TTP 95-08
April 1995

$\rho^0 - \omega$ mixing in chiral perturbation theory

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Abstract

In order to calculate the $\rho^0 - \omega$ mixing we extend the chiral couplings of the low-lying vector mesons in chiral perturbation theory to a lagrangian that contains two vector fields. We determine the p^2 dependence of the two-point function and recover an earlier result for the on-shell expression. We discuss the off-shell behaviour of the mixing.

1 Introduction

Chiral perturbation theory (CHPT) [1, 2, 3] is an effective low energy theory that describes first of all the interactions of the pseudoscalar meson octet. They are the Goldstone bosons of the spontaneously broken chiral symmetry in QCD with three light flavours. The chiral lagrangian is expanded perturbatively in the three small quark masses and in the derivatives of the fields. At lowest order p^2 the effective lagrangian is given by the non-linear σ-model coupled to external fields. In [2] the authors include the ρ meson in order to estimate the next to leading order coupling constants in CHPT via resonance exchange, in [4] this work is extended to general meson exchange in chiral $SU(3)$. The couplings of the resonances to the

pseudoscalar mesons are considered at lowest order in the chiral counting and linear in the resonance fields.
In the vector meson sector, (ρ^+, ρ^0, ρ^-) represents a (nearly) degenerated isospin triplet, whereas ω is an isospin singlet. However, isospin is not an exact symmetry, it is broken by the quark mass difference $m_u - m_d$ and the electromagnetic interaction and therefore ρ^0 and ω mix. We calculate this mixing in the framework of CHPT.

Here we determine the lowest order couplings of the pseudoscalar mesons to the vector mesons quadratic in the vector fields. We restrict ourselves to the couplings that we need for $\rho^0 - \omega$ mixing and represent the vector fields in the antisymmetric tensor notation [2, 4]. It has been proven [5] that this representation is equivalent to the vector field formulation, to the description that involves massive Yang-Mills fields [6] and to the model with hidden gauge vector bosons [7].
We give the two-point function for $\rho^0 - \omega$ mixing and find that the on-shell expression corresponds to the result given by Gasser and Leutwyler [8]. Numerically it can be estimated from the decay $\omega \to \rho^0 \to \pi^+\pi^-$. The mixing is dominated by the strong interaction, determined at this order by the quark mass ratio $R = (m_s - \hat{m})/(m_d - m_u)$ with $\hat{m} = (m_u + m_d)/2$. The branching ratio $B(\omega \to \pi^+\pi^-)$ has changed since 1982 from 1.4 to 2.2%, therefore we give an update of the value of R.
The vector mesons in the tensor representation can be related to the usual vector formalism by the definition (see section 2)

$$V_\mu = \frac{1}{M} \partial^\nu V_{\nu\mu}, \tag{1}$$

where M denotes the mass of the particle. With this definition, the couplings of the vector mesons are soft: for a vanishing four-momentum the vector resonances decouple. This implies that the $\rho^0 - \omega$ mixing amplitude has a zero at $p^2 = 0$. O'Connell et al. [9] have shown that a broad class of models contains this feature. However, the decoupling which occurs in two-point functions and in hadronic form factors [10] does not a priori show up in matrix elements without external vector mesons, e.g. nucleon-nucleon scattering with resonance exchange. This point will be discussed in section 4.
In section 2 we present the lagrangian of CHPT at leading order and give the properties of the vector mesons in the antisymmetric tensor notation. In section 3 we determine the couplings quadratic in the resonance fields and pin down the coupling constants by using large N_C arguments [11], the Zweig rule [12], and the quark model counting for the vector meson masses at first order in the quark mass expansion [8]. In section 4 we calculate the two-point function for $\rho^0 - \omega$ mixing and compare our numerical result for the on-shell expression with the calculations found in the literature. We evaluate the quark mass ratio R from the mixing amplitude and discuss the off-shell behaviour of the two-point function and its consequence on

nucleon-nucleon scattering involving $\rho^0 - \omega$ exchange.
For a introduction to the various topics we refer the reader to recent reviews of CHPT [13] and $\rho^0 - \omega$ mixing [14, 15].

