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SPECTRA OF BARYONS CONTAINING TWO HEAVY QUARKS

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ABSTRACT

The spectra of baryons containing two heavy quarks test the form of the QQ potential through the spin-averaged masses and hyperfine splittings. The mass splittings in these spectra are calculated in a nonrelativistic potential model and the effects of varying the potential studied. The simple description in terms of light quark and pointlike diquark is not yet valid for realistic heavy quark masses.

It is well-known that the quarkonium (Ψ and Υ) spectra test the QCD potential between quark and antiquark. In this case, a phenomenological flavor-independent potential and a QCD-derived hyperfine interaction can be used to describe the masses and splittings of the $c\bar{c}$ and $b\bar{b}$ mesons [1]. Furthermore, such models allow predictions to be made for the unobserved states in the spectra [2] and for the B_c system [3]. In the same way, the doubly-heavy baryons (those containing ccq or bbq) test the potential between two quarks. The one-gluon-exchange potential between heavy quarks differs from the quark-antiquark potential by a relative color factor 2. This relation does not hold at higher orders: the one-loop corrections to the QQ and $Q\overline{Q}$ potentials are not equal [4]. The confining potential, which is determined phenomenologically, should be studied for QQ interactions as it has been for $Q\overline{Q}$.

Tests of the QQ potential can be implemented not only in the spin-averaged spectra of the doubly-heavy baryons, but also by studying the hyperfine splittings of these particles. This is surprising, as the hyperfine interactions contain two types of terms: those describing the spin-spin interaction between the heavy quarks ($\sim 1/m_Q^2$), which depend on the QQ potential analogously to the quarkonium case, and those describing the spin-spin interactions between heavy and light quarks ($\sim 1/m_Q m_q$), which might be thought to depend on the light-heavy quark interaction and not significantly on the heavy-heavy quark potential. Nevertheless, the wave function of the light quark depends sufficiently on the heavy quark pair to provide information on the QQ binding.

Baryon spectroscopy with non-relativistic potential models is relatively accurate, although not perfect. The method is clearly limited by its use of a non-relativistic treatment of the light quarks. Nonetheless, such models provide a relatively good description of the baryon spectra and stable predictions for the hyperfine splittings $\Sigma_Q^* - \Sigma_Q$ and $\Sigma_Q - \Lambda_Q$ for the baryons containing one heavy quark [7]. One potential choice which provides a good fit to the observed baryon spectrum is the power-law form [8]:

$$V(r_{ij}) \; = \; \frac{1}{2}\sum_{i<j}\left(A + Br_{ij}^{\beta} + \frac{C}{m_i m_j}\delta(r_{ij})\vec{\sigma}_i\cdot\vec{\sigma}_j\right), \tag{1}$$

with

$$\beta = 0.1, \qquad A = -8.337\,\mathrm{GeV}, \qquad B = 6.9923\,\mathrm{GeV}^{1+\beta}, \qquad C = 2.572\,\mathrm{GeV}^2,$$
$$m_q = 0.3\,\mathrm{GeV}, \qquad m_s = 0.6\,\mathrm{GeV}, \qquad m_c = 1.895\,\mathrm{GeV}, \qquad m_b = 5.255\,\mathrm{GeV}.$$

The light and strange quark masses and the power β are held fixed at reasonable values, and the parameters A, B, and C are obtained from a fit to the masses of the N, Δ, and Ω^-. The charm and bottom quark masses are fit to the Λ_c and Λ_b. This potential, when rescaled by the relative color factor of 2 between $Q\overline{Q}$ and QQ one-gluon exchange, fits the J/Ψ and Υ spin-averaged spectra as well. Such power-law potentials are familiar from the description of $Q\overline{Q}$ bound states [9] and give the choice of β used above. The value of C given above is however significantly larger than that obtained from a fit to the J/Ψ–η_c hyperfine splitting, which is 1.172 [8].

