PITHA 94/36

TTP 94–18

September 1994

Some Properties of a Transient New Coherent Condition of Matter Formed in High–Energy Hadronic Collisions

Saul Barshay[a] and Patrick Heiliger[b]

[a]III. Physikalisches Institut (A), RWTH Aachen

D–52056 Aachen, Germany

[b]Institut für Theoretische Teilchenphysik

Universität Karlsruhe

D–76128 Karlsruhe, Germany

Abstract

We investigate the dynamical possibility for the formation of a transient new coherent condition of matter in high–energy hadronic collisions. The coherent bosonic amplitude is characterized by a non–zero momentum and is sustained by P–wave interactions of quasi–pions in a dense fermionic medium. We make quantitative estimates of several essential properties: the condensate momentum and the fermionic density, the size of the coherent amplitude and the negative energy density contributed by the condensate, a characteristic proper time for the system to exist prior to breakdown into a few pions, and a characteristic extension of the system over the plane perpendicular to the collision axis. These quantities then allow us to make definite estimates of new signals: a few pions with anomalously small transverse momenta ≤ 50 MeV/c; and a possible anomalous bremsstrahlung of very soft photons with characteristic transverse momenta as low as about 4 MeV/c.

There exist certain definite anomalous effects in current high–energy experiments involving collisions of hadrons [1, 2, 3, 4, 5]. These unusual effects can be connected through the transient existence of a new coherent condition of matter [1]. Such a hypothetical, relatively long–lived system, becoming spatially extended over the (impact–parameter) plane perpendicular to the collision axis can be formed in some of the real, intermediate inelastic states which contribute to the (imaginary) amplitude for the diffractive elastic scattering of hadrons. The coherent system can support an oscillatory behavior of matter with a fairly definite non–zero momentum [6] in the transverse plane. The relevant Fourier–Bessel transform of this spatial oscillatory behavior into the space of transverse momentum–transfer $\sqrt{-t} \simeq \frac{\sqrt{s}}{2}\theta$ (for small c.m. scattering angle θ), results in a localized structure in the differential cross section $\frac{d\sigma_{el}}{dt}$ around a fairly definite value of $\sqrt{-t}$. Such a localized structure exists [1] in the data from collider and fixed–target experiments up to the present highest center–of–mass energy $\sqrt{s} = 1800$ GeV. There must also be direct signals from the transient coherent system in inelastic processes. One signal that is a general consequence of the extension of the system to several fermis over the transverse plane is the breakdown into a few pions with anomalously small transverse momenta ≤ 50 MeV/c. Observation of this unusual effect against the background of many pions produced with the usual average transverse momentum of $(250 - 350)$ MeV/c requires measurements on individual events, in particular events with low multiplicity. In fact, this unusual phenomenon of small–p_T, few–particle emission is the main new empirical feature in a recent, detailed study of several hundred cosmic–ray showers "in the forwardmost small–angular region" [7]. If charged currents exist within the coherent system during its evolution, then it has been shown that soft photons will be emitted [8]. The occurrence of an anomalously high emission of soft photons has been and is today, an enigmatic feature of several different experiments at CERN [2, 3, 4, 5, 9].

In this paper we attempt to calculate approximately some properties of a transient new coherent condition of matter formed in high–energy hadronic collisions. We utilize general properties of a physical model in order to obtain numerical estimates for some definite properties of the hypothetical, new coherent condition. The model is patterned after the phenomenon of pion condensation in a sufficiently dense medium of nucleons [10, 11]. The pions are replaced by quasi–pions which are correlated quark–antiquark pairs with small $(\text{mass})^2$. The dense ("cold") medium of nucleons is replaced by a dense medium of "dressing" quarks and antiquarks formed in the collision. In this medium the quasi–pions propagate and interact with the fermions via strong, low–energy P–wave interactions, just as pions do in a medium of nucleons. In this model we examine the possibility for the occurrence of a coherent condition which contributes a negative energy density in the extending medium. The essential <u>dynamical</u> element which makes this possible is the existence of a pseudoscalar (quasi–) boson with quite small $(\text{mass})^2$ and strong, low–energy P–wave interaction with fermions. This allows for the possibility of cancellation of the positive, kinetic and $(\text{mass})^2$ terms by the attractive momentum–dependent interaction [10, 11]. The hypothetical coherent condition which is dynamically instigated following the collision, is evolving toward some (idealized) equilibrium situation, but it is limited by the intrinsic, though extended proper time for the system to break down into a few pions. Therefore one property that we seek a reason for and estimate of is the extended proper time for the system to exist prior to breaking down. In general, the center of mass of the developing coherent system can move rapidly along the collision axis in the collision center of mass (and more rapidly along the beam direction in the laboratory system). This longitudinal dimension is Lorentz–contracted; the intrinsic time interval is dilated. A second property that we estimate is the extension of the system over the transverse plane during the intrinsic time interval prior to its breakdown into a few pions. We estimate the magnitude of the momentum which characterizes the coherent oscillatory behavior of matter over the transverse plane, and we estimate the fermionic density which sustains this behavior. We calculate the negative energy density contributed by the coherent structure, which depends upon its squared amplitude. The latter quantity together with the square of the estimated transverse extension of the system makes possible an estimate of the amplitude for anomalous soft–photon emission. In the system, soft photons are emitted with a bremsstrahlung–like energy spectrum [8] ($\propto 1/p_\gamma$), at least down to p_γ of the order of $1/T$, where T is the intrinsic time interval before breakdown. In the laboratory the anomalous photons appear in a small angular interval (very small p_T) around the collision axis because of the assumed rapid motion of the system. The few pions formed when the system breaks down have anomalously small p_T, that is transverse momenta of the order of $1/R_b$ or less, where R_b is the transverse extension at T. In some events, these pions (in particular, $\pi^+\pi^-$ pairs) can be anomalously "bunched" in rapidity. In concluding we discuss possible energy dependence once $\sqrt{s}$ is such that sufficient fermionic densities are attainable following the collision; in particular the likelihood that the coherent system has some probability to break down into a large number of pions only when created in collisions with $\sqrt{s}$ well up in the TeV range. We also remark the possibility that a meson beam can be more effective at lower energies than a baryon beam in instigating the condensate.

