Two–Loop Radiative Corrections to the Upper Limit of the Lightest Higgs Boson Mass in the Minimal Supersymmetric Model

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Abstract. In the minimal supersymmetric model (MSSM) the upper limit of the lightest Higgs boson mass, m_{h^0}, depends strongly on the top quark mass, m_t. We have computed the dominant two-loop radiative corrections to this upper limit of m_{h^0} in order to eliminate large uncertainties due to QCD and m_t^6 corrections. It is shown that the QCD corrections significantly reduce the one-loop corrections. As a result, the SUSY parameter space accessible to LEP experiments is significantly increased.

In recent years supersymmetric theories have become maybe the most popular alternatives to the standard model (SM) of elementary particle physics. In the minimal supersymmetric extension of the SM (MSSM) [1] the Higgs sector contains only the two doublets, H_1 and H_2, required to give masses to up and down type fermions with all the quartic couplings related to the gauge couplings. This leads to various restrictions among the Higgs masses and couplings [2]. The most important consequence is the existence of a well defined tree-level upper limit for the mass of the lightest Higgs boson

$$m_{h^0} \leq \min\{m_{\rm z}, m_{A^0}\} \, |\cos 2\beta| \leq m_{\rm z} \,. \tag{1}$$

However, it has been shown recently that radiative corrections can significantly alter this prediction [3]. In particular, the experiments at LEP200 may not be able to detect h^0 or rule out the MSSM. The region in the SUSY parameter space, than can be ruled out at LEP experiments depends crucially on the top quark mass, m_t. However, there is a significant uncertainty in m_t due to large QCD corrections (*e.g.* the running mass differs from the pole mass by $\mathcal{O}(7\%)$ [4]). Eliminating this uncertainty by defining m_t as the pole mass at the one-loop level requires an explicit two-loop calculation of m_{h^0}.

The case $\tan\beta \to \infty$ is particularly interesting because here the tree-level constraint $m_{h^0} \leq m_{\rm z}$ is saturated as long as $m_{A^0} \gtrsim m_{\rm z}$ and thus we expect this case to yield the maximum Higgs mass. Note that m_{h^0} has another maximum in the limit $\tan\beta = 0$ (*i.e.*, $v_2 = 0$). However, the constraint that the Yukawa couplings do not develop a Landau-pole at high energies, together with the experimental lower bound on $m_t > 113$ GeV (95% CL) [5] require that $\tan\beta \gtrsim 0.5$ [6]. On the other hand, $\beta \approx \pi/2$ is theoretically very favorable since it would explain the large ratio of m_t/m_b (*e.g.* grand unified theories based on SO(10) predict $\tan\beta = m_t/m_b$).

The numerical analysis of the one-loop corrections [3;7;8] shows that the dominant contributions to m_{h^0} come from the top-stop sector due to an $g_t^2 m_t^2$ dependence (g_t is the top Yukawa coupling). Thus we expect the dominant two-loop corrections to be the contributions proportional to $g_t^4 m_t^2$ and $g_t^2 g_{\rm s}^2 m_t^2$ ($g_{\rm s}$ is the QCD gauge coupling). These terms can be obtained most easily in the approximation where we set g and g' to zero. In this case the tree-level potential of the lightest Higgs doublet reduces simply to

$$V^{(0)} = m_2^2 H_2^\dagger H_2 \,. \tag{2}$$

(Note that here $m_2 = m_{h^0} = 0$ at tree-level.) After shifting the neutral CP-even component by the vacuum expectation value (VEV) $[H_2 \to (h^0 + v_0)/\sqrt{2}]$ we obtain

$$V_{h^0}^{(0)} = t h^0 + \tfrac{1}{2} m_{h^0 0}^2 (h^0)^2 + \mathcal{O}[(h^0)^3] \,, \tag{3}$$

where $m_{h^0 0}^2 = m_2^2$ and $t = v_0 m_2^2$. If we eliminate m_2 in favor of t we end up with the relation $m_{h^0 0} = t/v_0$. Remember that $m_{h^0 0}$ is still an unrenormalized parameter which has to be expressed in terms of a physical observable. This is the physical mass of h^0 (without a subscript

