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Tau Decays into Four Pions

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Abstract

We compute branching ratios and invariant mass distributions of the tau decays into four pions. The hadronic matrix elements are obtained by starting from the structure of the hadronic current in chiral limit and then implementing low-lying resonances in the different channels with slowly varying coupling constants. Reasonable agreement with experiment is obtained both for the $\tau \to \nu_\tau + (4\pi)$ decay rates and the $e^+e^- \to (4\pi)$ cross sections. We derive a new prediction for the decay rate of $\tau^- \to \nu_\tau 2\pi^-\pi^+\pi^0$ which is significantly lower than a previous one. Furthermore we supply an interface to use our matrix elements within the Tauola Monte-Carlo program.

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1 Introduction

The last two years have provided us with a vast of new data on the physics of the τ lepton. (See [1] for a review of the most recent experimental results.) The overall branching ratio deficit that has been a problem for many years is getting smaller [1] and considerable improvement in the detection of small exclusive branching ratios involving π^0's and kaons has been achieved. In general agreement between theory and experiment is satisfactory.

In this paper we concentrate on a specific hadronic final state, namely the τ decays into four pions, $\tau^- \to \nu_\tau \pi^- \pi^- \pi^+ \pi^0$ and $\tau^- \to \nu_\tau \pi^0 \pi^0 \pi^0 \pi^-$. Of course these decays are related through CVC to corresponding e^+e^- data. The predictions [2, 3] from e^+e^- data for the total branching ratio are in good agreement with the τ–data [4, 5].

However, the prediction for the integrated rate for $\tau^- \to \nu_\tau \pi^- \pi^- \pi^+ \pi^0$ from the e^+e^- cross sections carries a rather large uncertainty due to the rather poorly known $e^+e^- \to \pi^+\pi^- 2\pi^0$ cross section. What is more important, this ansatz allows only to calculate the integrated rates. In this paper we will obtain a parametrization of the hadronic matrix element which allows us to describe differential decay rates. Furthermore, we are able to fix the parameters of our model without having to use the $e^+e^- \to \pi^+\pi^- 2\pi^0$ cross sections. There have been several similar attempts in the past, let us quote a typical example [6].

In our discussion we follow along the lines set in the phenomenological description for τ decays into three pseudoscalars [8]. Lorentz–invariance dictates the number of relevant formfactors. Then the resonances dominating these form factors are identified (in our case namely the ρ, ρ', ρ'', a_1 and ω mesons) and an expression for the form factors is constructed. Furthermore the expressions are restricted by assuming that only on mass-shell vertices contribute and that different vertices are related by Breit–Wigner propagators. In the case of the τ decays into three pseudoscalars, the coupling constants at the vertices were assumed to be constant, and the products of the coupling constants were fixed by the requirement that the formfactors have the correct chiral limit [9].

In the case of the four pion final states, we are forced to assume that these products are slowly varying functions of the momentum, because otherwise we cannot reproduce the experimental data.

An expression constructed from the chiral limit and including ρ resonances in all possible ways had been constructed and included in TAUOLA [7]. Unfortunately it yields a branching ratio off by about a factor of 2 to 4. In this paper we improve this result by including the 3π resonance, namely the a_1 meson, and obtain reasonable agreement with experiment.

In section 2 we discuss the general structure of the τ decay into four pions and we implement the low–lying resonances according to the different 4π final states into the hadronic matrix element. We compare the model predictions with a model–independent determination of the relative probabilities of the channels corresponding to the alternatives of charged and neutral π's [10]. In section 3 we discuss the two possible 4π final states in e^+e^- annihilation and the relationship of these processes to the τ decays into four pions. We discuss the numerical results as well as the invariant mass–distributions in the various 2π–subsystems in comparison to experiment in section 4.

2 The Four Pion Decay Mode of the Tau

There exist two 4π final states in τ^- decay:

i) $\tau^-(P) \to \nu(l)\pi^0(p_1)\pi^0(p_2)\pi^0(p_3)\pi^-(p_4)$

ii) $\tau^-(P) \to \nu(l)\pi^-(p_1)\pi^-(p_2)\pi^0(p_3)\pi^+(p_4)$

In the standard model [17] the following effective lagrangian is responsible for the semileptonic decays of the τ lepton [8]

$$L_{eff} = \frac{G_F U^{ij}}{\sqrt{2}} \overline{u}(l')\gamma_\mu(1-\gamma_5)u(l)H^\mu_{ij} \tag{1}$$

with G_F the Fermi constant, H^μ_{ij} and U^{ij} are the charged quark current and the quark mixing matrix respectively. In the following we present a model for the matrix element of the quark current, which reveals itself as being in agreement with general properties of a system of multi–pions. These considerations are consequences of a classification of various pion–subsystems according to isospin [10]. In our model the charged current consists of three parts which we explain in detail below.

