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Corrections of Order $\mathcal{O}(G_F\alpha_s m_t^2)$ to the Higgs Decay Rate $\Gamma(H \to b\bar{b})$ [1]

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Abstract

QCD corrections to the electroweak one-loop result for the partial width $\Gamma(H \to b\bar{b})$ are studied. For the decay channel into bottom quarks the rate is affected by a virtual top quark through electroweak interactions. The calculation of QCD corrections to this quantity is performed for an intermediate range Higgs mass in the heavy top mass limit. The leading correction of order $\mathcal{O}(G_F\alpha_s m_t^2)$ is estimated. Numerically the contribution is of comparable size as the electroweak correction, but of opposite sign.

[1] A complete postscript file, including figures, is available via anonymous ftp at ttpux2.physik.uni-karlsruhe.de (129.13.102.139) as /ttp94-07/ttp94-07.ps

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1 Introduction

At present high energy colliders experimental data have turned out to be in remarkable agreement with theoretical predictions of the Standard Model. Precision experiments have covered many aspects of both its electroweak and QCD sector. Although the formulation of the Standard Model as a $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge theory is firmly established, experimental evidence for the physical Higgs boson is still missing. Since the Higgs mechanism of spontaneous symmetry breaking and mass generation would reveal its structure through the properties of the Higgs boson, the high mass M_H of this scalar particle hides the Higgs sector completely from observation at present energies. Future accelerators like LHC and NLC, which are dedicated to the search and the study of the Higgs particle, may hopefully close the energy gap to its production threshold.

For intermediate Higgs masses $M_H < 2M_W$ the dominant decay channel is the decay into bottom quarks. A theoretical prediction for the partial width $\Gamma(H \to b\bar{b})$ is therefore needed for the physics analysis with high precision. Consequently much work has been spent in the past on the calculation of radiative corrections to this quantity. Excellent reviews on the Higgs phenomenology can be found for example in [1, 2].

Electroweak radiative corrections, which were studied at the one-loop level by several groups [3, 4, 5], are of particular interest for the process $H \to b\bar{b}$, since they are affected by the mass of the top quark. Virtual top states are possible due to bottom-top transitions mediated by charged Higgs ghosts $\Phi^\pm$ or $W^\pm$ bosons. The leading m_t^2 dependence originates from diagrams with an exchange of $\Phi^\pm$ due to the Yukawa coupling of the Higgs ghosts to the fermion line.

In this work QCD corrections to the electroweak one-loop result are calculated in the heavy top mass limit. For this purpose the hard mass procedure [6, 7, 8, 9] is employed. It results in a power series in the inverse heavy mass $1/m_t$, of which we compute the leading term of order $\mathcal{O}(G_F \alpha_s m_t^2)$.

It is convenient to consider the corresponding scalar current correlator $\Pi(q^2)$, since its absorptive part completely determines the partial decay rate

$$\Gamma(H \to b\bar{b}) = \frac{1}{M_H} \mathrm{Im}\Pi(M_H^2). \tag{1}$$

The calculation of $\Pi(q^2)$ involves multiloop propagator type integrals as well as massive tadpole integrals. Many of the computational tools, mostly based on the algebraic manipulation language FORM [10], have already been used previously for the calculation of $\mathcal{O}(G_F \alpha_s m_t^2)$ corrections to the partial width $\Gamma(Z \to b\bar{b})$ of the Z boson [11]. Nevertheless, the structures of the calculations for the processes $H \to b\bar{b}$ and $Z \to b\bar{b}$ differ in various respects. First, the generic diagram with a primordial decay of the Higgs into a pair of charged Higgs ghosts and their

Figure 1: The electroweak diagrams contributing to the self energy of the H boson. Internal dashed line: Higgs ghost, thin lines: bottom quark, thick lines: top quark.

subsequent transition into b quarks does not contribute to the considered order. This can be seen from the $H\Phi^+\Phi^-$ coupling and is explained on dimensional grounds. Second, the Yukawa couplings of the Higgs to the scalar fermion currents are proportional to the quark masses. Combined with mass terms from the fermion traces the decay rate is proportional to the square of the bottom mass and would vanish in the limit $m_b = 0$. Third, the quadratic dependence of the correction term in the considered order on the bottom as well as the top mass affects the renormalization of the scalar correlator. Wheras for the Z decay the sum of the three loop diagrams was finite, this is not the case for the Higgs rate. The renormalization of the quark masses induces contributions from lower order diagrams which have to be included to arrive at a finite result.