0) which is identified in the usual way as the pole of the propagator

$$m^2_{h^0} = m^2_{h^0 0} + \mathrm{Re}\ A_{h^0 h^0}(m^2_{h^0})\,. \qquad (4)$$

The self-energies and one-point functions (tadpoles) are defined in fig. 1. Note that h^0 is a stable particle at tree-level (the decay of h^0 into gluons is induced at the one-loop level, and we can assume that $m_{h^0} \leq 2\min\{m_t, m_{\tilde{t}_1}, m_{\tilde{t}_2}\}$). Thus, eq. (4) is valid even at the two-loop level. We now demand that v_0 is the true VEV to all orders in perturbation theory. This means that the tadpoles corresponding to a Higgs field disappearing into the vacuum are absent. This reads $t + A_{h^0}(0) = 0$ or $m^2_{h^0 0} = m^2_2 = -A_{h^0}(0)/v_0$. Thus the two-loop renormalized Higgs mass can be written as

$$m^2_{h^0} = A_{h^0 h^0}(m^2_{h^0}) - \frac{A_{h^0}(0)}{v_0}\,. \qquad (5)$$

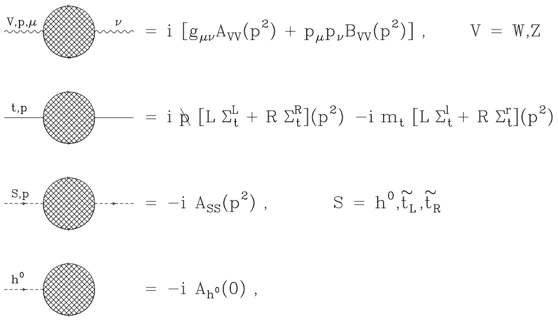

Fig. 1. The definition of the gauge boson top quark and scalar self energies, and the Higgs tadpole. The left and right handed projection operators are $R, L = (1 \pm \gamma_5)/2$.

The calculation of the two-loop diagrams can be simplified considerably by using the fact that scalar one-particle irreducible n-point Green functions with zero external momenta can be obtained as derivatives from the effective potential, $V_{\rm eff}$, with respect to the corresponding fields, *e.g.*

$$\begin{aligned} \frac{\mathrm{d}V_{\rm eff}}{\mathrm{d}v_0} &= m^2_2 v_0 + A_{h^0}(0)\,, \\ \frac{\mathrm{d}^2V_{\rm eff}}{\mathrm{d}^2v_0} &= m^2_2 + A_{h^0h^0}(0)\,. \end{aligned} \qquad (6)$$

Furthermore, it is easy to show that

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}v_0^2} - \frac{\mathrm{d}}{v_0\mathrm{d}v_0}\right) V_{\rm eff} = 4\frac{m^4_{t0}}{v_0^2}\left(\frac{\mathrm{d}}{\mathrm{d}m^2_{t0}}\right)^2 V_{\rm eff}\,, \qquad (7)$$

where $V_{\rm eff}$ on the right hand side is a function of the masses and coupling constants, but not of v_0. Thus we arrive at

$$m^2_{h^0} = 4\frac{m^4_{t0}}{v_0^2}\left(\frac{\mathrm{d}}{\mathrm{d}m^2_{t0}}\right)^2 V_{\rm eff} + A_{h^0h^0}(p^2)\Big|^{m^2_{h^0}}_{p^2=0}\,. \qquad (8)$$

The advantage of using the effective potential is that we only have to compute the difference $A_{h^0h^0}(m^2_{h^0}) - A_{h^0h^0}(0)$. Note that in a two-loop calculation it is sufficient to compute only the one-loop contributions to the Higgs self energies since m_{h^0} here is generated at one-loop. In particular, we have to keep the $\mathcal{O}(\epsilon)$ terms of the one-loop radiatively corrected Higgs mass, m_{h^0}, on the right hand side of eq. (8), since it occurs in a divergent expression. In addition, we have to compute the effective potential to second order $[V_{\rm eff} = V^{(0)} + V^{(1)} + V^{(2)}]$. The results of this calculation are presented in Appendix B. In the terms derived from $V^{(2)}$ we are again allowed to replace the unrenormalized quantities by the physical ones. However, $V^{(1)}$ depends on the unrenormalized top quark mass, m_{t0}, the left- and right-handed top squark masses, $M_{\tilde{t}_P 0}$ ($P = L, R$; we ignore the possibility of L-R mixing due to trilinear Higgs-squark-squark coupling, A_t, which yields only non-logarithmic and thus negligible corrections), and the VEV, v_0, at the one–loop level. Thus, we have to express the bare quantities in terms of physical quantities to first order. The unrenormalized masses can easily be expressed in terms of their pole masses. For the renormalization of the VEV we choose the tree-level relation $v_0 = 2^{-1/4}G_\mu^{-1/2}$, where $G_\mu = (1.166\,39 \pm 0.000\,02) \times 10^{-5}\ \mathrm{GeV}^{-2}$ [9] is the fermi-constant (note that the only contributions to the μ–decay come through the W self-energy). Therefore, our renormalization conditions are