Although this potential fits the known baryon spectrum, it is not convenient for study of the effects on the spectra of variation of the QQ potential. In particular, it is interesting to be able to vary separately the Coulomb-like small distance contribution to the potential and the long-distance confining term. The potential between light and heavy quarks is therefore left as in Eq. (1), while the QQ interaction is replaced with the Cornell form [10]:

$$\begin{aligned} V_{QQ}(r) &= \frac{1}{2}\left(-\frac{k}{r}+ar+c+\frac{C}{m_Q^2}\delta(r)\vec{\sigma}_1\cdot\vec{\sigma}_2\right), \qquad (2) \\ a = 0.2005\,\mathrm{GeV}^2, \qquad k &= 0.4274, \qquad c = -1.0455\,\mathrm{GeV}. \end{aligned}$$

The parameters a, k, and c are obtained from a fit to the J/Ψ and Υ spectra, with the quark masses and C held fixed to the values given above. The effects on the hyperfine splittings of small variations in a and k are then studied. These variations, as well as the power-law type potential given above, are plotted in Figure 1 for comparison.

In the limit of heavy quark mass, the heavy QQ pair is more easily excited to higher-energy states than the light quark. In the harmonic oscillator potential, for example, the ratio of the excitation energies for light and heavy degrees of freedom is $\sqrt{(2m_h+m_l)/3m_l}$ [11]. The states considered here are those containing two identical heavy quarks (cc and bb rather than bc) in a relative S-wave state, so that Fermi statistics require that the heavy quarks be in a spin-1 (symmetric) state. In the standard notation the lowest-energy states for QQu or QQd are Ξ_{QQ} (Ξ^*_{QQ}) for spin 1/2 (3/2), and for QQs they are denoted by Ω_{QQ} (Ω^*_{QQ}). The heavy-heavy hyperfine terms cancel in the difference between spin-3/2 and 1/2 states as the expectation value of the spin-spin operator, $<\vec{\sigma}_1\cdot\vec{\sigma}_2>$, is identical for these two cases. The hyperfine splittings discussed below thus originate solely from the light-heavy quark interactions. Three different mass splittings are considered here for each combination of heavy and light quark flavors. The first is the difference between the centers-of-mass of the states in which the QQ pair is in 2S and 1S states. The other two are the hyperfine splittings between the spin-3/2 and spin-1/2 baryons for the 2S and 1S QQ configurations.

Once the potentials are set, the solution of the three-body problem can be approached using a variety of numerical methods [11]. The Born-Oppenheimer method is an efficient solution method for the case of two heavy and one light quark. The wavefunction is split into heavy- and light-quark degrees of freedom,

$$\Psi(\rho,\lambda)=\sum_n \phi_n(\rho)f_n(\rho,\lambda), \qquad (3)$$

where ρ is the distance between the two heavy quarks (m_1 and m_2), and $\vec{\lambda}$ =

$(\vec{r}_3 - \vec{R}_{12})\sqrt{m_3(m_1+m_2)/\mu_{12}(m_1+m_2+m_3)}$ is proportional to the distance between the light quark m_3 and the center-of-mass $\vec{R}_{12}$ of the heavy-quark pair. This choice of variables separates the non-relativistic kinetic energy terms: $p_1^2/(2m_1) + p_2^2/(2m_2) + p_3^2/(2m_3) = [p_\rho^2 + p_\lambda^2]/(2\mu_{12})$, where μ_{12} is the reduced mass of the two heavy quarks. The dependence on the light-quark mass has been absorbed into the definition of λ. The light quark wavefunction f is found for fixed ρ

$$\left[\frac{1}{2\mu_{12}}p_\lambda^2 + V(r_{13}) + V(r_{23})\right] f_n(\rho,\lambda) \quad = \quad \epsilon_n(\rho) f_n(\rho,\lambda), \tag{4}$$

where r_{i3} is the distance between the light quark and heavy quark i. The energy of the QQq state is approximated by:

$$\left[\frac{1}{2\mu_{12}}p_\rho^2 + V_{QQ}(\rho) + \epsilon_0(\rho) + < f_0|\frac{1}{2\mu_{12}}p_\rho^2|f_0 >\right]\phi_0(\rho) = E\phi_0(\rho). \tag{5}$$

Keeping only the zeroth term in the expansion of Ψ corresponds to the so-called "uncoupled adiabatic" [12] approximation.

