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R_{had} at the B-factory*

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Abstract

A systematic theoretical evaluation of the cross section for hadron production in electron-positron collisions in the energy range just below the B meson threshold is presented which includes charm and bottom mass effects and is accurate to order α_s^3. The corresponding measurement in the energy region several GeV above the threshold is also discussed.

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The hadronic decay rate of the Z boson provides one of the most accurate values for the strong coupling constant α_s. This measurement is free from uncertainties and ambiguities which are inherent in the precise determination of α_s from shape variables like thrust distribution, energy-energy correlation or jet multiplicities and which originate from the hadronization models, ambiguities in the jet definition etc. With increasing statistics and precision at LEP the uncertainty in α_s from this measurement alone can be reduced to ± 0.002. It would be highly desirable to test the evolution of the strong coupling as predicted by the beta function through a determination of α_s from essentially the same observable, however, at lower energy. The region from several GeV above charm threshold (corresponding to the maximal energy of BEPC around 5.0 GeV) to just below the B meson threshold at around 10.5 GeV corresponding to the "off resonance" measurements of CESR seems particularly suited for this purpose. As a consequence of the favorable error propagation, the accuracy in the measurement (compared to 91 GeV) may decrease by factor of about 3 or even 4 at 10 and 5.6 GeV respectively, to achieve comparable precision in Λ_{QCD}:

$$\delta\alpha_s(s) = \frac{\alpha_s^2(s)}{\alpha_s^2(M_Z^2)}\delta\alpha_s(M_Z^2).$$

Most of the results derived in [1, 2] for massless quarks are applicable also for the case under consideration. However, two additional complications arise:
(i) Charm quarks effects cannot be ignored completely and should be taken into consideration through an expansion in the ratio m_c^2/s, employing the results of [3, 4] for terms of order m_c^2/s and m_c^4/s^2.
(ii) Contributions involving virtual bottom quarks are presented, starting from order α_s^2. Their contribution depends in a nontrivial manner on m_b^2/s. In order α_s^2 these can be calculated in closed form and are shown to be small. Estimates for the corresponding contributions of order α_s^3 indicate that these are under control and can be safely neglected, provided that one works within the correctly defined effective four quark theory.

Through most of this paper the results will be formulated in the $\overline{\text{MS}}$ scheme [5] for a theory with $n_f = 4$ effective flavours and with the corresponding definitions of the coupling constant and the quark mass. The relation to a formulation with $f = 5$ appropriate for the measurements above the $b\bar{b}$ threshold will be given at the end of the paper.

We shall now list the independent contributions and their relative importance. Neglecting for the moment the masses of the charmed quark and a forteriori of the u, d and s quarks one predicts in order α_s^3

$$R_{NS} = \sum_{f=u,d,s,c} 3Q_f^2 \left[1 + \frac{\alpha_s}{\pi} + 1.5245\left(\frac{\alpha_s}{\pi}\right)^2 - 11.52033\left(\frac{\alpha_s}{\pi}\right)^3\right] \qquad (1)$$

for the nonsinglet contribution [1, 2]. The second and the third order coefficients are evaluated with $n_f = 4$ which means that the bottom quark loops are absent. In order α_s^2 the bottom quark loops can be taken into consideration with heir full mass dependence. However, as shown in [6] the leading term of order α_s^2/m_b^2, which has also been calculated in [7] in the framework of an effective $n_f = 4$ theory, provides a fairly accurate description even up to the very threshold $s = 4m_b^2$. Hence one has to add a correction term

$$\delta R_{m_b} = \sum_{f=u,d,s,c} 3Q_f^2 \left(\frac{\alpha_s}{\pi}\right)^2 \frac{s}{\bar{m}_b^2}\left[\frac{44}{675} + \frac{2}{135}\log\frac{\bar{m}_b^2}{s}\right]. \tag{2}$$

For the singlet term one obtains [2].

$$R_S = -\left(\frac{\alpha_s}{\pi}\right)^3 (\sum_{u,d,s,c} Q_f)^2\, 1.239 = -0.55091\left(\frac{\alpha_s}{\pi}\right)^3. \tag{3}$$

The bottom quark is absent in this sum. In view of the smallness of the $\alpha_s^2 s/m_b^2$ correction (even close to the $\bar{b}b$ threshold!) also all other terms of $\mathcal{O}(\alpha_s^3)$ from virtual b quarks are neglected. In the same spirit it is legitimate to use the scale invariant value of the b quark mass $\bar{m}_b = \bar{m}_b(\bar{m}_b^2)$ as defined in the 5 quark theory. Starting from a pole mass of 4.7 GeV and using the results derived in [8, 9] we get $\bar{m}_b = 4.10$ GeV if $\alpha_s^{(5)}(M_Z) = 0.120$. (Values of $m_b = 4.59 \pm 0.04$ GeV [10], $m_b = 4.72 \pm 0.05$ GeV [11] and $m_b = 4.7 \pm 0.2$ GeV [12] are currently in use for the pole mass.)