$$\begin{aligned} v_0^2 &\equiv 2^{-1/2}G_\mu^{-1}\,(1 - a_{\rm W})\,, \\ m^2_{t0} &= m^2_t\,(1 - a)\,, \\ M^2_{\tilde{t}_P 0} &= M^2_{\tilde{t}_P}\,(1 - a_P)\,, \qquad P = L, R\,, \end{aligned} \qquad (9)$$

where we have introduced the abbreviations

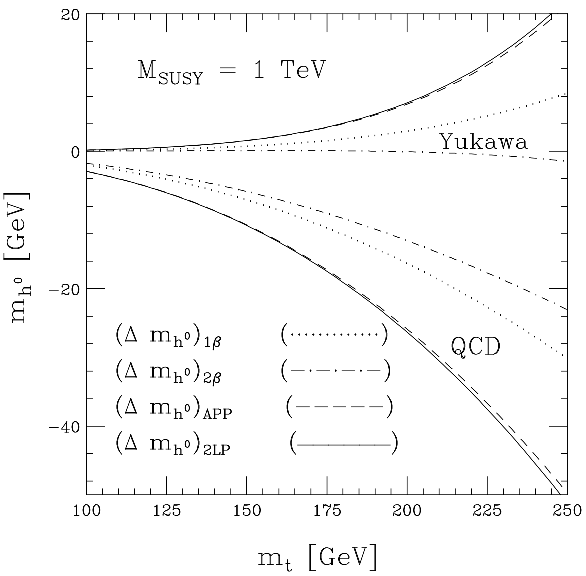

Fig. 2. The two-loop shift in the Higgs mass, $\left(\Delta m^2_{h^0}\right)$, as a function of the top mass. We present the terms to order $g_t^4 m_t^2$ (Yukawa) and $g_t^2 g_s^2 m_t^2$ (QCD) separately.

$$a_P = \frac{\mathrm{Re}\, A_{\tilde{t}_P\tilde{t}_P}(\tilde{u}_P)}{\tilde{u}_P}\,, \quad a_{\mathrm{W}} \equiv \frac{\mathrm{Re}\, A_{\mathrm{WW}}(0)}{m^2_{\mathrm{W}}}\,,$$
$$a \equiv \mathrm{Re}\left[\Sigma_t^l(u) + \Sigma_t^r(u) - \Sigma_t^L(u) - \Sigma_t^R(u)\right], \tag{10}$$

and we have defined $u \equiv m_t^2$ and $\tilde{u}_P \equiv M^2_{\tilde{t}_P}$ $(P = L, R)$. It is now straightforward to derive an expression for the two-loop radiatively corrected Higgs mass in terms of physical quantities

$$\begin{aligned}
\left(m^2_{h^0}\right)_{\mathrm{2LP}} &= h\left(1 + a_{\mathrm{W}} - 2a\right)\\
&+ 4\sqrt{2}N_c u^2 G_\mu \kappa^{-1}\Bigg\{(2a - a_R - a_L)\\
&+ \epsilon\left[\left(a - \tfrac{1}{2}a_{\mathrm{W}} - \tfrac{1}{2}a_h\right)\right.\\
&\times\left((\overline{\ln}\,\tilde{u}_R)^2 + (\overline{\ln}\,\tilde{u}_L)^2 - 2(\overline{\ln}\,u)^2\right)\\
&\left. + a_L\overline{\ln}\,\tilde{u}_L + a_R\overline{\ln}\,\tilde{u}_R - 2a\overline{\ln}\,u\right]\Bigg\}\\
&+ 4\sqrt{2}u^2 G_\mu\left(\frac{\mathrm{d}}{\mathrm{d}u}\right)^2 V^{(2)} + A_{h^0h^0}(p^2)\Big|^{h}_{p^2=0}\,,
\end{aligned} \tag{11}$$

where $\kappa \equiv 16\pi^2$ and we have defined

$$a_h \equiv \left.\frac{\mathrm{d}A_{h^0h^0}(p^2)}{\mathrm{d}p^2}\right|_{p^2=h}\,. \tag{12}$$

We have denoted the one-loop radiatively corrected squared Higgs mass in the approximation introduced above by

$$h \equiv 8\sqrt{2}N_c u^2 G_\mu \kappa^{-1} t_0\,. \tag{13}$$

Here we have defined $t_0 \equiv \ln(M^2_{\mathrm{SUSY}}/m_t^2)$ where we found it convenient to parameterize our numerical results by $M^2_{\mathrm{SUSY}} \equiv M_{\tilde{t}_L}M_{\tilde{t}_R}$ and $r \equiv M_{\tilde{t}_L}/M_{\tilde{t}_R}$. The analytic results for the relevant self-energies and the expression for the one-loop and the two-loop effective potential are given in Appendix A and B, respectively. Here we will simply present an approximate result in the case of heavy, mass-degenerate superpartners $M^2_{\tilde{t}_L} = M^2_{\tilde{t}_R} = M^2_{\tilde{g}} \gg m_t^2$

$$\begin{aligned}
\left(m^2_{h^0}\right)_{\mathrm{APP}} &= h\Big\{1 - 6g_s^2 C_F\kappa^{-1}\left[t_0 + 2\right]\\
&+ \tfrac{1}{6}g_t^2\kappa^{-1}\left[9t_0 + 18 + 7N_c + \left(15 - \pi^2\right)t_0^{-1}\right]\Big\}\,.
\end{aligned} \tag{14}$$