A high density of bare quarks and antiquarks is likely to occur just after the high–energy collision of the extended structured hadrons, with an initial extension of the order of $1/m_\pi = 1.4$ fm over the impact–parameter plane. These entities start to dress themselves via QCD self–interactions, and to become correlated toward forming pions as constituent quarks and antiquarks. In the latter process it is usually tacitly assumed that interaction signals occur with nearly the velocity of light over distances of the order of $1/m_\pi$. Therefore the sequential emission of 4 to 8 pions might involve a proper time of about $4/m_\pi - 8/m_\pi = (2-4)\cdot 10^{-23}$ *sec.* However as the dynamical degrees of freedom switch from those of QCD to constituent quarks and to quasi–pions – correlated quark–antiquark pairs with small $(\text{mass})^2$ – a coherent condition can begin to evolve over the impact–parameter plane instigated by the strong, low–momentum P–wave interaction of quasi–pions with "dressing" quarks and antiquarks. As we calculate below, the effective signal velocity in the extending coherent medium is not close to that of light. The proper time for the breakdown of the system into some physical pions can be significantly extended. This has direct physical consequences. Since the center of mass of the coherent system generally can move with a high speed along the collision axis in the collision c.m., it will arrive at a longitudinal point rather distant from the collision point before breakdown, during the Lorentz–dilated proper time. The entire spatial domain involves evolution of the coherent system. The longitudinal extent of the effective interaction region is much greater than is usual in $p(\overline{p}) - p$ and $\pi - p$ collisions, in those inelastic events in which the system occurs. If charged currents within the system are emitting bremsstrahlung [8], these photons can have very low energies, a few MeV or down to of order of the inverse of the above extended proper time, before they cut off [12].

We examine the possibility of a zero in the following dispersion relation, some time after the collision when the dynamics is assumed to have largely become that relevant to quasi–pions with effective mass μ, interacting coherently with a dense medium of constituent quarks and antiquarks with effective mass m. The fermionic medium is represented by a Fermi sea with momenta from 0 to p_F. If physical conditions for a zero in the dispersion relation are attained then the amplitude for a coherent boson field in the medium can emerge. (When appearing as numbers, all energies and momenta are henceforth to be understood as in units of μ.)

$$\omega^2 = 1 + k^2 - k^2 F^2(k) \Pi(k, p_F) \Longrightarrow 0 \tag{1}$$

The $(1 + k^2)$ terms are from the mass and kinetic energy; the negative third term results from the successive attractive P–wave interactions of nearly zero–energy ($\omega \to 0$) quasi–pions with the fermionic medium. The essential dynamical element is contained in the fact that the strong P–wave scattering interaction is attractive and starts out as k^2; we have included a phenomenological [F 1] form factor $F(k)$ to damp the interaction strength for $k \geq 3$. The "self–energy" function Π, for the pion in the medium is given explicitly by the following generalization of the Lindhard function [13] utilized in solids [14] for the interaction of phonons with the electron Fermi sea

$$\Pi(k, p_F) = \frac{12}{\pi^2}\frac{f^2}{4\pi} \int_{L_x}^{p_F} 2\,dx \int_{L_y}^{p_F} y\,dy \int_0^{\overline{\varphi}} 2\,d\varphi \cdot \frac{1}{(x^2+y^2+k^2+2yk\cos\varphi+m^2)^{1/2} - (x^2+y^2+m^2)^{1/2}} \tag{2}$$

$$\text{with} \quad \overline{\varphi} = \cos^{-1}\left(\frac{-k^2+2p_F^2-y^2-x^2}{2yk}\right)$$

$$L_y = \sqrt{2p_F^2 - x^2} - k$$

$$L_x = \begin{cases} 0 & \text{for } k > (\sqrt{2}-1)p_F \\ (2p_F^2 - (p_F+k)^2)^{1/2} & \text{for } k < (\sqrt{2}-1)p_F > 0 \end{cases}$$

We are evaluating the interaction function in cylindrical coordinates. The coherent interactions take place in the plane transverse to the collision axis. A quasi–pion with a small momentum $\vec{k}$ directed in this plane interacts with a fermion with transverse–momentum component $\vec{y}$ for $|\vec{y}| < p_F$, resulting in the intermediate–state fermion with a transverse momentum $(\vec{y}+\vec{k})$ and a total (squared) momentum $(\vec{x}^2+|\vec{y}+\vec{k}|^2) > 2p_F^2$ (a "hole" in the Fermi sea of momentum states). Coherence probably cannot develop with momentum $\vec{k}$ in the longitudinal direction because the system generally moves with considerable velocity along the collision axis in the collision c.m. These coherent interactions tend to be impeded by (relatively small) differences in this longitudinal motion in the parts of the system. The "strength" of Π is essentially governed by the coupling parameter [F 2] $(f^2/4\pi) \sim 0.08$, and by p_F^2 which is related to the fermionic density [F 3] by $\rho = \dfrac{3p_F^3}{\pi^2}\left[0.36\left(\dfrac{\mu}{m_\pi}\right)^3 fm^{-3}\right]$. Thus this density is of the order of 1 fm^{-3} for p_F of the order of 2 with $\mu = m_\pi$. Alternatively, $p_F \leq 3.5$ corresponds to approximately one dressing quark (or one antiquark) of a given color in a volume of about 1 (fm)^3. After expanding the denominator in eq. (2) in the k–dependent terms, we have evaluated Π with numerical integration and have examined the zero–condition in eq. (1). With [F 1] $F(k) = e^{-(k/6.2)}$ there are in fact zeros in the sensible domains $k < k_0 < p_F$ and $p_F < 3.5$. In Table 1 we list several paired values of k_0, p_F. Note that k_0 increases as p_F decreases, as is intuitively expected. This is because in the interaction term in eq. (1) we are in a region of k where the k^2 factor still controls the strength, rather than $F^2(k)$. For each fixed p_F, we have gradually increased k until the minimum in the dispersion relation is reached. The most negative value of ω^2 is $(\omega^2)_{min} = -|\omega^2_{min}|$, occurring at $k = k_c$. These quantitites are also given for the examples in Table 1.