2 Field properties and dynamics

The chiral lagrangian can be expanded in derivatives of the Goldstone fields and in the masses of the three light quarks. One derivative counts as a quantity of $O(p)$, the masses are of order $O(p^2)$ and the fields themselves are $O(p^0)$. The effective lagrangian starts at $O(p^2)$, denoted by $\mathcal{L}_2$. It is the non-linear σ-model lagrangian coupled to external fields, respects chiral symmetry $SU(3)_R \times SU(3)_L$ and is invariant under P and C transformations [2, 3],

$$\mathcal{L}_2 = \frac{1}{4} F_0^2 \langle d^\mu U^\dagger d_\mu U + \chi U^\dagger + \chi^\dagger U \rangle. \tag{2}$$

The brackets $\langle \cdots \rangle$ denote the trace in flavour space, U is a unitary 3×3 matrix and incorporates the fields of the eight pseudoscalar mesons,

$$\begin{aligned} U &= exp(i\Phi/F_0), \qquad \det U = 1, \\ \Phi &= \sqrt{2} \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix}. \end{aligned} \tag{3}$$

$d_\mu U$ is a covariant derivative incorporating the external vector and axialvector currents v_μ and a_μ, respectively,

$$d_\mu U = \partial_\mu U - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu), \tag{4}$$

disregarding singlet (axial)vector currents, we put tr $v_\mu =$ tr $a_\mu = 0$. χ represents the coupling of the mesons to the scalar and pseudoscalar currents s and p, respectively, and s incorporates the mass matrix

$$\begin{aligned} \chi &= 2B_0(s + ip), \\ s &= \mathcal{M} + \cdots = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix} + \cdots \end{aligned} \tag{5}$$

F_0 is the pion decay constant in the chiral limit ($\mathcal{M} = 0$), $F_\pi = F_0\,[1 + O(m_q)]$ and B_0 is related to the quark condensate $\langle 0|\bar{u}u|0\rangle = -F_0^2 B_0\,[1 + O(m_q)]$. The transformation properties under $SU(3)_R \times SU(3)_L$ are

$$\begin{array}{rcl} U & \to & g_R U g_L^\dagger \\ v_\mu + a_\mu & \to & g_R(v_\mu + a_\mu)g_R^\dagger + i g_R \partial_\mu g_R^\dagger \\ v_\mu - a_\mu & \to & g_L(v_\mu - a_\mu)g_L^\dagger + i g_L \partial_\mu g_L^\dagger \\ s + ip & \to & g_R(s+ip)g_L^\dagger \\ g_{R,L} & \in & SU(3)_{R,L}. \end{array} \qquad (6)$$

To be consistent in the chiral counting, v_μ and a_μ count as O(p), s and p are of order $O(p^2)$. For the representation of the vector mesons we follow [2, 4, 5], using the antisymmetric tensor notation $V_{\mu\nu}$. The lagrangian for free particles is given by

$$\mathcal{L}^V_{free} = -\frac{1}{2}\partial^\mu V_{\mu\nu}\partial_\rho V^{\rho\nu} + \frac{1}{4}M^2 V_{\mu\nu}V^{\mu\nu}, \qquad (7)$$

from where we derive the equation of motion,

$$\partial^\mu\partial_\rho V^{\rho\nu} - \partial^\nu\partial_\rho V^{\rho\mu} + M^2 V^{\mu\nu} = 0, \qquad (8)$$

and the free propagator

$$\begin{array}{c} \langle 0|T\, V_{\mu\nu}(x)V_{\rho\sigma}(y)|0\rangle = i\int \frac{d^4k}{(2\pi)^4} e^{-ik(x-y)}\Delta_{\mu\nu\rho\sigma}, \\ \begin{array}{rcl} \Delta_{\mu\nu\rho\sigma} & = & \frac{1}{M^2}\left(G_{\mu\nu\rho\sigma} + \frac{1}{M^2-k^2}P_{\mu\nu\rho\sigma}\right) \\ G_{\mu\nu\rho\sigma} & = & g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho} \\ P_{\mu\nu\rho\sigma} & = & g_{\mu\rho}k_\nu k_\sigma - g_{\mu\sigma}k_\nu k_\rho - g_{\nu\rho}k_\mu k_\sigma + g_{\nu\sigma}k_\mu k_\rho. \end{array} \end{array} \qquad (9)$$