The values of the higgsino mass parameter, μ, and of the heavy Higgs doublet, m_{H^0}, turn out to be irrelevant.

We will now proceed to compare our diagrammatic two-loop result with the second order terms obtained by a renormalization group (RG) approach. Here we reintroduce the Higgs self coupling, λ, in the potential of eq. (2) which we treat as a running low energy effective parameter rather than a bare parameter. In this case we find in general that $\lambda \neq 0$ at any energy scale $\sqrt{s} < M_{\mathrm{SUSY}}$ even in the case $g = g' = 0$. The RG improved Higgs mass can be written as

$$\left(m^2_{h^0}\right)_{\mathrm{RG}} = \lambda v_0^2\,, \tag{15}$$

where the running quartic Higgs self coupling evaluated at the scale $s = m^2_{h^0}$ is

$$\lambda = \int_{\ln M^2_{\mathrm{SUSY}}}^{\ln m_t^2} \mathrm{d}t\,\beta_\lambda\,, \tag{16}$$

By using the β function to two-loop order, β_λ, given in ref. 10 and 11, we obtain the leading and next-to-leading log radiative corrections to the Higgs mass summed to all orders in perturbation theory. In order to obtain an analytic result we will solve the RG equations (RGE) iteratively.

The second order contributions to order g_t^6 and $g_s^2 g_t^4$ are

$$\begin{aligned}\left(m^2_{h^0}\right)_{1\beta} &= h\left\{1 - t_0\kappa^{-1}\left[6g_s^2 C_F - \tfrac{3}{2}g_t^2\right]\right\}\\ \left(m^2_{h^0}\right)_{2\beta} &= \left(m^2_{h^0}\right)_{1\beta} + h\kappa^{-1}\left[4g_s^2 C_F - 5g_t^2\right],\end{aligned} \tag{17}$$

where the subscript 1β (2β) indicates the use of one-loop (two-loop) β functions. It is instructive to compare eq. (17) to eq. (14). We see that the coefficients of t_0^2 obtained by solving the one-loop RGEs to second order [8] are in agreement as they should be. However, the terms linear in t_0 obtained by solving the two-loop RGEs to first order [7] are different. This is not surprising since the RG approach does not include any threshold effects. For example, the QCD threshold corrections to m_t are [4]

$$m_t = m_t^{\rm run}\left(1 + C_F\frac{\alpha_{\rm s}}{\pi}\right), \tag{18}$$

where the running top quark mass is $m_t^{\rm run} = 2^{-3/4}G_\mu^{-1/2}g_t(m_t)$ and we use $\alpha_{\rm s} = g_{\rm s}^2/4\pi = 0.11$. We now investigate the origin of the m_t^4 dependence of the one-loop corrections to m_{h^0} more closely. From eq. (15), (16) and (18) we obtain

$$m^2_{h^0} \propto g_t^4 G_\mu^{-1} t_0 \propto m_t^4 G_\mu t_0\left(1 - 4C_F\frac{\alpha_{\rm s}}{\pi}\right). \tag{19}$$

The $\alpha_{\rm s}$-term in eq. (19) coming from finite threshold corrections to m_t are of the same order as the next-to-leading log term of eq. (17). If we add both terms together we indeed recover the coefficient proportional to $g_{\rm s}^2$ in eq. (14).

We will now present the numerical results of our two-loop calculation. It is contrasted with the second order leading log result and next-to-leading log result obtained from a simple analysis of the one-loop RGEs [8] and two-loop RGEs [7] not including QCD threshold corrections [eq. (17)]. We set $\mu = m_{H^0} = 200$ GeV, $r = 1$ and $M^2_{\tilde{b}_L} = M^2_{\tilde{g}} = M^2_{\rm SUSY} - m_t^2$. In fig. 2 we present the two-loop contribution to the Higgs mass obtained by different methods. Presented are the results from our diagrammatic two-loop calculation (solid curve), the second order contributions from a RG approach using one-loop and two-loop β functions (dotted curve and dot-dashed curve, respectively) and result of our approximate expansion (dashed curve). We