The quantity ω^2_{min} determines the negative energy density contributed by the coherent quasi–pion condition in the fermionic medium according to an estimate carried out in the following manner. We minimize the condensate energy with respect to the amplitude A of a trial (static [F4]) wave function of the (running–wave [F5]) form $\phi(\vec{r}) = Ae^{ik_cx}$ (for π^- with Ae^{-ik_cx} for π^+; x lies in the transverse plane). For the expectation value of the Hamiltonian in the condensate we use

$$\begin{aligned}
\langle H \rangle \; &= \; \Big\langle \frac{1}{2}\int d^3\vec{r}\phi_i^\dagger(\mu^2 - \vec{\nabla}^2)\phi_i \\
&+ \frac{1}{2}\int dk_x dk_y dk_z \{k_x^2 F(k_x^2)\Pi(k_x, p_F)\}\phi_i^\dagger(\vec{k})\phi_i(\vec{k}) \\
&- \frac{1}{16F^2}\int d^3\vec{r}\left\{4(\partial_\mu\phi_i^\dagger\phi_i)(\phi_j^\dagger\partial^\mu\phi_j) + 2\mu^2(\phi_i^\dagger\phi_i)(\phi_j\phi_j^\dagger)\right\}\Big\rangle \\
\text{where} \quad & \phi(\vec{k}) = A\delta(k_x - k_c)\delta(k_y)\delta(k_z) \qquad (3)
\end{aligned}$$

The indices i, j are summed over π^+ and π^- and we have modelled a (momentum–dependent) quartic interaction between quasi–pions by the form suggested by chiral dynamics [15], with $F \sim (\mu/m_\pi)\, F_\pi$ ($F_\pi = 95$ MeV). This term gives an effective interaction which is repulsive, and stabilizes the condensate [10, 11]. The minimization condition then reads as follows

$$0 = (-|(\omega^2)_{min}| + 8|A|^2\lambda)V \tag{4}$$

with $\lambda = (4k_c^2 - 2\mu^2)/16F^2$ and V the volume of the system. The result for $|A|^2$ is thus

$$|A|^2 = |(\omega^2)_{min}|/8\lambda \sim 0.07\mu^2 \tag{5}$$

Substitution of (5) into eq. (3) makes possible an estimate of the condensate energy density [F 5]

$$E_c/V = -|(\omega^2_{min})|^2/16\lambda \sim -1.5\frac{\text{MeV}}{\text{fm}^3} \tag{6}$$

In this estimate we have used $\omega^2_{min} \simeq -0.9$ and $\lambda \sim 1.7$ for $k_c \sim 1.9$, with $\mu \sim m_\pi$.

As a check on the approximate stability of the results in Table 1, we have examined the domains of k_0, p_F for zeros in the dispersion relation in eq. (1), using the standard analytical form for the Lindhard function as evaluated in a spherical system [14]. This amounts to replacing eq. (2) by

$$\begin{aligned}\Pi_L(k, p_F) \quad &= \quad \frac{12}{\pi^2}\left(\frac{f^2}{4\pi}\right)(\pi m p'_F) \\ &\left[1 + \frac{p'_F}{k}\left\{1 - \left(\frac{k}{2p'_F}\right)^2\right\} ln\left|\frac{1+\left(\frac{k}{2p'_F}\right)}{1-\left(\frac{k}{2p'_F}\right)}\right|\right]\end{aligned} \tag{7}$$

with p'_F related to the p_F in the cylindrical generalization in eq. (2) by $p'_F = \sqrt{2}p_F$. We use $F(k) = e^{-(k/6.2)}$. In Table 2 we list several paired values of k_0, p'_F. Again for each fixed p'_F we gradually increase k until the minimum in the dispersion relation is reached. These negative values of ω^2_{min} are given in Table 2 together with the values of $k = k_c$ at which they occur. Somewhat smaller values of k_0, p'_F occur than in the cylindrical generalization, but the minimum is less pronounced. In the cylindrical evaluation we have observed that the purely phenomenological function $F(k)$ which damps the interaction at high k cannot be made too weak i.e. like $e^{-(k/B)}$ with

$B > 6.2$. This is because then a minimum value ω^2_{min} is not readily obtained for fixed p_F with moderate increase of k ($\leq p_F$), within the present approximations for Π in the dispersion relation eq. (1). Of course, even more pronounced values of ω^2_{min} than those discussed here imply increased metastability of the hypothetical coherent system. For the present development we feel that the essential element is in showing the occurrence of the zeros in the dispersion relation for sensible domains of k_0, p_F; and this with minimal dependence upon the arbitrary form factor $F(k)$.

Consider a situation in the collision when, as an "initial" condition, a degree of coherence has been set up in the transverse plane; assume that the dynamics is near the condition $\omega^2_{min}(k_c)$. How does the coherence extend in the plane during the intrinsic time available before the system breaks down ? We estimate this behavior by using the following approximate expression for $\delta\omega(\delta k_x)$ near to $|\omega^2_{min}|^{1/2}$ for a change δk_x near to $k_x = k_c$.

$$\begin{aligned}
\omega^2 &\simeq -|\omega^2_{min}| + \frac{d^2\omega^2}{d^2k}|_{k_c} \frac{(\delta k_x)^2}{2} \\
\Rightarrow \omega(\delta k_x) &= i\{|\omega^2_{min}|^{1/2} - D(\delta k_x)^2\} = i|\omega^2_{min}|^{1/2} + \delta\omega(\delta k_x) \\
\text{with} \qquad & D = \frac{d^2\omega^2}{d^2k}|_{k_c} \frac{1}{4|\omega^2_{min}|^{1/2}}
\end{aligned} \tag{8}$$