This corresponds to the normalization $\langle 0|V_{\mu\nu}|V(p)\rangle = i(p_\mu\epsilon_\nu - p_\nu\epsilon_\mu)/M$, where ϵ_μ represents the polarization vector. With the definition

$$V_\mu = \frac{1}{M}\partial^\nu V_{\nu\mu} \qquad (10)$$

we obtain from (8) the Proca equation, $\partial_\rho(\partial^\rho V^\mu - \partial^\mu V^\rho) + M^2 V^\mu = 0$, and the two-point function

$$\langle 0|T\, V_\mu(x)V_\nu(y)|0\rangle = i\int \frac{d^4k}{(2\pi)^4} e^{-ik(x-y)}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{M^2}\right)\frac{1}{M^2-k^2}. \qquad (11)$$

The normalization is given by $\langle 0|V_\mu|V(p)\rangle = \epsilon_\mu$.

The fields $V_{\mu\nu}$ carry non-linear realizations of $SU(3)$, the transformation under the chiral group $G = SU(3)_R \times SU(3)_L$ is [4]

$$\begin{array}{rcll} V_{\mu\nu} & \xrightarrow{G} & h(\varphi)V_{\mu\nu}h^\dagger(\varphi) & \text{octet}, \\ \omega_{1\,\mu\nu} & \xrightarrow{G} & \omega_{1\,\mu\nu} & \text{singlet}. \end{array} \qquad (12)$$

The non-linear realization $h(\varphi)$ is defined by determining the action of G on an element $u(\varphi)$ of the coset space $[SU(3)_R \times SU(3)_L]/SU(3)_V$ [16]

$$u(\varphi) \xrightarrow{G} g_R u(\varphi) h^\dagger(\varphi) = h(\varphi) u(\varphi) g_L^\dagger, \tag{13}$$

where φ represents the Goldstone bosons and $U = u^2(\varphi)$. For the octet part, a covariant derivative is defined as

$$\nabla^\rho V_{\rho\mu} = \partial^\rho V_{\rho\mu} + [\Gamma^\rho, V_{\rho\mu}] \tag{14}$$

with

$$\Gamma^\rho = \frac{1}{2}\left\{u^\dagger[\partial^\rho - i(v^\rho + a^\rho)]u + u[\partial^\rho - i(v^\rho - a^\rho)]u^\dagger\right\} \tag{15}$$

and $V_{\mu\nu}$ in the matrix notation

$$V_{\mu\nu} = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0_{\mu\nu} + \frac{1}{\sqrt{6}}\omega_{8\,\mu\nu} & \rho^+_{\mu\nu} & K^{*+}_{\mu\nu} \\ \rho^-_{\mu\nu} & -\frac{1}{\sqrt{2}}\rho^0_{\mu\nu} + \frac{1}{\sqrt{6}}\omega_{8\,\mu\nu} & K^{*0}_{\mu\nu} \\ K^{*-}_{\mu\nu} & \bar{K}^{*0}_{\mu\nu} & -\frac{2}{\sqrt{6}}\omega_{8\,\mu\nu} \end{pmatrix}. \tag{16}$$

The covariant derivative is transformed like the field itself

$$\nabla^\rho V_{\rho\mu} \xrightarrow{G} h(\varphi)\nabla^\rho V_{\rho\mu} h^\dagger(\varphi). \tag{17}$$

Denoting the octet (singlet) mass in the chiral limit by M_V (M_{ω_1}), the kinetic part of the lagrangian takes the form

$$\begin{aligned} \mathcal{L}^V_{kin} &= -\frac{1}{2}\langle\nabla^\rho V_{\rho\mu}\nabla_\sigma V^{\sigma\mu} - \frac{1}{2}M_V^2 V_{\mu\nu}V^{\mu\nu}\rangle \\ &\quad -\frac{1}{2}\partial^\rho\omega_{1\,\rho\mu}\,\partial_\sigma\omega_1^{\sigma\mu} + \frac{1}{4}M^2_{\omega_1}\omega_{1\,\mu\nu}\omega_1^{\mu\nu}. \end{aligned} \tag{18}$$

