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$\mathcal{O}(\alpha\alpha_s \ln m_t^2)$ Non-Universal Corrections to the Decay Rate $\Gamma(Z \to b\bar{b})^*$

A.Kwiatkowski[†] , M.Steinhauser

Institut für Theoretische Teilchenphysik
Universität Karlsruhe
Kaiserstr. 12, Postfach 6980

D-76128 Karlsruhe, Germany

Abstract

The partial decay rate $\Gamma(Z \to b\bar{b})$ is significantly influenced by the mass of the top quark due to electroweak radiative corrections. The leading $\sim m_t^2$ and the next-to-leading contribution $\sim \ln m_t^2$ are known to be numerically of similar size. In this work we calculate the QCD corrections to the logarithmic correction using the heavy top mass expansion. The $\mathcal{O}(\alpha\alpha_s \ln m_t^2)$ corrections are of the same order as the QCD corrections to the quadratic top mass term, but of different sign.

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[†]Address after 15. October: Lawrence Berkeley Laboratory
Physics Division
Theoretical Physics Group
1 Cyclotron Road
Berkeley, CA 94720, USA

1 Introduction

Although the direct observation of the top quark is out of range for the experiments at LEP, several observables are affected by the top quark through virtual states in higher order radiative corrections.

High precision measurements and the comparison of these quantities with the theoretical predictions allow to extract bounds on the top mass. Present analysis of e^+e^- collisions at the Z peak estimates the top mass in the range $m_t = 173^{+12+13}_{+18-21}$ GeV [1], which is in agreement with top masses of $m_t = 174 \pm 10^{+13}_{-12}$ GeV from $\bar{p}p$ collisions at TEVATRON [2]. Of particular interest for deducing the limits on m_t from LEP data is the partial decay rate $\Gamma(Z \to b\bar{b})$. On the one hand this quantity exhibits a strong sensitivity on the top mass [3] as the leading term of the electroweak corrections is quadratic in m_t and the next-to-leading logarithmic contribution $\ln m_t^2$ is numerically of the same size. On the other hand the already small uncertainty of the measurement of $R_{b\bar{b}} = \Gamma(Z \to b\bar{b})/\Gamma(Z \to \text{hadrons}) = 0.2192 \pm 0.0018$ [1] is expected to be reduced below 1% in the future. As a consequence the determination of QCD corrections to the electroweak one loop result became increasingly important.

For the quadratic top mass contribution these $\mathcal{O}(\alpha\alpha_s m_t^2)$ corrections were calculated by four independent groups [4, 5, 6, 7]. In this work we present the QCD corrections of order $\mathcal{O}(\alpha\alpha_s \ln m_t^2)$ to the next-to-leading top mass contribution.

Our calculation is performed using dimensional regularisation in the $\overline{\text{MS}}$-scheme with anticommuting γ_5 in the 't Hooft-Feynman gauge. As in our previous work [7] we employ the hard mass procedure [8, 9, 10, 11] to derive the next-to-leading order in the inverse top mass expansion. In order to handle large expressions during this expansion we use the algebraic manipulation program FORM [12]. Massless multiloop integrals are evaluated with the help of the software package MINCER [13].

If we consider the properly resummed propagator of the Z boson we get in the case $q^2 \approx M_Z^2$ for the transverse part

$$D_T^{\mu\nu}(q^2) = -\frac{1}{i}\frac{1}{1+\Pi'(M_Z^2)}\frac{g_{\mu\nu}}{M_Z^2 - q^2 - \frac{i\,\mathrm{Im}\;\Pi(M_Z^2)M_Z}{1+\Pi'(M_Z^2)}} \tag{1}$$

where M_Z is the renormalized Z boson mass and $\Pi(q^2)$ is the transverse part of the polarisation tensor $\Pi_{\mu\nu}(q^2)$. One can get $\Pi(q^2)$ through contracting with $1/(D-1)(g_{\mu\nu} - q_\mu q_\nu/q^2)$ where $D = 4 - 2\epsilon$ is the space-time dimension. $\Pi'(M_Z^2)$ is defined as $d\ \mathrm{Re}\Pi(q^2)/dq^2$ also evaluated for $q^2 = M_Z^2$. Thus we can write the partial decay rate $\Gamma(Z \to b\bar{b})$ in the following form

$$\Gamma(Z \to b\bar{b}) = \left[\Gamma_0\left(v^2 + a^2\right) + \Delta\Gamma\right]\frac{1}{1+\Pi'(M_Z^2)} \tag{2}$$