Formally doing a momentum–space Fourier transform over $u = \delta k_x$ for large times, using the steepest–descent approximation, gives

$$\begin{aligned}
&\int_{-\infty}^{+\infty} du\, e^{iux} e^{-i\delta\omega(u)t}\, \Pi \simeq \int_{-\infty}^{+\infty} du\, e^{iux - Du^2 t}\, \Pi \\
\simeq\ & e^{-x^2/4Dt} \cdot \sqrt{\frac{\pi}{Dt}} \cdot \Pi(k_c(x), p_F(x)) \propto e^{-x^2/4R_x^2(t)}
\end{aligned} \tag{9}$$

We have used a local density approximation ($p_f(x)$) in Π and have taken $u_{sad} = ix/2Dt$. Eq. (9) indicates that the x–extension of the interaction is in part controlled by a function of time $R_x = \sqrt{Dt}$: The extension has the possibility to become large only as $\sqrt{t}$ where large t is taken from the "initial" collision condition, and x is added to the initial transverse extension in the collision, $\sim 1/m_\pi$. An effective velocity for the increase of R_x may be defined from

$$\begin{aligned} R_x &= v(t)t \\ \Rightarrow v(t) &= \sqrt{D/t} \end{aligned} \tag{10}$$

An average velocity during an "apriori" proper time T for the system is thus

$$\overline{v}(T) = 1/T \int_0^T dt\ v(t) = 2\sqrt{\frac{D}{T}} \tag{11}$$

This velocity generally is significantly less than unity (as estimated below). If viewed as an upper limit for signal propagation as the dynamical degrees of freedom switch from those of QCD to those of the coherent interactions, it implies an extension of the proper time interval prior to breakdown, to of the order of $T_p \sim T/\overline{v}$. In this extended intrinsic time interval, a characteristic transverse extension is then

$$\overline{R}_x(T_p) = 1/T_p \int_0^{T_p} dt\ R_x(t) = \frac{2}{3}\sqrt{DT_p} \simeq \frac{1}{3}(4D)^{1/4}(T)^{3/4} \tag{12}$$

The above argument is heuristic. The physical points are probably relevant for such an evolving dynamical system. Since $4D = \frac{d^2\omega^2}{d^2k}|_{k_c}/|(\omega^2_{min})|^{1/2}$ is of order unity [F 6] (in unit $1/\mu$) one can readily get a feeling for the numbers: for $T \simeq 8/m_\pi = 4 \cdot 10^{-23}\ sec$, $\overline{v}(T) \sim 0.35$, $T_p \sim 22.5/m_\pi$, $\overline{R}_x(T_p) \sim 1.6/m_\pi$ (for $\mu \sim m_\pi$). Adding the initial $\sim 1/m_\pi$ to $\overline{R}_x$ gives $2.6/m_\pi$. This number and that for T_p have immediate physical consequences [1], [F 7]. (1) The breakdown of the coherent system results in a few pions with anomalously small transverse momenta $\sim 1/\overline{R}_x \simeq 50$ MeV/c. (2) Very soft photons can be emitted in the system, down to energies of the order of $1/T_p \simeq 6$ MeV, appearing with laboratory transverse momenta ≤ 4 MeV/c.

We turn now to a more detailed discussion [8] of the possible emission of soft photons by a hypothetical, new coherent condition of matter formed in high–energy collisions [1]. We assume that charged currents exist during the evolution of the system prior to its breakdown into some pions. The relevant approximate amplitudes are given by Ae^{ik_0x} for π^- and Ae^{-ik_0x} for π^+, with values for A and k_0 that we have estimated. Using also our estimate for T_p and $\overline{R}_x(T_p)$, we calculate the soft–photon energy spectrum and estimate the emission strength.

In the coherent system, the matrix element [8] for a charged quasi–pion to propagate to a point in the medium, to emit there a very soft photon with momentum $\vec{p}_\gamma$ and polarization $\hat{\epsilon}_\gamma$, and to propagate further in the medium, is

$$
\begin{aligned}
|\mathcal{M}_\gamma| \quad = \quad & \Bigg| \left(2e\left[(2\overline{R}_x)A\right]^2\right)\ (2\vec{k}_0\cdot\hat{\epsilon}_\gamma)\Big/ \\
& \Bigg\{\left(\omega - \left[\mu^2 + \left(\vec{k}_0+\frac{\vec{p}_\gamma}{2}\right)^2 + \overline{\Pi}\left(\omega,\vec{k}_0+\frac{\vec{p}_\gamma}{2},p_F\right)\right]^{1/2}\right) \\
& \cdot\left(\omega - \left[\mu^2 + \left(\vec{k}_0-\frac{\vec{p}_\gamma}{2}\right)^2 + \overline{\Pi}\left(\omega,\vec{k}_0-\frac{\vec{p}_\gamma}{2},p_F\right)\right]^{1/2}\right)\Bigg\}\Bigg| \\
\omega, p_\gamma \overset{\simeq}{\Rightarrow} 0 \quad & \left|\frac{\left(2e\left[(2\overline{R}_x)A\right]^2\right)(1/c)\left(2\vec{k}_0\cdot\vec{\epsilon}_\gamma\right)}{\left(\left(\vec{p}_\gamma\cdot\vec{k}_0\right)^2+\left(\frac{Im\,\overline{\Pi}}{c}\right)^2\right)^{1/2}}\right| \qquad (13)
\end{aligned}
$$

where in terms of the Π defined by eqs. (1, 2), we have here $\overline{\Pi}(k,p_F) = k^2F^2(k) \cdot \Pi(k,p_F)$, and $|c| = |1+\frac{\overline{\Pi}}{k_0^2}| \sim \frac{\mu^2}{k_0^2}$. We have evaluated $|\mathcal{M}_\gamma|$ at $\omega^2(k_0) \Rightarrow 0$, the limit of a usual perturbative treatment with oscillatory behavior of amplitudes in time. Continuation to $\omega^2 < 0$ involves that in $|A|^2 \propto |\omega^2| \neq 0$. In eq. (13) we have retained the leading terms as p_γ formally goes to zero. The approximate strength (in units of charge e) is evident in the overall factor $[(2\overline{R}_x)A]^2$ which arises at the vertex for bremsstrahlung from the charged–pion condensate [F 8]; the additional factor of 2 occurs from adding coherently the contribution of oppositely–directed π^- and π^+ current filaments [F 9]. Our previous estimates $A \sim 0.26\mu$ and $2\overline{R}_x \sim (5/\mu)$, allow an immediate feeling for the significant probability factor E

$$E \sim \left(2\left[(2\overline{R}_x)A\right]^2\right)^2 \simeq 12 \qquad (14)$$

