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QCD Corrections from Top Quark to Relations between Electroweak Parameters to Order α_s^2†

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Abstract

The vacuum polarization functions $\Pi(q^2)$ of charged and neutral gauge bosons which arise from top and bottom quark loops lead to important shifts in relations between electroweak parameters which can be measured with ever-increasing precision. The large mass of the top quark allows approximation of these functions through the first two terms of an expansion in M_Z^2/M_t^2. The first three terms of the Taylor series of $\Pi(q^2)$ are evaluated analytically up to order α_s^2. The first two are required to derive the approximation, the third can be used to demonstrate the smallness of the neglected terms. The paper improves earlier results based on the leading term $\propto G_F M_t^2 \alpha_s^2$. Results for the subleading contributions to Δr and the effective mixing angle $\sin^2\bar{\Theta}$ are presented.

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1 Introduction

The precision achieved in recent measurements of electroweak observables [1] has surpassed by far earlier expectations. The predictions for these quantities and the relations between them, which are based on the Standard Model (SM), are strongly affected by radiative corrections. Particularly important are those induced by virtual contributions from the heavy top quark [2]. Their verification provides an important test of the theory and its quantum corrections. The agreement between the value of M_t suggested by the CDF and D0 collaborations [3] of $M_t = 176 \pm 8 \pm 10$ GeV and $M_t = 199^{+19}_{-21} \pm 22$ GeV, respectively, and deduced indirectly from precision measurements of $M_t = 173^{+12+18}_{-13-20}$ GeV [1], constitutes a triumph of the SM and a verification of the quantum corrections with increasing precision. A more refined understanding of these effects is on the agenda, including two- and even the dominant three-loop contributions [4, 5].

As far as contributions from the top-bottom multiplet are concerned, the perturbative results for the self-energies $\Pi_{WW}(q^2), \Pi_{ZZ}(q^2), \Pi_{\gamma\gamma}(q^2)$ and $\Pi_{\gamma Z}(q^2)$ are available in the literature for arbitrary top and bottom masses up to order α_s [6, 7, 8].

This allows evaluation not only of the leading corrections, which are governed by the ρ parameter and increase with M_t^2, but also of the subleading terms. These are required for a complete calculation of Δr (entering the relation between G_F, M_W^2, M_Z^2 and α) or of the effective mixing angle $\sin^2\bar{\Theta}$ (governing asymmetries in Z production and decay). Recently also three loop QCD corrections to the ρ parameter of $\mathcal{O}(G_F M_t^2 \alpha_s^2)$ have been calculated [4, 5]. These, in turn, control the dominant terms of order $G_F M_t^2 \alpha_s^2$ in Δr and $\sin^2\bar{\Theta}$.

The technique described in [4, 5] can be employed to obtain also the Taylor series coefficients of $\Pi(q^2)$ around $q^2 = 0$, in principle to arbitrary orders in q^2 and in second order in α_s. It will be demonstrated below that the two lowest terms in the expansion of Δr in M_Z^2/M_t^2 provide an excellent approximation to the full answer in one- and two-loop approximation, corresponding to α_s^0 and α_s^1 corrections. This justifies the expectation that also in order α_s^2 the leading terms $\propto G_F M_t^2 \alpha_s^2$ plus the subleading terms provide an adequate description of the complete answer for Δr and $\sin^2\bar{\Theta}$. This is verified by calculating the terms $\propto M_Z^2/M_t^2$ which indeed turn out to be negligible. The complete result for Δr and $\sin^2\bar{\Theta}$ to order α_s^2 is therefore at hand.

2 The $M_W - M_Z$ connection and the effective mixing angle

It has become customary to express the magnitude of radiative corrections in the relation between M_W, M_Z, G_F and α through the quantity Δr, defined through [9]

$$M_W^2 = \frac{M_Z^2}{2}\left(1 + \sqrt{1 - \frac{4\pi\alpha}{\sqrt{2} M_Z^2 G_F (1 - \Delta r)}}\right). \tag{1}$$

The influence of a heavy quark doublet can be expressed through the transversal parts of the gauge boson self energies:

$$\Delta r_{tb} = \frac{c^2}{s^2}\mathrm{Re}\left(\frac{\Pi_{WW}(M_W^2)}{M_W^2} - \frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2}\right) + \tilde{\Pi}_{\gamma\gamma}(0) + \frac{1}{M_W^2}\left(\Pi_{WW}(0) - \mathrm{Re}\Pi_{WW}(M_W^2)\right) \quad (2)$$

where $\tilde{\Pi}_{\gamma\gamma}(q^2) = \Pi_{\gamma\gamma}(q^2)/q^2$. In terms of the transversal parts of vector and axial current correlators Π^V and Π^A the building blocks for Δr are given by

$$\begin{aligned}
\Pi_{WW} &= \frac{g^2}{8}\Big[\Pi^V(q^2,m_t,m_b) + \Pi^A(q^2,m_t,m_b)\Big], \\
\Pi_{ZZ} &= \frac{g^2}{16c^2}\sum_{i=t,b}\Big[v_i^2\Pi^{V,i}(q^2,m_i) + \Pi^{A,i}(q^2,m_i)\Big] + \frac{g^2}{16c^2}\Pi^{A,S}(q^2,m_t,m_b), \\
\Pi_{\gamma\gamma} &= g^2s^2\sum_{i=t,b}Q_i^2\Pi^{V,i}(q^2,m_i), \qquad (3)\\
\Pi_{\gamma Z} &= -\frac{g^2 s}{4c}\sum_{i=t,b}Q_i v_i \Pi^{V,i}(q^2,m_i),
\end{aligned}$$

with $v_i = 2I_3^i - 4s^2Q_i$. The sin and cos of the weak mixing angle are denoted by s and c. The "singlet" contribution to the axial current correlator $\Pi^{A,S}$ which originates from double triangle diagrams occurs first in order α_s^2 and has been displayed separately.

