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Corrections of order $\mathcal{O}(G_F M_t^2 \alpha_s^2)$ to the ρ parameter

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Abstract

The three-loop QCD corrections to the ρ parameter from top and bottom quark loops are calculated. The result differs from the one recently calculated by Avdeev et al. As function of the pole mass the numerical value is given by $\delta\rho = \frac{3G_F M_t^2}{8\sqrt{2}\pi^2}(1 - 2.8599\,\frac{\alpha_s}{\pi} - 14.594\,(\frac{\alpha_s}{\pi})^2)$.

1 Introduction

The precision of electroweak observables measured at LEP, SLC and the TEVATRON has stimulated a variety of theoretical calculations. These are required to match the experimental accuracy and to pin down the parameters of the Standard Model, in particular the top and the Higgs mass and to search for new physics.

A cornerstone in this analysis is the evaluation of top quark contributions to the ρ parameter. With the high value of M_t as suggested by the CDF-collaboration [1] and the strong sensitivity of ρ to small variations of M_t through the quadratic dependence [2] precise theoretical predictions become mandatory.

Top mass corrections to the $Zb\bar{b}$ vertex, the only other place with a strong dependence on M_t [3, 4], are specific to this reaction, while those to the ρ parameter enter numerous relations between observables. This is the second justification for a precision calculation.

In addition to the $\mathcal{O}(G_F M_t^2 \alpha_s)$ two loop contribution [5] (for a related calculation based on dispersion relations see [6], for a recent review see [7]), the two-loop electroweak corrections have also been evaluated, for vanishing [8] and even for arbitrary M_H [9].

In this paper the three-loop result of order $G_F M_t^2 \alpha_s^2$ is presented. A similar calculation has been performed by Avdeev, Fleischer, Mikhailov and Tarasov [10]. However, our calculation disagrees with their formula. In the following section 2 details of the calculation will be described, in section 3 the result will be presented and a brief numerical discussion given.

2 The Calculation

Quantum corrections to the ρ parameter can be connected to the gauge boson self-energies through

$$\delta\rho = \frac{\Pi^Z(0)}{M_Z^2} - \frac{\Pi^W(0)}{M_W^2}. \tag{1}$$

Here $\Pi(0)$ denotes the transverse part of the polarisation tensor $\Pi^{\mu\nu}(q)$ for vanishing momentum q. The evaluation of these self-energy diagrams is performed for $m_b = 0$ and within the framework of dimensional regularisation. Large intermediate expressions are treated with the help of FORM 2.0 [11]. For $D \neq 4$ anticommuting γ_5 was used, except for the double triangle diagram. In order to evaluate this diagram, which is related to the axial anomaly, the definition of 't Hooft and Veltman [12], formalized in [13], was applied. Its contribution is finite and the result coincides with a previous calculation [14] where $D = 4$ from the very beginning. A covariant gauge with arbitrary gauge parameter for the gluon propagator was chosen.

The tadpole integrals required to calculate the one- or two-loop corrections are easy to evaluate even for arbitrary powers of the propagators. This does not hold true for the three-loop case. After performing the traces the reduction to scalar integrals is performed by decomposing the scalar products of the numerator in appropriate combinations of the denominator. Subsequently, recurrence relations provided by the integration-by-parts (IP) method [15] are used in order to reduce every scalar Feynman integral to a small number of so-called master diagrams which have to be calculated explicitly. The IP method was first applied in [16] to three-loop tadpole integrals. There the subclass of those diagrams which contain a continuous massive quark line and which are relevant for the Z boson self-energy was considered. For the calculation of the ρ parameter the method has to be extended to a second class of integrals originating from the W self-energies. One thus arrives at three master integrals