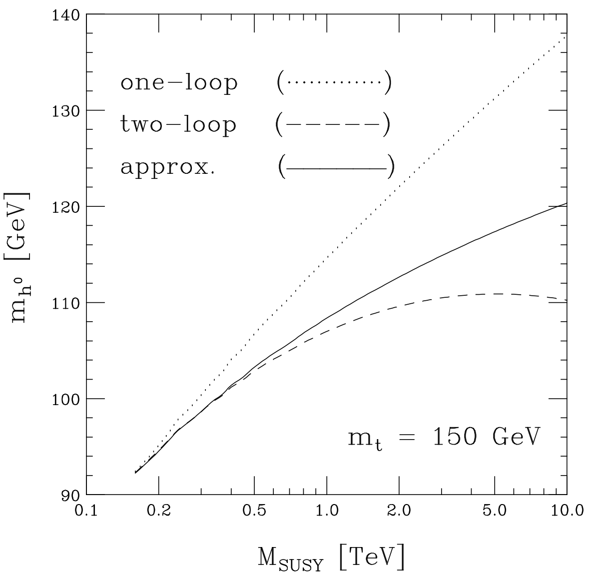

Fig. 3. The radiatively corrected Higgs mass, m_{h^0}, as a function of $M_{\rm SUSY}$. Included are terms to order $g_t^4 m_t^2$ and $g_t^2 g_{\rm s}^2 m_t^2$.

have parameterized the results by a linear mass shift defined as

$$\left(\Delta m_{h^0}\right)_x = \frac{(m^2_{h^0})_x - h}{2m_{\rm z}}, \tag{20}$$

where x = 2LP, 1β, 2β, APP. The contributions to order $g_t^4 m_t^2$ (Yukawa) and $g_t^2 g_{\rm s}^2 m_t^2$ (QCD) are plotted separately. The dependence of Δm_{h^0} on the remaining parameters, r, $M_{\tilde{g}}$, μ and m_{H^0} changes the result by less than 1 GeV as long as $M_{\tilde{g}}, \mu < 3M_{\rm SUSY}$ and $0.1 < r < 10$.

Sofar we have focused our attention on the case of large $\tan\beta$ where one obtains the upper limit for m_{h^0}. We have seen that the diagrammatic result differs significantly from the RGE results [eq. (17)]. However, the agreement with our large $M_{\rm SUSY}$ expansion [eq. (14)] is excellent. Furthermore, the two-loop result is dominated by the QCD corrections which are even larger than those one-loop electroweak corrections which are not enhanced by powers of m_t and thus never exceed a few GeV [12].

In recent years it has become standard in experimental analyses to only include the dominant m_t^4 term at the leading log level via the CP-even mass matrix in electroweak eigenstates

$$\begin{aligned}\mathcal{M}^2 = m^2_{A^0}&\begin{pmatrix}\sin^2\beta & -\sin\beta\cos\beta\\ -\sin\beta\cos\beta & \cos^2\beta\end{pmatrix}\\ &+ m_{\rm z}^2\begin{pmatrix}\cos^2\beta & -\sin\beta\cos\beta\\ -\sin\beta\cos\beta & \sin^2\beta + \epsilon m_{\rm z}^{-2}\end{pmatrix},\end{aligned} \tag{21}$$

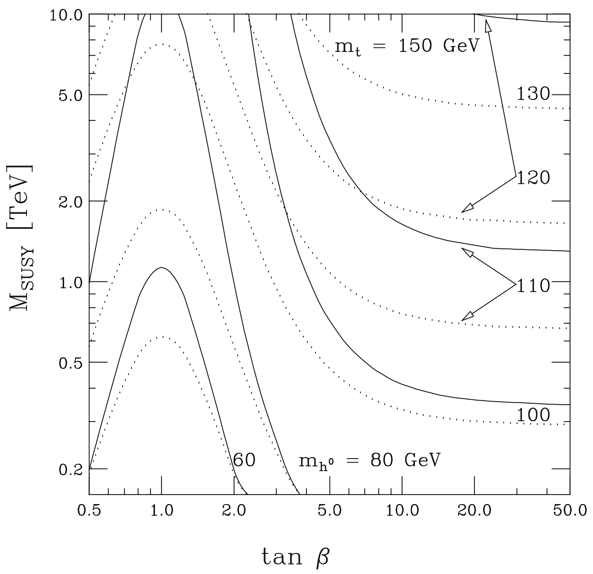

Fig. 4. Contours of constant Higgs mass, m_{h^0} in the $\tan\beta$–M_{SUSY} plane. We take $m_t = 150$ GeV and consider the limit $m^2_{A^0} \gg m^2_{\mathrm{z}}$.