The Born-Oppenheimer method integrates over the distances between the heavy quarks in the second step (Eq. (5)), so that the above approximation differs from that in which the heavy-quark pair behaves as a spin-one pointlike diquark, as it would for infinitely heavy quark mass. Because this method does not depend on the limit of infinite m_Q, it works well already for the charmed baryons. In the limit $m_{1,2} \gg m_3$ these equations approach the expected form: The distances r_{i3} in Eq. (4) are independent of the quark masses, and the term $p_\lambda^2/2\mu_{12} \to p_3^2/2m_3$, so that the light-quark wavefunction is independent of the distance ρ between the heavy quarks. Eq. (5) then reduces to the usual non-relativistic two-body equation. In the limit of infinite m_Q, the QQq spectra can be simply related to the meson spectra [5]. The splittings obtained in this way are however significantly smaller than those obtained from the potential model calculation, and the masses of the spin-averaged states somewhat higher. This limit gives the values: $M_{\Xi^*_{cc}} = 3.811\,\mathrm{GeV}$, $M_{\Xi_{cc}} = 3.742\,\mathrm{GeV}$, $M_{\Xi^*_{bb}} = 10.484\,\mathrm{GeV}$, and $M_{\Xi_{bb}} = 10.460\,\mathrm{GeV}$ [6].

The Coulomb and linear terms in the QQ potential are now varied by introducing parameters x_a and x_k:

$$V_{QQ}(r) \quad = \quad \frac{1}{2}\left(-x_k\frac{k}{r} + x_a a r + c + \frac{C}{m_Q^2}\delta(r)\vec{\sigma}_1\cdot\vec{\sigma}_2\right), \tag{6}$$

which are varied between 0.8 and 1.2 to determine the effect on the mass differences M(2S)–M(1S), ΔM(1S), and ΔM(2S). Variations in c do not affect the splittings, only the masses themselves. Variations in C obviously cause proportional changes in the hyperfine splittings. The effect of C on the 2S–1S mass difference comes from the

$1/m_Q^2$ hyperfine terms only, and is small. These results of the x_a and x_k variations are presented in Table 1, along with the percent change $100(\Delta\mathrm{M}(x_i=1.2) - \Delta\mathrm{M}(x_i=0.8))/\Delta\mathrm{M}(x_i=1)$. The values for $x_a = x_k = 1$ are in agreement with previous calculations [14] when the power-law type potential is used for the QQ interaction.

The hyperfine splittings are analogous to those in a heavy-light meson system, so that in the limit of infinitely heavy m_Q they should approach δ/m_Q, where δ is independent of heavy-quark flavor. The mass splittings in the table give $\delta(\mathrm{ccq,1S}) = 0.242$ and $\delta(\mathrm{bbq,1S}) = 0.305$. These states are not yet heavy enough to be accurately described by the simple heavy-quark limit. The values of δ extracted for these and for heavier masses are shown in Fig. 2, where δ may be seen to approach the heavy-mass limit for $m_Q \gtrsim$ 15–20 GeV. This is in contrast to what is known from meson spectroscopy, where the simple $1/m_Q$ description of the splittings works even for the strange quark. Although the masses of the diquarks are quite large ($\sim 2m_Q$), the radii for these states are also large for realistic quark masses, so that the simple application of the infinite-mass limit is not yet valid here. Typical radii $<r>$ for QQ states are 2.4 GeV^{-1} for cc 1S, and 1.5 GeV^{-1} for bb 1S. For $Q\overline{Q}$ states, typical radii are 1.7 GeV^{-1} for $c\bar{c}$, and 1.0 GeV^{-1} for $b\bar{b}$.