$$
\begin{aligned}
&\iiint \frac{d^Dp\, d^Dk\, d^Dl}{(\pi^{D/2})^3} \frac{1}{l^2k^2(M^2+p^2)(M^2+(p+k)^2)(M^2+(p+l)^2)(M^2+(p+k+l)^2)} \\
&= \left(\frac{\mu^2}{M^2}\right)^{3\epsilon} \left[\frac{2}{\epsilon}\zeta(3) + 6\zeta(3) - 9\zeta(4) + 2B_4\right] \\
&\iiint \frac{d^Dp\, d^Dk\, d^Dl}{(\pi^{D/2})^3} \frac{1}{(M^2+p^2)(M^2+k^2)(M^2+l^2)(p+k+l)^2} \\
&= \left(\frac{\mu^2}{M^2}\right)^{3\epsilon} M^4 \left[\frac{1}{\epsilon^3} + \frac{15}{4\epsilon^2} + \frac{3}{2\epsilon}\zeta(2) + \frac{65}{8\epsilon} + \frac{81}{4}S_2 - \zeta(3) + \frac{45}{8}\zeta(2) + \frac{135}{16}\right] \\
&\iiint \frac{d^Dp\, d^Dk\, d^Dl}{(\pi^{D/2})^3} \frac{1}{l^2k^2p^2(M^2+(p+k)^2)(M^2+(p+l)^2)(M^2+(p+k+l)^2)} \\
&= \left(\frac{\mu^2}{M^2}\right)^{3\epsilon} \left[\frac{2}{\epsilon}\zeta(3) + D_3\right]
\end{aligned}
$$

(The same variables and conventions as those of [10] are adopted. Following standard $\overline{\mathrm{MS}}$ practice we discarded terms proportional to γ_E and $\log 4\pi$ in the r.h.s.) The first master integral has been calculated analytically in [16]; the results for the last two integrals are given [10] (D_3 is presently only known numerically.). We checked the result for B_4 numerically. For S_2 we reproduced the analytical result. The evaluation of D_3 is described in [17]. Employing a different method, we obtain the result given below, which is consistent with [17]. The values for the constants B_4, S_2 and D_3 are as follows:

$$
\begin{aligned}
B_4 &= 16\,\mathrm{Li}_4\left(\frac{1}{2}\right) + \frac{2}{3}\log^4 2 - \frac{2}{3}\pi^2\log^2 2 - \frac{13}{180}\pi^4 = -1.76280\ldots \\
S_2 &= \frac{4}{9\sqrt{3}}\mathrm{Cl}_2\left(\frac{\pi}{3}\right) = 0.260434\ldots \\
D_3 &= -3.02700\ldots
\end{aligned}
$$

3 Results and Discussion

After performing mass and charge renormalization in the $\overline{\mathrm{MS}}$ scheme the following result for the W boson propagator is obtained:

$$
\begin{aligned}
\Pi^W(0) = {} & 12 x_t M_W^2 \\
& \Bigg\{ -\frac{1}{2\epsilon} - \frac{1}{4} - \frac{1}{2}l + \frac{\alpha_s}{4\pi} C_F \Big(\frac{3}{2\epsilon^2} - \frac{5}{4\epsilon} - \frac{13}{8} + \zeta(2) - l - \frac{3}{2}l^2 \Big) \\
& + \Big(\frac{\alpha_s}{4\pi}\Big)^2 \Bigg[C_F \Big(-\frac{44}{3} - \frac{243}{2}S_2 + \frac{92}{3}\zeta(3) + \frac{7}{3}\zeta(2) \Big) \\
& + C_F^2 \Big(-\frac{3}{\epsilon^3} + \frac{3}{\epsilon^2} - \frac{6}{\epsilon}\zeta(3) + \frac{119}{24\epsilon} + \frac{1025}{72} + \frac{1053}{4}S_2 - D_3 - 18\zeta(3)l - \frac{379}{3}\zeta(3) \\
& \qquad + 26\zeta(4) + 6\zeta(2)l + \frac{259}{18}\zeta(2) + \frac{95}{8}l + \frac{21}{4}l^2 - 3l^3 - 8B_4 \Big) \\
& + C_F C_A \Big(-\frac{11}{6\epsilon^3} + \frac{83}{12\epsilon^2} + \frac{3}{\epsilon}\zeta(3) - \frac{77}{12\epsilon} - \frac{869}{48} - \frac{1053}{8}S_2 + \frac{1}{2}D_3 + 9\zeta(3)l \\
& \qquad + 47\zeta(3) - 21\zeta(4) + \frac{11}{3}\zeta(2)l + \frac{73}{6}\zeta(2) - \frac{137}{8}l - \frac{119}{12}l^2 - \frac{11}{6}l^3 + 4B_4 \Big) \\
& + C_F n_f \Big(\frac{1}{3\epsilon^3} - \frac{5}{6\epsilon^2} + \frac{2}{3\epsilon} + \frac{73}{24} + 4\zeta(3) - \frac{2}{3}\zeta(2)l - \frac{7}{3}\zeta(2) + \frac{9}{4}l + \frac{7}{6}l^2 + \frac{1}{3}l^3 \Big) \Bigg] \Bigg\}
\end{aligned} \tag{2}
$$

