilon} - \frac{3}{8}\ln m_\mu^2 - \frac{5}{8}\ln m_t^2 - \frac{1}{4}\ln M_Z^2\right)\Bigg]\end{aligned} \tag{21}$$

where we neglected terms $\mathcal{O}(M_Z^2/m_t^2)$ which proved to be small.

In the remaining diagrams we have a scalar particle coupling to the fermion loop, and therefore we only consider top loops. If the Z boson in the diagram 1(d) is replaced by the neutral Goldstone boson we obtain

$$\Delta C^{\rm ferm}_{1d(G)}(t) = -\frac{16}{5} - \frac{8}{5}\ln\frac{m_t^2}{M_Z^2} \tag{22}$$

Finally, there is the diagram 1(f) containing the Higgs boson. It is the only non-negligible contribution of the Higgs boson among two loop fermion diagrams. We consider three cases, depending on the hierarchy of masses of the top quark and the Higgs. For $M_H \ll m_t$ we get

$$\begin{aligned} \Delta C^{\rm ferm}_{1f}(t) &= -\frac{104}{45} - \frac{16}{15}\ln\frac{m_t^2}{M_H^2} \\ \to \Delta a_\mu^H &= -2.1\times 10^{-11} \quad (\text{for } M_H = 60 \text{ GeV}) \end{aligned} \tag{23}$$

For the case of $M_H \gg m_t$ we observe stronger suppression of this amplitude

$$\begin{aligned} \Delta C^{\rm ferm}_{1f}(t) &= -\frac{m_t^2}{M_H^2}\left[\frac{24}{5} + \frac{8}{15}\pi^2 + \frac{8}{5}\left(\ln\frac{M_H^2}{m_t^2} - 1\right)^2\right] \\ \to \Delta a_\mu^H &= -1.6\times 10^{-11} \quad (\text{for } M_H = 300 \text{ GeV}) \end{aligned} \tag{24}$$

In order to estimate the size of the Higgs diagram in the case of similar top and Higgs masses we put $m_t = M_H$. In this case we find

$$\begin{aligned} \Delta C^{\rm ferm}_{1f}(t) &= -\frac{32}{5}\left[1 - \frac{1}{\sqrt{3}}{\rm Cl}_2\left(\frac{\pi}{3}\right)\right] \qquad (M_H = m_t) \\ &\to \Delta a_\mu^H = -1.2\times 10^{-11} \end{aligned} \tag{25}$$

where ${\rm Cl}_2$ is the Clausen function [22]. The contribution of the Higgs boson is small and we approximate it by $-1.5(\pm 1.0)\times 10^{-11}$.

A few words are in order to explain why several diagrams with neutral bosons have been omitted in fig. 1. Diagrams with two scalar bosons (HH, HG^0, G^0G^0) coupling to the muon line are at most of the order m_μ^4/M_Z^4. The remaining diagrams (e.g. ZH) are either exactly zero, or are suppressed by the vector coupling of the Z boson to the muon (factor $1 - 4s_W^2$).

Some large two loop corrections can be absorbed in the one loop result if it is parametrized in terms of G_μ determined from the muon's life time. This corresponds to the replacement of bare parameters

$$\frac{e^2}{8 s_W^2 M_W^2} \to \frac{G_\mu}{\sqrt{2}}(1-\Delta) \tag{26}$$

where Δ is determined by studying electroweak corrections to the muon decay width. Because we are interested in fermion loops, only quark and lepton corrections to the W propagator need be included. They induce the following counterterm which cancels the divergences we encountered in the charged boson diagrams (eq. 15) and the Z boson vacuum polarization (eq. 21)

$$\Delta C^{\rm ferm}_{c.t.} = -\frac{2}{s_W^2} + \frac{2}{5 s_W^2 c_W^2}\left(1 - 2 s_W^2 + \frac{8}{3} s_W^4\right)$$
$$-\frac{3}{4 s_W^2}\frac{m_t^2}{M_W^2}\left(-\frac{1}{2\varepsilon} + \frac{1}{2}\ln m_t^2 - \frac{1}{5}\ln M_Z^2 + \ln M_W^2 - \frac{3}{10}\ln m_\mu^2 - \frac{79}{60}\right) \quad (27)$$

The final result for the fermion loop effect on muon $g-2$ is obtained by adding the contributions given by equations (13,14,15,19,21,22,27) and the contribution of the Higgs boson diagram $\Delta C^{\rm ferm}_{1f}(t)$ given by approximating eq. (23, 24,25). Our final formula is

$$\begin{aligned} C^{\rm ferm} &= -\frac{18}{5}\ln\frac{(m_u m_c M_Z)^{4/3}}{(m_d m_s m_b)^{1/3} m_\mu^2 m_\tau} \\ &\quad -\frac{3}{16}\frac{m_t^2}{s_W^2 M_W^2} - \frac{3}{10 s_W^2}\ln\frac{m_t^2}{M_W^2} \\ &\quad -\frac{8}{5}\ln\frac{m_t^2}{M_Z^2} - \frac{41}{5} - \frac{7}{10 s_W^2} + \frac{8}{15}\pi^2 + \Delta C^{\rm ferm}_{1f}(t). \end{aligned} \quad (28)$$

This is our main result which replaces the old estimate of $C^{\rm ferm}$ given in (12). We dropped all terms suppressed by negative powers of m_t. We checked explicitly that their numerical impact is negligible.