Throughout this paper the mass of the bottom quark will be neglected. To circumvent the mass singularity which originates from the bottom loop contribution $\tilde{\Pi}_{\gamma\gamma}(0)$ is replaced by $\tilde{\Pi}_{\gamma\gamma}(q^2 = M_Z^2)$. The difference is accounted for by dispersion relations with input from the actual measurement of $\sigma(e^+e^- \to hadrons)$ in the low energy region [10].

The evaluation of $\Pi(q^2)$ up to three-loop diagrams involving massless diagrams only is performed with the FORM [11] package MINCER [12] implementing an algorithm developed in [13]. For diagrams involving a heavy top quark the approximation based on a Taylor series of $\Pi(q^2)$ around $q^2 = 0$, which is equivalent to an expansion in M_Z^2/M_t^2, leads to an adequate approximation:

$$\begin{aligned}
\Delta\bar{r}_{tb} &\equiv \Delta r_{tb} - \tilde{\Pi}_{\gamma\gamma}(0) + \mathrm{Re}\tilde{\Pi}_{\gamma\gamma}(M_Z^2) \\
&= \frac{c^2}{s^2}\left(\frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2}\right) \qquad (4)\\
&\quad + \frac{c^2}{s^2}\left(\Pi'_{WW}(0) - \Pi'_{ZZ}(0)|_t - \mathrm{Re}\frac{\Pi_{ZZ}(M_Z^2)|_b}{M_Z^2}\right) \\
&\qquad\qquad + \tilde{\Pi}_{\gamma\gamma}(0)|_t + \mathrm{Re}\tilde{\Pi}_{\gamma\gamma}(M_Z^2)|_b - \Pi'_{WW}(0) \\
&\quad + \frac{c^2}{s^2}\left(\Pi''_{WW}(0)\frac{M_W^2}{2} - \Pi''_{ZZ}(0)|_t\frac{M_Z^2}{2}\right) + \tilde{\Pi}'_{\gamma\gamma}(0)|_t M_Z^2 - \Pi''_{WW}(0)\frac{M_W^2}{2} \\
&\quad + \dots
\end{aligned}$$

The first line in this expansion is determined by the ρ parameter, leads to terms $\propto G_F M_t^2$, and has been recently calculated up to order α_s^2.

The second line leads to terms of order $G_F M_Z^2$ and $G_F M_Z^2 \ln M_t^2/M_Z^2$ and will be calculated below up to order α_s^2. The subsequent terms are of order $G_F M_Z^4/M_t^2$. Even after charge and mass renormalization $\Pi(q^2)$ exhibits (in dimensional regularization) $1/\epsilon$ singularities which cancel in the combination (4).

The reliability of the M_Z^2/M_t^2 expansion is demonstrated in Fig. 1 for the one- and two-loop results respectively. The full answer (solid line) is compared to the approximation based on the term quadratic im M_t (dotted line) and the approximation including constant plus $\ln M_Z^2/M_t^2$ terms (dashed line). The latter provides an excellent approximation in the range $M_t > 150$ GeV; corrections of order $G_F M_Z^4/M_t^2$ may safely be ignored (dash-dotted line).

The leading terms of the expansion are given by [14] ($X_t = G_F M_t^2/(8\sqrt{2}\pi^2)$)

$$
\begin{aligned}
\Delta\bar{r}_{tb} &= -3X_t\frac{c^2}{s^2}\Bigg\{1+\frac{\alpha_s}{\pi}C_F\left(-\frac{1}{2}-\zeta(2)\right)+\frac{M_Z^2}{M_t^2}\Bigg[\ln\frac{M_Z^2}{M_t^2}\left(-\frac{2}{3}+\frac{8}{9}s^2\right)+\frac{1}{3}-\frac{16}{27}s^2 \\
&\quad +\frac{\alpha_s}{\pi}C_F\Bigg(\ln\frac{M_Z^2}{M_t^2}\left(-\frac{1}{2}+\frac{2}{3}s^2\right)-2\zeta(3)+\frac{4}{9}\zeta(2)+\frac{1}{2} \\
&\quad +s^2\left(\frac{8}{3}\zeta(3)-\frac{8}{9}\zeta(2)-\frac{5}{3}\right)\Bigg)\Bigg]+\frac{M_Z^4}{M_t^4}\Bigg[-\frac{2}{5}+\frac{103}{90}s^2-s^4 \\
&\quad +\frac{\alpha_s}{\pi}C_F\left(-\frac{523}{3240}+\frac{3}{2}s^2\zeta(2)+\frac{827}{3888}s^2-s^4\zeta(2)-\frac{25}{24}s^4-\frac{1}{2}\zeta(2)\right)\Bigg]\Bigg\}. \qquad (5)
\end{aligned}
$$

The full result is given e.g. in [7, 14].

