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Perturbative QCD Corrections to the Z Boson Width and the Higgs Decay Rate [1]

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Abstract

Radiative QCD corrections significantly influence the theoretical predictions for the decay rates of the Z and the Higgs boson. The status of the QCD calculations to the hadronic Z width is reviewed. The role of mass corrections from bottom quark final states is emphasized. An estimate of the theoretical uncertainties is given. New results for quartic mass terms of order $\mathcal{O}(\alpha_s^2)$ are presented. The impact of secondary radiation of bottom quarks on the determination of $\Gamma(Z \to b\bar{b})$ is discussed. Second order QCD corrections to the partial decay rate $\Gamma(H \to b\bar{b})$ are also presented in this talk. A recent result for the flavour singlet contribution to this quantity is presented. It includes quark mass effects and completes the otherwise massless calculations of order $\mathcal{O}(\alpha_s^2)$.

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[1] Summary of contributed talks at
1) XXIXth Rencontres de Moriond, QCD and High Energy Interactions, Méribel, March 19 – 26, 1994
2) Workshop: QCD at LEP, Aachen, April 11, 1994
3) Zeuthen Workshop on Elementary Particle Theory: Physics at LEP200 and Beyond, Teupitz, April 10 – 15, 1994.

1 Introduction

Since experiments at the e^+e^- storage ring LEP started data taking a few years ago, more than 7 million events have been collected at the Z resonance [1]. Besides the electroweak sector of the Standard Model, LEP provides an ideal laboratory for the investigation of strong interactions. Due to their purely leptonic initial state events are very clean from both theoretical and experimental point of view and represent the "golden" place for testing QCD. From cross section measurements as well as the analysis of event topologies the strong coupling constant can be extracted. Other observables which are measurable with very high precision are the (partial) Z decay rates into hadrons $\Gamma_{\rm had}/\Gamma_e = 20.763 \pm 0.049$ and bottom quarks $\Gamma_{b\bar{b}}/\Gamma_{\rm had} = 0.2200 \pm 0.0027$ [1]. The program of experimentation at LEP is still not completed. The prospect of additional 60 pb^{-1} per experiment means for example that the relative uncertainty of the partial decay rate into b quarks $\Delta\Gamma_b/\Gamma_b$ may become even smaller than one percent and an experimental error for α_s of 0.002 may be achieved [2].

Also at lower energies significant improvements can be expected in the accuracy of cross section measurements. The energy region of around 10 GeV just below the $B\bar{B}$ threshold will be covered with high statistics at future B meson factories. The cross section between the charm and bottom thresholds can be measured at the BEPC storage ring. These measurements could provide a precise value for α_s and — even more important — a beautiful proof of the running of the strong coupling constant.

In view of this experimental situation theoretical predictions for the various observables with comparable or even better accuracy become mandatory and higher order radiative corrections are required. Significant improvements in these calculations have been achieved in the last years and will be reviewed in this talk. The discussion includes singlet as well as nonsinglet corrections to the vector and the axial vector correlator. The corresponding ratios R^V and R^A enter both the formulae for the e^+e^- annihilation cross section and the hadronic Z boson width. Estimates of the theoretical uncertainties, based on a variation of the renormalization scale, will be reviewed in Section 2. Particular emphasis is put on mass corrections from bottom quark final states. The quadratic mass corrections are of relevance for Z decays and at lower energies. Quartic mass terms, however, are of particular importance for the low energy region, especially below and above the $b\bar{b}$ threshold. Corrections of order m^2/s and new results of order $\alpha_s^2 m^4$ [3] will be reviewed in Section 3. The precise measurement of the rate $\Gamma(Z \to b\bar{b})$ allows for a determination of the mass of the top through its impact on the effective $Zb\bar{b}$ vertex. Secondary $b\bar{b}$ radiation as well as the assignment of singlet contributions are covered in Section 4.