In eq. (13), we have utilized a necessary, phenomenological imaginary–part addition to $\overline{\Pi}$. We write

$$\begin{aligned} Im\,\overline{\Pi} &= \left\{\left(Im\,\overline{\Pi}_s\right)^2 + \left(Im\,\overline{\Pi}_t\right)^2\right\}^{1/2} \\ Im\,\overline{\Pi}_t &= (\mu/T_p) \sim m_\pi^2/22.5 \\ Im\,\overline{\Pi}_s &\simeq \left(1/\pi R_x^2(t)\right) \simeq 4u/\pi t \sim 4m_\pi p_\gamma/\pi \end{aligned} \tag{15}$$

The essential lower limit [12] on the anomalous p_γ–spectrum is incorporated through the characteristic intrinsic (long) time T_p for the ultimate breakdown of the coherent system into some pions; this is in $Im\,\overline{\Pi}_t$, and to be definite we have used our numerical estimate for T_p and put $\mu = m_\pi$. There is another physical effect which inhibits development of the condensate over a time interval t following the instigation of a coherent condition: this is scattering interactions, with small momentum transfer, in the fermionic density over the transverse plane. This effect is incorporated through $Im\,\overline{\Pi}_s$, where we have assumed the probability for such scatterings to be approximately proportional to the inverse of the effective transverse area at t. We have used eq. (10) giving $R_x^2(t) \propto t$, and have set $1/t \sim p_\gamma$ as a measure of the progressive smallness of the relevant momenta–transfer as time increases. From eqs. (13–15) we calculate the differential probability for anomalous soft–photon bremsstrahlung in the phase–space element $d\rho_\gamma$ where, for simplicity, we have used a polar angle θ of $\vec{p}_\gamma$ with respect to $\vec{k}_c$ (a direction in the transverse plane in each event).

$$\begin{aligned} |\mathcal{M}_\gamma|^2\,d\rho_\gamma &= (E/c^2)(e^2/2\pi^2)(dp_\gamma/p_\gamma)(d\phi/2\pi)d(\cos\theta) \\ &\quad \cdot \frac{k_0^2\sin^2\theta}{\left[k_0^2\cos^2\theta + \frac{\left\{(Im\,\overline{\Pi}_s)^2 + (Im\,\overline{\Pi}_t)^2\right\}}{c^2 p_\gamma^2}\right]} \end{aligned} \tag{16}$$

The p_γ spectrum is bremsstrahlung–like [8]. It is nearly proportional to $(1/p_\gamma)$ until it is cut–off by the part of the denominator involving $Im\,\overline{\Pi}_t \propto 1/T_p$. For smaller p_γ the probability approaches zero as p_γ in accord with a basic theorem [12]. The cut–off clearly occurs for p_γ so low that the following condition holds,

$$\begin{aligned} & Im\,\overline{\Pi}_s/p_\gamma < Im\,\overline{\Pi}_t/p_\gamma \\ & \Rightarrow p_\gamma < (\pi/4)\,(m_\pi/22.5) \sim 5\ MeV \end{aligned} \tag{17}$$

This value for p_γ based upon our definite numerical estimate in eq. (15) is low on the usual hadronic scale of the order of $m_\pi/2 = 70$ MeV. The anomalous photons are quite soft. Integrating over the angles in eq. (16) gives the differential p_γ probability, with the overall weight–factor $(\frac{EK}{c^2}) \geq 1$.

$$\begin{aligned} \frac{dP(\gamma)}{dp_\gamma} &= \left(\frac{EK}{c^2}\right)\left(\frac{e^2}{\pi^2}\right)\left(\frac{1}{p_\gamma}\right) \\ \text{where } K &= \left\{\frac{(1+B)}{B^{1/2}}\right\}\tan^{-1}\left(\frac{1}{B^{1/2}}\right) - 1 \\ \text{with } B &= \frac{1}{c^2}\frac{1}{k_0^2 p_\gamma^2}\left\{(Im\,\overline{\Pi}_s)^2 + (Im\,\overline{\Pi}_t)^2\right\} \end{aligned} \tag{18}$$

Where could these soft photons appear in the laboratory system, say in an experiment with 280 GeV π^- on a proton target [4, 5] ? Since the ϕ distribution in eq. (16) is uniform (this azimuthal angle is in a plane perpendicular to the transverse plane), in the coherent system the photons have comparable components of momentum along the collision axis and perpendicular to it. An immediate consequence is the possibility of observing anomalous photons with exceedingly low transverse momenta in the laboratory; a characteristic value being of the order of $(5/\sqrt{2})$ MeV/c. We assume [1] that the center of mass of the coherent system tends to move rapidly along the collision axis in the collision c.m., in particular along the π^-–beam direction. (Note our reason for the π^- direction stated in the next–to–the–last paragraph of the paper.) In addition there is the motion of the collision c.m. with respect to the laboratory system. With an overall $\gamma(v) = (1 - v^2)^{-1/2} \sim 280$, the low–$p_T$ photons appear inside a cone of about 10 milliradians around the beam direction, with laboratory energies of a few hundred MeV. Their center–of–mass–rapidities are between about +2 and +4.