with $v = -1 + \frac{4}{3}s_W^2, a = -1$ and $\Gamma_0 = \alpha M_Z/48c_W^2 s_W^2$. $\Delta\Gamma$ contains all vertex corrections and the γZ-mixing. Expressing the result in terms of G_F (instead of α) introduces a factor $(1 - \Delta r)$. The above equation can be written in the form

$$\Gamma(Z \to b\bar{b}) = \left[\frac{G_F M_Z^3}{8\sqrt{2}\pi}\left(v^2 + a^2\right) + \Delta\Gamma\right]\frac{1 - \Delta r}{1 + \Pi'(M_Z^2)}. \tag{3}$$

The top mass dependence of the last term is of universal nature. This contribution — as well as the universal part of $\Delta\Gamma$ — can be expressed through the universal ρ-parameter [14, 15]. In this work we calculate the non-universal part of order $\mathcal{O}(\alpha\alpha_s \ln m_t^2)$.

The outline of the paper is as follows: Section 2 contains further details of the calculation and in section 3 we present the results in the $\overline{\text{MS}}$ and OS-scheme.

2 Calculation of the Order $\mathcal{O}(\alpha\alpha_s \ln m_t^2)$

In the decay rate $\Gamma(Z \to b\bar{b})$ the top mass dependence is due to the appearance of the top quark as a virtual particle, i.e. it is induced by $t - b$-transitions due to the exchange of charged Higgs ghosts $\Phi^\pm$ or W bosons. One can distinguish between seven classes of diagrams as listed in figure 1 from which the imaginary part has to be calculated. QCD corrections are obtained by attaching gluon lines in all possible ways. This results in 58 topologically different diagrams.

We perform our calculation in the large top mass limit $m_t \to \infty$, i.e. we apply an asymptotic expansion in the inverse heavy mass $1/m_t$ [8, 9, 10, 11]. In practice one isolates all possible hard subgraphs and expands them w.r.t. the small masses and (external) momenta. Afterwards this expansion is inserted as an effective vertex into the remaining diagram.

The hard mass procedure was already used in our previous work [7] to evaluate the $\mathcal{O}(\alpha\alpha_s m_t^2)$ corrections. In this order only diagrams with Higgs ghost exchange contributed. The calculation of the next-to-leading $\mathcal{O}(\alpha\alpha_s \ln m_t^2)$ corrections is extended in three different ways. First, the diagrams with W boson exchange need to be taken into account. Second, additional hard subgraphs of the Higgs ghost diagrams contribute to the considered order. Third, the subgraphs already considered in [7] must be expanded to the next higher order.

We find that the next-to-leading order results are finite separately for each class of diagrams which contain a Higgs ghost. Third order poles $1/\epsilon^3$ compensate in the sum of all possible hard subgraphs of a diagram and second order poles $1/\epsilon^2$ cancel after replacing the bare top mass by the $\overline{\text{MS}}$ renormalized one

$$m_t^{\text{bare}} = \bar{m}_t\left(1 - \frac{\alpha_s}{\pi}\frac{1}{\epsilon}\right). \tag{4}$$

The remaining simple divergence drops out in the imaginary part of the diagram.

The class of W diagrams and their renormalized contribution $R\Pi^{(\mathrm{W})}(q^2)$ remain to be calculated. Here R denotes the so-called Bogoliubov-Parasiuk R-operation [16] adopted to the minimal subtraction scheme [17]. This procedure determines the finite renormalized value of a given regularized Feynman integral. It works on a graph-by-graph basis and removes all subdivergencies together with the overall UV divergence in a way compatible with adding local counterterms to the Lagrangian.

We apply this prescription to subtract all subdivergencies of each W diagram, whereas the overall divergence again is eliminated by taking the imaginary part.

The pole part $\Delta(\gamma)$ of a divergent subgraph γ of a Feynman diagram Γ is defined as the overall divergence of the subgraph γ and can be written as a polynomial in possible masses and external momenta of γ with coefficients being