A second quantity of practical interest is the effective weak mixing angle, which governs in particular the asymmetries [15] in Z boson production and decay. It is related to $\sin^2\Theta \equiv 1 - M_W^2/M_Z^2$ through a correction factor

$$
\sin^2\bar{\Theta} = (1+\Delta\kappa)\sin^2\Theta \qquad (6)
$$

which in turn is influenced by the polarization functions of Eq. (3)

$$
\begin{aligned}
\Delta\kappa_{tb} &= -\frac{c}{s}\frac{\Pi_{\gamma Z}(M_Z^2)}{M_Z^2}-\frac{c^2}{s^2}\mathrm{Re}\left(\frac{\Pi_{WW}(M_W^2)}{M_W^2}-\frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2}\right) \\
&= -3X_t\frac{c^2}{s^2}\Bigg\{-1+\frac{\alpha_s}{\pi}C_F\left(\frac{1}{2}+\zeta(2)\right)+\frac{M_Z^2}{M_t^2}\Bigg[\ln\frac{M_Z^2}{M_t^2}\left(\frac{2}{3}-\frac{4}{9}s^2\right)-\frac{1}{3}+\frac{8}{27}s^2 \\
&\quad +\frac{\alpha_s}{\pi}C_F\Bigg(\ln\frac{M_Z^2}{M_t^2}\left(\frac{1}{2}-\frac{1}{3}s^2\right)+2\zeta(3)-\frac{4}{9}\zeta(2)-\frac{1}{2} \\
&\quad +s^2\left(-\frac{4}{3}\zeta(3)+\frac{4}{9}\zeta(2)+\frac{5}{6}\right)\Bigg)\Bigg]+\frac{M_Z^4}{M_t^4}\Bigg[\frac{2}{5}-\frac{37}{45}s^2+\frac{1}{2}s^4 \\
&\quad +\frac{\alpha_s}{\pi}C_F\left(\frac{523}{3240}-s^2\zeta(2)-\frac{713}{1944}s^2+\frac{1}{2}s^4\zeta(2)+\frac{25}{48}s^4+\frac{1}{2}\zeta(2)\right)\Bigg] \\
&\quad +i\pi\frac{M_Z^2}{M_t^2}\Bigg[-\frac{4}{9}s^2+\frac{16}{27}s^4+\frac{\alpha_s}{\pi}C_F\left(-\frac{1}{3}s^2+\frac{4}{9}s^4\right)\Bigg]\Bigg\}. \qquad (7)
\end{aligned}
$$

The full analytic result and the approximation based on the M_Z/M_t^2 expansion are compared in Fig. 2, adopting the same notation as in Fig. 1. From these figures and from Eqs. (5,7) it is evident that (for $M_t \approx 180$ GeV) the next-to-leading corrections amount to about 25% of the $G_F M_t^2$ terms. The next-to-next-to-leading terms of order $G_F M_Z^2/M_t^2$ are below 1.2%. These considerations justify the restriction of α_s^2 corrections to the first two or at most three terms in the M_Z^2/M_t^2 expansion.

3 Three-Loop Corrections

In addition to the massless diagrams there are two further types of integrals: The evaluation of the derivatives of $\Pi(q^2)$ resulting from diagrams where the massive top quark is coupled to the W or Z is reduced to the evaluation of tadpole integrals discussed in [4, 5]. The derivatives are obtained through Taylor expansion of the respective integrands up to $\mathcal{O}(q^2)$

$$\Pi_{\mu\nu}(q) = g_{\mu\nu}\Pi(q^2) + q_\mu q_\nu \Pi_2(q^2) \tag{8}$$

and projecting out the transverse part. Following [13, 16] the resulting tadpole integrals are subsequently reduced to the master set listed in [4, 5]. Diagrams involving external massless quark loops and internal top loops (and the anomaly graph) are treated with the large mass expansion technique [17].

Setting $C_A = 3, C_F = 4/3$ and $\mu^2 = \bar{m}_t^2$ a fairly compact form for the subleading parts of $\Delta\bar{r}_{tb}$ and $\Delta\kappa_{tb}$ is obtained ($x_t = G_F\bar{m}_t^2/(8\sqrt{2}\pi^2), \bar{l}_Z = \ln M_Z^2/\bar{m}_t^2$):