It is clear that apart from a possible, conceptually important soft–photon signal, a signal from events with a few pions that have anomalously small transverse momenta ($\leq$ 50 MeV/c) should occur. Anomalous soft–photon events would be correlated with these; it is possible that photon emission instigates the breakdown. The estimate that the coherent system typically breaks down into few pions is simply based upon the estimate of its transverse extent, $(2\overline{R}_x) \sim 5/m_\pi$. This dimension accomodates 2 or 4 pions (not overlapping), with $2 < r_\pi^2 >^{1/2} \sim 1/m_\pi$ [F 10]. The observational problem is that there may often be some pions with the usual characteristic transverse momentum of about 300 MeV/c also present in the events; therefore the average will be only somewhat lower. One must look at the distribution of pion transverse momenta in individual events; preferably those with low multiplicity (compared to $< n(s) >$). It is noteworthy that the recent, comprehensive study of several hundred individual cosmic–ray events [7] has concluded that there exists a class of showers that are highly collimated along the projectile direction and that carry a large energy fraction with a penetrating, hadronic character. The authors emphasize [7] the unusual feature that these showers seem to often involve only few particles (hadrons and photons) at production, but with anomalously small transverse momenta, of the order of 10 MeV/c.

It seems likely that once $\sqrt{s}$ is high enough to produce high densities (in a particular collision $A + B$) that allow instigation of the coherent effect and which, although falling, can sustain it for a time, the growth with energy is moderate. For example in $p(\overline{p}) - p$ collisions, it may grow as the central opacity which is a measure of an initial, interacting–matter density induced by the collision of fully–overlapping hadrons. According to a good parameterization [16] the opacity in head–on $p(\overline{p}) - p$ collisions grows by about 40% between $\sqrt{s}$ = 53 and 546 GeV, and by another 20% between 546 and 1800 GeV. The complete inelastic cross section grows [16] in about the same way percentage–wise; thus formation of the coherent system occurs as approximately a fixed fraction of σ_{inel} (but a somewhat decreasing fraction of σ_{total}). It is possible that an energetic boson projectile (π, K) is more effective at lower $\sqrt{s}$ in instigating the quasi-pion condensate. Such a boson projectile clearly carries much energy into a potential collision involving (quasi-)bosons [F 11]; this is generally not the case for $p(\overline{p}) - p$ collisions, or $\sqrt{s}$ must be higher. The dynamical circumstance which might become more probable at $\sqrt{s}$ in the multi–TeV range is the breakdown into a large number of pions ($\geq< n(s) >$). This is possible if T_p and $\overline{R}_x(T_p)$ become significantly larger [1]. If this were to happen, then events could occur with the proverbial [7] large number of $\pi^- \simeq \pi^+$ and few or no π^0, or vice versa, since the coherent system is either principally [17] a charged condensate (π^-, π^+), or principally a π^0 condensate. In general, the relative probability for such an "unmistakable burst" is low, and thus this feature of the system is difficult to use for establishing its presence [18, 19, 20] [F 12].

This paper has given dynamical reasons for the possibility of the transient formation of a new coherent condition of matter in high–energy hadronic collisions. We have quantitatively estimated a number of properties of the system and have enumerated their related observational consequences for current experiments. It is possible that there is already evidence for the formation of such a new condition [1, 2, 3, 4, 5, 7, 9]. This is a physical idea which must be guided by experiments.

p_F	k_0	$(\omega^2)_{min}$	k_c
2.83	1.9	-0.89	2.8
3.03	1.8	-1.67	3.0
3.44	1.7	-3.61	3.3

Table 1. Some definite values in the domains of k_0, p_F for zeros in the dispersion relation eq. (1), using the interaction in eq. (2) and $F(k) = e^{-(k/6.2)}$. The minimum value $(\omega^2)_{min}$ which occurs at k_c for the fixed p_F is also tabulated, with k_c.

p'_F	k_0	$(\omega^2)_{min}$	k_c
1.9	1.0	-0.49	1.7
2.0	0.9	-0.89	1.8
2.2	0.8	-1.52	2.0

Table 2. Some definite values in the domains of k_0, p'_F for zeros in the dispersion relation eq. (1), using the interaction in eq. (7) and $F(k) = e^{-(k/6.2)}$. The minimum value $(\omega^2)_{min}$ which occurs at k_c for the fixed p'_F is also tabulated, with k_c.

Appendix

We give in this appendix a summary of the origin of the phenomenological formula which we have recently used to represent the anomalous behavior in the diffractive, elastic scattering of hadrons [1]. The derivation is carried out within the general geometric framework for high–energy scattering [21]. The specific dynamical parameters utilized [1] are related to those estimated in the present paper to be characteristic of the new coherent condition. We consider the occurrence of this system in some of the real, inelastic intermediate states which contribute to the amplitude for diffractive scattering, as the dynamical origin of anomalous structure in the amplitude.

The structure of Glauber's formula [21] for the imaginary, high–energy amplitude $\overline{F}(\vec{k}',\vec{k})$, for scattering from initial momentum $\vec{k}$ to final $\vec{k}'$, is given in terms of the eikonal Ω, by

$$\begin{aligned}\overline{F}(\vec{k}',\vec{k}) &= \frac{F(\vec{k}',\vec{k})}{k} \\ &= i\int\frac{d^2\vec{b}}{2\pi}\int_{-\infty}^{+\infty} dz e^{-i\vec{k}'\cdot(\vec{b}+\vec{z})}\Omega(\vec{b}+\vec{z})\left\{e^{i\vec{k}\cdot(\vec{b}+\vec{z})-\int_{-\infty}^{z}dz'\Omega(\vec{b}+\vec{z}')}\right\} \qquad (A1)\end{aligned}$$