where $\epsilon = h/\sin^2\beta$. Note that at this level our two-loop calculation can be directly generalized to the case of arbitrary $\tan\beta$ by using

$$\epsilon \equiv \frac{3\sqrt{2}}{2\pi^2}\frac{m_t^4}{\sin^2\beta}G_\mu t_0\left[1+\frac{2\alpha_{\mathrm{s}}}{\pi}(t_0+2)\right]^{-1}, \tag{22}$$

where $t_0 \equiv \ln(M_{\tilde{t}_1}M_{\tilde{t}_2}/m_t^2)$. In fig. 3 we compare this approximation (solid curve) with the one-loop result, $\sqrt{m_{\mathrm{z}}^2+h}$, (dotted curve) and the two-loop result, $\sqrt{m_{\mathrm{z}}^2+(m^2_{h^0})_{2\mathrm{LP}}}$, (dashed curve) in the limit of large $\tan\beta$. We plot m_{h^0} as a function of M_{SUSY} for $m_t = 150$ GeV and $r = 1$, and we set $m^2_{H^0} = \mu^2 = M^2_{\tilde{b}_L} = M^2_{\tilde{g}} = M^2_{\mathrm{SUSY}} - m_t^2$. We see that in the few TeV region the two-loop corrections are comparable to the one-loop result in magnitude due to large logarithms. Those terms have to be summed over using the RGEs. We have moved the QCD corrections into the denominator which is in better agreement with the RG improved result [13]. In particular, the approximation of eq. (21) and (22) increases monotonically even for very large t_0.

In fig. 4 we show contours of constant $m_{h^0} = 60$, 80, 100, 110, 120, 130 GeV in the $\tan\beta$–M_{SUSY} plane in the approximation from eq. (21). We compare the results including QCD corrections (solid curves) and without QCD corrections (dashed curves) in the limit $m^2_{A^0} \gg m^2_{\mathrm{z}}$ where we obtain the upper limit of m_{h^0}.

We have shown that the QCD corrections to m_{h^0} are large and negative and thus lead to a reduction of the one-loop result by $\mathcal{O}(30\%)$ for $M_{\mathrm{SUSY}} = 1$ TeV. Consequently the region in the SUSY parameter space accessible at LEP experiments increases significantly. The effects are particularly important in the case $\tan\beta = 1$ where the tree-level Higgs mass vanishes. The implications for the large $\tan\beta$ regime depend on whether it is possible to detect a Higgs boson with a mass, $m_{h^0} < m_{\mathrm{z}}$. Consider *e.g.* the scenario where $m_t = 150$ GeV and the experimental lower limit of $m_{h^0} \geq 110$ GeV. Then the QCD corrections shift the lower limit of M_{SUSY} form 670 GeV to 1.3 TeV .

Acknowledgement : *We would like to thank H. Kühn for suggesting this project.*

Appendix A: One-Loop Self Energies

In this appendix we present the results of the one-loop self energies of top quark, top squarks, the Higgs boson and W boson [fig. 5] in the approximation that $g = g' = A_U = 0$. We have done all the calculations in dimensional reduction in order to preserve SUSY (*i.e.*, we set the number of space-time dimension to $n = 4 - 2\epsilon$, but keep the number of components of all other tensors fixed) [14]. We have suppressed the arbitrary renormalization scale. The results are presented in terms of the following integrals formulated in Minkowski space using the Bjorken-Drell metric

$$\begin{aligned} A_0(m^2) &= -i\kappa\int\frac{\mathrm{d}^n k}{(2\pi)^n}\frac{1}{k^2-m^2+i\delta} \\ &= m^2\left[\frac{1}{\epsilon}-\overline{\ln}(m^2)+1+\epsilon\left(1-\overline{\ln}\,m^2+\frac{\pi^2}{12}+\tfrac{1}{2}\overline{\ln}^2 m^2\right)\right]+\mathcal{O}(\epsilon^2)\,, \end{aligned}$$

$$B_0(q^2,m_1^2,m_2^2) = -i\kappa\int\frac{\mathrm{d}^n k}{(2\pi)^n}\frac{1}{(k^2-m_1^2+i\delta)[(k+q)^2-m_2^2+i\delta]} = \frac{1}{\epsilon}-\int_0^1\mathrm{d}x\,\overline{\ln}\,M^2+\mathcal{O}(\epsilon)\,,$$

$$q_\mu B_1(q^2, m_1^2, m_2^2) = -i\kappa \int \frac{\mathrm{d}^n k}{(2\pi)^n} \frac{k_\mu}{(k^2 - m_1^2 + i\delta)[(q+k)^2 - m_2^2 + i\delta]} \, ,$$
$$B_1(q^2, m_1^2, m_2^2) = \frac{1}{2q^2} \left[A_0(m_1^2) - A_0(m_2^2) - (q^2 + m_1^2 - m_2^2) B_0(q^2, m_1^2, m_2^2 \right] \, , \tag{A.1}$$

where we have used $\Gamma(1)'' - \Gamma(1)'^2 = \pi^2/6$ and we have defined

$$\begin{aligned} M^2 &\equiv m_2^2 x + m_1^2 (1-x) - q^2 x(1-x) \\ \overline{\ln}\, m^2 &\equiv \ln m^2 - \Gamma(1)' - \ln(4\pi) \, . \end{aligned} \tag{A.2}$$

We will use the expansion of $A_0(m^2)$ to first order in ϵ in Appendix B.