The coefficient of $C_F^2\zeta(4)$ differs from the recent result of [10]. Its value 26 has to be compared with 88/5. This leads to a significant modification of the numerical predictions to be discussed below. In this expression $n_f = 6$ denotes the total number of quark species and $l \equiv \log \mu^2/\bar{m}_t^2$. The result is expressed in terms of the $\overline{\mathrm{MS}}$ renormalized top mass $\bar{m}_t(\mu^2)$. The variable x_t is defined as

$$
x_t(\mu^2) = \frac{G_F \bar{m}_t^2(\mu^2)}{8\sqrt{2}\pi^2}. \tag{3}
$$

From now on the explicit μ-dependence both in x_t and α_s will be suppressed. From the context it should be evident which scale is adopted.

For the Z boson propagator the following result is obtained

$$
\begin{aligned}
\Pi^Z(0) = {} & 12 x_t M_Z^2 \\
& \Bigg\{ -\frac{1}{2\epsilon} - \frac{1}{2}l + \frac{\alpha_s}{4\pi} C_F \Big(\frac{3}{2\epsilon^2} - \frac{5}{4\epsilon} - \frac{1}{8} + \frac{1}{2}l - \frac{3}{2}l^2 \Big) \\
& + \Big(\frac{\alpha_s}{4\pi}\Big)^2 \Bigg[C_F \Big(-2 - 12\zeta(3) \Big) \\
& + C_F^2 \Big(-\frac{3}{\epsilon^3} + \frac{3}{\epsilon^2} - \frac{6}{\epsilon}\zeta(3) + \frac{119}{24\epsilon} + \frac{51}{16} - 18\zeta(3)l - 36\zeta(3) + 27\zeta(4) + \frac{101}{8}l \\
& \qquad + \frac{39}{4}l^2 - 3l^3 - 6B_4 \Big)
\end{aligned} \tag{4}
$$

$$C_FC_A\Big(-\frac{11}{6\epsilon^3}+\frac{83}{12\epsilon^2}+\frac{3}{\epsilon}\zeta(3)-\frac{77}{12\epsilon}+3+9\zeta(3)l+\frac{28}{3}\zeta(3)-\frac{27}{2}\zeta(4)-\frac{85}{24}l$$
$$-\frac{43}{6}l^2-\frac{11}{6}l^3+3B_4\Big)$$
$$+C_Fn_f\Big(\frac{1}{3\epsilon^3}-\frac{5}{6\epsilon^2}+\frac{2}{3\epsilon}-\frac{1}{12}+\frac{8}{3}\zeta(3)+\frac{5}{12}l+\frac{2}{3}l^2+\frac{1}{3}l^3)\Big]\Big\}.$$

in agreement with [10]. Both Π^W and Π^Z are independent of the gauge parameter. As expected this holds true even before mass and charge renormalization are performed.

Eqs. (2) and (4) immediately lead to $\delta\rho$ in terms of the $\overline{\rm MS}$ renormalized top mass. The poles which are still present in the Z and W self-energies individually cancel.