In Section 2.1 the calculation of the three loop propagator diagrams is described. Subsequently the relation between renormalized and bare quark masses including corrections up to the two loop level is combined with the Born graph as well as pure electroweak and QCD diagrams. The corresponding contributions to the order $\mathcal{O}(G_F\alpha_s m_t^2)$ corrections are discussed in Section 2.2. In Section 3 universal corrections to $\Gamma(H \to b\bar{b})$ are added to the flavour specific corrections. The numerical size of the effects is evaluated.

2 Description of the Calculation

2.1 Three-Loop Diagrams

All diagrams of order $\mathcal{O}(G_F\alpha_s m_t^2)$ are obtained from two different generic graphs (see Figure 1), to which a gluon has to be attached in all possible ways. The calculation is performed with dimensional regularisation in the on-shell scheme. In $D = 4 - 2\epsilon$ dimensions the definition of the Hermitian, anticommuting γ_5 matrix is used. After combining pairs of γ_5 to unity, traces with single γ_5 vanish identically. We work in the t'Hooft gauge with an arbitary QCD gauge parameter ξ_S. The verification of gauge invariance serves as an internal check of our result.

Being interested in the leading correction of the power series with respect to the inverse heavy mass $1/m_t$, we apply the hard mass procedure as developed and elaborated in [6, 7, 8, 9]. Those subgraphs are selected, which comprise all top propagators and become one-particle-irreducible after contracting the heavy lines to a point. They are expanded with respect to external momenta and small masses. This formal Taylor expansion is then inserted as an effective vertex into the remaining diagram which has to be evaluated. In the heavy top limit the bottom mass as well as the W mass and the Higgs mass are considered small as compared to the top mass. Since the expansion is performed only to the first nonvanishing order, the Higgs ghost propagators are always contained in the hard subgraphs. For the leading term M_W is therefore set to zero. As a consequence the electroweak gauge parameter drops out trivially. Although the bottom mass is also a small expansion parameter, it cannot be neglected throughout, since that would result in an identically vanishing decay rate. The bottom mass is not only introduced as an overall factor through the Yukawa coupling, but is also present in fermion traces with an odd number of Dirac matrices for $m_b = 0$. Besides the expansion of the hard subgraphs as prescribed by the hard mass procedure also the remaining diagram is expanded with respect to m_b^2. After all expansions are performed, the leading piece can be isolated. Its contribution $\Delta\Gamma_{\alpha\alpha_s}$ to the partial Higgs decay rate reads

$$\begin{aligned}\Delta\Gamma_{\alpha\alpha_s} &= \Gamma_0 m_b^2 x_t \frac{\alpha_s}{\pi}\left\{9\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\left(19 + 12\ln\frac{\mu^2}{m_t^2} - 15\ln\frac{M_H^2}{\mu^2}\right)\right. \\ &\quad + \frac{199}{3} - 24\zeta(3) - \frac{23}{6}\pi^2 + 16\ln\frac{\mu^2}{m_t^2} - 41\ln\frac{M_H^2}{\mu^2} \\ &\quad \left. + 9\ln^2\frac{\mu^2}{m_t^2} + \frac{27}{2}\ln^2\frac{M_H^2}{\mu^2} - 18\ln\frac{\mu^2}{m_t^2}\ln\frac{M_H^2}{\mu^2}\right\}\end{aligned} \tag{2}$$

with

$$\begin{aligned}\Gamma_0 &= \frac{G_F}{4\sqrt{2}\pi} N_C M_H \\ x_t &= \frac{G_F m_t^2}{8\sqrt{2}\pi^2}.\end{aligned} \tag{3}$$

According to the conventions of the multiloop integration package MINCER [12], which we used for the calculation of massless integrals, terms with γ_E and $\ln(4\pi)$ are suppressed. In addition for each loop integration a factor $e^{\zeta(2)\epsilon^2/2}$ is included for convenience. It is clear, that physical results are not influenced by this convention, since only terms of order ϵ^2 are affected. A consistent convention holds also for the calculation of massive tadpole integrals [13]. Our result is obtained in the limit of large top masses and is applicable only in the range of intermediate Higgs masses, in particular in the regime $M_H < 2m_t$ below the top threshold.