$$\begin{aligned}
\Delta\bar{r}_{tb}^{\overline{\rm MS}} = & -\frac{c^2}{s^2}\delta\rho_{\overline{\rm MS}} - 3\frac{c^2}{s^2}x_t\frac{M_Z^2}{\bar{m}_t^2} \\
& \Bigg\{\frac{1}{3} + \frac{8}{9}s^2\bar{l}_Z - \frac{16}{27}s^2 - \frac{2}{3}\bar{l}_Z \\
& + \frac{\alpha_s}{4\pi}\left(\frac{88}{9} + \frac{128}{9}s^2\zeta(3) - \frac{128}{27}s^2\zeta(2) + \frac{32}{9}s^2\bar{l}_Z - \frac{496}{27}s^2 - \frac{32}{3}\zeta(3) + \frac{64}{27}\zeta(2) - \frac{8}{3}\bar{l}_Z\right) \\
& + \left(\frac{\alpha_s}{4\pi}\right)^2\Bigg[n_f\Bigg(-\frac{4480}{243} + \frac{256}{27}s^2\zeta(3)\bar{l}_Z - \frac{2944}{81}s^2\zeta(3) + \frac{448}{27}s^2\zeta(2) - \frac{352}{27}s^2\bar{l}_Z + \frac{32}{27}s^2\bar{l}_Z^2 \\
& \quad + \frac{2696}{243}s^2 - \frac{64}{9}\zeta(3)\bar{l}_Z + \frac{2080}{81}\zeta(3) - \frac{176}{27}\zeta(2) + \frac{88}{9}\bar{l}_Z - \frac{8}{9}\bar{l}_Z^2\Bigg) \\
& \quad + \frac{94957}{243} + 856S_2s^2 - 428S_2 + \frac{256}{81}D_3s^2 - \frac{128}{81}D_3 - \frac{4480}{27}s^2\zeta(3)\bar{l}_Z + \frac{12016}{27}s^2\zeta(3) \\
& \quad + \frac{12032}{81}s^2\zeta(4) - \frac{3200}{27}s^2\zeta(5) - \frac{52816}{243}s^2\zeta(2) - \frac{256}{27}s^2B_4 + \frac{688}{3}s^2\bar{l}_Z - \frac{560}{27}s^2\bar{l}_Z^2 \\
& \quad - \frac{24164}{81}s^2 + \frac{1120}{9}\zeta(3)\bar{l}_Z - \frac{8117}{18}\zeta(3) - \frac{6016}{81}\zeta(4) + \frac{800}{9}\zeta(5) \\
& \quad + \frac{22736}{243}\zeta(2) + \frac{128}{27}B_4 - \frac{1400}{9}\bar{l}_Z + \frac{116}{9}\bar{l}_Z^2\Bigg]\Bigg\},
\end{aligned} \tag{9}$$

$$
\begin{aligned}
\Delta\kappa_{tb}^{\overline{\rm MS}} = {} & \frac{c^2}{s^2}\delta\rho_{\overline{\rm MS}} - 3\frac{c^2}{s^2}x_t\frac{M_Z^2}{\bar{m}_t^2} \\
& \Bigg\{ -\frac{1}{3} - \frac{4}{9}s^2\bar{l}_Z + \frac{8}{27}s^2 + \frac{2}{3}\bar{l}_Z \\
& + \frac{\alpha_s}{4\pi}\Bigg(-\frac{88}{9} - \frac{64}{9}s^2\zeta(3) + \frac{64}{27}s^2\zeta(2) - \frac{16}{9}s^2\bar{l}_Z + \frac{248}{27}s^2 + \frac{32}{3}\zeta(3) - \frac{64}{27}\zeta(2) + \frac{8}{3}\bar{l}_Z\Bigg) \\
& + \left(\frac{\alpha_s}{4\pi}\right)^2 \Bigg[n_f\Bigg(\frac{4480}{243} - \frac{128}{27}s^2\zeta(3)\bar{l}_Z + \frac{1472}{81}s^2\zeta(3) - \frac{224}{27}s^2\zeta(2) + \frac{176}{27}s^2\bar{l}_Z \\
& \quad - \frac{16}{27}s^2\bar{l}_Z^2 - \frac{1348}{243}s^2 + \frac{64}{9}\zeta(3)\bar{l}_Z - \frac{2080}{81}\zeta(3) + \frac{176}{27}\zeta(2) - \frac{88}{9}\bar{l}_Z + \frac{8}{9}\bar{l}_Z^2\Bigg) \\
& \quad - \frac{94957}{243} - 428S_2s^2 + 428S_2 - \frac{128}{81}D_3s^2 + \frac{128}{81}D_3 + \frac{2240}{27}s^2\zeta(3)\bar{l}_Z \\
& \quad - \frac{6008}{27}s^2\zeta(3) - \frac{6016}{81}s^2\zeta(4) + \frac{1600}{27}s^2\zeta(5) + \frac{26408}{243}s^2\zeta(2) + \frac{128}{27}s^2B_4 - \frac{344}{3}s^2\bar{l}_Z \\
& \quad + \frac{280}{27}s^2\bar{l}_Z^2 + \frac{12082}{81}s^2 - \frac{1120}{9}\zeta(3)\bar{l}_Z + \frac{8117}{18}\zeta(3) + \frac{6016}{81}\zeta(4) \\
& \quad - \frac{800}{9}\zeta(5) - \frac{22736}{243}\zeta(2) - \frac{128}{27}B_4 + \frac{1400}{9}\bar{l}_Z - \frac{116}{9}\bar{l}_Z^2\Bigg] \\
& + i\pi\Bigg[-\frac{4}{9}s^2 + \frac{16}{27}s^4 + \frac{\alpha_s}{4\pi}\Bigg(-\frac{16}{9}s^2 + \frac{64}{27}s^4\Bigg) \\
& \quad + \left(\frac{\alpha_s}{4\pi}\right)^2 \Bigg(n_f\Bigg(\frac{176}{27}s^2 - \frac{32}{27}s^2\bar{l}_Z - \frac{128}{27}s^2\zeta(3) - \frac{704}{81}s^4 + \frac{128}{81}s^4\bar{l}_Z + \frac{512}{81}s^4\zeta(3)\Bigg) \\
& \quad\quad - \frac{344}{3}s^2 + \frac{560}{27}s^2\bar{l}_Z + \frac{2240}{27}s^2\zeta(3) + \frac{1376}{9}s^4 - \frac{2240}{81}s^4\bar{l}_Z - \frac{8960}{81}s^4\zeta(3)\Bigg)\Bigg]\Bigg\}
\end{aligned}
\tag{10}
$$