The derivation [21] of the form of the wave function embodied by the factor in brackets $\{\ldots\}$, involves the restriction to sufficiently small (c.m.) scattering angles θ such that $\{\frac{k\theta^2}{2}\}z_m < 1$, where the bracketed quantity is the approximated longitudinal momentum transfer ($\simeq (-t)/\sqrt{s}$) and z_m is a characteristic, maximal longitudinal dimension within which the primary collision dynamics occurs (as parameterized by Ω). Without this constraint, additional phase factors occur in the exponentiated quantity. Consistent with the same constraint, one usually replaces the factor $e^{i(\vec{k}-\vec{k}')\cdot\vec{z}}$ by unity; this brings the z integration to that of an exact differential, with the textbook result [21],

$$\begin{aligned}
\overline{F}(\vec{k}',\vec{k}) &= i\int_0^{\infty} db\, b\, J_0(\sqrt{-t}\, b)\,(1-e^{-\Omega(b,s)}) \\
\text{with }\ \Omega(b,s) &= \int_{-\infty}^{+\infty} dz'\ \Omega(\vec{b}+\vec{z}'),\ \ \text{and}\ \ -t=(\vec{k}'-\vec{k})^2 \simeq \left(\frac{\sqrt{s}}{2}\,\theta\right)^2 \\
& \qquad\qquad k'=k=\frac{\sqrt{s}}{2} \qquad\qquad (A2)
\end{aligned}$$

We treat, perturbatively, formation of the condensate as an occasional, dynamical consequence of the initial stage of the collision, and add phenomenologically $\{\Delta\Omega\}$ to the eikonal. We neglect this addition in the bracketed quantity in eq. $(A1)$, and extend the upper limit of the z' integration to $z'=\infty$. The principal contribution in this integration domain occurs around $z'=0$ since the primary, overlapping configuration of interacting partons involved in the collision is strongly Lorentz contracted along the collision axis at high energies. The addition to the diffractive amplitude is then

$$\Delta\overline{F}(\vec{k}',\vec{k},\vec{k}_c) \simeq i\int \frac{d^2\vec{b}}{2\pi}\int_{-\infty}^{\infty} dz e^{-i\vec{k}'\cdot(\vec{b}+\vec{z})}\{\Delta\Omega(\vec{b}+\vec{z})\}e^{i(\vec{k}-\vec{k}_c)\cdot(\vec{b}+\vec{z})}e^{-\Omega(b)} \qquad (A3)$$

In eq. $(A3)$, the momentum $(\vec{k}-\vec{k}_c)$ which characterizes the "incoming wave–function" is the initial (collision–c.m.) momentum reduced by the momentum $\vec{k}_c$ associated with formation of the condensate where $\vec{k}_c = \{\sqrt{-t_c}\,\hat{b} + (\frac{-t_c}{\sqrt{s}})\,\hat{z}\}$ and is thus largely transverse. The magnitude $|\vec{k}-\vec{k}_c|$ is effectively the momentum of the coherent system, including initial projectile. An oscillatory behavior in the transverse plane with a small wave number k_c appears through the factor $e^{-i\vec{k}_c\cdot\vec{b}}$. We alternatively use $\{\Delta\Omega(\vec{b}+\vec{z})\}\,e^{-i\vec{k}_c\cdot\vec{b}} \Rightarrow \{f(z)f(b)\}J_0(k_c b)$ in eq. $(A3)$, leading to

$$\begin{aligned}
\Delta\overline{F} &\simeq i\int_{-\infty}^{\infty} dz\, e^{i\{(\vec{k}'-\vec{k})-\vec{k}_c\}\cdot\vec{z}}\, f(z)\int_0^{\infty} db\, b\, J_0(yb)\, J_0(y_c b)\, f(b)\, e^{-\Omega(b)} \\
\text{where} &\qquad y=\sqrt{-t},\ \ y_c=\sqrt{-t_c} \qquad\qquad (A4)
\end{aligned}$$

Here we do not replace the exponential in z by unity because the oscillations in this factor can be relevant over the extended interaction–domain of z in which the condensate formation takes place. This occurs out to some distant z_h where the coherent system breaks down into a few pions after a relatively long proper time (which is Lorentz dilated).

Using as a first approximation (condensate formation is favored in central collisions where $\Omega(b \sim 0)$ is large),

$$\{f(z)f(b)\} \simeq [\delta(z-z_h)+\delta(z+z_h)](N\,e^{\Omega(b)})$$

in eq. $(A4)$, gives

$$\begin{aligned} \Delta\overline{F}(t,s) &= i\left[2\cos\left\{(y^2-y_c^2)\left(\frac{T_h}{2m(s)}\right)\right\}\right]\left(N\frac{\delta(y-y_c)}{y_c}\right) \\ &\sim iN\left[\cos\left\{(y^2-y_c^2)\left(\frac{T_h}{2m(s)}\right)\right\}\right]\left(\frac{R_x}{\sqrt{\pi}y_c}\,e^{-(y-y_c)^2R_x^2/4}\right) \qquad (A5) \end{aligned}$$

Generally a sum of weighted δ–functions could represent different points $z_h' < z_h$, with the largest weight at z_h where the condensate is most fully developed just before breakdown; z_h relates to the most rapid oscillatory behavior in $\Delta\overline{F}$. For a more realistic phenomenological form [1] of the local structure near $y = y_c$, we have smeared the δ–function maintaining the overall normalization (in general y_c fluctuates on an event basis). In the argument of the oscillatory function the longitudinal momentum–transfer is $\simeq y^2/\sqrt{s} \simeq \sqrt{s}\theta^2/4$ and the Lorentz–dilated time interval (leading to z_h) is $(\sqrt{s}T_h/2m(s))$, where $m(s)$ is an effective mass–like parameter which includes $1/\lambda(s)$ where $\lambda(s)$ is the fraction of $\sqrt{s}/2$ involved in the (assumed) rapid motion of the center of mass of the coherent system in the collision c.m. The parameters y_c, T_h, R_x have been estimated in this paper [F 13]. The size of the anomaly is controlled by the dimensionless $N \simeq 2|A|^2\overline{\sigma}$, where $|A|^2$ has been estimated in eq. (5) of this paper and $\overline{\sigma}$ is a characteristic cross section associated with the initial stage of condensate formation in a particular collision system $A+B$; empirically [1] $\overline{\sigma} \sim 1$ mb.