Our model is based on an amplitude derived [9] in the chiral limit, supplemented by vector dominance to correctly describe the resonances in the 4π mass distributions and a slow Q^2 dependence of the form factors. In this frame it is quite natural to calculate $\tau \to 4\pi\nu_\tau$ via sequential processes like $\tau \to \nu_\tau\rho,\ \rho \to a_1\pi,\ a_1 \to \rho\pi,\ \rho \to \pi\pi$ and $\tau \to \nu_\tau\rho,\ \rho \to (\pi\pi)_{s-wave}\rho,\ \rho \to \pi\pi$. To construct the covariant amplitudes corresponding to Fig. 1 we need the appropriate vertices and propagators. Following the successful reasoning in a similar analyses concerning tau decay into three pseudoscalar mesons [8] we assume the vertex functions to be given by their on shell structure and to be transverse, e.g.

$$p^\nu_{a_1}\Gamma_{\nu\sigma}(a_1 \to \rho\pi) = p^\sigma_\rho\Gamma_{\nu\sigma}(a_1 \to \rho\pi) = p^\tau_\rho\Gamma_\tau(\rho \to \pi\pi) = 0 \tag{2}$$

With the pions all on mass shell, the vertex factors may thus be written [18]

$$\begin{aligned}\Gamma^{\mu\nu}\left(a_1(q)\to\rho(k)\pi(p)\right) &= f_{a_1\rho\pi}(q^2,k^2)\left(-g^{\mu\nu}+\frac{k^\mu k^\nu}{k^2}+\frac{q^\mu q^\nu}{q^2}-\frac{k\cdot q}{k^2q^2}q^\mu k^\nu\right)\\ \Gamma^{\nu}\left(\rho(k)\to\pi_1(p_1)\pi_2(p_2)\right) &= -if_{\rho\pi\pi}(k^2)(p_1^\nu-p_2^\nu)\end{aligned}\tag{3}$$

where the formfactors are real and assumed to be only slowly varying with the momentum. In the first case, we have neglected a further possible formfactor which is of higher order in momenta.
The resonances are in general described by normalized Breit–Wigner resonance propagators $BW_R(Q^2=s)$ with $BW(0)=1$ and

$$BW_R(s)=\frac{-M_R^2}{s-M_R^2+i\sqrt{s}\Gamma_R(s)}\tag{4}$$

where the energy dependent width $\Gamma_R(s)$ is computed from its usual definition

$$\Gamma_R(s)=\frac{1}{2\sqrt{s}}|M_{R\to f_i}|^2d\Phi\delta(Q-\sum p_i)\quad \Gamma_R(s)_{s=M_R^2}=\Gamma_R\tag{5}$$

with $\Gamma_R(s)=0$ for $s<$ threshold. For R decaying into two particles with masses M_1,M_2, this formula is simplified to

$$\begin{aligned}\Gamma_R(s) &= \Gamma_R\frac{M_R^2}{s}\left(\frac{p}{p_R}\right)^{2n+1}\\ p &= \frac{1}{2\sqrt{s}}\sqrt{(s-(M_1+M_2)^2)(s-(M_1-M_2)^2)}\\ p_R &= \frac{1}{2M_R}\sqrt{(M_R^2-(M_1+M_2)^2)(M_R^2-(M_1-M_2)^2)}\end{aligned}\tag{6}$$

where n is the power of $|p|$ in the matrix element. In the Breit–Wigner of the a_1 we use

$$\begin{aligned}m_{a_1}=1.251\text{ GeV},\qquad &\Gamma_{a_1}=0.599\text{ GeV}\\ \sqrt{s}\,\Gamma_{a_1}(s) &= m_{a_1}\Gamma_{a_1}\frac{g(s)}{g(m_{a_1}^2)}\end{aligned}\tag{7}$$

where the function $g(s)$ is given by [19]

$$g(s)=\left(\begin{array}{cc} 4.1(s-9m_\pi^2)^3(1-3.3(s-9m_\pi^2)+5.8(s-9m_{\pi^2}^2)^2) & \text{if } s<(m_\rho+m_\pi)^2\\ s(1.623+\frac{10.38}{s}-\frac{9.32}{s^2}+\frac{0.65}{s^3}) & \text{else}\end{array}\right)\tag{8}$$