with [4]

$$
\begin{aligned}
B_4 &= 16\,\mathrm{Li}_4\left(\frac{1}{2}\right) + \frac{2}{3}\log^4 2 - \frac{2}{3}\pi^2\log^2 2 - \frac{13}{180}\pi^4 = -1.76280\ldots \\
S_2 &= \frac{4}{9\sqrt{3}}\mathrm{Cl}_2\left(\frac{\pi}{3}\right) = 0.260434\ldots \\
D_3 &= -3.02700\ldots.
\end{aligned}
$$

The formula for $\delta\rho_{\overline{\rm MS}}$ can be found in [5].

This result is formulated in terms of the $\overline{\rm MS}$ coupling α_s and the mass $\bar{m}_t$. Employing the two-loop relation between the $\overline{\rm MS}$ mass and the pole mass M_t [18] the $\overline{\rm MS}$ results are easily expressed in terms of M_t. The final result, after setting $n_f = 6$, reads ($\mu^2 = M_t^2$):

$$
\begin{aligned}
\Delta\bar{r}_{tb}^{OS} = {} & -\frac{c^2}{s^2}3X_t\ \Bigg\{ 1 + \frac{M_Z^2}{M_t^2}\Big(0.3333 - 0.6667\,l_Z + (-0.5926 + 0.8889\,l_Z)s^2\Big) \\
& + \left(\frac{M_Z^2}{M_t^2}\right)^2\Big(-0.4 + 1.144s^2 - s^4\Big)
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{\alpha_s}{\pi}\Big[-2.8599+\frac{M_Z^2}{M_t^2}\Big(-1.5640-0.6667\,l_Z+(0.1022+0.8889\,l_Z)s^2\Big)\\
&\quad+\left(\frac{M_Z^2}{M_t^2}\right)^2\Big(-1.312+3.573s^2-3.582s^4\Big)\Big]\\
&+\left(\frac{\alpha_s}{\pi}\right)^2\Big[-14.594+\frac{M_Z^2}{M_t^2}\Big(-17.224+0.08829\,l_Z+0.4722\,l_Z^2\\
&\quad+(22.6367+1.2527\,l_Z-0.8519\,l_Z^2)s^2\Big)+\left(\frac{M_Z^2}{M_t^2}\right)^2\Big(-7.7781-0.07226\,l_Z\\
&\quad+0.004938\,l_Z^2+(21.497+0.05794\,l_Z-0.006584\,l_Z^2)s^2-21.0799s^4\Big)\Big]\Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
\Delta\kappa_{tb}^{OS}=-\frac{c^2}{s^2}3X_t\,\Bigg\{&-1+\frac{M_Z^2}{M_t^2}\Big(-0.3333+0.6667\,l_Z+(0.2963-0.4444\,l_Z)s^2\Big)\\
&+\left(\frac{M_Z^2}{M_t^2}\right)^2\Big(0.4-0.8222s^2+0.5s^4\Big)\\
&+\frac{\alpha_s}{\pi}\Big[2.8599+\frac{M_Z^2}{M_t^2}\Big(1.564+0.6667\,l_Z+(-0.051103-0.4444\,l_Z)s^2\Big)\\
&\quad\left(\frac{M_Z^2}{M_t^2}\right)^2\Big(1.312-2.682s^2+1.791s^4\Big)\Big]\\
&+\left(\frac{\alpha_s}{\pi}\right)^2\Big[14.594+\frac{M_Z^2}{M_t^2}\Big(17.224-0.08829\,l_Z-0.47222\,l_Z^2\\
&\quad+(-11.3184-0.6263\,l_Z+0.4259\,l_Z^2)s^2\Big)+\left(\frac{M_Z^2}{M_t^2}\right)^2\Big(7.7781+0.072263\,l_Z\\
&\quad-0.004938\,l_Z^2+(-16.0186-0.02897\,l_Z+0.003292\,l_Z^2)s^2+10.54s^4\Big)\Big]\\
&+i\frac{M_Z^2}{M_t^2}\Bigg[-1.396s^2+1.862s^4+\frac{\alpha_s}{\pi}(-1.396s^2+1.862s^4)\\
&\quad+\left(\frac{\alpha_s}{\pi}\right)^2\Big[(-1.968+2.676\,l_Z)s^2+(2.6235-3.5682\,l_Z)s^4\\
&\qquad+\frac{M_Z^2}{M_t^2}\Big((-0.09102+0.02069\,l_Z)s^2+(0.1214-0.02758\,l_Z)s^4\Big)\Big]\Bigg]\Bigg\},
\end{aligned}
$$

where $l_Z=\ln M_Z^2/M_t^2$. In these formulae also the terms of order $G_F\alpha_s^2M_Z^4/M_t^2$ are given. As shown in Fig. 1 and 2 and Table 1 their effect of the numerical result is extremely small and can safely be neglected.