$$\delta\rho_{\overline{\rm MS}}=3x_t \tag{5}$$
$$\Big\{1+\frac{\alpha_s}{4\pi}C_F\Big(6-4\zeta(2)+6l\Big)$$
$$+\Big(\frac{\alpha_s}{4\pi}\Big)^2\Big[C_F\Big(\frac{152}{3}+486S_2-\frac{512}{3}\zeta(3)-\frac{28}{3}\zeta(2)\Big)$$
$$+C_F^2\Big(-\frac{1591}{36}-1053S_2+4D_3+\frac{1084}{3}\zeta(3)+4\zeta(4)-24\zeta(2)l$$
$$-\frac{518}{9}\zeta(2)+3l+18l^2+8B_4\Big)$$
$$C_FC_A\Big(\frac{1013}{12}+\frac{1053}{2}S_2-2D_3-\frac{452}{3}\zeta(3)+30\zeta(4)-\frac{44}{3}\zeta(2)l-\frac{146}{3}\zeta(2)$$
$$+\frac{163}{3}l+11l^2-4B_4\Big)$$
$$+C_Fn_f\Big(-\frac{25}{2}-\frac{16}{3}\zeta(3)+\frac{8}{3}\zeta(2)l+\frac{28}{3}\zeta(2)-\frac{22}{3}l-2l^2\Big)\Big]\Big\}$$

With the help of the relation between the OS and running top mass [18] for $\mu^2=M_t^2$

$$\bar m_t(M_t^2) = M_t\Big[1-\frac{\alpha_s}{\pi}C_F+\Big(\frac{\alpha_s}{\pi}\Big)^2\Big(C_Fn_f\Big(\frac{1}{4}\zeta(2)+\frac{71}{192}\Big)+C_FC_A\Big(\frac{1}{2}\zeta(2) \tag{6}$$
$$-\frac{1111}{384}-\frac{1}{4}\pi^2\log 2+\frac{3}{8}\zeta(3)\Big)+C_F\Big(-\frac{3}{4}\zeta(2)+\frac{3}{8}\Big)$$
$$+C_F^2\Big(\frac{1}{2}\pi^2\log 2-\frac{3}{4}\zeta(3)-\frac{15}{8}\zeta(2)+\frac{7}{128}\Big)\Big)\Big]$$

the result is easily expressed in terms of the OS mass:

$$\delta\rho_{OS}=3X_t \tag{7}$$
$$\Big\{1+\frac{\alpha_s}{4\pi}C_F\Big(-2-4\zeta(2)\Big)$$
$$+\Big(\frac{\alpha_s}{4\pi}\Big)^2\Big[C_F\Big(\frac{188}{3}+486S_2-\frac{512}{3}\zeta(3)-\frac{100}{3}\zeta(2)\Big)$$

$$
\begin{aligned}
&+C_F^2\Big(-\frac{238}{9}-1053S_2+4D_3+\frac{1012}{3}\zeta(3)+4\zeta(4)\\
&\quad-\frac{770}{9}\zeta(2)+8B_4+96\zeta(2)\log 2\Big)\\
&+C_FC_A\Big(-\frac{49}{6}+\frac{1053}{2}S_2-2D_3-\frac{416}{3}\zeta(3)+30\zeta(4)-\frac{98}{3}\zeta(2)-4B_4\\
&\quad-48\zeta(2)\log 2-\frac{22}{3}\log\frac{\mu^2}{M_t^2}-\frac{44}{3}\zeta(2)\log\frac{\mu^2}{M_t^2}\Big)\\
&+C_Fn_f\Big(-\frac{2}{3}-\frac{16}{3}\zeta(3)+\frac{52}{3}\zeta(2)+\frac{4}{3}\log\frac{\mu^2}{M_t^2}+\frac{8}{3}\zeta(2)\log\frac{\mu^2}{M_t^2}\Big)\Big]\Big\}.
\end{aligned}
$$

Here M_t is the pole mass and $X_t = G_F M_t^2/8\sqrt{2}\pi^2$. The residual $\log\mu$ terms are cancelled by the μ-dependence of α_s.