The vector resonances for more meson channels allow a richer structure, containing the ρ–meson and some exitations. With energy dependent widths we take

$$T_\rho^{(i)}(s) = \left\{ \frac{\sum_R \beta_R^{(i)} BW_R^{(i)}(s)}{\sum_R \beta_R^{(i)}} \right\} \tag{9}$$

The index i indicates the fact that for rho like resonances which couple to different particles, the relative contributions $\beta_R^{(i)}$ of the different radial exitations need not be the same. For $a_1 \to \rho\pi$ we use the parametrization which was derived in [19] and also used in [7, 8]:

$$\begin{aligned} m_\rho &= 0.773 \text{ GeV}, \qquad & \Gamma_\rho &= 0.145 \text{ GeV} \\ m_{\rho'} &= 1.370 \text{ GeV}, \qquad & \Gamma_{\rho'} &= 0.510 \text{ GeV} \\ \beta_{\rho'}^{a_1\to\rho\pi} &= -0.145 & & \end{aligned} \tag{10}$$

For all other rho-like resonances we take for the moment a common T_ρ. We fix its parameters (masses, widths and β_R's) by fitting the cross section for the process $e^+e^- \to 2\pi^+2\pi^-$ to experimental data (see Sec. 4).

These effective vertices and propagators allow to construct the full amplitude for the τ decays into 4π final states using the structure of the chiral currents [9], and assuming a dominant role of the a_1 meson in these decays, as indicated by the data. Explicitly we have for this part of the hadronic current in eq.(1) the following matrix element:

$$\begin{aligned} < 4\pi|H^{\mu(1)}|0 > \;\sim\;& T_\rho(Q^2)\,\Gamma^{\mu\nu}\left(\rho(Q)\to a_1(k)\pi\right)\, BW_{a_1}(k^2) \\ & \cdot\Gamma^{\nu\lambda}\left(a_1(k)\to\rho(l)\pi\right)\, BW_\rho(l^2)\,\Gamma^\lambda(\rho(l)\to\pi\pi) \end{aligned} \tag{11}$$

In the chiral limit, where the meson masses tend to zero and then the limit $s \to 0$ is performed, the hadronic current should have the form [9]

$$\begin{aligned} J_\mu(\pi^-\pi_1^0\pi_2^0\pi_3^0) &= \frac{2\sqrt{3}}{f_\pi^2}\sum_{i=1}^{3}(q_{0i}-q_-)^\nu A_{\mu\nu}^{i-} \\ J_\mu(\pi_1^-\pi_2^-\pi^+\pi^0) &= \frac{2\sqrt{3}}{f_\pi^2}[2(q_+-q_0)^\nu A_{\mu\nu}^{+0} + (q_{-1}-q_+)^\nu A_{\mu\nu}^{1+} + (q_{-2}-q_+)^\nu A_{\mu\nu}^{2+}] \\ \text{with} & \\ A_{\mu\nu}^{ik} &= g_{\mu\nu} - \sum_{l\neq i,k}\frac{(Q-2q_l)_\mu(Q-q_l)_\nu}{(Q-q_l)^2} \end{aligned} \tag{12}$$

Explicit calculation shows that the decay chain $\tau \to \nu_\tau\rho, \rho \to a_1\pi, a_1 \to \rho\pi, \rho \to \pi\pi$ alone does not have the correct chiral limit. This can be corrected by adding additional contributions, see Fig. 1 (b), which can be interpreted as being induced by two pions in the s–wave which possibly induce $H^{\mu(2)}$ given by

$$< 4\pi|H^{\mu(2)}|0 > \sim\ T_\rho(Q^2)\left(g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2}\right) T_\rho(k^2)\Gamma^\nu(\rho(k) \to \pi\pi)(BW_{f_0}((Q-k)^2)) \tag{13}$$

Thus a second decay chain of the type $\tau \to \nu_\tau \rho, \rho \to (\pi\pi)_s\rho, \rho \to \pi\pi$ is built up; where the total charge of the two pions is zero, we introduce a Breit–Wigner for $(\pi\pi)_s$ resonance: f_0. In its Breit-Wigner we use

$$m_{f_0} = 1300\,\mathrm{MeV} \qquad \Gamma_{f_0} = 600\,\mathrm{MeV} \tag{14}$$