2.2 Mass Renormalization and Induced Contributions

The result of eq.(2) is still divergent. Renormalization of the quark masses induces additional contributions which lead to a finite expression for the Higgs decay rate. The relation between the bare mass m_b and the renormalized mass m_b^{OS} of the bottom quark in the on-shell scheme may be written as follows

$$m_b^{OS} = m_b \left\{ 1 + C_\alpha x_t + C_{\alpha_s} \frac{\alpha_s}{\pi} + C_{\alpha\alpha_s} x_t \frac{\alpha_s}{\pi} \right\} \tag{4}$$

where only corrections of orders relevant for our problem are taken into account. The corresponding coefficients C_α, C_{α_s} and $C_{\alpha\alpha_s}$ can be derived from the fact that the renormalized (pole) mass m_b^{OS} is defined through the location of the pole of the quark propagator

$$S_F = \frac{1}{i} \frac{1}{m_b - \not{p} + \Sigma(p)}. \tag{5}$$

The selfenergy $\Sigma(p)$ of the bottom quark is decomposed in the form

$$\Sigma(p) = m_b \Sigma_2 \left(\frac{m_b^2}{p^2} \right) + (m_b - \not{p}\delta) \Sigma_1 \left(\frac{m_b^2}{p^2} \right). \tag{6}$$

The functions $\Sigma_{1,2}$ receive contributions of the order $\mathcal{O}(\alpha_s)$ from diagram 2a with $\delta = 1$, of order $\mathcal{O}(G_F m_t^2)$ from diagram 2b with $\delta = (1 - \gamma_5)$ and of order $\mathcal{O}(G_F \alpha_s m_t^2)$ from diagrams 2c–2e with $\delta = (1 - \gamma_5)$. For the calculation of the quark selfenergies we again employ the hard mass procedure. As suggested in [14] remaining standard scalar integrals may be simplified by an expansion in m_b^2/p^2 around 1. Neglecting all higher order terms only integrals on the bare mass shell need to be evaluated:

$$m_b^{OS} = m_b \left[1 + \Sigma_2(1) - \Sigma_2(1) \left(2\Sigma_2'(1) + \Sigma_1(1) \right) + \cdots \right] \tag{7}$$

The derivatives $\Sigma_2' \equiv \partial \Sigma_2 / \partial (m_b^2/p^2)$ may be conveniently obtained through derivations with respect to the bare mass, thus raising the power in the denomi-

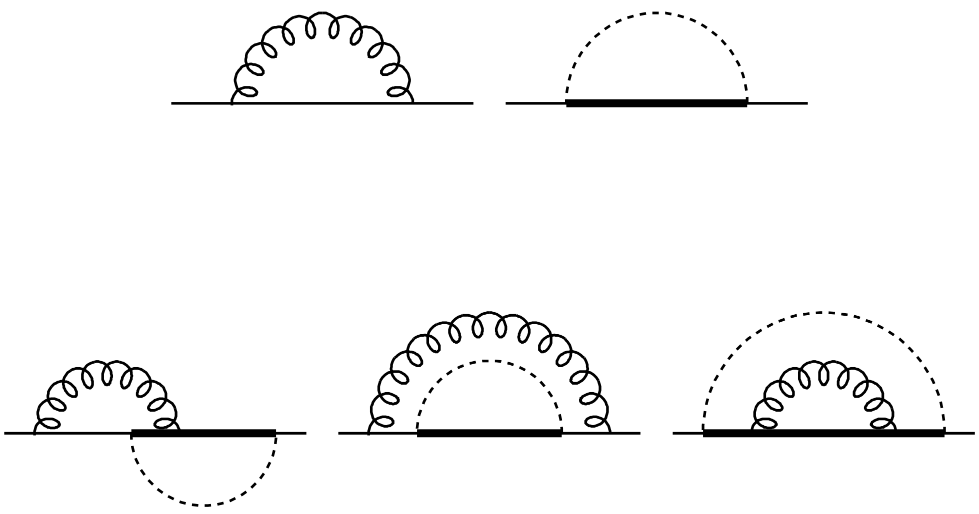

Figure 2: Diagrams for the renormalization of the bottom quark.

nator of the quark propagator. One obtains

$$
\begin{aligned}
C_{\alpha} &= \frac{3}{2\epsilon} + \frac{5}{4} + \frac{3}{2} \ln \frac{\mu^2}{m_b^2} \\
C_{\alpha_s} &= \frac{1}{\epsilon} + \frac{4}{3} + \ln \frac{\mu^2}{m_b^2} \\
C_{\alpha\alpha_s} &= 3\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\left(\frac{19}{4} + \frac{9}{2}\ln\frac{\mu^2}{m_t^2} + \frac{3}{2}\ln\frac{\mu^2}{m_b^2}\right) \\
& \frac{95}{8} + \frac{1}{3}\pi^2 + \frac{25}{4}\ln\frac{\mu^2}{m_t^2} + \frac{13}{4}\ln\frac{\mu^2}{m_b^2} \\
& +\frac{15}{4}\ln^2\frac{\mu^2}{m_t^2} + \frac{3}{4}\ln^2\frac{\mu^2}{m_b^2} + \frac{3}{2}\ln\frac{\mu^2}{m_t^2}\ln\frac{\mu^2}{m_b^2}.
\end{aligned}
\tag{8}
$$