In contrast to the bottom mass the effects of the charmed mass can be incorporated through an expansion in m_c^2/s. Quadratic mass corrections have been calculated [3] up to order α_s^3, quartic mass terms up to order α_s^2 [4]. Since m_c^2/s is in itself a small expansion parameter, the order $\alpha_s^2 m_c^4/s^2$ terms should be sufficient for the present purpose.

The results for these corrections read

$$\begin{aligned}
&\delta R_{m_c} = \\
&3\,Q_c^2\, 12\,\frac{m_c^2}{s}\frac{\alpha_s}{\pi}\left[1 + 9.097\frac{\alpha_s}{\pi} + 53.453\left(\frac{\alpha_s}{\pi}\right)^2\right] \\
&-3\sum_{f=u,d,s,c} Q_f^2\frac{m_c^2}{s}\left(\frac{\alpha_s}{\pi}\right)^3 6.476 \\
&+3\,Q_c^2\frac{m_c^4}{s^2}\left[-6 - 22\frac{\alpha_s}{\pi} + \left(141.329 - \frac{25}{6}\ln(\frac{m_c^2}{s})\right)\left(\frac{\alpha_s}{\pi}\right)^2\right] \\
&+3\sum_{f=u,d,s,c} Q_f^2\frac{m_c^4}{s^2}\left(\frac{\alpha_s}{\pi}\right)^2\left[-0.4749 - \ln\left(\frac{m_c^2}{s}\right)\right] \\
&-3\,Q_c^2\frac{m_c^6}{s^3}\left[8 + \frac{16}{27}\frac{\alpha_s}{\pi}\left(6\ln(\frac{m_c^2}{s}) + 155\right)\right],
\end{aligned} \tag{4}$$

where $n_f = 4$ has been adopted everywhere. Note that terms of order $\alpha_s^3 m_c^2/s$ are more important than those of order $\alpha_s^2 m_c^4/s^2$ in the whole energy region under consideration. For completeness also m_c^6/s^3 and $\alpha_s m_c^6/s^3$ terms are listed, which, however, are insignificant amd will be ignored in the numerical analysis.

The charm quark mass is to be taken as $m_c = \bar{m}_c^{(4)}(s)$ and is to be evaluated in the 4 flavour theory via the standard RG equation with the initial value $\bar{m}_c(\bar{m}_c) =$ 1.11 GeV corresponding to a pole mass of 1.46 GeV in the case of $\alpha_s^{(5)}(M_Z) =$ 0.120 [10].

A similar line of reasoning could be pursued for bottom mass terms in the region several GeV above the B meson threshold. It has been argued in [4, 13] (see also [14]) that mass effects of order α_s are very well parametrized by taking leading terms of the expansion in m_q^2/s. This holds true not only in the high energy region but even a few GeV above the threshold. With this motivation in mind the terms of order $\alpha_s^2 m_b^2/s$ and $\alpha_s m_b^6/s^3$ will be included in the formula below. The result is expected to provide a reliable answer for $\sqrt{s}$ around 15 GeV and perhaps even down to 13 GeV. The corresponding formula reads as follows