The percent changes in the mass splittings are given in Fig. 3 for QQq. The sensitivity of the QQs states to changes in the potential is almost exactly that of the QQq states. M(2S)–M(1S) is more sensitive to changes in the QQ potential than either of the hyperfine splittings. This mass difference is essentially the mass splitting between the 1S and 2S states of the two-body QQ potential, so that the strong dependence here is unsurprising. The dependence of M(2S)–M(1S) on x_k is about one-quarter(half) that on x_a for the cc (bb) states. The bb pair is more closely bound than the cc so that these states depend more strongly on the Coulomb (x_k) part of the potential.

The hyperfine splitting in the 1S states, ΔM(1S), depends more strongly on x_a than on x_k for the cc case, and on x_k than on x_a for bb. The strong dependence of the bb 1S splitting on the Coulomb term in the potential reflects the closer binding of the b quarks. On the other hand, ΔM(2S) depends in both cases more strongly on x_a than on x_k. This is a reflection of the larger radius ($< r > \sim 3.0$ GeV^{-1}) of the bb 2S state.

In conclusion, the spectra of doubly-heavy baryons have been studied in a non-relativistic potential model, with emphasis on the sensitivity of the mass splittings of these states to variations in the interaction between the two heavy quarks. Although the hyperfine splittings are between the two possible spin states of the light quark relative to a heavy spin-1 system, it was found that dependence on the QQ interaction remains. This dependence is however rather small. The 2S–1S mass difference is more sensitive to variations in the potential than the hyperfine splittings. A comparison of the splittings between the center-of-mass of the 2S and 1S states and the hyperfine splittings in these levels exhibits different dependences on the strengths of the separate parts of the potential (Coulomb vs. confining terms). More importantly, it is demonstrated that the use of the infinitely heavy m_Q limit, where the heavy quarks form a pointlike diquark and the baryon as a whole a meson-like system, is not yet valid for realistic heavy quark masses.