A replacement of the bare bottom and top masses through the renormalized ones according to eq.(4) in the results for the Born graph, the electroweak and the QCD corrected diagrams leads to induced contributions to the order $\mathcal{O}(G_F\alpha_s m_t^2)$. Combined with eq.(2) one obtains the finite result for the decay rate.

3 Discussion

In the previous section the non-universal corrections to the partial width $\Gamma(H \to b\bar{b})$ have been discussed. Combining all contributions one obtains the following result for these process dependent corrections:

$$\Delta\Gamma^{non-univ.}_{H\to b\bar{b}} = -6x_t\Gamma_0 m_b^2 \left[1 + \frac{\alpha_s}{\pi}\left(\frac{8}{3} - 2\ln\frac{M_H^2}{m_b^2}\right)\right] \tag{9}$$

This result has been confirmed recently in [18].

In addition the partial rate is also influenced by universal corrections. Process independent terms of order $\mathcal{O}(G_F\alpha_s m_t^2)$ originate from Δr due to the use of the Fermi constant G_F instead of the electroweak coupling constant α and from the renormalization of the Higgs vacuum. They are given by [4]

$$\Delta\Gamma^{univ.}_{H\to b\bar{b}} = -\Gamma_0 m_b^2 \left[\frac{\Pi^{WW}(0)}{M_W^2} + \mathrm{Re}\Pi^{HH\prime}(M_H^2) + \cdots\right]\left(1 + \delta_{QCD}\frac{\alpha_s}{\pi}\right) \tag{10}$$

where the dots indicate terms of subleading order and the QCD correction factor[1] is given by $\delta_{QCD} = 3 - 2\ln(M_H^2/m_b^2)$.

For the calculation of the the (unrenormalized) selfenergies Π^{WW} and Π^{HH} again the hard mass procedure can be applied. Being interested in their real parts the leading hard subgraphs are constituted by the two loop diagrams themselves with the W selfenergy to be evaluated for zero external momentum. Since the hard mass procedure for the Higgs vacuum polarization is in fact an expansion of the graph with respect to the external momentum, the derivative $\mathrm{Re}\Pi^{HH\prime}$ is readily obtained from the next to leading contribution to the power expansion. After top mass renormalization the combination eq.(10) of the $W-$ and the $H-$ selfenergy is finite in their sum at order $\mathcal{O}(G_F\alpha_s m_t^2)$. The result reads

$$\Delta\Gamma^{univ.}_{H\to b\bar{b}} = \Gamma_0 m_b^2 x_t \left[7 - 2\frac{\alpha_s}{\pi}\left(3 + \frac{1}{3}\pi^2\right)\right]\left(1 + \delta_{QCD}\frac{\alpha_s}{\pi}\right). \tag{11}$$

The electroweak term in the first bracket is in agreement with [4, 5] and its QCD contribution reproduces the result obtained by [15] (see also [16, 17]).

In the sum we arrive at the final result for the partial decay rate

$$\Gamma_{H\to b\bar{b}} = \Gamma_0 m_b^2 \left\{1 + x_t\left[1 + \frac{\alpha_s}{\pi}\left(-1 - \frac{2}{3}\pi^2 - 2\log\frac{M_H^2}{m_b^2}\right)\right]\right\}. \tag{12}$$

[1] We thank the authors of [18] for pointing out that in the earlier version of this paper this factor was missing in eq.(10). As a consequence the interference term of the 1-loop QCD corrections and the 1-loop universal electroweak corrections was erraneously not taken into account.

The QCD correction to the electroweak result is of comparable size as the electroweak correction itself, but of opposite sign. With $m_b = 4.7$ GeV, $m_t = 174$ GeV and $M_H = 60$ GeV (120 GeV) the electroweak contribution of 0.32% normalized to the Born width is combined with a QCD corrected term of -0.21% (-0.24%).

To conclude, in this work we have calculated the QCD corrections to the electroweak one-loop result for the partial Higgs decay rate $\Gamma(H \to b\bar{b})$ in the heavy top mass limit. For intermediate Higgs masses electroweak corrections are significantly reduced by the $\mathcal{O}(G_F\alpha_s m_t^2)$ corrections.

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