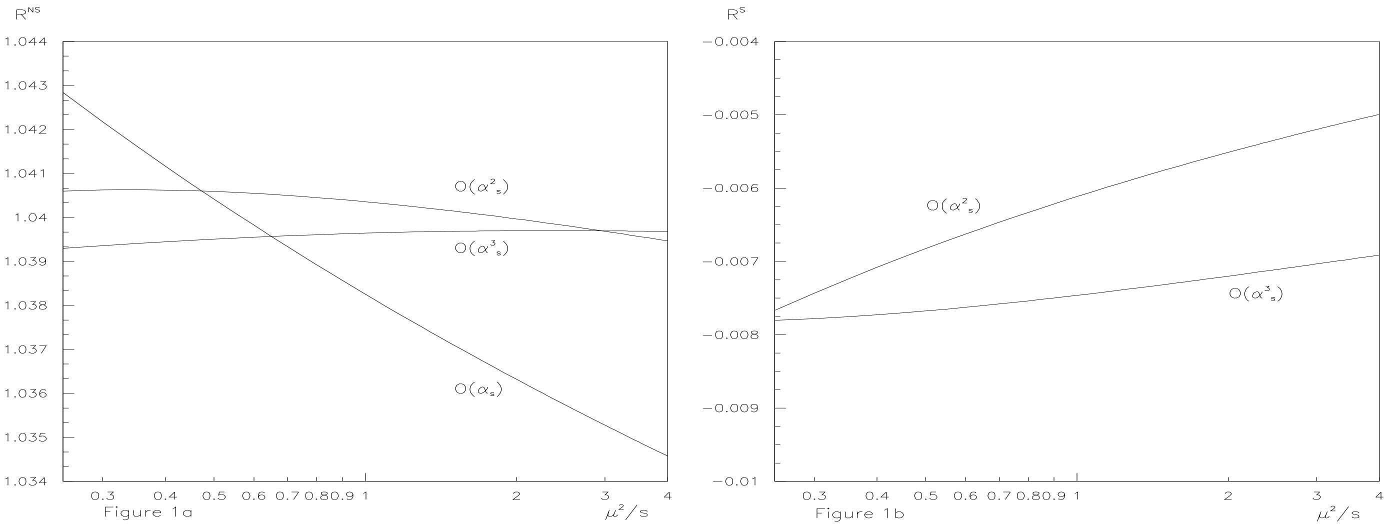

Figure 1: Renormalization scale dependence of the nonsinglet (a) and the axial singlet (b) massless QCD corrections. ($\alpha_s(M_Z^2) = 0.12$)

Despite the remarkable confirmation of the Standard Model by the precision tests at LEP and SLC, experimental evidence for the Higgs boson is still missing. Future colliders like LHC and NLC may provide enough energy for the production of the Higgs particle and open the experimental door to the electroweak symmetry breaking sector of the Standard Model. In Section 5 QCD corrections to the Higgs decay rate are discussed. With $\Gamma(H \to b\bar{b})$ being the dominant decay channel for intermediate Higgs masses, second order singlet corrections to this quantity are calculated in the heavy top mass limit. Leading quark mass terms contribute to the same order as nonsinglet graphs in the massless limit and are numerically sensitive on the ratio M_H^2/m_b^2.

2 The Z Boson Width

2.1 Results for Massless Quarks

Higher order QCD corrections to e^+e^- annihilation into hadrons were first calculated for the electromagnetic case in the the approximation of massless quarks. Considering the annihilation process through the Z boson, numerous new features and subtleties become relevant at the present level of precision. An important distinction, namely "nonsinglet" versus "singlet" diagrams, originates from two classes of diagrams with intrinsically different topology and resulting charge structure. The first class of diagrams consists of nonsinglet contributions with one fermion loop coupled to the external currents. All these amplitudes are proportional to the square of the quark charge. QCD corrections corresponding to these diagrams contribute a correction factor which is independent from the current under consideration. Singlet contributions arise from a second class of diagrams where two currents are coupled to two different fermion loops and hence can be cut into two parts by cutting gluon lines only. They cannot be assigned to the contribution from one individual quark species. In the axial vector and the vector case the first contribution of this type arises in order $\mathcal{O}(\alpha_s^2)$ and $\mathcal{O}(\alpha_s^3)$ respectively. Each of them gives rise to a charge structure different from the nonsinglet terms.

The nonsinglet terms have been calculated in [4] and [5] to second and third order in α_s respectively. As shown in Figure 1a the scale dependence of the result (evaluated in the $\overline{\mathrm{MS}}$-scheme and for a renormaliation scale μ^2 between $s/4$ and $4s$) is reduced considerably through the inclusion of higher order terms. The dependence of the result due to the renormalization scale is an indicator for the influence of yet uncalculated higher order QCD corrections. The remaining variation of R_{NS} of about $\pm 2 \cdot 10^{-4}$ for $\alpha_s(M_Z^2) = 0.12$ translates into an uncertainty in α_s of $\pm 0.7 \cdot 10^{-3}$. This error estimate is in itself strongly dependent on the actual value of α_s. For $\alpha_s(M_Z^2) = 0.135$ for example, it is amplified to ± 0.001. These numbers are quite comparable to the effect of an $(\alpha_s/\pi)^4$ term with a coefficient around 100 as advocated in [6].

Similar observations apply for the singlet contributions to the axial current correlator (see Figure 1b). The order $\mathcal{O}(\alpha_s^2)$ term [7] still exhibits a sizable scale dependence. Inclusion of the $\mathcal{O}(\alpha_s^3)$ term [8], however, leads to fairly stable answer. It should be emphasized that the contribution of this singlet term, which is effectively present in the axial $b\bar{b}$ rate only, is larger than the total contribution of the α_s^3 term.