At this point two consistency checks should be mentioned which were performed in order to test the correctness of our result. The first one is a different method of calculating $\delta\rho$ respectively the polarisation functions for the W and Z boson. It relies on the axial Ward identity which connects the axial part of the polarisation tensor with the pseudoscalar polarisation function. (The double triangle diagram was not considered in this context.) If the fermions in the loop have the masses m_1 and m_2 the following identity holds:

$$
q_\mu q_\nu \Pi^{\mu\nu,a}(q) = (m_1+m_2)^2\Pi^p(q) + (m_1+m_2) < 0|\bar{q}_1 q_1 + \bar{q}_2 q_2|0 > . \tag{8}
$$

The second term of the r.h.s. is independent of q and therefore not relevant in this context. The l.h.s. in lowest order is already $\mathcal{O}(q^2)$. $\Pi^p(q)$ was evaluated up to $\mathcal{O}(q^2)$. Because of the different tensor structure, different recurrence relations have to be applied to compute the ρ parameter. Π^p has to be expanded in q up to order q^2. Additional propagators are generated and different parts of the FORM programs become relevant. In addition it was checked that the q independent part of Π^p cancels against the second term such that the r.h.s. indeed is of $\mathcal{O}(q^2)$. $\delta\rho$ as given in eq. (5) was reproduced with this method.

The second check is also connected with an expansion in the external momentum. The polarisation tensor of the vector bosons was expanded up to order q^2. The external momentum was routed through the graphs in two different ways. Again the same result was obtained for every diagram although the intermediate steps are very different.

Substituting $C_A = 3$, $C_F = 4/3$ and $\mu^2 = \bar{m}_t^2$ or $\mu^2 = M_t^2$ in eqs. (5) and (7) respectively a fairly compact form for $\delta\rho$ is obtained:

$$
\begin{aligned}
\delta\rho_{\overline{\mathrm{MS}}} &= 3x_t \qquad (9)\\
&\Big\{1+\frac{\alpha_s}{4\pi}\Big(8-\frac{16}{3}\zeta(2)\Big)\\
&+\Big(\frac{\alpha_s}{4\pi}\Big)^2\Big[n_f\Big(-\frac{50}{3}-\frac{64}{9}\zeta(3)+\frac{112}{9}\zeta(2)\Big)
\end{aligned}
$$

$$+\frac{26459}{81}+882S_2-\frac{8}{9}D_3-\frac{5072}{27}\zeta(3)+\frac{1144}{9}\zeta(4)-\frac{25064}{81}\zeta(2)-\frac{16}{9}B_4\Big]\Big\}$$

$$\delta\rho_{OS} = 3X_t \tag{10}$$

$$\Big\{1+\frac{\alpha_s}{4\pi}\Big(-\frac{8}{3}-\frac{16}{3}\zeta(2)\Big)$$

$$+\Big(\frac{\alpha_s}{4\pi}\Big)^2\Big[n_f\Big(-\frac{8}{9}-\frac{64}{9}\zeta(3)+\frac{208}{9}\zeta(2)\Big)+\frac{314}{81}+882S_2-\frac{8}{9}D_3$$

$$-\frac{4928}{27}\zeta(3)+\frac{1144}{9}\zeta(4)-\frac{26504}{81}\zeta(2)-\frac{16}{9}B_4-\frac{64}{3}\zeta(2)\log 2)\Big]\Big\}$$

To evaluate these results numerically the values for B_4, S_2, D_3 and the ζ functions are inserted. For convenience of the reader we display separately four different contributions: (i) the contribution from the double triangle diagrams related to the axial anomaly [14], (ii) the contributions with exactly one fermion loop which together with the previous one would give the result in the "quenched" approximation, (iii) the contribution from light quark loops proportional to $n_l \equiv n_f - 1$, and finally (iv), the contribution with the light quarks replaced by top quarks.

$$\begin{aligned}\delta\rho_{\overline{\mathrm{MS}}} &= 3x_t\Big(1-0.19325\,\frac{\alpha_s}{\pi}\\ &\qquad +(-4.2072+1.4151-0.29652n_l+0.30516)\,\Big(\frac{\alpha_s}{\pi}\Big)^2\Big)\end{aligned}\tag{11}$$

$$\begin{aligned}\delta\rho_{OS} &= 3X_t\Big(1-2.8599\,\frac{\alpha_s}{\pi}\\ &\qquad +(-4.2072-19.416+1.7862n_l+0.098035)\,\Big(\frac{\alpha_s}{\pi}\Big)^2\Big).\end{aligned}\tag{12}$$

The coefficient in front of n_l in eq. (12) is in reasonable agreement with the numerical result in [19]. Eqs. (11) and (12) in particular the separation of the various contributions allow the use or test of a variety of optimization schemes which is left to the reader.