The interaction of the pseudoscalar mesons with one vector meson starts at order $O(p^2)$. The interaction lagrangian contains the octet fields only, there is no coupling to the singlet at this order [4],

$$\mathcal{L}^V_2 = \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{iG_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle, \tag{19}$$

where

$$\begin{aligned} f_+^{\mu\nu} &= uF_L^{\mu\nu}u^\dagger + u^\dagger F_R^{\mu\nu}u \\ F_{R,L}^{\mu\nu} &= \partial^\mu(v^\nu \pm a^\nu) - \partial^\nu(v^\mu \pm a^\mu) - i[v^\mu \pm a^\mu, v^\nu \pm a^\nu] \\ u^\mu &= iu^\dagger d^\mu U u^\dagger = u^{\dagger\,\mu}. \end{aligned} \tag{20}$$

We refer to [4] for a complete list of all the terms that can couple to the resonances and their properties under P and C transformations. F_V and G_V are real coupling constants that are not restricted by chiral symmetry [5]. For later use we include the electromagnetic interactions in $\mathcal{L}_2^V$ by adding the photon field A_μ to the vector current,

$$v_\mu \pm a_\mu \to v_\mu + QA_\mu \pm a_\mu, \tag{21}$$

where Q is the charge matrix of the three light quarks, $Q = e\,\mathrm{diag}(2/3, -1/3, -1/3)$. The corresponding kinetic part of the photons reads

$$\mathcal{L}_{kin}^\gamma = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} (\partial_\mu A^\mu)^2 \tag{22}$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and the gauge fixing parameter chosen to be $\lambda = 1$.

3 Interaction quadratic in the vector mesons

In this section we construct the lagrangian with two vector fields coupled to the pseudoscalar mesons. We restrict ourselves to the part that is relevant for $\rho^0 - \omega$ mixing at lowest order,

$$\begin{aligned} \mathcal{L}_2^{\rho\omega} &= v_8 \langle V_{\mu\nu} V^{\mu\nu} \chi_+ \rangle + \tilde{v}_8 \langle V_{\mu\nu} V^{\mu\nu} \rangle \langle \chi_+ \rangle \\ &\quad + v_{18}\, \omega_{1\,\mu\nu} \langle V^{\mu\nu} \chi_+ \rangle + v_1\, \omega_{1\,\mu\nu}\, \omega_1^{\mu\nu} \langle \chi_+ \rangle, \end{aligned} \tag{23}$$

where $\chi_+ = u^\dagger \chi u^\dagger + u \chi^\dagger u$. In order to pin down the coupling constants in $\mathcal{L}_2^{\rho\omega}$ we make the following assumptions.

(i) In the large N_C limit [11] the product of two traces is suppressed by at least one power of N_C. Therefore we will put $\tilde{v}_8 = 0$. Furthermore, for $N_C \to \infty$ the octet and singlet mesons are degenerated and thus $M_V = M_{\omega_1}$.

(ii) The Zweig rule [12] implies that the ω-meson is free of $s\bar{s}$, the $\phi - \omega$ mixing is ideal,

$$\begin{aligned} |\phi\rangle &= \cos\theta_V |\omega_8\rangle - \sin\theta_V |\omega_1\rangle, \\ |\omega\rangle &= \sin\theta_V |\omega_8\rangle + \cos\theta_V |\omega_1\rangle \end{aligned} \tag{24}$$

with $\tan\theta_V = 1/\sqrt{2}$. On the same footing we assume that there is no $\rho^0 - \phi$ mixing in the lagrangian $\mathcal{L}_2^{\rho\omega}$.

(iii) For the masses of the vector mesons at order $O(p^2)$ we invoke the quark counting rule [8]. We keep the eigenstates of the kinetic lagrangian $\mathcal{L}_{kin}^V$, there are no first order mass shifts from $\rho^0 - \omega$ mixing. In addition we work (for the masses only) in the isospin limit $\hat{m} = m_u = m_d$,

$$\begin{aligned} M_\rho &= M_\omega = M_V + 2\hat{m}, \\ M_{K^*} &= M_V + \hat{m} + m_s, \\ M_\phi &= M_V + 2m_s. \end{aligned} \tag{25}$$