3 Mass Corrections

3.1 Quadratic Mass Corrections

The calculation of higher order QCD corrections with massive quarks may be simplified for small masses by expanding in $m^2/s \ll 1$. The operator product expansion of current correlators, including subleading terms, provides the theoretical framework. It has been developed in [17, 18] and applied to the present problem in [13, 14]. The expansion is simultaneously performed in α_s and the

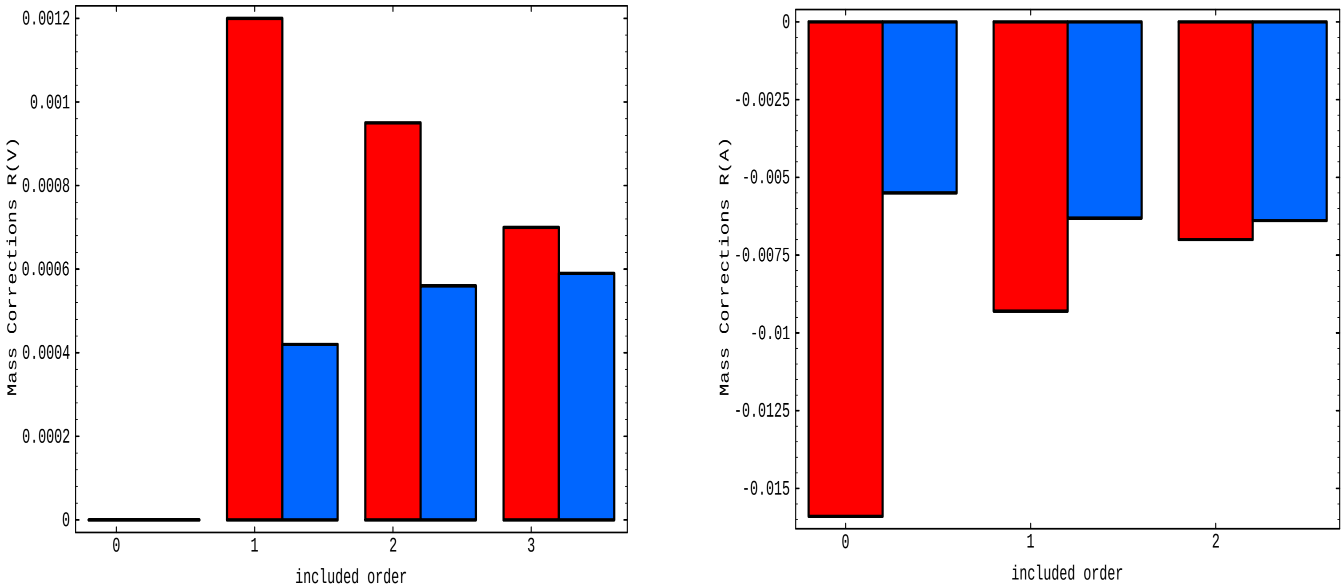

Figure 2: Mass Corrections to $R_V^{(1)}$ (left) and $R_A^{(1)}$ (right). The left bar represents the result in the on-shell-scheme, the right one is obtained in the $\overline{\mathrm{MS}}$-scheme.

quark mass:

$$
\begin{aligned}
R^{V/A} &= R^{(0)} + \frac{\bar{m}^2}{s} R^{(1)}_{V/A} + \left(\frac{\bar{m}^2}{s}\right)^2 R^{(2)}_{V/A} \\
&= 1 + \frac{\alpha_s}{\pi} + \ldots + \frac{\bar{m}^2}{s}\left[c_0^{V/A} + c_1^{V/A}\frac{\alpha_s}{\pi} + \ldots\right] + \left(\frac{\bar{m}^2}{s}\right)^2 \left[d_0^{V/A} + d_1^{V/A}\frac{\alpha_s}{\pi} + \ldots\right]
\end{aligned}
\tag{1}
$$

The calculation is conveniently performed in the $\overline{\mathrm{MS}}$-scheme [9] with the running mass, which has the remarkable property that no logarithms of the large ratio s/m^2 appear in c_i for arbitrary orders in α_s. (This does not hold true for the coefficients d_i.) For the vector induced rate the first coefficient c_0^V vanishes and the corrections c_1^V, c_2^V, c_3^V have been calculated in [10], [11][2] and [13] respectively. For $n_f = 5$ one obtains

$$
R_V^{(1)} = 12\frac{\bar{m}_b^2}{s}\frac{\alpha_s}{\pi}\left\{1 + 8.74\frac{\alpha_s}{\pi} + 45.15\left(\frac{\alpha_s}{\pi}\right)^2\right\} \tag{2}
$$

In Figure 2 it is demonstrated that the calculation is fairly stable and higher order corrections are small in the $\overline{\mathrm{MS}}$-scheme, in contrast to the large changes in the on-shell scheme.