$$
\begin{aligned}
\delta R_m &= \\
& 3(Q_c^2\frac{m_c^2}{s} + Q_b^2\frac{m_b^2}{s})12\frac{\alpha_s^{(5)}}{\pi}\left[1 + 8.736\frac{\alpha_s^{(5)}}{\pi} + 45.657\left(\frac{\alpha_s^{(5)}}{\pi}\right)^2\right] \\
& -3\sum_{f=u,d,s,c,b} Q_f^2(\frac{m_c^2}{s} + \frac{m_b^2}{s})\left(\frac{\alpha_s^{(5)}}{\pi}\right)^3 6.126 \\
& +3\,Q_c^2\frac{m_c^4}{s^2}\left[-6 - 22\frac{\alpha_s^{(5)}}{\pi} + \left(139.488 - \frac{23}{6}\ln(\frac{m_c^2}{s}) + 12\frac{m_b^2}{m_c^2}\right)\left(\frac{\alpha_s^{(5)}}{\pi}\right)^2\right] \\
& +3\,Q_b^2\frac{m_b^4}{s^2}\left[-6 - 22\frac{\alpha_s^{(5)}}{\pi} + \left(139.488 - \frac{23}{6}\ln(\frac{m_b^2}{s}) + 12\frac{m_c^2}{m_b^2}\right)\left(\frac{\alpha_s^{(5)}}{\pi}\right)^2\right] \\
& +3\sum_{f=u,d,s,c,b} Q_f^2\frac{m_c^4}{s^2}\left(\frac{\alpha_s^{(5)}}{\pi}\right)^2\left[-0.4749 - \ln\left(\frac{m_c^2}{s}\right)\right] \\
& +3\sum_{f=u,d,s,c,b} Q_f^2\frac{m_b^4}{s^2}\left(\frac{\alpha_s^{(5)}}{\pi}\right)^2\left[-0.4749 - \ln\left(\frac{m_b^2}{s}\right)\right] \\
& -3\,Q_b^2\frac{m_b^6}{s^3}\left[8 + \frac{16}{27}\frac{\alpha_s^{(5)}}{\pi}\left(6\ln(\frac{m_b^2}{s}) + 155\right)\right].
\end{aligned}
\tag{5}
$$

Above the B meson threshold it is more conveniently to express all quantities for $n_f = 5$ theory and thus in (5) all the coupling constant and quark masses are evaluated in the 5 flavour theory at the scale $\mu = \sqrt{s}$.

The transition from the 4 to 5 flavour theory is performed as follows: The charm mass is naturally defined in the $n_f = 4$ theory. In order to obtain the value of $m_c = \bar{m}_c^{(5)}(s)$ the initial value $\bar{m}_c^{(4)}(1\text{GeV})$ is evolved via the $n_f = 4$ RG equation to the point $\mu^2 = m_b^2$ and then from there up to $\mu^2 = s$, however, now with the $n_f = 5$ RG equation. The bottom mass, on the other hand, is naturally defined in the $n_f = 5$ theory irrespective of the characteristic momentum scale of the problem under consideration. As a numerical value we take the $\bar{m}_b(s)$ obtained from the scale invariant mass $\bar{m}_b(\bar{m}_b)$ after running the latter with the help of the $n_f = 5$ RG equation.

Finally, $\alpha_s^{(4)}$ and $\alpha_s^{(5)}$ are related through the following equation [15]

$$\frac{\alpha_s^{(4)}(\mu)}{\pi} = \frac{\alpha_s^{(5)}(\mu)}{\pi}\left[1 + \frac{1}{6}x\frac{\alpha_s^{(5)}(\mu)}{\pi} + \left(\frac{x^2}{36} + \frac{11x}{24} + \frac{7}{72}\right)\left(\frac{\alpha_s^{(5)}(\mu)}{\pi}\right)^2\right], \qquad (6)$$

with $x = \ln(m_b^2(\mu)/\mu^2)$. Given $\Lambda_{QCD}^{(5)}$, eq. (6) is employed to find $\Lambda_{QCD}^{(4)}$ by setting $\mu = \bar{m}_b(\bar{m}_b)$.

In tables 1 – 6 the predictions for R are listed for different values of $\alpha_s^{(5)}(M_Z)$ together with the values of $\alpha_s(s)$ and the running masses. (Note that our predictions are presented without QED corrections from the running α and initial state radiation.) Figure 1 shows the behavior of the ratio $R(s)$ as a function of energy below and above the bottom threshold, for $\alpha_s(M_Z) = 0.120, 0.125$ and 0.130. The light quark (u, d, s, c) contribution is displayed separately also above 10.5 GeV. It is evident that the predictions from the 4 and 5 flavour theories join smoothly. The additional contribution from the $b\bar{b}$ channel is presented down to 11.5 GeV, where resonances start to contribute and the perturbative treatment necessarily ceases to apply. Evidently the $b\bar{b}$ channel is present with full strength down to the resonance region – an important consequence of QCD corrections. From our discussion it should be evident that the theoretical prediction is well under control. Mass effects are small below the $b\,\bar{b}$ threshold as well as a few GeV above. An experimental test is of prime importance.

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Table 1: Values of $\Lambda^{(5)}_{\overline{MS}}$, $\Lambda^{(4)}_{\overline{MS}}$, $\alpha_s^{(4)}(s)$, $m_c^{(4)}(s)$ and $\bar{m}_b(\bar{m}_b)$ at $\sqrt{s} = 5.0$ GeV for different values of $\alpha_s^{(5)}(M_Z^2)$.