In the chiral limit, the product of the coupling constants and the relative contributions of the two decay chains are fixed. However, in order to reproduce the experimental data, we are forced to multiply the hadronic currents with an additional function $F(Q^2)$, which describes the energy dependence of the form factors. From the low energy theorem, we know that it is unity at zero momentum squared, $F(0) = 1$. However, we have found that $F(Q^2)$ can no longer be unity at $Q^2 \sim (1.5\,\mathrm{GeV})^2$. In order to be able to reproduce the experimental $e^+e^- \to 4\pi$ cross sections, we have to assume that $F(Q^2)$ is considerably lower than unity at high Q^2. Now the question arises which functional dependence of $F(Q^2)$ should be assumed. From experimental data [4] we know that in about 75% of the decays $\tau^- \to \pi^-\pi^-\pi^+\pi^0\nu_\tau$, the invariant mass of the four pions falls into the relatively small range of $Q^2 = (1.15\cdots 1.55\,\mathrm{GeV})^2$. Therefore we make the approximation that $F(Q^2)$ is constant in the relevant energy intervall:

$$F(Q^2) = \begin{cases} 1 & \text{for } Q^2 = 0 \\ \gamma & \text{for } Q^2 \approx (1.15\cdots 1.55\,\mathrm{GeV})^2 \end{cases} \tag{15}$$

So in fact we multiply the hadronic current by a constant γ, but we have to keep in mind that this might be a bad approximation outside the range indicated above. Furthermore the two decay chains need not be multiplied by the same function $F(Q^2)$. So we could multiply the first decay chain with the $a_1\pi$ intermediate state by γ_1 and the second decay chain with the $(\pi\pi)_s\rho$ intermediate state by a different γ_2. For simplicity, however, we use

$$\gamma = \gamma_1 = \gamma_2 \tag{16}$$

In addition to these two decay chains, the decay $\tau^- \to 2\pi^-\pi^+\pi^0$ has contributions from an anomalous part ($\rho\omega\pi$–coupling) [20] in Fig. 2; the matrix element of the corresponding hadronic current reads [7]

$$< 4\pi|H^{\mu\ anom}|0 > \;=\; \left(\frac{1}{m_\rho^2 - Q^2 - im_\rho\Gamma_\rho} + \sigma\frac{1}{m_{\rho'}^2 - Q^2 - im_{\rho'}\Gamma_{\rho'}}\right)$$

$$
\begin{aligned}
&\cos\Theta_c G_{\omega 3\pi} g_{\rho\omega\pi} F_\rho \left\{ \frac{1}{m_\omega^2 - (Q-p_2)^2 - i m_\omega \Gamma_\omega} \right.\\
&[p_{1\mu}\left((Q-p_2)\cdot p_3 p_4\cdot p_2 - (Q-p_2)\cdot p_4 p_3\cdot p_2\right)\\
&+p_{3\mu}\left((Q-p_2)\cdot p_4 p_1\cdot p_2 - (Q-p_2)\cdot p_1 p_4\cdot p_2\right)\\
&+p_{4\mu}\left((Q-p_2)\cdot p_1 p_3\cdot p_2 - (Q-p_2)\cdot p_3 p_1\cdot p_2\right)]\\
&\left. +(1\leftrightarrow 2)\right\}
\end{aligned}
\tag{17}
$$

As for the parameters σ, $m_{\rho'}$ and $\Gamma_{\rho'}$, we stick to the parametrization used in [7]. The product $G_{\omega 3\pi} g_{\rho\omega\pi}$ was taken to be

$$
G_{\omega 3\pi} g_{\rho\omega\pi} = 1476\,\mathrm{GeV}^{-3} 12.924\,\mathrm{GeV}^{-1} \tag{18}
$$

in [7], where the coupling constants have been extracted from experimental data. Note, however, that both numbers have some uncertainty. Especially the extraction of $g_{\rho\omega\pi}$ from the $\omega \to \gamma\pi$ decay rate is not free from theoretical uncertainties. The rate for the decay $\tau \to \pi\omega\nu_\tau$ obtained with the above product disagrees somewhat with the experimental number [4]. Therefore we write

$$
G_{\omega 3\pi}\, g_{\rho\omega\pi} = g_\omega \; \cdot (1476 \cdot 12.924)\,\mathrm{GeV}^{-4} \tag{19}
$$

where g_ω is a number of order one to be determined below.