The $\mathcal{O}(m_t^2/M_W^2)$ term in eq. (28) is related to the ρ parameter that appears in the ratio of weak neutral to charged current amplitudes and comparisons of the $W^\pm$ and Z masses. It can be viewed as an induced correction brought about by our renormalization of the one loop Z contribution in terms of G_μ, a charged current parameter. We also note that except for their incomplete cancellation in the anomaly diagrams of fig. 1(d), no other effect of the 2 light fermion generations resides in our final result.

For the numerical evaluation of the remaining terms which contain the weak mixing angle we take $s_W^2 = 0.223$. We obtain

$$C^{\rm ferm} = -50(6) \quad (29)$$

which means that the correction to muon anomalous magnetic moment a_μ from the fermion loops is $-(23\pm 3)\times 10^{-11}$. The theoretical uncertainty has several sources: the unknown mass of the Higgs boson, uncertainty in the masses of the light quarks which parametrize the hadronic effects, and the large experimental error in the present value of m_t. Finally, higher order three loop contributions remain unknown. Altogether we estimate these effects to yield an uncertainty at the level of 3×10^{-11}, more than an order of magnitude below the predicted experimental precision.

Including the fermionic two loop corrections and partial two loop bosonic effects, we obtain the updated theoretical predictions

$$\begin{aligned} a_\mu^{\rm EW} &= (152(3) + 0.45 R_b)\times 10^{-11} \\ a_\mu^{\rm th} &= (116591802(153) + 0.45 R_b)\times 10^{-11} \end{aligned} \quad (30)$$

What remains is to compute R_b and lower the hadronic loop uncertainty. Work on both is in progress.

After completing this calculation we learned about ref. [23] which contains an analysis of quark contributions to the diagram in fig. 1(d). For the light quarks their numerical result obtained using the chiral perturbation theory is the same as our evaluation using constituent quark masses. The large difference between their final evaluation of the fermionic loops and our result is due to diagrams of fig. 1(a-c,e,f) which were not considered in [23].

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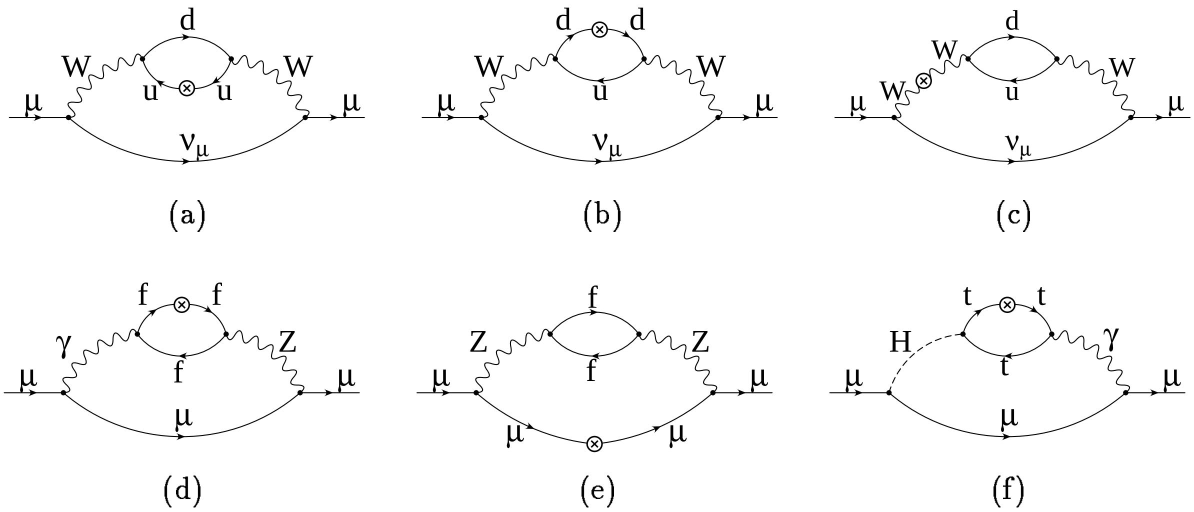

Figure 1: Fermion loop diagrams contributing to the muon anomalous magnetic moment. Crossed circles denote interactions with an external photon.