In the lowest diagrams of Fig. 1 and 2 the $\mathcal{O}(\alpha_s^2)$ corrections of $\Delta\bar{r}_{tb}$ and $\Delta\kappa_{tb}$ are presented as functions of M_t and the quality of the M_Z^2/M_t^2 expansion is confirmed. The difference between the quadratic term (dotted line) and the constant plus log term (dashed line) amounts to about 25%. Adding the subsequent term proportional M_Z^2/M_t^2 (dashed dotted line) barely affects the answer. The numerical effects on M_W and $\sin^2\bar{\Theta}$ are given in Table 1. The numbers are obtained with the following input data: $\alpha_s(M_t^2)=0.1092$ (corresponding $\alpha_s^{(5)}(M_Z^2)=0.120$), $M_t=175$ GeV, $M_Z=91.188$ GeV, $M_H=300$ GeV,

	M_t^2	M_t^2 + const.	M_t^2 + const. $+1/M_t^2$
$\delta M_W/M_W$ (OS)	0.00677	0.00838	0.00825
$\delta\sin^2\bar{\Theta}/\sin^2\bar{\Theta}$ (OS)	-0.01618	-0.01832	-0.01814
$\delta M_W/M_W$ ($\overline{\rm MS}$)	0.00674	0.00833	0.00821
$\delta\sin^2\bar{\Theta}/\sin^2\bar{\Theta}$ ($\overline{\rm MS}$)	-0.01610	-0.01823	-0.01804

Table 1: Numerical results for the M_t^2, the constant plus logarithmic and the $1/M_t^2$ contributions. For the numerical evaluation of $\delta\sin^2\bar{\Theta}/\sin^2\bar{\Theta}$ only the real part of $\Delta\kappa_{tb}$ is taken.

δM_W in MeV	α_s^0	α_s^1	α_s^2
M_t^2	611.9	-61.3	-10.9
const.	136.6	-6.0	-2.6
$1/M_t^2$	-9.0	-1.0	-0.2

Table 2: The change in M_W separated according to powers in α_s and M_t in the on-shell scheme.

$\alpha = 1/137.04$ and $G_F = 1.16639\,10^{-5}$ GeV^{-2}. In each column the terms of order α_s^0, α_s^1 and α_s^2 belonging to the corresponding expansion in the top mass are added up. For Δr in Eq. (1) we used $\Delta r = \Delta\alpha + \Delta\bar{r}_{tb} + \delta_{rem}$ with $\Delta\alpha = 0.05940$ [10]. δ_{rem} contains all contributions of order α which are not present in the other two pieces and can e.g. be found in [19]. In Table 2 the contributions are listed separately according to powers of α_s and M_t. One observes that the absolute prediction for the W mass is changed by -10.9 MeV if the $\alpha_s^2 M_t^2$ term is added to the full two-loop result. This increases to -13.7 MeV if also the constant and the $1/M_t^2$ suppressed terms terms are added. (For a fictitious top mass of 100 GeV the numbers would be -4.2 MeV and -7.9 MeV respectively.)

Summary: Top mass dependent corrections to relations between electroweak parameters have been calculated up to order α_s^2, with the help of an expansion in M_Z^2/M_t^2. The quality of the approximation has been confirmed in one-, two- and three-loop approximations.

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References

[1] D. Schaile (CERN), CERN-PPE-94-162, talk given at 27th International Conference on High Energy Physics (ICHEP), Glasgow, Scotland, 20-27 Jul 1994.

[2] M. Veltman, Nucl. Phys. B123 (1977) 89.

[3] CDF Collaboration, F. Abe, et al., FERMILAB-PUB-94/022-E, hep-ex/9503002; D0 Collaboration, S. Abachi, et al., FERMILAB-PUB-94/028-E, hep-ex/9503003.

[4] L. Avdeev, J. Fleischer, S. Mikhailov and O. Tarasov, Phys. Lett. B336 (1994) 560, erratum (to appear).

[5] K.G. Chetyrkin, J.H. Kühn and M. Steinhauser, TTP95-03, hep-ph/9502291, accepted for publication in Phys.Lett.B.

[6] A. Djouadi and C. Verzegnassi, Phys. Lett. B195 (1987) 265;
A. Djouadi, Il Nuova Cimento 100A (1988) 357.

[7] B.A. Kniehl, Nucl. Phys. B347 (1990) 65.

[8] B.A. Kniehl, J.H. Kühn and R.G. Stuart, Phys. Lett. B214 (1988) 621.

[9] A. Sirlin, Phys. Rev. D22 (1980) 971.

[10] S. Eidelman and F. Jegerlehner, PSI-PR-95-1, hep-ph/9502298.

[11] J.A.M. Vermaseren, Symbolic Manipulation with FORM, CAN (Amsterdam, 1991).

[12] S.A. Larin, F.V. Tkachov und J.A.M. Vermaseren, Preprint NIKHEF-H/91-18 (1991).

[13] K.G. Chetyrkin and F.V. Tkachov, Nucl. Phys. B192 (1981) 159.