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Figures

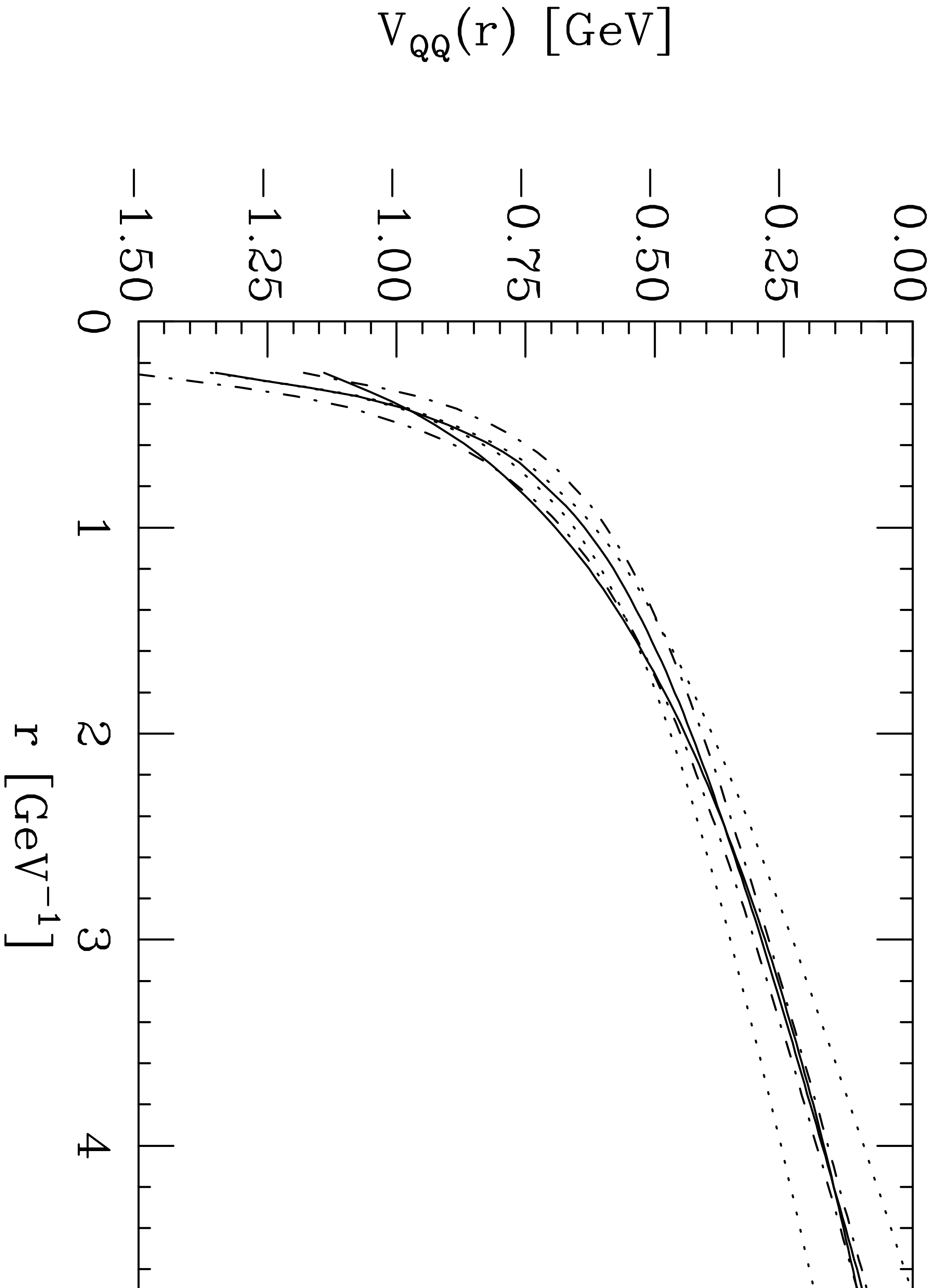
Fig. 1. The QQ potentials used in this paper. Solid lines denote the power-law (Eq. (1)) and Coulomb-plus-linear potentials (Eq. (2)) without any variation. The dot-dashed (dotted) lines show the effect of the variation of x_k (x_a) between 0.8 and 1.2.

Fig. 2. The quantity $\delta = m_Q \Delta$M(1S) as a function of heavy-quark mass.