For the final result and after setting $n_l = 5$ we obtain

$$\delta\rho_{\overline{\mathrm{MS}}} = 3x_t\left(1-0.19325\,\frac{\alpha_s}{\pi}-3.9696\,\Big(\frac{\alpha_s}{\pi}\Big)^2\right)\tag{13}$$

$$\delta\rho_{OS} = 3X_t\left(1-2.8599\,\frac{\alpha_s}{\pi}-14.594\,\Big(\frac{\alpha_s}{\pi}\Big)^2\right).\tag{14}$$

The coefficient in front of α_s^2 in the $\overline{\mathrm{MS}}$ result differs significantly from [10]: $-3.969\ldots$ versus $+0.07111\ldots$. The OS results differ by the same amount $-14.59\ldots$ versus $-10.55\ldots$.

In [10] it was mentioned that the contribution from the double triangle diagram, associated with the axial anomaly [14], alone amounts to about 40% of the total three-loop correction. Here a fraction of approximately 30% is still traceable back to this single diagram in the OS scheme. In the $\overline{\rm MS}$ scheme the corrections are completely dominated by this diagram.

Finally the numerical effect on the prediction for M_W and $\sin^2\Theta_{\rm eff}$ from α, G_F and M_Z will be discussed. If sub-leading terms are neglected the following relations hold [20]:

$$\begin{aligned} M_W^2 &= \frac{\rho M_Z^2}{2}\left(1+\sqrt{1-\frac{4A^2}{\rho M_Z^2}\frac{1}{1-\Delta\alpha}}\right) \\ \sin^2\Theta_{\rm eff} &= 1-\frac{M_W^2}{\rho M_Z^2} = \frac{1}{2}\left(1-\sqrt{1-\frac{4A^2}{\rho M_Z^2}\frac{1}{1-\Delta\alpha}}\right). \end{aligned}$$

Here $A = \sqrt{\pi\alpha/\sqrt{2}G_F} = 37.2802$ GeV, $\Delta\alpha \approx 0.06$ and $\rho = 1/(1-\delta\rho)$. The relative size of the one-, two- and three-loop corrections is given in Table 1. The first row indicates the size of $\delta\rho$ itself. Rows two and three give the relative contribution of the one-, two- and three-loop corrections with respect to the Born result in the OS and $\overline{\rm MS}$ scheme. The numbers are obtained with the following input data: $\alpha_s(M_t^2) = 0.1092$, $\bar{m}_t(M_t^2) = 164$ GeV, $M_t = 174$ GeV, $M_Z = 91.188$ GeV and $G_F = 1.16639\,10^{-5}$ GeV^{-2}. The value of $\alpha_s(M_t^2) = \alpha_s^{(6)}(M_t^2)$ was obtained from $\alpha_s^{(5)}(M_Z^2) = 0.120$ with the help of the three-loop β function [21] and the matching condition between $\alpha_s^{(5)}(\mu^2)$ and $\alpha_s^{(6)}(\mu^2)$ at the scale $\mu^2 = M_t^2$ [22]. (The numbers in brackets indicate thereby the numbers of active flavours.) The $\overline{\rm MS}$ top mass $\bar{m}_t(M_t^2)$ was derived from eq. (6). This value serves as starting point in order to calculate $\bar{m}_t(\mu^2)$ using the corresponding three-loop evolution equation [23]. (Unlike the coupling constant the running top quark mass is only defined in full $n_f = 6$ theory.)

OS	+1-loop	+2-loop	+3-loop
$\delta\rho$	0.00949	0.00854	0.00838
$\delta M_W/M_W$	0.00682	0.00614	0.00601
$\delta\sin^2\Theta_{\rm eff}/\sin^2\Theta_{\rm eff}$	-0.01349	-0.01216	-0.01192

$\overline{\rm MS}$	+1-loop	+2-loop	+3-loop
$\delta\rho$	0.00843	0.00837	0.00833
$\delta M_W/M_W$	0.00605	0.00601	0.00598
$\delta\sin^2\Theta_{\rm eff}/\sin^2\Theta_{\rm eff}$	-0.01200	-0.01191	-0.01186

Table 1: Numerical results including successively higher orders.