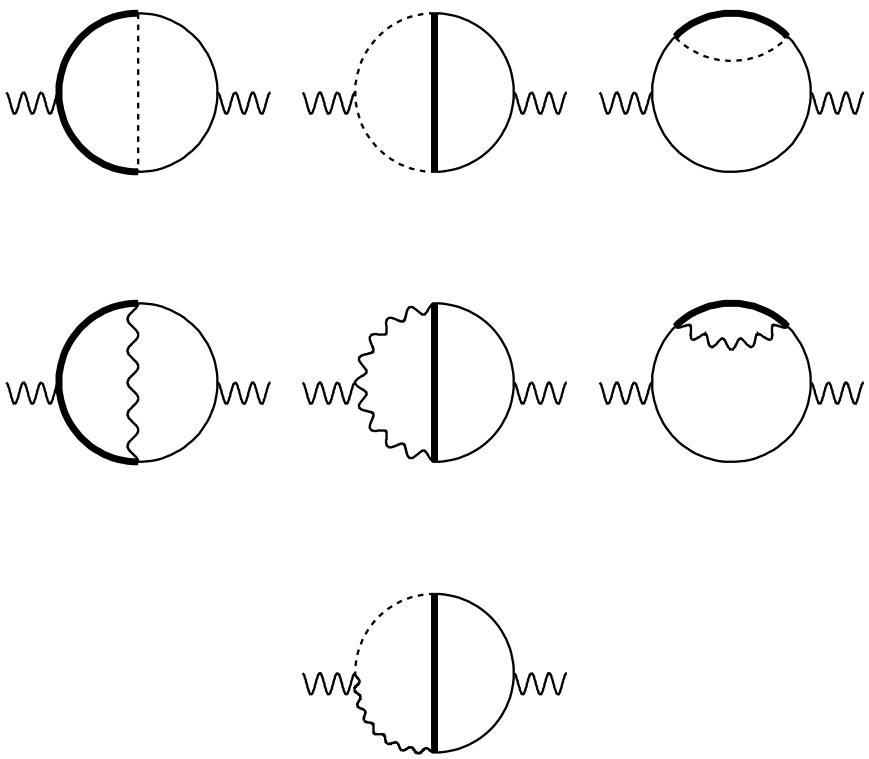

Figure 1: The seven distinct classes of electroweak diagrams, which contain the top quark. Dashed lines: Higgs ghost; wavy (internal) lines: W boson; thin lines: b quark; thick lines: t quark.

pure poles in ϵ. Since the ϵ poles themselves are independent of masses and momenta, their determination can be simplified by setting masses or momenta zero. However, care must be taken to guaranty that no spurious IR singularities are introduced this way. The subtracted Feynman integral $\Delta(\gamma)\langle\Gamma/\gamma\rangle$ is obtained by replacing the subgraph γ through $\Delta(\gamma)$. For the calculation of the remaining integrals $\langle\Gamma/\gamma\rangle$ the heavy mass expansion can be employed.

There are several checks for the correctness of our result. The first one is the fact that after renomalization the seven classes of diagrams are separately finite.

The second check is the gauge invariance with respect to QCD. The calculation is done with an arbitrary QCD gauge parameter ξ_S which is introduced through the gluon propagator. As expected each class is separately independent of ξ_S if the sum of all diagrams is taken.

The third check is the invariance of the gauge parameter ξ_W from the elektroweak theory. ξ_W appears through the propagator of the W boson and the $\Phi^{\pm}$. The total sum of all diagrams is indeed independent of ξ_W.

3 Results and Discussion

In order to calculate explicitely the $\mathcal{O}(\alpha\alpha_s \ln m_t^2)$ correction to the $Z \to b\bar{b}$ vertex, we have employed the hard mass procedure as an expansion up to the next-to-leading order in the $1/m_t$ series. As a consequence the leading order result is automatically reproduced. Furthermore it became necessary to repeat the calculation up to next-to-leading order for the case without QCD corrections (see [18]) since they induce corrections in first order α_s through top mass renormalization.

Let us first recall this purely electroweak result for the non-universal decay rate.

$$\begin{aligned}
\Delta\Gamma^{\mathrm{non-univ.}}_{\mathrm{Z}\to\mathrm{b\bar{b}}} \quad = \quad & -\Gamma_0 \frac{G_F}{\sqrt{2}\pi^2}\left(1-\frac{2}{3}s_W^2\right)\left\{\bar{m}_t^2\right. \\
& + M_Z^2\left[\ln\frac{\mu^2}{\bar{m}_t^2}\left(-3+6s_W^2-3s_W^4\right)\right. \\
& \left.\left. + \ln\frac{M_W^2}{\bar{m}_t^2}\left(\frac{1}{6}-\frac{10}{3}s_W^2+3s_W^4\right)\right]\right. \qquad (5) \\
\overset{\mu^2=M_W^2}{=} \quad & -\Gamma_0 \frac{G_F}{\sqrt{2}\pi^2}\left(1-\frac{2}{3}s_W^2\right) \\
& \left\{\bar{m}_t^2 + M_Z^2 \ln\frac{M_W^2}{\bar{m}_t^2}\left(-\frac{17}{6}+\frac{8}{3}s_W^2\right)\right\} \qquad (6)
\end{aligned}$$

