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Polarization in Top Pair Production and Decay near Threshold †

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Abstract

Theoretical results are presented for top quarks produced in annihilation of polarized electrons on positrons. Polarization studies for $t\bar{t}$ pairs near threshold are free from hadronization ambiguities. This is due to the short lifetime of the top quark. Semileptonic decays are discussed as well as their applications in studying polarization dependent processes involving top quarks. The Green function formalism is applied to $t\bar{t}$ production at future e^+e^- colliders with polarized beams. Lippmann–Schwinger equation is solved numerically for the QCD chromostatic potential given by the two-loop formula at large momentum transfers and Richardson ansatz at intermediate and small ones. The polarization dependent momentum distributions of top quarks and their decay products are calculated.

1 Introduction

The top quark is the heaviest fermion of the Standard Model. Its large mass allows to probe deeply into the QCD potential for nonrelativistic $t\bar{t}$ system produced near energy threshold. Such a system will provide a unique opportunity for a variety of novel QCD studies. The lifetime of the top quark is shorter than the formation time of top mesons and toponium resonances. Therefore top decays intercept the process of hadronization at an early stage and practically eliminate associated nonperturbative effects.

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The analysis of polarized top quarks and their decays has recently attracted considerable attention, see [1, 2] and references cited therein. The reason is that this analysis will result in determination of the top quark coupling to the W and Z bosons either confirming the predictions of the Standard Model or providing clues for physics beyond. The latter possibility is particularly intriguing because m_t plays an exceptional role in the fermion mass spectrum.

The polarization fourvector s^μ of the top quark can be determined from the angular-energy distributions of the charged leptons in semileptonic t decays. In the t quark rest frame this distribution is in Born approximation the product of the energy and the angular distributions[3]:

$$\frac{\mathrm{d}^2\Gamma}{\mathrm{d}E_\ell\,\mathrm{d}\cos\theta} = \frac{1}{2}\,[1\,+\,S\cos\theta]\,\frac{\mathrm{d}\Gamma}{\mathrm{d}E_\ell} \tag{1}$$

where $s^\mu = (0, \vec{s}\,)$, $S = |\,\vec{s}\,|$ and θ is the angle between $\vec{s}$ and the direction of the charged lepton. QCD corrections essentially do not spoil factorization[4]. Thus, the polarization analyzing power of the charged lepton energy-angular distribution remains maximal. There is no factorization for the neutrino energy-angular distribution which is therefore less sensitive to the polarization of the decaying top quark. On the other hand it has been shown [5] that the angular-energy distribution of neutrinos from the polarized top quark decay will allow for a particularly sensitive test of the V-A structure of the weak charged current.

A number of mechanisms has been suggested that will lead to polarized top quarks. However, studies at a linear electron-positron collider are particularly clean for precision tests. Moreover, close to threshold and with longitudinally polarized electrons one can study decays of polarized top quarks under particularly convenient conditions: large event rates, well identified rest frame of the top quark, and large degree of polarization. At the same time, thanks to the spectacular success of the polarization program at SLC [6], the longitudinal polarization of the electron beam will be an obvious option for a future linear collider[1]. In this article some results are presented of a recent calculation [9] of top quark polarization for the reaction $e^+e^- \to t\bar{t}$ in the threshold region. In Sect.2 we discuss the dependence of the top quark polarization on the longitudinal polarizations of the beams. Due to restricted phase space the amplitude is dominantly S wave and the electron and positron polarizations are directly transferred to the top quark. For a quantitative study this simple picture has to be extended and the modifications originating from $S-P$ wave interference should be taken into account. In Sect.3 all these corrections are calculated from numerical solutions of Lippmann-Schwinger equations.

[1]Another proposed and closely related facility is a photon linear collider. At such a machine the high energy photon beams can be generated via Compton scattering of laser light on electrons accelerated in the linac. The threshold behaviour of the reaction $\gamma\gamma \to t\bar{t}$ has been reviewed in [7] and the top quark polarization has been recently considered in [8].