Similar considerations apply for the axial current induced two point function. In particular one can again demonstrate the absence of large logarithms. The expansion starts in this case with c_0^A already. Nonsinglet terms have been evaluated

[2]Note that an erratum for the $\zeta(3)$ term turned out to be wrong and the originally published result proved to be correct (see [12]).

in [14] to order $\mathcal{O}(\alpha_s^2)$, the leading singlet terms of order $\mathcal{O}(\alpha_s^2)$ were calculated in [15, 16]:

$$
\begin{aligned}
R_A^{(1)} = & -6\frac{\bar{m}_b^2}{s}\frac{\alpha_s}{\pi}\left\{1+\frac{11}{3}\frac{\alpha_s}{\pi}+\left(\frac{\alpha_s}{\pi}\right)^2\left(11.296+\ln\frac{s}{m_t^2}\right)\right\} \\
& -10\frac{\bar{m}_b^2}{m_t^2}\left(\frac{\alpha_s}{\pi}\right)^2\left(\frac{8}{81}-\frac{1}{54}\ln\frac{s}{m_t^2}\right)
\end{aligned}
\tag{3}
$$

As shown in Figure 2 the calculation in the on-shell scheme exhibits significant changes with the inclusion of coefficients with large logarithms. In the $\overline{\mathrm{MS}}$-scheme the expansion is remarkably stable. The size of the corrections is comparable to the anticipated experimental precision.

3.2 Quartic Mass Corrections

The second order calculation of quartic mass corrections presented below is based on [3]. The operator product expansion included power law suppressed terms up to operators of dimension four induced by nonvanishing quark masses. Renormalisation group arguments similar to those employed already in [13, 14] allowed to deduce the $\alpha_s^2 m^4$ terms. The calculation was performed for vector and axial vector current nonsinglet correlators. The first one is of course relevant for electron positron annihilation into heavy quarks at arbitrary energies, the second one for Z decays into b quarks and for top production at a future linear collider. Below only the results for R_V will be presented.

QCD corrections to the vector current correlator in order α_s and for arbitrary m^2/s were derived in [10]. These are conventionally expressed in terms of the pole mass denoted by m in the following. It is straightforward to expand these results in m^2/s and one obtains

$$
\begin{aligned}
R_V = & \; 1-6\frac{m^4}{s^2}-8\frac{m^6}{s^3} \\
& +\frac{\alpha_s}{\pi}\left[1+12\frac{m^2}{s}+\left(10-24\ln\left(\frac{m^2}{s}\right)\right)\frac{m^4}{s^2}\right. \\
& \left.-\frac{16}{27}\left(47+87\ln\left(\frac{m^2}{s}\right)\right)\frac{m^6}{s^3}\right].
\end{aligned}
\tag{4}
$$

The approximations to the correction function for the vector current correlator (including successively higher orders and without the factor α_s/π) are compared to the full result in Fig.3. As can be seen in Fig.3, for high energies, say for $2m_b/\sqrt{s}$ below 0.3, an excellent approximation is provided by the constant plus the m^2 term. In the region of $2m/\sqrt{s}$ above 0.3 the m^4 term becomes increasingly important. The inclusion of this term improves the agreement significantly and

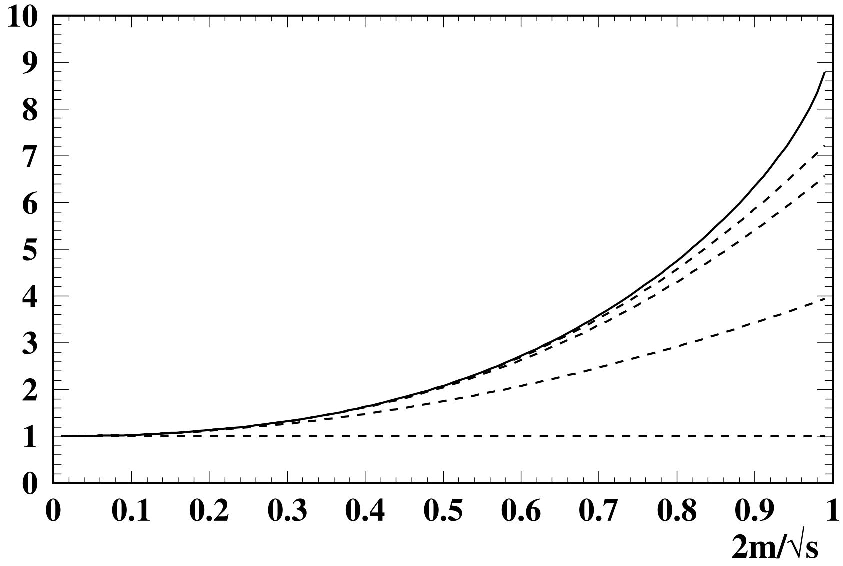

Figure 3: Comparison between the complete $\mathcal{O}(\alpha_s)$ correction function (solid line) and approximations of increasing order (dashed lines) in m^2 for vector induced rates.