The top quark self energies are

$$\begin{aligned} \kappa \Sigma_t^{\mathrm{L}}(u) &= -g_t^2 \left[B_1(u, \tilde{h}, \tilde{u}_R) + B_1(u, u, 0) \right] \\ &+ g_{\mathrm{s}}^2 C_F \left[-2B_1(u, \tilde{g}, \tilde{u}_L) \right. \\ &\left. + (1+\xi) B_0(u, 0, u) + 2B_1(u, 0, u) \right] \, , \end{aligned} \tag{A.3}$$

$$\begin{aligned} \kappa \Sigma_t^{\mathrm{R}}(u) &= -g_t^2 \left[B_1(u, \tilde{h}, \tilde{u}_L) + B_1(u, \tilde{h}, \tilde{d}_L) \right. \\ &\left. + B_1(u, u, 0) + B_1(u, 0, 0) \right] \\ &+ g_{\mathrm{s}}^2 C_F \left[-2B_1(u, \tilde{g}, \tilde{u}_R) \right. \\ &\left. + (1+\xi) B_0(u, 0, u) + 2B_1(u, 0, u) \right] \, , \end{aligned} \tag{A.4}$$

$$\kappa \Sigma_t^{\mathrm{l}}(u) = g_{\mathrm{s}}^2 C_F (3+\xi) B_0(u, 0, u) \, , \tag{A.5}$$

$$\kappa \Sigma_t^{\mathrm{r}}(u) = g_{\mathrm{s}}^2 C_F (3+\xi) B_0(u, 0, u) \, . \tag{A.6}$$

The top squark self energies are

$$\begin{aligned} \kappa A_{\tilde{t}_L \tilde{t}_L}(\tilde{u}_L) &= g_{\mathrm{s}}^2 C_F \left\{ -A_0(\tilde{u}_L) \right. \\ &+ \left[4u_{\tilde{L}} B_0(\tilde{u}_L, 0, \tilde{u}_L) - A_0(\tilde{u}_L) \right] \\ &\left. + 2\left[(u + \tilde{g} - \tilde{u}_L) B_0(\tilde{u}_L, \tilde{g}, u) + A_0(\tilde{g}) + A_0(u) \right] \right\} \\ &+ g_t^2 \left\{ \left[A_0(u) + A_0(\tilde{h}) + (u + \tilde{h} - \tilde{u}_L) B_0(\tilde{u}_L, \tilde{h}, u) \right] \right. \\ &- 2u B_0(\tilde{u}_L, \tilde{u}_L, 0) - u B_0(\tilde{u}_L, \tilde{d}_L, 0) \\ &\left. - \tilde{h} B_0(\tilde{u}_L, \tilde{u}_R, H) - A_0(\tilde{u}_R) \right\} \, , \end{aligned} \tag{A.7}$$

$$\begin{aligned} \kappa A_{\tilde{t}_R \tilde{t}_R}(\tilde{u}_R) &= g_{\mathrm{s}}^2 C_F \left\{ -A_0(\tilde{u}_R) \right. \\ &+ \left[4u_{\tilde{R}} B_0(\tilde{u}_R, 0, \tilde{u}_R) - A_0(\tilde{u}_R) \right] \\ &\left. + 2\left[(u + \tilde{g} - \tilde{u}_R) B_0(\tilde{u}_R, \tilde{g}, u) + A_0(\tilde{g}) + A_0(u) \right] \right\} \\ &+ g_t^2 \left\{ \left[A_0(u) + A_0(\tilde{h}) + (u + \tilde{h} - \tilde{u}_R) B_0(\tilde{u}_R, \tilde{h}, u) \right] \right. \end{aligned}$$