It is interesting to mention that the scheme dependence decreases enormously when taking higher loop corrections successively into account.

One observes that the three-loop corrections are (at least for M_W) not at all small compared with the two-loop QCD contribution. Furthermore they are approximately of the same order of magnitude as the two-loop electroweak result because for $m_H/m_t = 1.5$ the contribution from the two-loop electroweak correction amounts -0.018% [10, 9] to be compared with the three-loop QCD corrections of -0.013%. It is also possible to translate the $\mathcal{O}(\alpha_s^2)$ corrections directly in a change of the top mass contained in X_t. In the on-shell scheme this corresponds to a change of approximately -1.5 GeV.

The dependence of the result on the renormalization scale μ^2 is shown in Figs. 1a and 1b for $\delta\rho_{OS}$ and $\delta\rho_{\overline{\rm MS}}$ respectively. The same input parameters have been used as before. The dotted line in Fig. 1b gives the one-loop prediction (which is constant for $\delta\rho_{OS}$ and completely off-scale in Fig. 1a), dashed and solid lines represent the two- and three-loop results. The prediction is clearly stabilized through inclusion of higher orders.

Another possibility would be to absorb the α_s^2 contribution in the choice of an effective

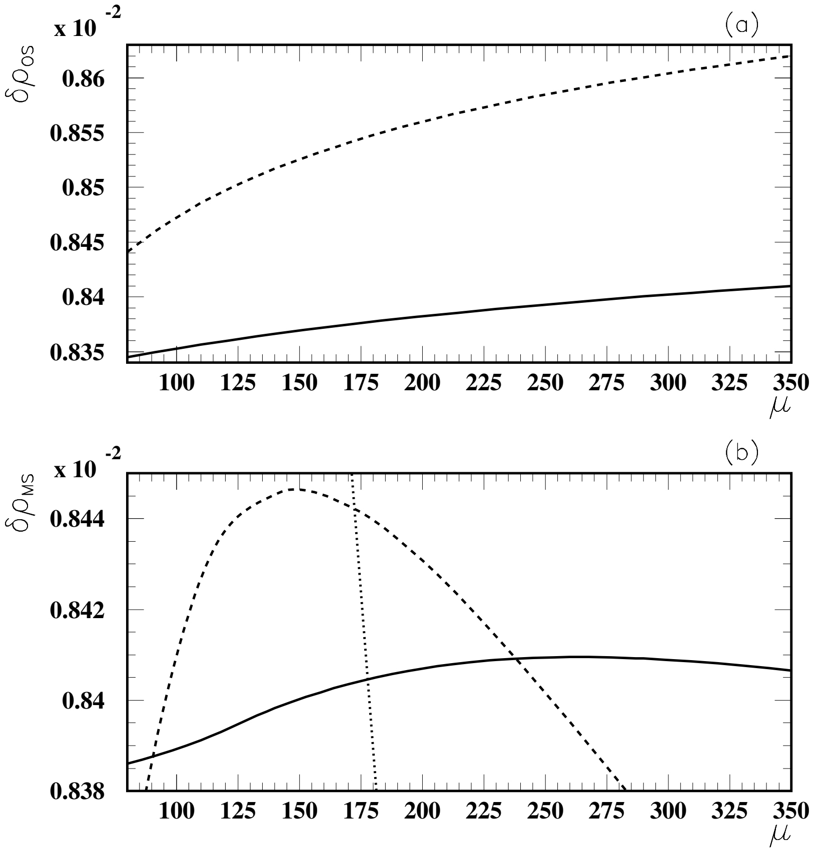

Figure 1: Renormalization scale dependence of $\delta\rho_{OS}$ (a) and $\delta\rho_{\overline{\rm MS}}$ (b).

scale of the α_s correction:

$$\delta\rho_{OS} = 3X_t \left(1 - 2.8599 \, \frac{\alpha_s((0.302 M_t)^2)}{\pi}\right). \tag{15}$$

To summarize: The evaluation of the three-loop QCD correction to the ρ parameter has been repeated with a result different from the one of [10]. The numerical difference is sizeable.

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