$\alpha_s^{(5)}(M_Z^2)$	$\Lambda^{(5)}_{\overline{MS}}$	$\Lambda^{(4)}_{\overline{MS}}$	$\alpha_s^{(4)}(s)$	$m_c^{(4)}(s)$	$\bar{m}_b(\bar{m}_b)$
0.1150	175 MeV	246 MeV	0.202	0.824 GeV	4.16 GeV
0.1175	203 MeV	280 MeV	0.210	0.777 GeV	4.13 GeV
0.1200	233 MeV	317 MeV	0.218	0.725 GeV	4.10 GeV
0.1225	266 MeV	357 MeV	0.227	0.666 GeV	4.07 GeV
0.1250	302 MeV	399 MeV	0.236	0.599 GeV	4.04 GeV
0.1275	341 MeV	444 MeV	0.245	0.522 GeV	4.01 GeV
0.1300	383 MeV	493 MeV	0.255	0.433 GeV	3.98 GeV

Table 2: Predictions for $R(s)$ at $\sqrt{s} = 5.0$ GeV; the contributions to δR_{m_c} are shown separately for every power of the quark mass.

$\alpha_s^{(5)}(M_Z^2)$	R_{NS}	R_S	$\delta R_{m_c^2}$	$\delta R_{m_c^4}$	δR_{m_b}	R
0.1150	3.558	-0.00015	0.05	-0.0066	0.0012	3.603
0.1175	3.567	-0.00016	0.047	-0.0052	0.0013	3.610
0.1200	3.576	-0.00018	0.044	-0.0040	0.0014	3.618
0.1225	3.586	-0.00021	0.040	-0.0028	0.0015	3.624
0.1250	3.596	-0.00023	0.034	-0.0018	0.0017	3.630
0.1275	3.606	-0.00026	0.028	-0.0011	0.0019	3.634
0.1300	3.617	-0.00029	0.020	-0.0005	0.0020	3.638

Table 3: Values of $\Lambda^{(5)}_{\overline{MS}}$, $\Lambda^{(4)}_{\overline{MS}}$, $\alpha_s^{(4)}(s)$, $m_c^{(4)}(s)$ and $\bar{m}_b(\bar{m}_b)$ at $\sqrt{s} = 10.5$ GeV for different values of $\alpha_s^{(5)}(M_Z^2)$.

$\alpha_s^{(5)}(M_Z^2)$	$\Lambda^{(5)}_{\overline{MS}}$	$\Lambda^{(4)}_{\overline{MS}}$	$\alpha_s^{(4)}(s)$	$m_c^{(4)}(s)$	$\bar{m}_b(\bar{m}_b)$
0.1150	175 MeV	246 MeV	0.166	0.739 GeV	4.16 GeV
0.1175	203 MeV	280 MeV	0.171	0.694 GeV	4.13 GeV
0.1200	233 MeV	317 MeV	0.176	0.644 GeV	4.10 GeV
0.1225	266 MeV	357 MeV	0.182	0.589 GeV	4.07 GeV
0.1250	302 MeV	399 MeV	0.187	0.527 GeV	4.04 GeV
0.1275	341 MeV	444 MeV	0.193	0.457 GeV	4.01 GeV
0.1300	383 MeV	493 MeV	0.199	0.376 GeV	3.98 GeV

Table 4: Predictions for $R(s)$ at $\sqrt{s} = 10.5$ GeV; the contributions to δR_{m_c} are shown separately for every power of the quark mass.

$\alpha_s^{(5)}(M_Z^2)$	R_{NS}	R_S	$\delta R_{m_c^2}$	$\delta R_{m_c^4}$	δR_{m_b}	R
0.1150	3.518	-0.000081	0.0068	-0.00022	0.0022	3.526
0.1175	3.523	-0.000089	0.0063	-0.00017	0.0024	3.532
0.1200	3.530	-0.000097	0.0057	-0.00013	0.0026	3.538
0.1225	3.536	-0.00011	0.0050	-0.000088	0.0028	3.543
0.1250	3.542	-0.00012	0.0042	-0.000056	0.0030	3.549
0.1275	3.548	-0.00013	0.0033	-0.000032	0.0032	3.555
0.1300	3.555	-0.00014	0.0023	-0.000015	0.0034	3.560

Table 5: Values of $\Lambda^{(5)}_{\overline{MS}}$, $\Lambda^{(5)}_{\overline{MS}}$, $\alpha_s^{(5)}(s)$, $m_c^{(5)}(s)$ and $m_b^{(5)}(s)$ at $\sqrt{s} = 13$ GeV for different values of $\alpha_s^{(5)}(M_Z^2)$.