2 Top quark polarization

We adopt the conventions of ref.[10] and describe the longitudinal polarization of the e^+e^- system in its center-of-mass frame as a function of the variable

$$\chi = \frac{P_{e^+} - P_{e^-}}{1 - P_{e^+} P_{e^-}} \tag{2}$$

where $P_{e^\pm}$ denote the polarizations of $e^\pm$ with respect to the directions of e^+ and e^- beams, respectively[2]. In the absence of phases from final state interaction, which can be induced by higher orders in α_s and will be considered elsewhere [9], the top quark polarization is in the production plane. $P_\parallel$ and $P_\perp$ denote the longitudinal and the transverse components of top polarization vector (in its rest frame) with respect to the electron beam. The angle ϑ denotes the angle between e^- and the top quark. In the threshold region the top quark is nonrelativistic (with velocity $\beta = p/m_t \sim \alpha_s$) and the kinetic energy of the $t\bar{t}$ system $E = \sqrt{s} - 2m_t$ is of the order $\mathcal{O}(\beta^2)$. Retaining only the terms up to $\mathcal{O}(\beta)$ one derives the following expressions for the components of the polarization vector:

$$P_\parallel = C_\parallel^0(\chi) + C_\parallel^1(\chi)\Phi(E)\cos\vartheta \tag{3}$$
$$P_\perp = C_\perp(\chi)\Phi(E)\sin\vartheta \tag{4}$$

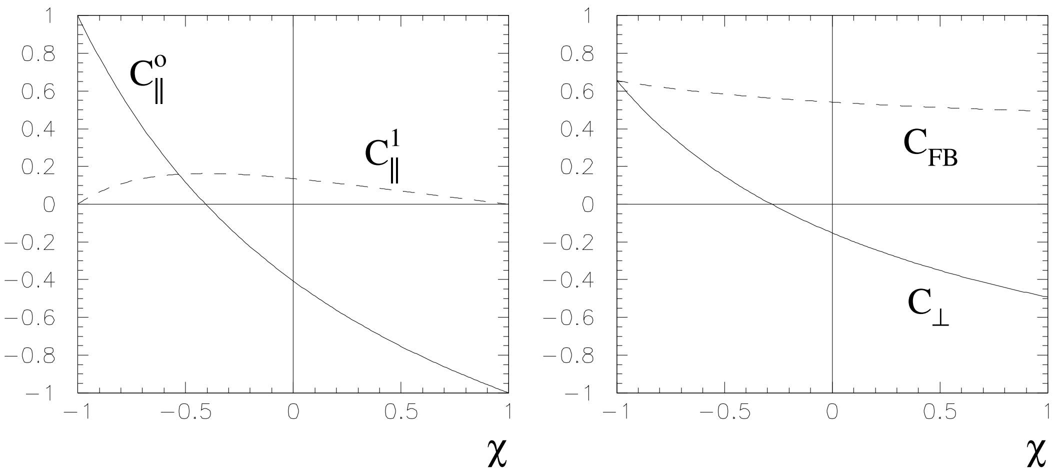

Figure 1: Coefficient functions: a) $C_\parallel^0(\chi)$ – solid line and $C_\parallel^1(\chi)$ – dashed line, b) $C_\perp(\chi)$ – solid line and $C_{FB}(\chi)$ – dashed line.

The coefficients $C_\parallel^0(\chi)$, $C_\parallel^1(\chi)$ and $C_\perp(\chi)$ depend on the polarization χ, the electroweak coupling constants, the Z mass and the center-of-mass energy $\sqrt{s} \approx 2m_t$. They are plotted in Fig.1 for $m_t = 174$ GeV. $C_\parallel^0(\chi)$ and $C_\parallel^1(\chi)$ are

[2] It is conceivable that for a future linear e^+e^- collider $P_{e^+} = 0$, $P_{e^-} \neq 0$ and then $\chi = -P_{e^-}$.

shown in Fig.1a as the solid and the dashed lines, respectively, and $C_\perp(\chi)$ as the solid line in Fig.1b.

The function $\Phi(E)$ describes the complicated dynamics of the $t\bar{t}$ system near threshold. In particular it includes effects of the would-be toponium resonances and Coulomb enhancement. Nevertheless, it is possible to calculate this function using the Green function method. The same function $\Phi(E)$ also governs the forward-backward asymmetry in $e^+e^- \to t\bar{t}$

$$A_{FB} = C_{FB}(\chi)\Phi(E) \tag{5}$$

where C_{FB} is shown as the dashed line in Fig.1b. Eqs.(3) and (4) extend the results of [11] into the threshold region.

3 Lippmann-Schwinger equations

The Green function method has become a standard tool for studying e^+e^- annihilation in the threshold region [12, 13, 14, 15]. We follow the momentum space approach of [15] and solve the Lippmann-Schwinger equations numerically for the S-wave and P-wave Green functions

$$\begin{aligned} G(p,E) &= G_0(p,E) + G_0(p,E)\int \frac{\mathrm{d}^3q}{(2\pi)^3}\tilde{V}\left(\,|\vec{p}-\vec{q}\,|\,\right)G(q,E) && (6)\\ F(p,E) &= G_0(p,E) + G_0(p,E)\int \frac{\mathrm{d}^3q}{(2\pi)^3}\frac{\vec{p}\cdot\vec{q}}{p^2}\tilde{V}\left(\,|\vec{p}-\vec{q}\,|\,\right)F(q,E) && (7)\end{aligned}$$

where $p = |\,\vec{p}\,|$ is the momentum of the top quark in $t\bar{t}$ rest frame,

$$G_0(p,E) = \left(E - p^2/m_t + \mathrm{i}\Gamma_t\right)^{-1}. \tag{8}$$

Γ_t denotes the top width and $\tilde{V}(p)$ is the QCD potential in momentum space; see [15, 9] for details. The function $\Phi(E)$ is related to $G(p,E)$ and $F(p,E)$:

$$\Phi(E) = \frac{\left(1-\frac{4\alpha_s}{3\pi}\right)\frac{1}{m_t}\int_0^{p_m}\mathrm{d}p\,p^3\mathcal{R}e\left(\,G\,F^*\,\right)}{\left(1-\frac{8\alpha_s}{3\pi}\right)\int_0^{p_m}\mathrm{d}p\,p^2\,|\,G\,|^2} \tag{9}$$

where p_m has been introduced in order to cut off a logarithmic divergence of the numerator. The denominator remains finite for $p_m \to \infty$. In experimental analyses the contributions of very large intrinsic momenta will be automatically suppressed by separation of $t\bar{t}$ events from the background. In our calculation we use $p_m = m_t/3$. The function $\Phi(E)$ is plotted in Fig.2a for $m_t = 174$ GeV. The QCD potential depends on $\alpha_s(m_Z)$ and our results have been obtained for $\alpha_s = 0.12$. For a comparison in Fig.2b the annihilation cross section $\sigma(e^+e^- \to t\bar{t}\,)$ is shown in units of the annihilation cross section $\sigma(e^+e^- \to \mu^+\mu^-)$.

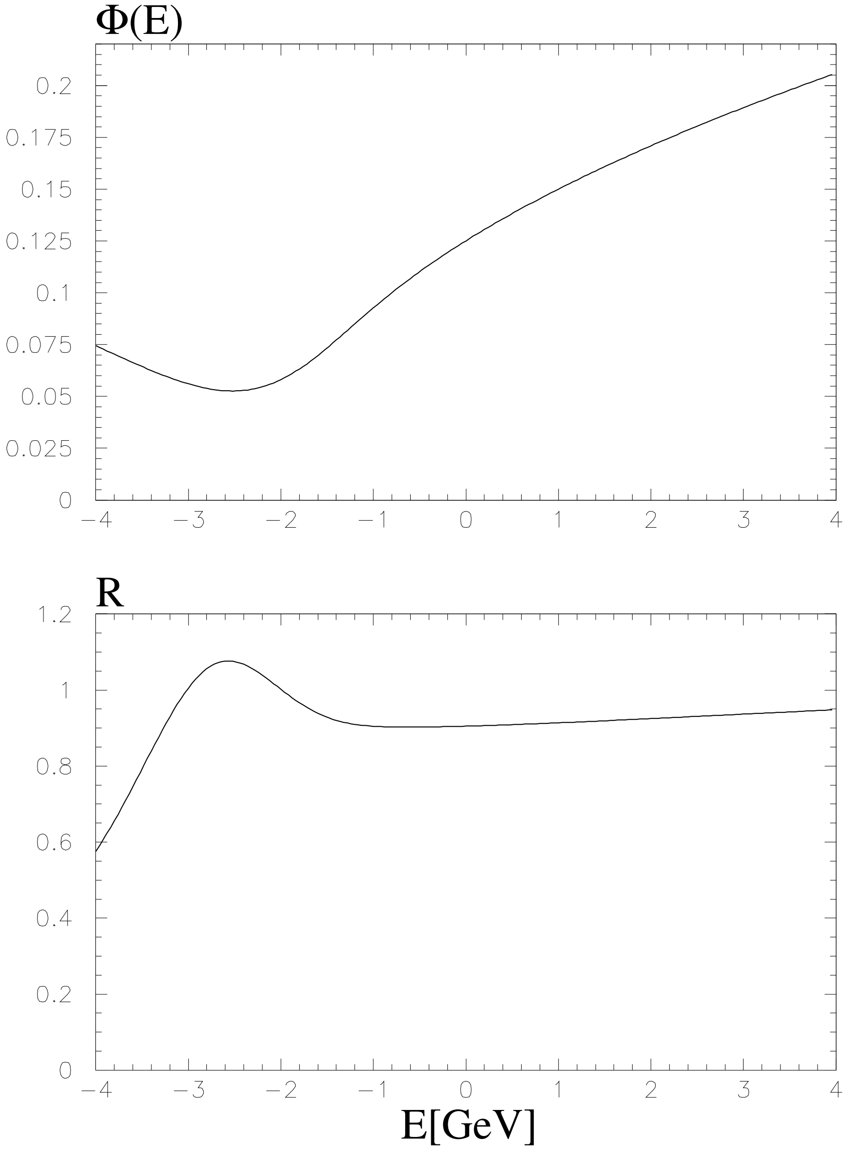

Figure 2: Energy dependence in the threshold region of: a) $\Phi(E)$ and b) $R = \sigma(e^+e^- \to t\bar{t})/\sigma(e^+e^- \to \mu^+\mu^-)$ for $m_t = 174$ GeV and $\alpha_s(m_Z) = 0.12$.

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References

[1] J.H. Kühn, "Top Quark at a Linear Collider", in *Physics and Experiments with Linear e^+e^- Colliders*, eds. F.A. Harris et al., (World Scientific, Singapore, 1993), p.72.

[2] M. Ježabek, *Top Quark Physics*, in proceedings of Zeuthen workshop *Physics at LEP 200 and Beyond*, to appear in Nucl. Phys. B(1994) Suppl.; Karlsruhe preprint TTP94-09.

[3] M. Ježabek and J.H. Kühn, Nucl. Phys. B320 (1989) 20.

[4] A. Czarnecki, M. Ježabek and J.H. Kühn, Nucl. Phys. B351 (1991) 70;
A. Czarnecki and M. Ježabek, Nucl. Phys. B427 (1994) 3.

[5] M. Ježabek and J.H. Kühn, Phys. Lett. B329 (1994) 317.

[6] M. Woods, *Polarization at SLAC*, in these proceedings.

[7] P.M. Zerwas (ed.), e^+e^- Collisions at 500 GeV: The Physics Potential, DESY Orange Reports DESY 92-123A, DESY 92-123B and DESY 93-123C.

[8] V.S. Fadin, V.A. Khoze and M.I. Kotsky, Z. Phys. C64 (1994) 45.

[9] R. Harlander, M. Ježabek, J.H. Kühn and T. Teubner, to be published.

[10] G. Alexander et al. (eds.), *Polarization at LEP*, CERN 88-06, Geneva 1988.

[11] J.H. Kühn, A. Reiter and P.M. Zerwas, Nucl. Phys. B272 (1986) 560.

[12] V.S. Fadin and V.A. Khoze, JETP Lett. 46 (1987) 525; Sov. J. Nucl. Phys. 48 (1988) 309.

[13] J.M. Strassler and M.E. Peskin, Phys. Rev D43 (1991) 1500.

[14] Y. Sumino, K. Fujii, K. Hagiwara, H. Murayama and C.-K. Ng, Phys. Rev. D47 (1993) 56; H. Murayama and Y. Sumino, Phys. Rev. D47 (1993) 82.

[15] M. Ježabek, J.H. Kühn and T. Teubner, Z. Phys. C56 (1992) 653;
M. Ježabek and T. Teubner, Z. Phys. C59 (1993) 669.