The logarithms have their origins in different classes of diagrams. From the diagrams with Higgs ghost exchange logarithms $\ln\mu^2/\bar{m}_t^2$ and $\ln\mu^2/M_W^2$ occur

such that in the combination the μ drops out and $\ln M_W^2/\bar{m}_t^2$ are left. The $\ln \mu^2/\bar{m}_t^2$ only comes from the W graphs. Choosing the scale $\mu^2 = M_W^2$ reproduces the result known from [3]. Since we are only interested in top mass effects, i.e. in the difference of a heavy and a light top quark, we are allowed to set $\mu^2 = M_W^2$ because the difference $\Gamma(Z \to b\bar{b})_{\text{heavy top}} - \Gamma(Z \to b\bar{b})_{\text{light top}}$ is independent of μ and only contains logarithms of the form $\ln m_t^2/M_W^2$.

Including first order QCD corrections we obtain the non-universal part of $\Delta\Gamma$ in the $\overline{\text{MS}}$ scheme:

$$\begin{aligned}
\Delta\Gamma^{\text{non-univ.}}_{\text{Z}\to\text{b}\bar{\text{b}}} &= -\Gamma_0 \frac{G_F}{\sqrt{2}\pi^2}\left(1-\frac{2}{3}s_W^2\right)\frac{\alpha_s(\mu^2)}{\pi} \\
&\quad M_Z^2\Bigg\{\ln\frac{\mu^2}{\bar{m}_t^2}\left(\frac{8}{3}+\frac{2}{3}s_W^2-3s_W^4\right)+\frac{7}{81}\ln\frac{M_Z^2}{\bar{m}_t^2}\left(-1+\frac{2}{3}s_W^2\right) \\
&\qquad +\ln\frac{M_W^2}{\bar{m}_t^2}\left(\frac{1}{6}-\frac{10}{3}s_W^2+3s_W^4\right) \\
&\qquad +\Delta\Gamma_{\text{rem}}^{\overline{\text{MS}}}\Bigg\}.
\end{aligned} \tag{7}$$

$\Delta\Gamma_{\text{rem}}$ contains the constant terms of the non-universal diagrams containing the top quark and also terms from graphs without top quark which we have not calculated. Via the relation

$$\bar{m}(\mu^2) = m_{OS}\left(1-\frac{\alpha_s}{\pi}\left[\frac{4}{3}+\ln\frac{\mu^2}{m_{OS}^2}\right]\right) \tag{8}$$

we transform this result into the OS-scheme:

$$\begin{aligned}
\Delta\Gamma^{\text{non-univ.}}_{\text{Z}\to\text{b}\bar{\text{b}}} &= -\Gamma_0 \frac{G_F}{\sqrt{2}\pi^2}\left(1-\frac{2}{3}s_W^2\right)\frac{\alpha_s(\mu^2)}{\pi} \\
&\quad M_Z^2\Bigg\{\ln\frac{\mu^2}{m_t^2}\left(-3+6s_W^2-3s_W^4\right) \\
&\qquad +\frac{7}{81}\ln\frac{M_Z^2}{m_t^2}\left(-1+\frac{2}{3}s_W^2\right) \\
&\qquad +\ln\frac{M_W^2}{m_t^2}\left(\frac{1}{6}-\frac{10}{3}s_W^2+3s_W^4\right)+\Delta\Gamma_{\text{rem}}^{\text{OS}}\Bigg\}
\end{aligned} \tag{9}$$

$$\begin{aligned}
&\overset{\mu^2=M_W^2}{=} -\Gamma_0 \frac{G_F}{\sqrt{2}\pi^2}\left(1-\frac{2}{3}s_W^2\right)\frac{\alpha_s}{\pi} \\
&\quad M_Z^2\Bigg\{\ln\frac{M_W^2}{m_t^2}\left(-\frac{17}{6}+\frac{8}{3}s_W^2\right) \\
&\qquad +\frac{7}{81}\ln\frac{M_Z^2}{m_t^2}\left(-1+\frac{2}{3}s_W^2\right)+\Delta\Gamma_{\text{rem}}^{\text{OS}}\Bigg\}
\end{aligned} \tag{10}$$

As a consequence of dimensional regularisation the renormalization scale μ appears in the calculation. For the diagrams with Higgs ghost exchange again the logarithms $\ln \mu^2/m_t^2, \ln \mu^2/M_W^2$ and $\ln \mu^2/M_Z^2$ are combined such that μ^2 disappears. Since these logarithms are connected with the pole structure of the result, this cancellation is expected from the finiteness of these classes of diagrams.