3 Relation between the Tau Decays and the e^+e^- Annihilation Cross Sections

There are two possible 4π final states in e^+e^- annihilation, $2\pi^-2\pi^+$ and $\pi^+\pi^-2\pi^0$ (the $4\pi^0$ channel is forbidden by charge–conjugation invariance). They are accessible by a different I_3 component of the same $I = 1$ weak current describing τ decay. This allows to relate these processes to the τ decays [3]

$$
\begin{aligned}
\frac{\Gamma(\tau^- \to \nu_\tau \pi^- 3\pi^0)}{\Gamma(\tau^- \to \nu_\tau e^- \overline{\nu}_e)} &= \frac{3\cos^2\theta_c}{2\pi\alpha^2 m_\tau^8} \int_0^{m_\tau^2} dQ^2 Q^2 (m_\tau^2 - Q^2)^2 (m_\tau^2 + 2Q^2)\\
&\quad \cdot \left[\frac{1}{2}\sigma_{e^+e^-\to 2\pi^-2\pi^+}(Q^2)\right]\\
\frac{\Gamma(\tau^- \to \nu_\tau 2\pi^-\pi^+\pi^0)}{\Gamma(\tau^- \to \nu_\tau e^- \overline{\nu}_e)} &= \frac{3\cos^2\theta_c}{2\pi\alpha^2 m_\tau^8} \int_0^{m_\tau^2} dQ^2 Q^2 (m_\tau^2 - Q^2)^2 (m_\tau^2 + 2Q^2)\\
&\quad \cdot \left[\frac{1}{2}\sigma_{e^+e^-\to 2\pi^-2\pi^+}(Q^2) + \sigma_{e^+e^-\to\pi^+\pi^-2\pi^0}(Q^2)\right]
\end{aligned}
\tag{20}
$$

These relations can be inverted (in the range of Q^2 covered by the tau) to

$$
\sigma_{e^+e^-\to 2\pi^+2\pi^-}(Q^2) = \frac{2\pi\alpha^2}{3\cos^2\theta_c} \frac{m_\tau^2}{Q^3(1-Q^2/m_\tau^2)^2(1+2Q^2/m_\tau^2)}
$$

$$\cdot \left[\frac{\Gamma_{\pi^- 3\pi^0}}{\Gamma_e} \frac{1}{N_{\pi^- 3\pi^0}} \frac{dN_{\pi^- 3\pi^0}(Q)}{dQ} \right]$$

$$\sigma_{e^+e^- \to \pi^+\pi^- 2\pi^0}(Q^2) = \frac{\pi\alpha^2}{3\cos^2\theta_c} \frac{m_\tau^2}{Q^3(1-Q^2/m_\tau^2)^2(1+2Q^2/m_\tau^2)}$$
$$\cdot \left[\frac{\Gamma_{\pi^+ 2\pi^- \pi^0}}{\Gamma_e} \frac{1}{N_{\pi^+ 2\pi^- \pi^0}} \frac{dN_{\pi^+ 2\pi^- \pi^0}(Q)}{dQ} - \frac{\Gamma_{\pi^+ 2\pi^- \pi^0}}{\Gamma_e} \frac{1}{N_{\pi^+ 2\pi^- \pi^0}} \frac{dN_{\pi^+ 2\pi^- \pi^0}(Q)}{dQ} \right] \tag{21}$$

(Here Q denotes the square root of the four momentum squared, $Q := \sqrt{Q_\mu Q^\mu}$.)

Thus any measurement of, and any model for differential distributions of the tau decays into four pions has implications for the electron positron annihilation cross sections. Whereas from a measurement of the tau decays there are of course only implications for cross sections up to $Q^2 < m_\tau^2$, within a model for tau decays one can formally assume a larger mass of the tau and deduce predictions for the cross sections even at higher Q^2.

Before confronting the model with experiment and making more detailed predictions, it is worth noting that any model for the description of a multipion final state has to satisfy quite general conditions of charge correlations following from isospin considerations [10]. They fix the relative probabilities of the channels corresponding to the various alternatives of charged and neutral π's. In the case of the tau decays into four pions these considerations amount to the statement that

$$\frac{\Gamma(\tau^- \to 2\pi^-\pi^+\pi^0)}{\Gamma(\tau^- \to 3\pi^0\pi^-)} \geq \frac{3}{2} \tag{22}$$

which is well satisfied in our model, as will be seen in the next section.