leads to an excellent approximation even up to $2m/\sqrt{s} \approx 0.7$ or 0.8. For the narrow region between 0.6 and 0.8 the agreement is further improved through the m^6 term. The logarithms accompanying the m^4 terms can also be absorbed through a redefinition of m in terms of the $\overline{MS}$ mass [19, 20] at scale s and one obtains ($\overline{m} \equiv \overline{m}(s)$)

$$\begin{aligned} R_V &= 1 - 6\frac{\overline{m}^4}{s^2} - 8\frac{\overline{m}^6}{s^3} \\ &\quad + \frac{\alpha_s}{\pi}\left[1 + 12\frac{\overline{m}^2}{s} - 22\frac{\overline{m}^4}{s^2} - \frac{16}{27}\left(6\ln(\frac{\overline{m}^2}{s}) + 155\right)\frac{\overline{m}^6}{s^3}\right]. \end{aligned} \tag{5}$$

This resummation is possible for the second and fourth powers of m in first order α_s and in fact for m^2 corrections to R_V and R_A in all orders of α_s. However, logarithmic terms persist in the m^4 corrections, starting from $\mathcal{O}(\alpha_s^2)$.

Motivated by the fact that the first few terms in the m^2/s expansion provide already an excellent approximation to the complete answer in order α_s, the three loop corrections have been calculated. The calculation is based on the operator product expansion of the T-product of two vector currents $J_\mu = \overline{u}\gamma_\mu d$. Here u and d are just two generically different quarks with masses m_u and m_d. Quarks which are not coupled to the external current will influence the result in order α_s^2 through their coupling to the gluon field. The result may be immediately

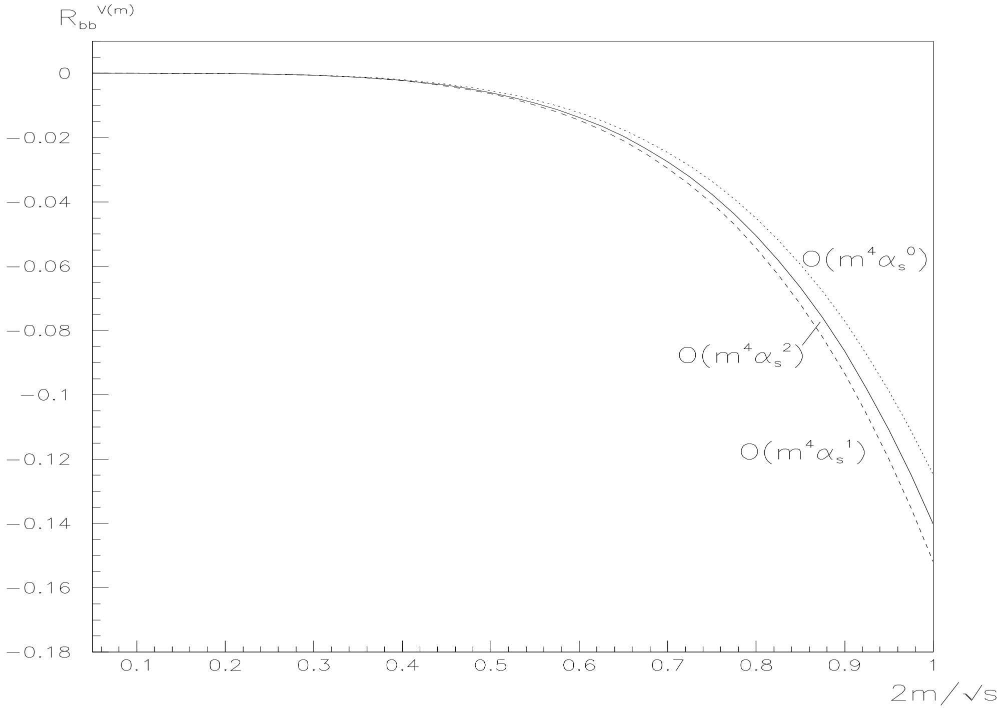

Figure 4: Contributions to R^V from m^4 terms including successively higher orders in α_s (order α_s^0/ α_s^1/ α_s^2 corresponding to dotted/ dashed/ solid lines) as functions of $2m_{\rm pole}/\sqrt{s}$.

transformed to the case of the electromagnetic current of a heavy, say, t (or b) quark.

The asymptotic behaviour of the transverse part of this (operator valued) function for $Q^2 = -q^2 \to \infty$ is given by an OPE of the following form. (Different powers of Q^2 may be studied separately and only operators of dimension 4 are displayed.)

$$i \int T(J_\mu(x) J_\nu^+(0)) \mathrm{e}^{iqx} \mathrm{d}x \underset{q^2\to\infty}{=} \frac{1}{Q^4} \sum_n (q_\mu q_\nu - g_{\mu\nu} q^2) \ \overset{T}{C}_n (Q^2, \mu^2, \alpha_s) O_n + \ldots \quad (6)$$

Only the gauge invariant operators $G_{\mu\nu}^2, m_i \bar{q}_j q_j$ and a polynomial of fourth order in the masses contributes to physical matrix elements. The coefficient functions were calculated in [17], the vacuum expectation values of the relevant operators in [21, 18, 22]. Employing renormalization group arguments the vacuum expectation value of $\sum_n C_n O_n$ is under control up to terms of order α_s as far as the constant terms are concerned and even up to α_s^2 for the logarithmic terms proportional to $\ln Q^2/\mu^2$. Only these logarithmic terms contribute to the absorptive part. Hence one arrives at the full answer for $\alpha_s^2 m^4/s^2$ corrections. Internal quark