For the classes of the W boson exchange graphs with their explicit calculation of counterterms in the $\overline{\rm MS}$ scheme a logarithm $\ln \mu^2/\bar{m}_t^2$ remains left.

One can observe that for the QCD corrections the coefficients of the $\ln \mu^2/\bar{m}_t^2$ and the $\ln M_W^2/\bar{m}_t^2$ terms in eq.(9) are identically the same as in the pure electroweak result of eq.(5). Thus these logarithms are characterized by the same correction factor $(1+\alpha_s/\pi)$ as it is known from the pure QCD corrections to the Born decay rate. With the electroweak result being well established in the literature, the explicit μ dependence in the OS result eq.(5) is known to be compensated by the running electroweak coupling constant, thus giving a RG invariant result. The common correction factor $(1+\alpha_s/\pi)$ therefore also guaranties the μ-cancellation in our $\mathcal{O}(\alpha\alpha_s \ln m_t^2)$ result.

Having gained more experience and insight in the heavy mass expansion as a practical approach of an effective theory since our previous work [7], let us add an additional comment concerning the appropriate choice of scale as argument for the running mass and coupling constant.

The leading $\mathcal{O}(\alpha\alpha_s m_t^2)$ calculation exhibits a remarkable structure. All integrals factorize and can be classified into two cathegories, both of which are separately finite and gauge invariant. The first one is characterized by a factorization into a two loop massive tadpole integral and a massless one loop p-integral, where the gluon is part of the former. Since this integration is affected by only one mass scale namely the top quark mass, it is instructive to evaluate α_s at the scale m_t^2 for these contributions. Contrary to this the second cathegory comprises all corrections which factorize into a one loop massive tadpole and a two loop massless p-integral, with the gluon lines contained in the latter. The only scale in the massless p-integral is the energy regime of the process under consideration and it seems to be appropriate to calculate α_s at this scale.

The result for the $\mathcal{O}(\alpha\alpha_s m_t^2)$ term can now be written as a factorized form, where both scales are separated and higher order QCD corrections of order α_s^2 are neglected.

Recalling that pure QCD corrections to $\Gamma(Z \to b\bar{b})$ are given by $(1+\alpha_s(s)/\pi)$ the following interpretation is at hand. The virtual top quark is of purely electroweak origin and its effect can be accounted for by effective vertices. QCD corrections enter twofold. On the one hand they may be part of the effective vertex with the relevant scale being m_t^2. On the other hand these QCD corrected vertices are dressed by a virtual gluon, thus being multiplied by the correction

factor $(1+\alpha_s(s)/\pi)$. The scale of the top mass is in both cases $\mu^2 = m_t^2$ because the overall m_t^2 factor always results from the tadpole integral. We have not found such a factorization in the $\ln m_t^2$ term, mainly because the two classes are not separately finite.

To summrize all contributions discussed in this paper, we use the common parameters $\rho = 1 + \delta\rho$ and $\kappa = 1 + \delta\kappa$ through which the Z decay rate into two b quarks can be expressed:

$$\Gamma(Z \to b\bar{b}) = \frac{G_F M_Z^3}{12\sqrt{2}\pi} N_C \rho \left[1 - \frac{4}{3}\kappa s_W^2 + \frac{8}{9}\kappa^2 s_W^4\right] \tag{11}$$

$$\begin{aligned}
\delta\rho^{\text{non-univ.}} &= -\frac{G_F}{2\sqrt{2}\pi^2} \\
&\left\{ m_t^2 \left(1 - \frac{\pi^2}{3}\frac{\alpha_s(m_t^2)}{\pi}\right)\left(1 + \frac{\alpha_s(M_Z^2)}{\pi}\right)\right. \\
&+ M_Z^2 \ln\frac{M_W^2}{m_t^2}\left(-\frac{17}{6} + \frac{8}{3}s_W^2\right) \\
&+ \frac{\alpha_s}{\pi} M_Z^2 \left[\ln\frac{M_W^2}{m_t^2}\left(-\frac{17}{6} + \frac{8}{3}s_W^2\right) + \frac{7}{81}\ln\frac{M_Z^2}{m_t^2}\left(-1 + \frac{2}{3}s_W^2\right)\right. \\
&\left.\left. \qquad + \Delta\Gamma_{\text{rem}}^{\text{OS}}\right]\right\} \\
\delta\kappa^{\text{non-univ.}} &= -\frac{1}{2}\delta\rho^{\text{non-univ.}}
\end{aligned} \tag{12}$$