Footnotes

[F 1] This purely phenomenological form factor includes, in addition to the cut–off of the interaction of quasi–pions with dressing quarks at high k, the diminishing amplitude for a quark and antiquark to be correlated as a quasi–pion with high overall momentum k.

[F 2] In our numerical work we use an estimate for this coupling parameter given by the number relevant for the low–energy, P–wave pion–nucleon interaction. There $f^2_{\pi N}/4\pi = (m_\pi/2m_N)^2(g^2_{\pi N}/4\pi)$, where $g^2_{\pi N}/4\pi \sim 14.5$ is the usual pseudoscalar coupling. In the present case $f^2/4\pi = (\mu/2m)^2(g^2/4\pi)$; we use $(\mu/m)\, g \sim (m_\pi/m_N)\, g_{\pi N}$, and $m \sim 2.5\mu$ in eq. (2).

[F 3] We use the weight of two spin orientations and three colors for quarks and antiquarks.

[F 4] As we have stated, the system is evolving in time up to the time of breakdown. This treatment is an approximation in a time interval following the onset of condensate structure as the breakdown time is approached.

[F 5] The standing–wave form $A \sin k_x x$ which could give a more negative condensate energy density may evolve later, in particular for a "π^0" condensate.

[F 6] For the examples listed in Table 1, the approximate value is $4D \sim 2$.

[F 7] In [1] we have utilized these numerical estimates in a phenomenological representation of the localized structure in the differential cross section for diffractive, elastic scattering.

[F 8] Formally in eq. (3), $\delta(k_x - k_c) \Rightarrow (2\overline{R}_x)$ for a system whose transverse extension is not arbitrarily large.

[F 9] In this paper we do not address the attractive, magnetic interaction–energy in such configurations. This energy appears to favor the $\pi^- - \pi^+$ condensate over the π^0 condensate.

[F 10] The variable effective mass of the system includes the negative condensate energy; thus it seems likely to be spread from near $2m_\pi$ to below $\sim 4m_\pi$. In a sense, the system appears like a variable–mass coherent excitation of a "dressing" constituent quark.

[F 11] If such a bosonic collision is involved at the first stage, the transverse deflection of the incident pion may sometimes determine the direction of the condensate momentum in individual events.

[F 12] Of course observation of one such event in highly–controlled circumstances [20] may be enough. Reference [20] seeks a signal from a hypothetical, transient change in the "vacuum". A dynamical reason for instigation of this change is absent. In the present paper we have discussed a coherent dynamical structure of <u>matter</u> which gives rise to a non–zero expectation value of a pion–like field.

[F 13] The parameter y_c is approximately twice the condensate momentum calculated in the text (i.e. $2k_0 < y_c < 2k_c$), because the oscillatory behavior of matter in the transverse plane involves the square of the condensate amplitude: $\sim |A\,e^{ik_cx} + Ae^{-ik_cx}|^2 = 2|A|^2\{1 + \cos 2k_cx\}$. The first term is incorporated in the mean distribution and the second term represents the matter oscillations associated with the condensate.

References

[1] S. Barshay and P. Heiliger, "A Transient New Coherent Condition of Matter: The Signal for New Physics in Hadronic Diffractive Scattering", Aachen, Karlsruhe preprint PITHA 94/15, TTP 94–03, March 1994, appears in Z. Phys. C 64 (1994).

[2] P. V. Chliapnikov et al., Phys. Lett. B 141 (1984) 276.

[3] NA 22 Collab., F. Botterweck et al., Z. Phys. C 51 (1991) 54.

[4] SOPHIE/WA 83 Collab., S. Banerjee et al., Phys. Lett. B 305 (1993) 182.

[5] SOPHIE/WA 83 Collab., "Some Aspects of Anomalous Soft Photon Production in $\pi - p$ Interactions at 280 GeV", Report by T. J. Brodbeck (1993).

[6] S. Barshay and R. Arnold, Nuovo Cim. 35 A (1976) 457.

[7] Chacaltaya and Pamir Collab., L. T. Baradzei et al., Nucl. Phys. B 370 (1992) 365.

[8] S. Barshay, Phys. Lett. B 227 (1989) 274.

[9] EMC Collab., J. J. Aubert et al., Phys. Lett. B 218 (1989) 258.

[10] A. B. Migdal, Sov. Phys. JETP 34 (1972) 1184; 36 (1973) 1052.

[11] S. Barshay, G. E. Brown and G. Vagradov, Phys. Lett. B 43 (1973) 359.

[12] F. E. Low, Phys. Rev. 110 (1958) 974.

[13] J. Lindhard, Kgl. Danske Videnskab, Selskab, Mat. Fys. Medd. 28 (1954) 8.

[14] J. R. Schrieffer, "Theory of Superconductivity", W. A. Benjamin, Inc., Publishers, New York, Amsterdam 1964, p. 139.

[15] S. Weinberg, Comm. in Nuclear and Particle Physics 2 (1968) 28.

[16] S. Barshay, P. Heiliger and D. Rein, Z. Phys. C 56 (1992) 77.

[17] S. Barshay, Particle World 3 (1994) 180.

[18] UA1 Collab., G. Arnison et al., Phys. Lett. B 122 (1983) 189;
UA5 Collab., K. Alpgard et al., Phys. Lett. B 115 (1982) 71;
UA5 Collab., G. H. Alner et al., Phys. Lett. B 180 (1986) 415.

[19] JACEE Collab., J. Lord and J. Awai, paper 515, Int. Conf. on HEP, Dallas, Texas, Aug. 1992; J. Awai, Univ. of Washington preprint UWSEA 92–06.

[20] Minimax proposal T 864, J. Bjorken and C. Taylor, Fermilab, April 1993.

[21] R. J. Glauber, Lectures on theoretical physics, Vol. 1, p. 315. W. E. Brittin et al. (eds.), New York, Interscience 1959.