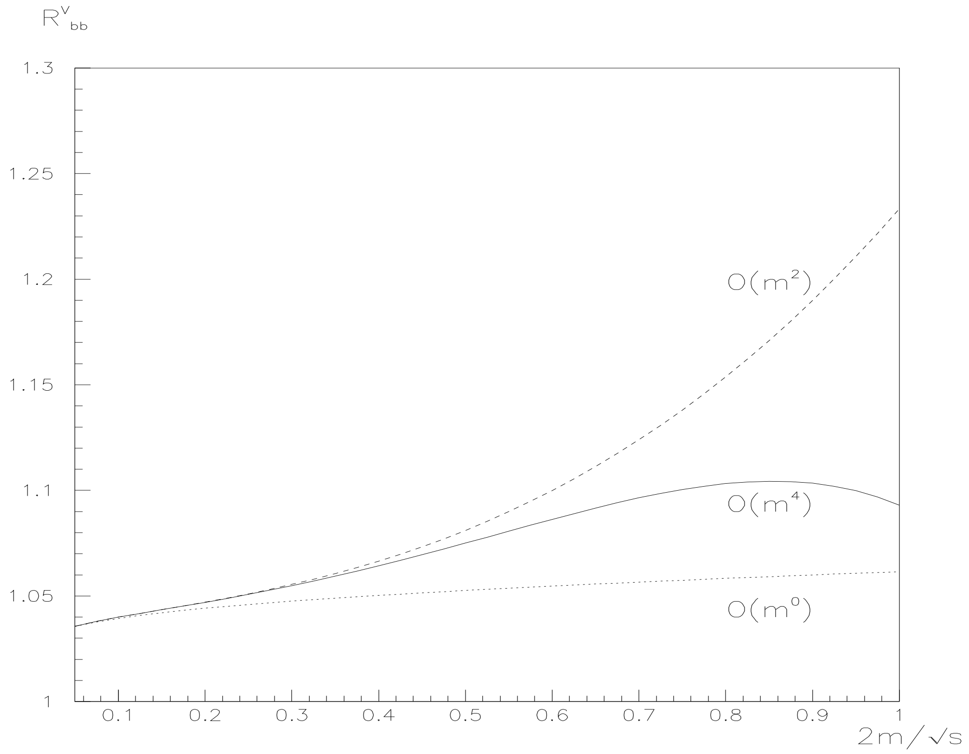

Figure 5: Predictions for R^V including successively higher orders in m^2.

loops contribute in this order, giving rise to the terms proportional to $\sum m_i^2$ and $\sum m_i^4$ below.

The result reads (below we set for brevity the $\overline{\mathrm{MS}}$ normalization scale $\mu = \sqrt{s}$ and $\overline{m}_u(s) = \overline{m}_d(s) = \overline{m}$; $a \equiv \alpha_s/\pi$)

$$\begin{aligned} R_V \;\; = \;\; & 1 + O(\overline{m}^2/s) - 6\frac{\overline{m}^4}{s^2}\left(1 + \frac{11}{3}a\right) \\ & + a^2\frac{\overline{m}^4}{s^2}\left[f\left(\frac{1}{3}\ln\left(\frac{\overline{m}^2}{s}\right) - 1.841\right)\right. \\ & - \frac{11}{2}\ln\left(\frac{\overline{m}^2}{s}\right) + 136.693 + 12\sum_i \frac{\overline{m}_i^2}{\overline{m}^2} \\ & \left. - 0.475\sum_i \frac{\overline{m}_i^4}{\overline{m}^4} - \sum_i \frac{\overline{m}_i^4}{\overline{m}^4}\ln\left(\frac{\overline{m_i}^2}{s}\right)\right], \end{aligned} \tag{7}$$

Note that the sum over i includes also the quark coupled to the external current and with mass denoted by m. Hence in the case with one heavy quark u of mass m ($d \equiv u$) one should set $\sum_i \frac{\overline{m}_i^4}{\overline{m}^4} = 1$ and $\sum_i \frac{\overline{m}_i^2}{\overline{m}^2} = 1$. In the opposite case when one considers the correlator of light (massless) quarks the heavy quark appears only through its coupling to gluons. There one finds:

$$R_V = 1 + O(\overline{m}^2/s) + a^2\frac{\overline{m}^4}{s^2}\left[\frac{13}{3} - \ln\left(\frac{\overline{m}^2}{s}\right) - 4\zeta(3)\right]. \quad (8)$$

The Z decay rate is hardly affected by the m^4 contributions. The lowest order term evaluated with $\overline{m} = 2.6$ GeV leads to a relative suppression (enhancement) of about 5×10^{-6} for the vector (axial vector) current induced $Z \to b\bar{b}$ rate. Terms of increasing order in α_s become successively smaller. It is worth noting, however, that the corresponding series, evaluated in the onshell scheme, leads to terms which are larger by about one order of magnitude and of oscillatory signs. The m_b^4 correction to $\Gamma(Z \to q\bar{q})$ which starts in order α_s^2 is evidently even smaller. From these considerations it is clear that m^4 corrections to the Z decay rate are well under control — despite the still missing singlet piece — and that they can be neglected for all practical purposes.