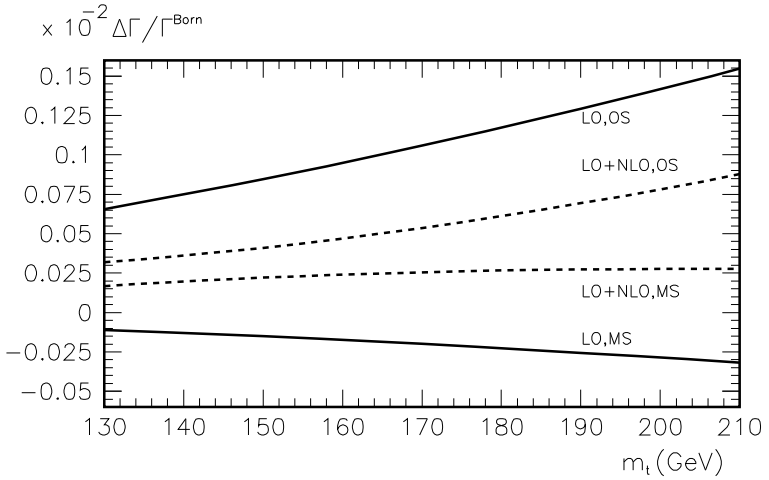

Figure 2: Leading order and sum of leading and next-to-leading order QCD corrections.

The leading m_t^2 QCD correction is positive in the OS and negative in the $\overline{\rm MS}$ scheme. In both cases the corresponding $\ln m_t^2$ term has the opposite sign. The leading order OS result is reduced by about a factor one half. The modulus of the next-to-leading term in the $\overline{\rm MS}$ scheme is slightly bigger than the m_t^2 term, so that the sum is small and positive.

In Figure 2 the leading order and the sum of the leading and next-to-leading order corrections are plotted against the top mass. It is easy to see that the inclusion of the next-to-leading correction reduces the difference of the predictions between both schemes considerably.

Figure 3 compares the QCD corrections with the pure electroweak result both in the $\overline{\rm MS}$ and in the OS scheme. One can recognize that in the $\overline{\rm MS}$ scheme the corrections are numerically tiny ($2.5 \cdot 10^{-4}$).

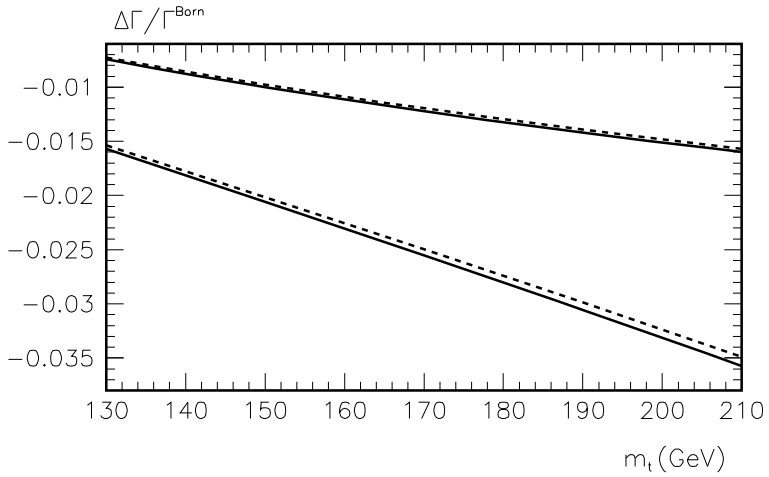

Figure 3: Pure electroweak corrections (solid line), electroweak + QCD corrections (dashed line). The upper curves belong to the $\overline{\rm MS}$ scheme.

To conclude, we calculated QCD corrections to the known electroweak result for the partial width $\Gamma(Z \to b\bar{b})$ in the limit of a heavy top quark. We used an expansion in the inverse top mass and calculated the next-to-leading term of order $\mathcal{O}(\alpha\alpha_s \ln m_t^2)$.

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