4 Numerical Results in Comparison to Experiment

We still have to determine the following parameters of our model, g_ω, $\beta_{\rho'}$, $\beta_{\rho''}$, $m_{\rho'}$, $\Gamma_{\rho'}$ and $\gamma \equiv F(Q^2 \approx m_{\rho'}^2)$. (The parameters of the $\rho'' \equiv \rho(1700)$, which turns out to contribute only very little, are kept at their particle data book values [11].) The ω does not contribute to $e^+e^- \to 2\pi^+2\pi^-$, so the cross section for this process is independent of g_ω. We determine the other parameters by fitting the cross-section obtained with our model to experimental data, see Fig. 3. Our best fit is

$$\begin{aligned} \beta_{\rho'} &= 0.08 & \beta_{\rho''} &= -0.0075 \\ m_{\rho'} &= 1.35\,\mathrm{GeV} & \Gamma_{\rho'} &= 0.3\,\mathrm{GeV} \\ m_{\rho''} &\equiv 1.70\,\mathrm{GeV} & \Gamma_{\rho''} &\equiv 0.235\,\mathrm{GeV} \\ \gamma &= 0.38 \end{aligned} \tag{23}$$

where the ρ'' parameters have been stated for completeness. If we fix $\gamma = 1$ in order to retain the assumption of constant coupling constants, it is not possible to obtain a reasonable fit within our model. We have tried various modifications of our model in order to be able to obtain a good fit with $\gamma = 1$. For example, we allowed for different resonance parameters for the different $T_\rho^{(i)}$ involved, modified the $T_\rho^{a_1\to\rho\pi}$, or assumed that the dominance by the a_1 occurs only for the decay of the $\rho(770)$, whereas for the ρ' we fixed the a_1 resonance factor equal to unity. However, in all these cases, the best fit for the cross section is by about a factor of two or three too large. Furthermore, when keeping $\gamma = 1$, the fits tend to prefer unphysical values for the resonance parameters. Therefore we are convinced that when going from $Q^2 = 0$ to $Q^2 = (1.5\,\mathrm{GeV})^2$, the couplings at the vertices can not be assumed to be constant.

The result of our fit in Fig. 3 lies somewhat below the data in the range $\sqrt{Q^2} =$ 1.9–2.2 GeV. This could possibly be cured by including the $\rho(2150)$ in the fit. As our main goal is to describe the tau decays, where Q^2 can of course not be larger than m_τ^2, we do not do this. Furthermore one has to remember that at such high Q^2, $F(Q^2)$ is presumably no longer constant. There is also some disagreement between our fit and the Nowosibirsk data [12] in the region $\sqrt{Q^2} \approx 1.3\,\mathrm{GeV}$, but as the Nowosibirsk and the new Orsay data [14] do not match well, this might be due to errors in the experimental data.

Next we have fixed g_ω by requiring that the experimental results for $\tau \to \nu_\tau\omega\pi$ [4] are reproduced reasonably (see Tab. 1 below). This results in

$$g_\omega = 1.4 \tag{24}$$

Having fixed the parameters from $e^+e^- \to 2\pi^+2\pi^-$ and from $\tau \to \nu_\tau\omega\pi$, we obtain predictions for the cross section for $e^+e^- \to \pi^+\pi^-2\pi^0$ and for the other tau decays into four pions.

Our prediction for the $e^+e^- \to \pi^+\pi^-2\pi^0$ cross section is shown in Fig. 4. The experimental data for this state are much poorer than those for the other mode, and the data from different experiments do not agree very well. Note that our prediction is rather low, but still compatible with the data. It is interesting that our prediction favours the new Orsay data [14], which is in fact also true for the process $e^+e^- \to 2\pi^+2\pi^-$, see Fig. 3.

Now we will discuss the τ decays. Our results for the integrated decay rates are given in Tab. 1, where we compare with the version 2.4 of Tauola and with experimental data. The branching ratios have been calculated assuming a tau lifetime of

$$\tau_\tau = 295.7\,\mathrm{fs} \tag{25}$$

Note that we use physical masses for the pions, $m_{\pi^+} \neq m_{\pi^0}$, in the phase space, and a tau mass of

$$m_\tau = 1.7771\,\mathrm{GeV} \tag{26}$$

This explains why the numbers in Tab. 1 for Tauola 2.4 do not agree exactly with those in Tabs. 4 and 5 of [7]. Whereas the numbers for the $\tau \to \nu_\tau \omega \pi$ mode have been fitted by adjusting g_ω, the other values are predictions, and they agree quite well with experiment.