The situation is different in the low energy region, say several GeV above the charm or the bottom threshold. For definiteness the second case will be considered and for simplicity all other masses will be put to zero The contributions to R^V from m^4 terms are presented in Fig.4 as functions of $2m/\sqrt{s}$ in the range from 0.05 to 1. As input parameters $m_{\text{pole}} = 4.7$GeV and $\alpha_s(m_Z^2) = 0.12$ have been chosen. Corrections of higher orders are added successively. The prediction is fairly stable with increasing order in α_s as a consequence of the fact that most large logarithms were absorbed in the running mass. The relative magnitude of the sequence of terms from the m^2 expansion is displayed in Fig.5. The curves for m^0 and m^2 are based on corrections up to third order in α_s with the m^2 term starting at first order. The m^4 curve receives corrections from order zero to two.

Of course, very close to threshold, say above 0.75 (corresponding to $\sqrt{s}$ below 13 GeV) the approximation is expected to break down, as indicated already in Fig.3. Below the $b\bar{b}$ threshold, however, one may decouple the bottom quark and consider mass corrections from the charmed quark within the same formalism.

Also $R_{q\bar{q}}$ where q denotes a massless quark coupled to the external current is affected by virtual or real heavy quark radiation. The m^2 terms have been calculated in [13] and start from order α_s^3:

$$\delta R = -\left(\frac{\alpha_s}{\pi}\right)^3\frac{4\overline{m}^2}{s}(15 - \frac{2}{3}f)(\frac{4}{3} - \zeta(3)) \quad (9)$$

The terms of order $\alpha_s^2 m^4$ were given above. Both lead to corrections of $\mathcal{O}(10^{-4})$, (evaluated at an energy $\sqrt{s}$ of 10 GeV) and can be neglected for all practical purposes.

4 Secondary $b\bar{b}$ Production

The formulae for the QCD corrections to the total rate $\Gamma_{\rm had}$ have a simple, unambiguous meaning. The theoretical predictions for individual $q\bar{q}$ channels, however, require additional interpretation. In fact, starting from order $\mathcal{O}(\alpha_s^2)$ it is no longer possible to assign all hadronic final states to well specified $q\bar{q}$ channels in a unique manner. For definiteness we shall try to isolate and define $\Gamma(Z \to b\bar{b})$. The thorough understanding of QCD and mass corrections is mandatory to disentangle weak corrections with a variation of about 1% for m_t between 150 and 200 GeV from QCD effects.

1. *Non Singlet Contribution*
The vector and axial vector induced rates receive (non singlet) contributions from the diagrams, where the heavy quark pair is radiated off a light $q\bar{q}$ system. The rate for this specific contribution to the $q\bar{q}b\bar{b}$ final state is given by [23]

$$
\begin{aligned}
R^{NS}_{q\bar{q}b\bar{b}} = \frac{\Gamma^{NS}_{q\bar{q}b\bar{b}}}{\Gamma^{\rm Born}_{q\bar{q}}} &= \left(\frac{\alpha_s}{\pi}\right)^2 \frac{1}{27}\left[\ln^3\frac{s}{m_b^2} - \frac{19}{2}\ln^2\frac{s}{m_b^2} + \left(\frac{146}{3} - 12\zeta(2)\right)\ln\frac{s}{m_b^2}\right. \\
&\left. -\frac{2123}{18} + 38\zeta(2) + 30\zeta(3)\right] \\
&= \left(\frac{\alpha_s}{\pi}\right)^2 \cdot (0.922/0.987/1.059) \text{ for } m_b = (4.9/4.7/4.5)\text{ GeV}
\end{aligned}
\tag{10}
$$

Despite the fact that b quarks are present in the four fermion final state, the natural prescription is to assign these contributions to the rate into the $q\bar{q}$ channel. Therefore those events with primary light quarks and secondary bottom quarks must be subtracted experimentally from the partial rate $\Gamma_{b\bar{b}}$. This should be possible, since their signature is characterized by a large invariant mass of the light quark pair and a small invariant mass of the bottom system, which is emitted collinear to the light quark momentum. If this subtraction is not performed, the $b\bar{b}$ rate is overestimated by (for $m_b = 4.7$ GeV and $\alpha_s = 0.115\ldots0.18$)

$$
\Delta \equiv \frac{\sum\limits_{q=u,d,s,c} \Gamma^{NS}_{q\bar{q}b\bar{b}}}{\Gamma_{b\bar{b}}} \approx 0.005\ldots0.013 \tag{11}
$$

2. *Singlet Contribution*
The situation is more complicated for the four fermion final state from the singlet contribution which originates from the interference term $R^S_{q\bar{q}b\bar{b}}$ between the $q\bar{q}$ and $b\bar{b}$ induced amplitudes. It cannot be assigned in an unambiguous way to an individual $q\bar{q}$ partial rate.