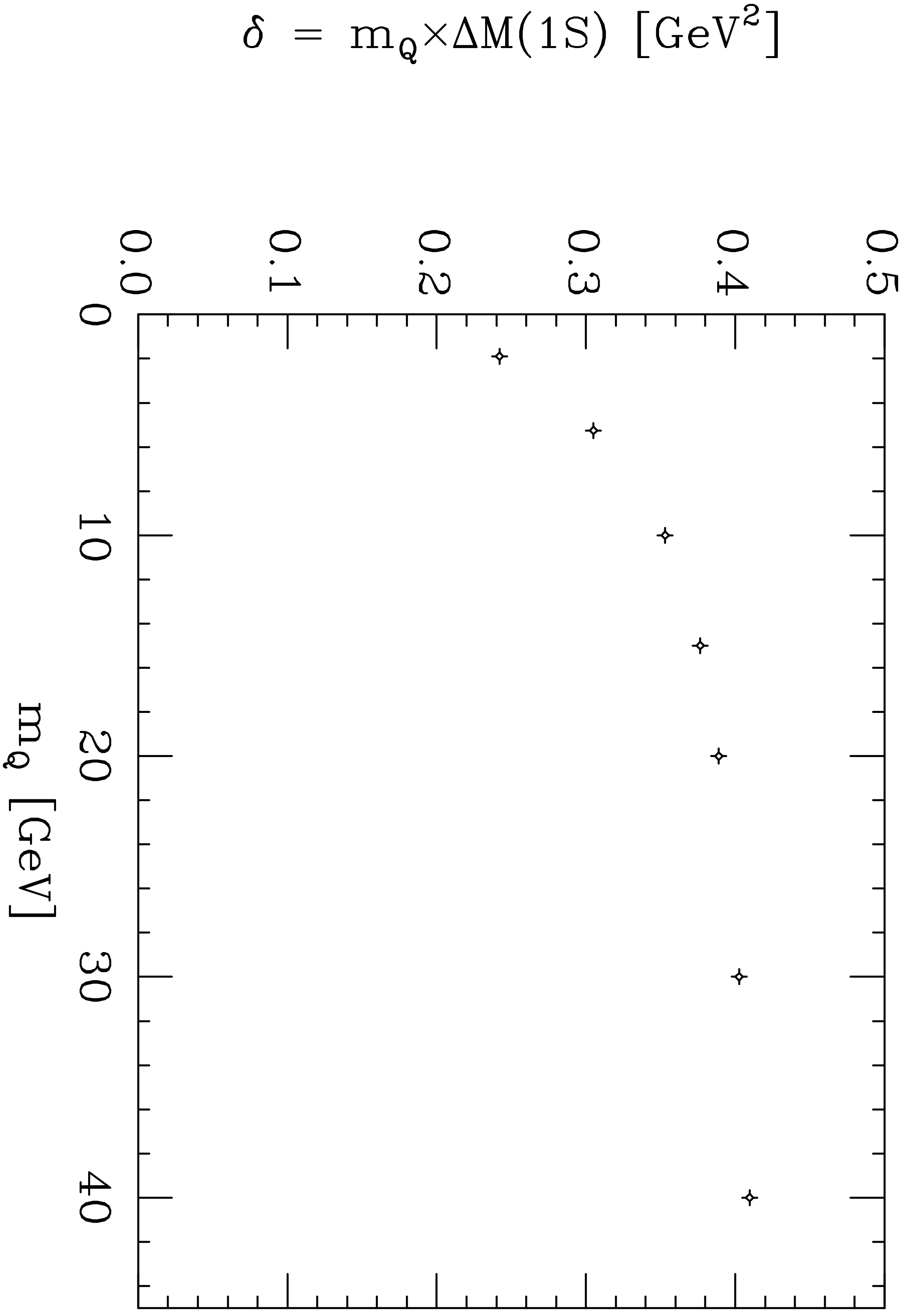
Fig. 3. The percent changes in the mass splittings $\delta\mathcal{M}(x_i) = 100(\mathcal{M}(x_i)/\mathcal{M}(1)-1)$. The labels 1, 2, 3 refer to M(2S)–M(1S), ΔM(1S), and ΔM(2S), respectively, while a and k indicate variations in x_a or x_k.

	x_a	M(2S)-M(1S)	ΔM(1S)	ΔM(2S)
ccq	0.80	0.394	0.125	0.0802
	1.00	0.431	0.128	0.0832
	1.20	0.467	0.130	0.0861
	% change	16.9	4.2	7.0
bbq	0.80	0.280	0.0571	0.0417
	1.00	0.304	0.0581	0.0430
	1.20	0.328	0.0590	0.0441
	% change	15.7	3.3	5.6
	x_k			
ccq	0.80	0.423	0.126	0.0828
	1.00	0.431	0.128	0.0832
	1.20	0.440	0.129	0.0837
	% change	3.8	2.3	1.1
bbq	0.80	0.294	0.0565	0.0425
	1.00	0.304	0.0581	0.0430
	1.20	0.316	0.0593	0.0435
	% change	7.4	4.8	2.4