Our predictions are also to be compared to the earlier predictions in [3],

$$\begin{aligned} \frac{\Gamma(\tau^- \to \nu_\tau 2\pi^-\pi^+\pi^0)}{\Gamma_e} &= 0.275 \ (0.185\text{–}0.305) \\ \frac{\Gamma(\tau^- \to \nu_\tau \pi^- 3\pi^0)}{\Gamma_e} &= 0.055 \end{aligned} \tag{27}$$

Whereas our prediction for the channel $\pi^- 3\pi^0$ is practically identical to that of [3], our prediction for the $2\pi^-\pi^+\pi^0$ mode is significantly lower than theirs. This is due to our rather small prediction for the $e^+e^- \to \pi^+\pi^- 2\pi^0$ cross section, which is supported, as we have stated before, by the new Orsay data which where not available for the analysis in [3].

Note that actually this lower prediction does not depend on details of our model. According to Eqn. 20, this decay rate depends strongly on the cross section for $e^+e^- \to \pi^+\pi^- 2\pi^0$. The calculation by Gilman and Rhie used a fit to this cross section which reproduced the Nowosibirsk data at low Q, and estimated the uncertainty from variations of the fit in the high Q region. However, as can be seen from Fig. 3, the Nowosibirsk data do not fit well to the Frascati data. The new Orsay data now strongly support the impression that the Nowosibirsk data are probably about a factor of two too large at $Q = 1.4\,\mathrm{GeV}$. Taking this into account brings the prediction for the tau decay rate down to a value much below the central value given by Gilman and Rhie.

A detailed recent analysis of e^+e^- cross sections and τ decay rates has been performed in [15] where data have been analyzed in terms of partitions as defined by Pais [10] rather than by a dynamical model. From these results, the authors deduce values of the rates into definite charge states in τ decay: For $\Gamma(\pi^\pm 3\pi^0)/\Gamma_e$ they obtain a value of 0.050 ± 0.002, for $\Gamma(\pi^\pm\pi^+\pi^-\pi^0)/\Gamma_e$ their result is 0.190 ± 0.009 or 0.223 ± 0.011, according to different data sets.

Our results for the differential distributions in the decay $\tau \to \nu_\tau 2\pi^-\pi^+\pi^0$ are given in Figs. 5–7. We compare with the Argus data [4] and with the 2.4 version of Tauola. For the Argus data we have performed a rebinning into larger bins.

Note that we display distributions

$$\frac{1}{N}\frac{dN}{dm} \tag{28}$$

which are normalized to the total number of events N. We run the Monte-Carlos with very high statitics. Therefore the statistical errors of the Monte-Carlo results are very small and not shown in the figures. At this point it is important to stress

Table 1: Numerical results for the integrated decay rates of our model, compared to Tauola 2.4 and to experimental data

observable	present model	Tauola 2.4	experiment
$\Gamma(\tau^- \to \nu_\tau 2\pi^-\pi^+\pi^0)/\Gamma_e$	0.172	0.0832	
$\mathrm{BR}(\tau^- \to \nu_\tau 2\pi^-\pi^+\pi^0)$	3.11%	1.51%	$(4.4 \pm 1.6)\%$[16]
$\Gamma(\tau^- \to \nu_\tau \pi^- 3\pi^0)/\Gamma_e$	0.0540	0.0172	
$\mathrm{BR}(\tau^- \to \nu_\tau \pi^- 3\pi^0)$	0.98%	0.31%	$\tau \to h^- 3\pi^0$[1] : $1.00 \pm 0.15\%$
$\Gamma(\tau^- \to \nu_\tau \pi^- \omega(\pi^-\pi^+\pi^0))/\Gamma_e$	0.0664	0.0339	
$\mathrm{BR}(\tau^- \to \nu_\tau \pi^- \omega(\pi^-\pi^+\pi^0))$	1.20%	0.61%	$1.65 \pm 0.36\%$[4]
$\frac{\Gamma(\tau^- \to \nu_\tau \pi^- \omega(\pi^-\pi^+\pi^0))}{\Gamma(\tau^- \to \nu_\tau 2\pi^-\pi^+\pi^0)}$	0.39	0.41	0.33 ± 0.05[4]

that the predicted total rate by TAUOLA 2.4 is much too low, which is not visible in these normalized shapes.