For the vector current induced rate and after phase space integration this term vanishes as a consequence of charge conjugation (Furry's theorem).

For the axial current induced rate, however, this interference term gives a non-vanishing contribution which remains finite even in the limit $m_q \to 0$. Neglecting all masses one obtains

$$R^S_{b\bar{b}b\bar{b}} = -\frac{1}{2}R^S_{b\bar{b}u\bar{u}} = +\frac{1}{2}R^S_{b\bar{b}d\bar{d}} = -\frac{1}{2}R^S_{b\bar{b}c\bar{c}} = +\frac{1}{2}R^S_{b\bar{b}s\bar{s}} = -0.153\left(\frac{\alpha_s}{\pi}\right)^2$$

Among the final states from these singlet diagrams with at least one $b\bar{b}$ pair, only the $b\bar{b}b\bar{b}$ term remains after all compensations have been taken into account. Numerically it is tiny. Let us compare it with the other singlet contributions consisting of $b\bar{b}(g)$ jets. They can be assigned to $\Gamma_{b\bar{b}}$ in a unique way, although they are induced by the $b\bar{b}$ and the $t\bar{t}$ currents. These two cuts, in particular the leading term with two $b\bar{b}$ jets, dominate the four fermion final state by a factor of 20. Therefore it is suggestive to assign the total singlet contribution to $\Gamma_{b\bar{b}}$.

5 The Higgs Decay Rate

The partial width $\Gamma(H \to b\bar{b})$ is significantly affected by QCD radiative corrections. First order $\mathcal{O}(\alpha_s)$ corrections were studied in [24]. Besides the overall m_b^2 dependence due to the Higgs-bottom coupling, the otherwise massless corrections in second order $\mathcal{O}(\alpha_s^2)$ were obtained by [25]. The resulting expression for the decay rate in question reads

$$\Gamma(H \to b\bar{b}) = \frac{3G_F}{4\sqrt{2}\pi}M_H\bar{m}_b^2\left[1 + \Delta\Gamma_1\left(\frac{\alpha_s}{\pi}\right) + \Delta\Gamma_2\left(\frac{\alpha_s}{\pi}\right)^2\right], \qquad (12)$$

with $\Delta\Gamma_1 = \frac{17}{3}, \quad \Delta\Gamma_2 = 29.14$.

In this talk we present the calculation of an additional contribution to the "massless" $\mathcal{O}(\alpha_s^2)$ corrections, which were neglected in the literature so far. These singlet corrections are due to the "light-by-light" type diagram which we calculate for nonvanishing quark masses in the heavy top limit. The Yukawa couplings of the fermions yield a factor $m_b m_t$ and each fermion trace produces a factor m_b and m_t respectively. Together with a power supression of $1/m_t^2$ on dimensional grounds all mass factors are combined to m_b^2, which is of the same order as the otherwise massless $\mathcal{O}(\alpha_s^2)$ nonsinglet corrections.

The absorptive part of the triangle diagram is given by

$$\Delta\Gamma^{abs} = \frac{3G_F}{4\sqrt{2}\pi}M_H m_b^2\left(\frac{\alpha_s}{\pi}\right)^2\frac{1}{3}\left[\frac{28}{3} - 2\ln\frac{M_H^2}{m_t^2}\right]. \qquad (13)$$

Here also the decay mode into two gluons is included. It is separately known [26] and should be subtracted in order to arrive at the singlet correction for the bottom final state:

$$\Delta\Gamma^S_{H\to b\bar b} = \frac{3G_F}{4\sqrt{2}\pi} m_b^2 N_C \left(\frac{\alpha_s}{\pi}\right)^2 \frac{1}{3}\left[8 - \frac{\pi^2}{3} - 2\ln\frac{M_H^2}{m_t^2} + \frac{1}{3}\ln^2\frac{m_b^2}{M_H^2}\right]. \qquad (14)$$

At last, combining (12) and (14) we get

$$\Gamma(H\to b\bar b) = \frac{3G_F}{4\sqrt{2}\pi} M_H \bar m_b^2 \left[1 + \frac{17}{3}\left(\frac{\alpha_s}{\pi}\right) + (30.71 - \frac{2}{3}\ln\frac{M_H^2}{m_t^2} + \frac{1}{9}\ln^2\frac{\bar m_b^2}{m_H^2})\left(\frac{\alpha_s}{\pi}\right)^2\right]. \qquad (15)$$

It should be finally stressed that the separation between the decay channels into bottom quarks and gluons leads to a logarithmically enhanced $\ln^2(m_b^2/M_H^2)$ correction to the decay width.

References

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