The nearly flavour independent differences [17]

$$M_\rho^2 - M_\pi^2 = 0.58 \text{ GeV}^2, \qquad M_{K^*}^2 - M_K^2 = 0.55 \text{ GeV}^2, \tag{26}$$

and the lowest order mass formulae for the Goldstone bosons

$$M_\pi^2 = 2B_0\hat{m}, \qquad M_K^2 = B_0(\hat{m} + m_s), \tag{27}$$

lead to the relation $M_V \simeq B_0/2$. Taking all the assumptions into account, we arrive at

$$v_8 = \frac{1}{8}, \qquad \tilde{v}_8 = 0, \qquad v_{18} = \frac{1}{4\sqrt{3}}, \qquad v_1 = \frac{1}{24}. \tag{28}$$

The lagrangian takes the simple form (omitting terms that are not relevant for $\rho^0 - \omega$ mixing)

$$\begin{aligned} \mathcal{L}_{kin}^V + \mathcal{L}_2^{\rho\omega} &= -\frac{1}{2}\partial^\lambda \rho^0_{\lambda\mu}\, \partial_\sigma \rho^{0\,\sigma\mu} + \frac{1}{4}M_\rho^2 \rho^0_{\mu\nu}\rho^{0\,\mu\nu} \\ &\quad -\frac{1}{2}\partial^\lambda \omega_{\lambda\mu}\, \partial_\sigma \omega^{\sigma\mu} + \frac{1}{4}M_\omega^2\, \omega_{\mu\nu}\omega^{\mu\nu} + M_\rho(m_u - m_d)\rho^0_{\mu\nu}\omega^{\mu\nu}, \end{aligned} \tag{29}$$

where we have replaced M_V by M_ρ in the interaction term, but kept in the mass term the different notation for M_ω and M_ρ.

4 Mixing amplitude

The contributions to lowest order $\rho^0 - \omega$ mixing are the contact term in (29) and the one-photon exchange from the lagrangian $\mathcal{L}_2^V$ linear in the vector fields in (19). The Fourier transform of the two-point function in the tensor notation has the form

$$\begin{aligned} & i\int d^4x e^{ipx}\langle 0|T\, \rho^0_{\mu\nu}(x)\omega_{\rho\sigma}(0)e^{i\int d^4y\{\mathcal{L}_2^V + \mathcal{L}_2^{\rho\omega}\}}|0\rangle = \\ & \quad \frac{2M_\rho(m_u - m_d)}{M_\rho^2 M_\omega^2}\left\{G_{\mu\nu\rho\sigma} + \left[\frac{1}{M_\rho^2 - p^2} + \frac{M_\rho^2}{(M_\rho^2 - p^2)(M_\omega^2 - p^2)}\right]P_{\mu\nu\rho\sigma}\right\} \\ & \quad + \frac{1}{3}\frac{e^2F_V^2}{(M_\rho^2 - p^2)(M_\omega^2 - p^2)p^2}P_{\mu\nu\rho\sigma}. \end{aligned} \tag{30}$$

For the calculation of the on-shell amplitude, we consider the decay $\omega \to \rho^0 \to \pi^+\pi^-$. We find

$$\begin{aligned} \Gamma(\omega \to \pi^+\pi^-) &= \frac{\Theta_{\rho\omega}^2 \Gamma(\rho^0 \to \pi^+\pi^-)}{|M_\omega^2 - M_\rho^2 - i(M_\omega \Gamma_\omega - M_\rho \Gamma_\rho)|^2} \\ &\simeq \frac{\Theta_{\rho\omega}^2}{4M_\rho^2} \cdot \frac{\Gamma(\rho^0 \to \pi^+\pi^-)}{(M_\omega - M_\rho)^2 + \frac{1}{4}(\Gamma_\omega - \Gamma_\rho)^2}, \end{aligned} \tag{31}$$

where we introduced the widths Γ_ρ and Γ_ω in the propagator and

$$\begin{aligned} \Gamma(\rho^0 \to \pi^+\pi^-) &= \frac{1}{48\pi} \frac{G_V^2 M_\rho^3}{F_0^4} \left(1 - \frac{4M_\pi^2}{M_\rho^2}\right)^{3/2}, \\ \Theta_{\rho\omega} &= 2M_\rho(m_u - m_d) + \frac{1}{3} e^2 F_V^2. \end{aligned} \tag{32}$$