[14] F. Halzen, B.A. Kniehl and M.L. Stong, Z. Phys. C 58 (1993) 119.

[15] W. Hollik, Lectures Given at the 1989 CERN-JINR School of Physics, Egmond-Aan-Zee, Netherlands, Jun 25 - Jul 8, 1989. Published in CERN-JINR School Phys. (1989), pg. 50-102.

[16] D.J. Broadhurst, Z. Phys. C 54 (1992) 54.

[17] For a recent review see e.g.: V.A. Smirnov, MPI-PHT-94-82, hep-th/9412063. Talk given at Workshop on Advances in Nonperturbative and Perturbative Techniques, Ringberg Castle, Germany, 13-19 Nov 1994.

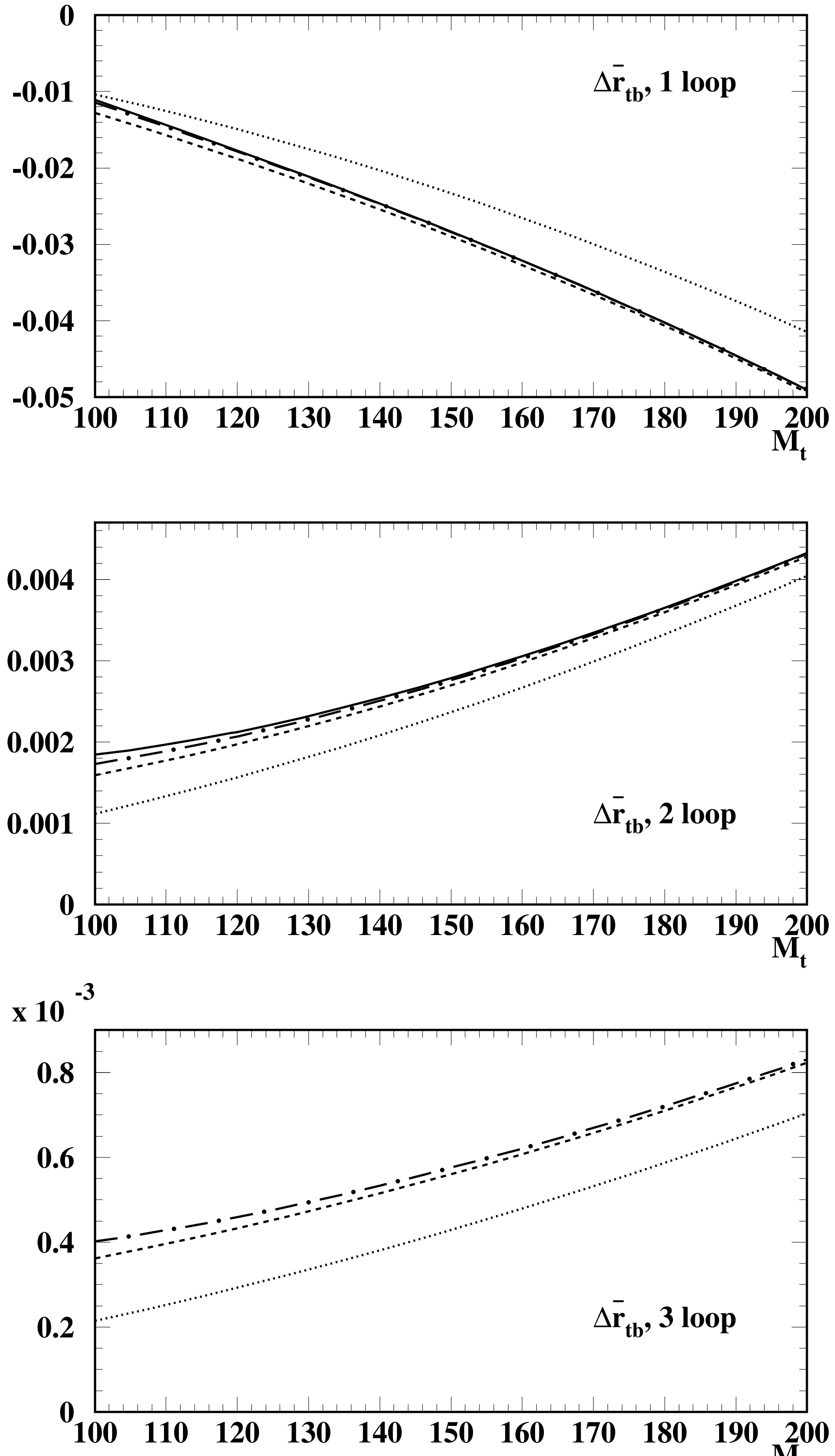
[18] D.J. Broadhurst, W. Grafe, N. Gray and K. Schilcher, Z. Phys. C48 (1990) 673.

[19] G. Burgers and F. Jegerlehner, PSI-PR-89-28, Workshop on Z Physics at LEP, Geneva, Switzerland, Feb 20, 1989. Published in Z Physics at LEP 1, CERN 89-08, pg. 55-88.

Figure Captions

Figure 1: $\Delta\bar{r}_{tb}$ as a function of M_t. The dotted, dashed and dash-dotted curves correspond to an increasing number of terms in the approximation. In the figures a value $s^2 = 0.2321$ is chosen.