Overall we find a reasonable agreement between our model and the experimental data. In the invariant mass of the four pion system (Fig. 5), our prediction for the peak of the distribution is a bit on the high side, but we do not consider this as very pronounced. We have tried to improve the fit by lowering the mass of the ρ'. This, however, inevitably leads to predictions for the e^+e^- cross sections which badly disagree with the experimental data.

In comparing the experimental three pion invariant mass distribution in Fig. 6 with our model one should remember the limited experimental mass resolution. Furthermore, we have chosen $g_\omega = 1.4$ such that both the ratio $\frac{\Gamma(\tau^- \to \nu_\tau \pi^- \omega(\pi^-\pi^+\pi^0))}{\Gamma(\tau^- \to \nu_\tau 2\pi^-\pi^+\pi^0)}$ and the absolute number for the branching ratio agree reasonably with the experimental numbers. However, in the line shapes obviously only the relative contribution of the ω channel is important, which comes out a bit on the high side with $g_\omega = 1.4$. A somewhat lower value of about 1.2 or 1.3 would result in a better fit here.

In the case of the two pion invariant mass distributions (Fig. 7 (a)–(e)), we get good agreement for the charged opposite sign combination (containg the ρ^0)(Fig. 7(a)) and the charged like sign one (Fig. 7(e)). In the case of the charged-neutral combinations, however, we find that we predict too many ρ^+ (Fig. 7(d)).

5 Conclusions

In this paper we have analyzed the four pion decay modes of the τ–lepton and the corresponding e^+e^- data related through CVC. We improved the hadronic matrix element in comparison to previous work by implementing low–lying resonances like ρ, ρ', ρ'', a_1 and ω mesons in the different channels into the chiral structure. In particular we respected the dominant role of the a_1 meson as indicated by the data. We fixed our parameters from $e^+e^- \to 2\pi^+2\pi^-$ and $\tau \to \nu_\tau \omega\pi$ and obtained predictions for the other four–pion decay modes of the τ–lepton (Table 1). We have given a new prediction for the normalized decay rate

$$\frac{\Gamma(\tau^- \to \nu_\tau 2\pi^-\pi^+\pi^0)}{\Gamma_e} = 0.172 \tag{29}$$

which is significantly lower than the prediction by Gilman and Rhie of 0.275 (0.185 –0.305) [3]. This prediction is supported both by the prediction of our model and independently by the new data [14] on the $e^+e^- \to \pi^+\pi^-2\pi^0$ cross section.

We have studied in detail two– three– and four–pion mass distributions based on a detailed dynamical model with slowly varying form factors and compared our predictions with available data. In general we find reasonable agreement between the TAUOLA Monte Carlo with our matrix elements and the experimental data.Fine tuning of the parameters is left to the experimentalists and requires more accurate data.

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6 Appendix

Our model is available in the form of a FORTRAN code suitable to be used with the Monte-Carlo Tauola. The file HCURR4PI.F contains three COMPLEX FUNCTIONs, viz. BWIGA1, BWIGEPS, FRHO4 and the SUBROUTINEs CURINF, CURINI and CURR. The latter is the main subroutine, which calls the other functions and routines and which replaces the original subroutine CURR of Tauola 2.4. Therefore the user has to delete the subroutine CURR in Tauola 2.4 and link our HCURR4PI instead. The subroutine CURINF prints some general information about the new routine CURR. The subroutine CURINI initializes the COMMON block /TAU4PI/ with the values we obtained in our fit. The meaning of the individual parameters is explained in Tab. 2. Alternatively, the user may choose to perform a new fit of these parameters to experimental data. Note that in the routine γ_1 and γ_2 may be chosen differently.

Table 2: Input parameters in the common block TAU4PI

Parameter	Meaning	Our fit
GOMEGA	g_ω	1.4
GAMMA1	γ_1	0.38
GAMMA2	γ_2	0.38
ROM1	$m_{\rho'}$ [GeV]	1.35
ROG1	$\Gamma_{\rho'}$ [GeV]	0.3
BETA1	β_1	0.08
ROM2	$m_{\rho''}$ [GeV]	1.70
ROG2	$\Gamma_{\rho''}$ [GeV]	0.235
BETA2	β_2	-0.0075

To obtain a copy of the file HCURR4PI.F, send an e-mail to M.F.:
finkemeier@vaxlnf.lnf.infn.it

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