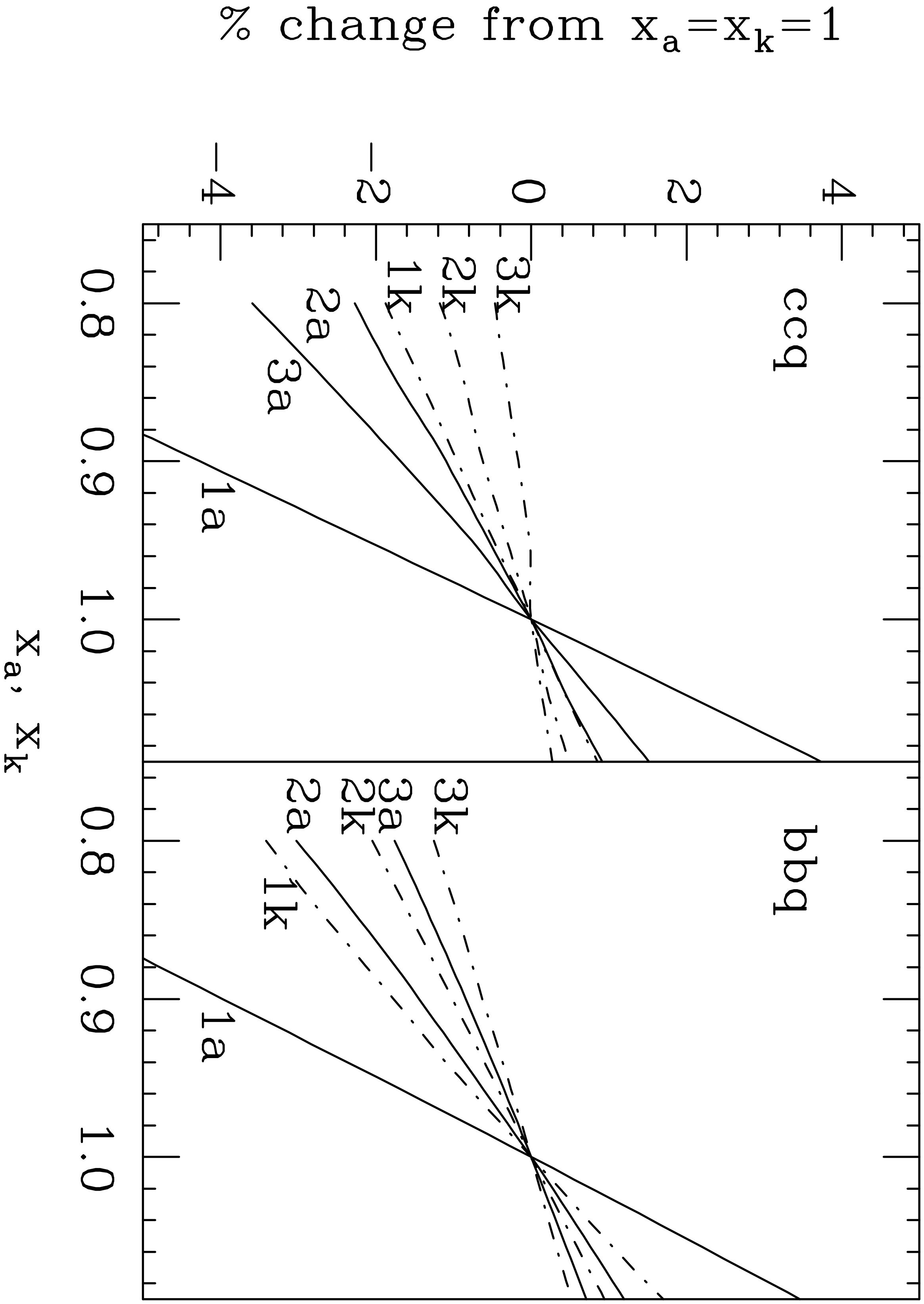
Table 1: Mass splittings (GeV) in the ccq and bbq systems as a function of the parameters x_a and x_k.



V_QQ(r) [GeV]
0.00
−0.25
−0.50
−0.75
−1.00
−1.25
−1.50
0
1
2
3
4
5
r [GeV^−1]



$\delta = m_Q \times \Delta M(1S)$ [GeV^2]
0.0
0.1
0.2
0.3
0.4
0.5
0
10
20
30
40
m_Q [GeV]


% change from x_a=x_k=1
x_a, x_k
ccq
bbq
3k
2k
1k
2a
3a
1a
0.8
0.9
1.0
−4
−2
0
2
4