Figure 2: $\Delta\kappa_{tb}$ as a function of M_t. The conventions are the same as for $\Delta\bar{r}_{tb}$. Here only the real part is plotted.



$\Delta\bar{r}_{tb}$, 1 loop
0
-0.01
-0.02
-0.03
-0.04
-0.05
100 110 120 130 140 150 160 170 180 190 200
M_t
$\Delta\bar{r}_{tb}$, 2 loop
0.004
0.003
0.002
0.001
0
100 110 120 130 140 150 160 170 180 190 200
M_t
x 10 -3
$\Delta\bar{r}_{tb}$, 3 loop
0.8
0.6
0.4
0.2
0
100 110 120 130 140 150 160 170 180 190 200
M_t

Figure 1:

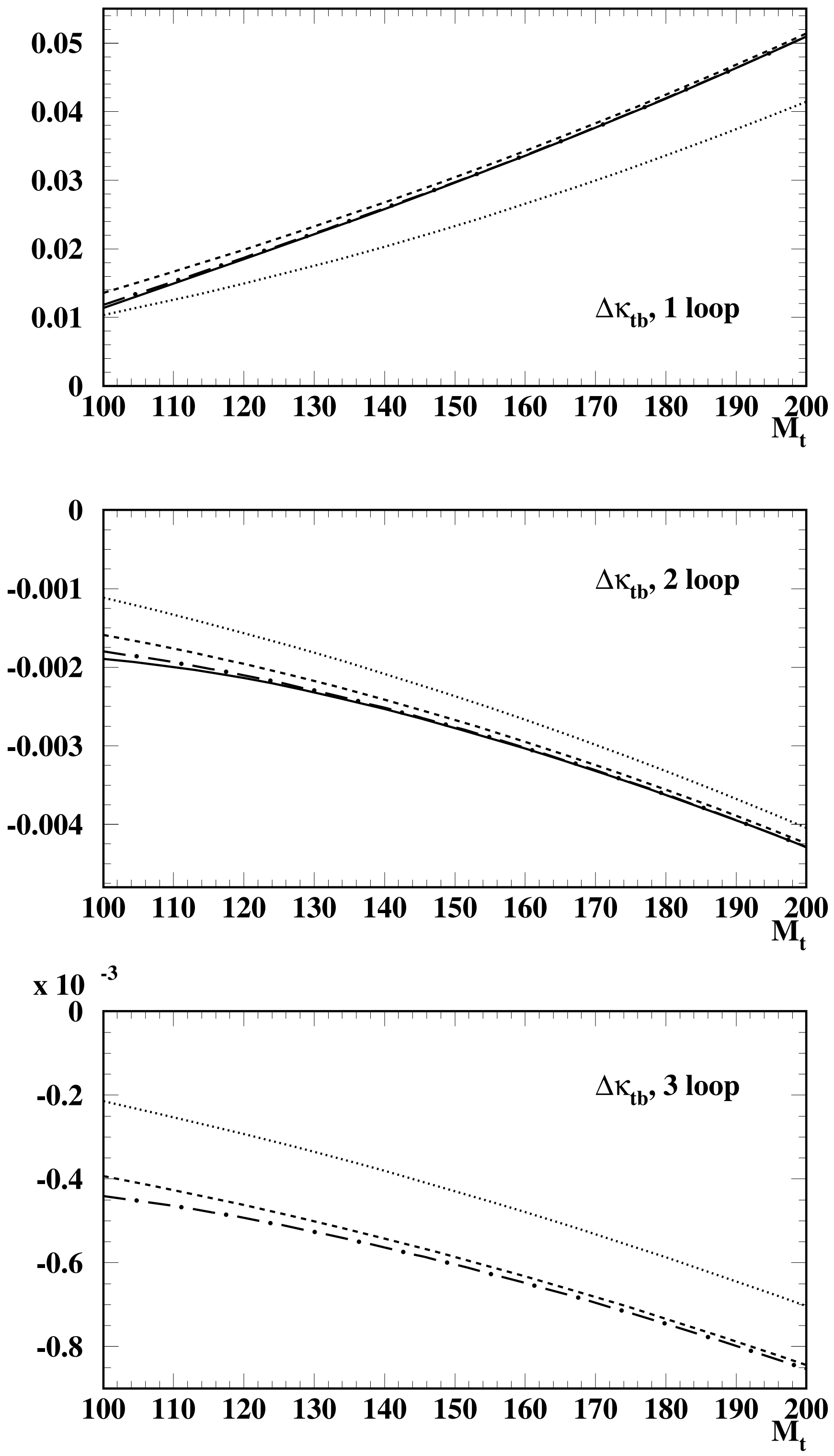

0.05
0.04
0.03
0.02
0.01
0
100 110 120 130 140 150 160 170 180 190 200
M_t
$\Delta\kappa_{tb}$, 1 loop
0
-0.001
-0.002
-0.003
-0.004
100 110 120 130 140 150 160 170 180 190 200
M_t
$\Delta\kappa_{tb}$, 2 loop
x 10^{-3}
0
-0.2
-0.4
-0.6
-0.8
100 110 120 130 140 150 160 170 180 190 200
M_t
$\Delta\kappa_{tb}$, 3 loop

Figure 2: