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QCD Corrections to Inclusive Distributions of Leptons in Decays of Polarised Heavy Quarks*

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Abstract

Compact analytic expressions have been obtained for the first order perturbative QCD corrections to the inclusive spectra of the leptons in the semileptonic decays of polarised heavy quarks.

Charmed and beautiful Λ baryons from Z^0 decays can be viewed as sources of highly polarised charm and bottom quarks. Charged leptons and neutrinos from Λ_b and Λ_c decays can be used in the polarisation studies for the corresponding heavy quarks. Thus our results are applicable for the b quark polarisation measurements at LEP.

Short lifetime enables polarisation studies for the top quark. The angular-energy spectra of the charged leptons are particularly useful in this respect whereas the distributions of the neutrinos are sensitive to deviations from the V-A structure of the charged weak current in the decay.

1. Introduction

Inclusive semileptonic decays of polarised charm and bottom quarks play important role in present day particle physics. With increasing statistics at LEP and good prospects for B-factories quantitative description of these processes may offer the most interesting tests of the standard quantum theory of particles. In fact the first measurement of b quark polarisation at LEP has been presented by the ALEPH collaboration at this conference [1]. At the high energy frontier semileptonic decays of the top quark will be instrumental in establishing its properties [2, 3, 4, 5].

In this article I present the results of calculations of the first order perturbative QCD corrections to semileptonic decays of polarised heavy quarks. Some of these results have been published in [6, 7]. In [8] compact analytic formulae have been obtained for the distributions of the charged lepton and the neutrino. These formulae agree with those given in [6] for the joint angular and energy distribution of the charged lepton in top quark decays and are much simpler.

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2. The formula and cross checks

The QCD corrected triple differential distribution of the charged lepton for the semileptonic decay of the polarised quark with the weak isospin $I_3 = \pm 1/2$ can be written in the following way [8]:

$$\begin{aligned}\frac{\mathrm{d}\Gamma^{\pm}}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}\cos\theta} \quad \sim \quad & \left[\,\mathrm{F}_0^{\pm}(x,y) + S\cos\theta\,\mathrm{J}_0^{\pm}(x,y)\,\right] \\ & - \frac{2\alpha_s}{3\pi}\left[\,\mathrm{F}_1^{\pm}(x,y) + S\cos\theta\,\mathrm{J}_1^{\pm}(x,y)\,\right]\end{aligned} \tag{1}$$

In the rest frame of the decaying heavy quark θ denotes the angle between the polarisation vector $\vec{s}$ of the heavy quark and the direction of the charged lepton, $S = |\,\vec{s}\,|$, $x = 2Q\ell/Q^2$ and $y = 2\ell\nu/Q^2$ where Q, ℓ and ν denote the four-momenta of the decaying quark, charged lepton and neutrino. Eq.(1) describes also the triple differential distribution of the neutrino for $I_3 = \mp 1/2$. In this case, however, $x = 2Q\nu/Q^2$ and θ denotes the angle between $\vec{s}$ and the three-momentum of the neutrino. The functions $\mathrm{F}_0^{\pm}(x,y)$ and $\mathrm{J}_0^{\pm}(x,y)$ corresponding to Born approximation read:

$$\mathrm{F}_0^{+}(x,y) \quad = \quad x(x_m - x) \tag{2}$$

$$\mathrm{J}_0^+(x,y) = \mathrm{F}_0^+(x,y) \quad (3)$$
$$\mathrm{F}_0^-(x,y) = (x-y)(x_m - x + y) \quad (4)$$
$$\mathrm{J}_0^-(x,y) = (x-y)(x_m - x + y - 2y/x) \quad (5)$$

where $x_m = 1 - \epsilon^2$, $\epsilon^2 = q^2/Q^2$ and q denotes the four-momentum of the quark originating from the decay. The functions $\mathrm{F}_1^{\pm}(x,y)$ and $\mathrm{J}_1^+(x,y)$ correspond to the first order QCD corrections and are given in [8].

Non-trivial cross checks are fulfilled by the polarisation independent parts of the distributions (1):

- the distributions $\mathrm{d}\Gamma^{\pm}/\mathrm{d}x\,\mathrm{d}y$ agree with the results for unpolarised decays which were obtained in [9]. The present formulae are simpler.
- in the four-fermion (Fermi) limit integration over y can be performed numerically. The resulting distributions $\mathrm{d}\Gamma^{\pm}/\mathrm{d}x$ also agree with those of [9]. Recently the results of [9] have been confirmed [10]. Thus an old conflict with other calculations [11] is solved and the agreement with [9] can be considered as a non-trivial cross check. Moreover, the analytic result of [9] for $\mathrm{d}\Gamma^+/\mathrm{d}x$ and $\epsilon = 0$ has been also confirmed [12].
- $$\mathrm{d}\Gamma^+/\mathrm{d}y = \mathrm{d}\Gamma^-/\mathrm{d}y$$
  and the analytic formula for this distribution exists [13] which at the same time describes the lifetime of the top quark as a function of its mass. This formula has been confirmed by a few groups, c.f. [14] and references therein.
- in the four-fermion limit the result for the total rate Γ derived from eq.(1) agrees with the results of [15] and the analytical formula of [16].

3. Applications

3.1. Polarised bottom and charm quarks

Polarisation studies for heavy flavors at LEP are a new interesting field of potentially fundamental significance, see [17, 1] for recent reviews. According to the Standard Model $Z^0 \to b\bar{b}$ and $Z^0 \to c\bar{c}$ decays can be viewed as sources of highly polarised heavy quarks. The degree of longitudinal polarisation is fairly large, amounting to $\langle P_b \rangle = -0.94$ for b and $\langle P_c \rangle = -0.68$ for c quarks [2]. The polarisations depend weakly on the production angle. QCD corrections to Born result are about 3% [18]. The real drawback is that due to hadronisation the net longitudinal polarisation of the decaying b and c quarks is drastically decreased. In particular these b quarks become depolarised which are bound in B mesons both produced directly and from $B^* \to B\gamma$ transitions. The signal is therefore significantly reduced. Only those b's (a few percent) which fragment directly into Λ_b baryons retain information on the original polarisation [19]. Polarisation transfer from a heavy quark Q to the corresponding Λ_Q baryon is 100% [20] at least in the limit $m_Q \to \infty$. Thus, a large net polarisation is expected for heavy quarks in samples enriched with these heavy baryons.

It has been proposed long ago [21] that distributions of charged leptons from semileptonic decays of beautiful hadrons can be used in polarisation studies for b quarks. Some advantages of neutrino distributions have been also pointed out [7, 22, 23]. Recently there has been considerable progress in the theory of the inclusive semileptonic decays of heavy flavor hadrons. It has been shown that in the leading order of an expansion in inverse powers of heavy quark mass $1/m_Q$ the spectra for hadrons coincide with those for the decays of free heavy quarks [24] and there are no Λ_{QCD}/m_Q corrections to this result away from the energy endpoint. Λ_{QCD}^2/m_Q^2 corrections have been calculated in [25, 26] for B mesons and in [26] for polarised Λ_b baryons. For some decays the results are similar to those of the well-known *ACCMM* model [27]. The corrections to charm decays are larger than for bottom and convergence of $1/m_Q$ expansion is poorer [28]. Perturbative first order QCD corrections contribute 10-20% to the semileptonic decays and for bottom are much larger than the nonperturbative ones.

3.2. Polarised top quarks

The analysis of polarised top quarks and their decays has recently attracted considerable attention, see [4, 5] and references cited therein. The reason is that this analysis will result in determination of the top quark coupling to the W and Z bosons either confirming the predictions of the Standard Model or providing clues for physics beyond. The latter possibility is particularly intriguing for the top quark because m_t plays an exceptional role in the fermion mass spectrum.

A number of mechanisms have been suggested that will lead to polarised top quarks. Studies at a linear electron-positron collider are particularly clean for precision tests. However, also $\gamma\gamma$ collisions with circular polarised photons and subsequent spin analysis of top quarks might reveal new information. Related studies may be performed in hadronic collisions which in this case are mainly based on the correlation between t and $\bar{t}$ decay products. However, single top production through Wb fusion at LHC may also be a useful source of polarised top quarks. Electron-positron collisions are the most efficient and flexible reactions producing polarised top quarks. A small component of polarisation transverse to the production plane is induced by final state interactions. The longitudinal polarisation P_L is large. P_L varies strongly with the production angle. Averaging over the production angle leads therefore to

a significant reduction of P_L with typical values of $\langle P_L \rangle$ around -0.2 [18].
All these reactions lead to sizable polarisation and can be used to obtain information on the production mechanism. However, two drawbacks are evident: production and decay are mixed in an intricate manner, and furthermore the degree of polarisation is relatively small and depends on the production angle. Top quark production with longitudinally polarised electron beams and close to threshold provides one important exception: the restricted phase space leads to an amplitude which is dominantly S-wave such that the electron (and positron) spin is directly transferred to the top quark. Close to threshold and with longitudinally polarised electrons one can study decays of polarised top quarks under particularly convenient conditions: large event rates, well identified rest frame of the top quark, and large degree of polarisation. Moreover, short lifetime of top quark practically eliminates nonperturbative corrections due to hadronisation.

In the rest frame of the decaying t quark distributions of the decay products are sensitive to its polarisation. Eq.(3) implies that in Born approximation the double differential angular-energy distribution of the charged lepton is the product of the energy distribution and the angular distribution. The latter distribution is of the following form

$$\frac{\mathrm{d}N}{\mathrm{d}\cos\theta} = \frac{1}{2}\,[\,1\,+\,S\cos\theta] \tag{6}$$

QCD corrections essentially do not spoil factorisation of the charge lepton distribution [6]. It is noteworthy that for S=1 the angular dependence in (6) is maximal because any larger coeffecient multiplying $\cos\theta$ would be in conflict with positivity of the decay rate. Thus the polarisation analysing power of the charged lepton energy-angular distribution is maximal †.

It follows from eqs. (4) and (5) that already in Born approximation there is no factorisation for the neutrino energy-angular distribution. Neutrino distributions are therefore less sensitive to the polarisation of the decaying top quark than charge lepton distributions On the other hand it has been shown [29] that the angular-energy distribution of neutrinos from the polarised top quark decay will allow for a particularly sensitive test of the V-A structure of the weak charged current. The effect of QCD correction can mimic a small admixture of V+A interaction. Therefore, inclusion of the radiative QCD correction to the decay distributions is necessary for a quantitative study.

† This is reversed for b decays and the polarisation analysing power is maximal for the neutrino distributions because the formulae for the neutrino distributions in down-type quark decay describe the charged lepton distributions for an up-type quark.

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