The amplitude $\Theta_{\rho\omega}$ has already been determined in [8], where the strong interaction part was derived in a quantum mechanical approach,

$$\Theta_{\rho\omega} = 2M_\rho \left[-\frac{m_d - m_u}{m_s - \hat{m}} (M_{K^*} - M_\rho) + e^2 \frac{F_\rho F_\omega}{2M_\rho} \right]. \tag{33}$$

If we identify $F_\rho = F_V$, $F_\omega = \frac{1}{3} F_V$ and insert the masses for the vector mesons from (25) we get the same expression for $\Theta_{\rho\omega}$. With the experimental decay widths [17] we find the value

$$\Theta_{\rho\omega} = (-3.91 \pm 0.30) \times 10^{-3}\ \mathrm{GeV}^2. \tag{34}$$

The sign is determined from the relative phase of the ω and ρ amplitudes in $e^+e^- \to \pi^+\pi^-$ near M_ρ [18]. The error in (34) is entirely due to the uncertainty in the decay width $\Gamma(\omega \to \pi^+\pi^-)$. If we neglect the mass difference $(M_\omega - M_\rho)$ and the width Γ_ω in (31), the mixing amplitude decreases to $\Theta_{\rho\omega} = -4.08 \times 10^{-3}$ GeV^2. Our result in (34) is in agreement with the values quoted in the literature, with the exception of the recent result of Friedrich and Reinhardt [21],

$\Theta_{\rho\omega}$ $\left[\times 10^{-3}\ \mathrm{GeV}^2\right]$	references
-4.52 ± 0.60	Coon and Barrett [19]
-3.74 ± 0.30 -4.23 ± 0.68 -3.67 ± 0.30	Bernicha et al. [20]
-3.97 ± 0.20	O'Connell et al. [15]
-4.85	Friedrich and Reinhardt [21]

(35)

From the expression in (33), Gasser and Leutwyler determined the quark mass ratio $R = (m_s - \hat{m})/(m_d - m_u)$ to $R = 44 \pm 3$ [8]. The branching ratio $B(\omega \to \pi^+\pi^-)$ changed in the meantime from 1.4 to 2.2%, motivation enough to give an update of R. The value for F_ρ can be obtained from the decay $\rho^0 \to e^+e^-$,

$$\begin{aligned} \Gamma(\rho^0 \to e^+e^-) &= \frac{1}{12\pi}\frac{e^4 F_\rho^2}{M_\rho}, \\ F_\rho &= 152 \pm 4 \text{ MeV}. \end{aligned} \tag{36}$$

This leads to the estimate

$$R = 40.7 \pm 3.0, \tag{37}$$

where we assumed again that $F_\rho/F_\omega = 3$, as it turns out from the lagrangian $\mathcal{L}_2^V$ in (19). Experimentally, this ratio is given by [17]

$$\frac{F_\rho}{F_\omega} = \left[\frac{M_\rho \Gamma(\rho^0 \to e^+e^-)}{M_\omega \Gamma(\omega \to e^+e^-)}\right]^{1/2} = 3.31 \pm 0.15, \tag{38}$$

which changes the meanvalue in (37) to $R = 41.3$. At this stage, there is poor information about the uncertainties in the contribution from the strong interaction part. We can ask whether or not the assumption in (25) is still a good approximation beyond leading order in the quark mass expansion. For comparison we consider the kaon and nucleon masses. One finds at the one loop level [3, 22, 23]

$$\begin{aligned} \frac{\partial M_K}{\partial \hat{m}} = \frac{\partial M_K}{\partial m_s} &= \frac{M_K}{2(\hat{m} + m_s)}(1 \pm 0.1), \\ \hat{m}\frac{\partial M_N}{\partial \hat{m}} = \sigma_{\pi N} &= \sigma_{\pi N}^{tree}\left(1 + \frac{6.8 \text{ MeV} + 0.9 \times 10^{-2} \overset{\circ}{m}}{\sigma_{\pi N}^{tree}}\right). \end{aligned} \tag{39}$$