$\alpha_s^{(5)}(M_Z)$	$\Lambda^{(5)}_{\overline{MS}}$	$\alpha_s^{(5)}(s)$	$m_c^{(5)}(s)$	$m_b^{(5)}(s)$
0.1150	175 MeV	0.162	0.720 GeV	3.52 GeV
0.1175	203 MeV	0.167	0.675 GeV	3.47 GeV
0.1200	233 MeV	0.172	0.626 GeV	3.42 GeV
0.1225	266 MeV	0.177	0.572 GeV	3.36 GeV
0.1250	302 MeV	0.183	0.511 GeV	3.30 GeV
0.1275	341 MeV	0.188	0.443 GeV	3.23 GeV
0.1300	383 MeV	0.194	0.364 GeV	3.17 GeV

Table 6: Predictions for $R(s)$ at $\sqrt{s} = 13$ GeV; the contributions to δR_m are shown separately for every power of the quark masses.

$\alpha_s^{(5)}(M_Z^2)$	R_{NS}	R_S	δR_{m^2}	δR_{m^4}	δR_{m^6}	R
0.1150	3.863	-0.000019	0.027	-0.012	-0.0016	3.876
0.1175	3.869	-0.000021	0.027	-0.011	-0.0015	3.884
0.1200	3.875	-0.000023	0.027	-0.011	-0.0014	3.891
0.1225	3.882	-0.000025	0.027	-0.0099	-0.0013	3.898
0.1250	3.888	-0.000027	0.027	-0.0092	-0.0011	3.905
0.1275	3.895	-0.000030	0.027	-0.0086	-0.0010	3.912
0.1300	3.902	-0.000032	0.026	-0.0079	-0.00091	3.919

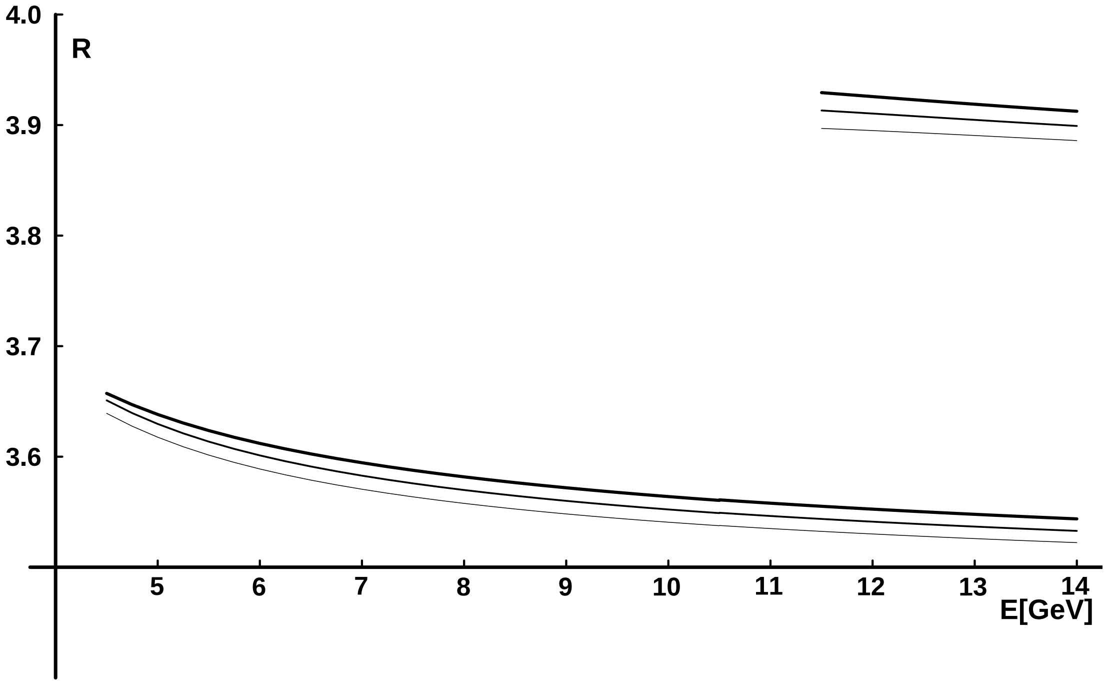

Figure 1: The ratio $R(s)$ below and above the b quark production threshold at 10.5 GeV for $\alpha_s(M_Z) = 0.120$, 0.125 and 0.130. The contributions from light quarks are displayed separately.