The second entry in the bracket shows the correction from the one-loop contribution. For $\sigma_{\pi N}$ we used a linear approximation (in $\overset{\circ}{m}$) of the result given in [23]. $\overset{\circ}{m}$ denotes the mass of the nucleon in the chiral limit and $\sigma_{\pi N}^{tree} = (92.7 \text{ MeV} - 7.4 \times 10^{-2} \overset{\circ}{m})$. In contrast to the kaon mass, the one-loop contribution to the σ term is not small, it is between 32%($\overset{\circ}{m} = 700$ MeV) and 57%(900 MeV). We conclude that for the vector mesons the mass formula (25) *could* underestimate the valence quark content at next to leading order by an overall factor 1.5. The meanvalue of the quark mass ratio would change to $R \sim 27$. We leave this guess as it stands, we only take the electromagnetic deviations from (38) into account, but keep in mind that there is a strong dependence on the assumption in (25). We correct the value for R by increasing the error bar and our estimate finally reads

$$R = 41 \pm 4. \tag{40}$$

For the discussion of the off-shell mixing we consider again the two-point function in (30). With the definition in (10) we arrive at

$$
\begin{aligned}
& i \int d^4 x e^{ipx} \langle 0 | T \, \rho^0_\mu(x) \omega_\nu(0) e^{i\int d^4 y \{\mathcal{L}_2^V + \mathcal{L}_2^{\rho\omega}\}} | 0 \rangle = \\
& \qquad \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{\Theta(p^2)}{(M_\rho^2 - p^2)(M_\omega^2 - p^2)}, \\
\Theta(p^2) \quad & = \quad \frac{p^2}{M_\rho M_\omega} \Theta_{\rho\omega},
\end{aligned}
\tag{41}
$$

where $\Theta_{\rho\omega}$ is given in (32). $\Theta(p^2)$ contains a zero at $p^2 = 0$ and is positive in the spacelike region. It has been shown by O'Connell et al. [9] that in a wide range of models the $\rho^0 - \omega$ mixing amplitude must vanish at $p^2 = 0$. Still we cannot conclude that matrix elements involving ρ^0 and ω as intermediate states show the same decoupling. Consider nucleon-nucleon scattering, where the baryons are again described by a 3×3 matrix that transforms under $SU(3)_R \times SU(3)_L$ in the same way as the meson resonances [24],

$$
B \xrightarrow{G} h(\varphi) B \, h^\dagger(\varphi). \tag{42}
$$

Chiral symmetry and the invariance under P and C transformations lead to the interaction lagrangian $\mathcal{L}_0^{B\bar{B}V}$ at lowest order with two baryons $B, \bar{B} = B^\dagger \gamma^0$ and one vector meson,

$$
\mathcal{L}_0^{B\bar{B}V} = a_1 \sigma^{\mu\nu} \langle \bar{B} \{ V_{\mu\nu}, B \} \rangle + a_2 \sigma^{\mu\nu} \omega_{1\,\mu\nu} \langle \bar{B} B \rangle, \tag{43}
$$

where we leave the coupling constants a_1, a_2 undetermined, since we are interested in the qualitative picture only and neglect the mass difference $(M_\rho - M_\omega)$ and the electromagnetic contribution in the following. The amplitude of nucleon-nucleon scattering with resonance exchange and $\rho^0 - \omega$ mixing from (30) has the form (see [24] for conventions)

$$
T_{NN}^{\rho\omega}(t) \sim a_{NN} \left(2 + \frac{t(2M_\rho^2 - t)}{(M_\rho^2 - t)^2} \right) = a_{NN} \frac{(M_\rho^2 - t)^2 + M_\rho^4}{(M_\rho^2 - t)^2}, \tag{44}
$$

with a_{NN} a flavour dependent constant (including $\Theta_{\rho\omega}$) and t the momentum transfer. We see a t dependence in the numerator which reaches its minimum at $t = M_\rho^2$ and is positive everywhere, therefore it does not change the sign in the spacelike region. Though the mixing amplitude $\Theta(p^2)$ in (41) is linear in the momentum squared, the numerator in the scattering amplitude (44) is not. However, the change in the numerator moving from $t = M_\rho^2$ to $t < 0$ is not negligible, a result which was first claimed by Goldmann et al. [25].

Acknowledgements

I thank A.Bramon, J.Gasser and H.Genz for helpful discussions, and U.Bürgi for providing me with numerical results of his diploma thesis.

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