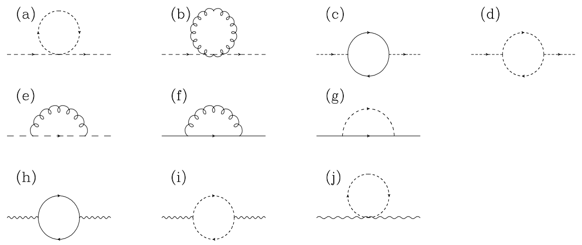

Fig. 5. The contributions to the one-loop self energies

$$\begin{aligned} &+ \left[A_0(\tilde{h}) + (\tilde{h} - \tilde{u}_R) B_0(\tilde{u}_R, \tilde{h}, 0) \right] \\ &- 2u B_0(\tilde{u}_R, \tilde{u}_R, 0) - \tilde{h} B_0(\tilde{u}_R, \tilde{d}_L, H) \\ &\left. - \tilde{h} B_0(\tilde{u}_R, \tilde{u}_L, H) - A_0(\tilde{u}_L) - A_0(\tilde{d}_L) \right\} \, . \end{aligned} \tag{A.8}$$

The Higgs self energy is

$$\begin{aligned} \kappa A_{h^0 h^0}(h) &= N_c g_t^2 \left\{ \left[(4u - h) B_0(h, u, u) \right. \right. \\ &\left. + 2A_0(u) \right] - 2u B_0(h, \tilde{u}_L, \tilde{u}_L) - 2u B_0(h, \tilde{u}_R, \tilde{u}_R) \\ &\left. - A_0(\tilde{u}_L) - A_0(\tilde{u}_R) \right\} \, . \end{aligned} \tag{A.9}$$

The W self energy is

$$\begin{aligned} &\kappa A_{WW}(0) = N_c m_{\mathrm{w}}^2 g_t^2 \\ &\times \left(\frac{1}{\epsilon} + \frac{1}{2} - \overline{\ln}\, u + \frac{\tilde{u}_L + \tilde{d}_L}{2u} - \frac{\tilde{u}_L \tilde{d}_L}{u^2} \ln \frac{\tilde{u}_L}{\tilde{d}_L} \right) \, , \end{aligned} \tag{A.10}$$

where we have used the following abbreviations for the gluino mass, the left-handed bottom squark, the Higgsino mass and the mass of the second Higgs doublet mass

$$\tilde{g} \equiv M_{\tilde{g}}^2 \, , \;\; \tilde{d}_L \equiv M_{\tilde{b}_L}^2 \, , \;\; \tilde{h} \equiv \mu^2 \, , \;\; H \equiv m_{H^0}^2 \, .$$

Appendix B: The Effective Potential

Here we present the calculation of the Feynman diagrams contributing to the effective potential. We have done all the calculations in dimensional reduction to preserve the supersymmetric Ward-identities. The QCD gauge sector has been calculated in arbitrary R_ξ gauge. The results for the one-loop and two-loop effective potential are presented in terms of the following integrals [11]

$$K(m^2) = i\int \frac{\mathrm{d}^n k}{(2\pi)^n}\ln(k^2 - m^2 + i\delta) = \int_0^{m^2} \frac{\mathrm{d}x}{\kappa} A_0(x)\,,$$

$$\kappa^2 J(m_1^2, m_2^2) = A_0(m_1^2)A_0(m_2^2) = m_1^2 m_2^2\left[\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\left(2 - \overline{\ln}\, m_1^2 - \overline{\ln}\, m_2^2\right) + 3\right.$$
$$\left. -2\overline{\ln}\, m_1^2 - 2\overline{\ln}\, m_2^2 + \tfrac{1}{2}\left(\overline{\ln}\, m_1^2 + \overline{\ln}\, m_2^2\right)^2 + \tfrac{1}{6}\pi^2\right]\,,$$

$$\kappa^2 I(m_1^2, m_2^2, m_3^2) = \kappa^2 \int \frac{\mathrm{d}^n k}{(2\pi)^n}\frac{\mathrm{d}^n p}{(2\pi)^n}(k^2 - m_1^2 + i\delta)^{-1}(p^2 - m_2^2 + i\delta)^{-1}[(k+p)^2 - m_3^2 + i\delta]^{-1}$$
$$= \sum_{i=1,2,3} m_i^2\left[-\frac{1}{2\epsilon^2} - \frac{3 - 2\overline{\ln}\, m_i^2}{2\epsilon} - \tfrac{7}{2} - \frac{\pi^2}{12} + 3\overline{\ln}\, m_i^2 - \tfrac{1}{2}(\overline{\ln}\, m_i^2)^2\right] + F(m_1^2, m_2^2, m_3^2)$$
$$+ (m_1^2 + m_2^2 - m_3^2)\overline{\ln}\, m_1^2\overline{\ln}\, m_2^2 + (m_2^2 + m_3^2 - m_1^2)\overline{\ln}\, m_2^2\overline{\ln}\, m_3^2 + (m_3^2 + m_1^2 - m_2^2)\overline{\ln}\, m_3^2\overline{\ln}\, m_1^2\,. \quad \text{(B.1)}$$

With the definition $S(a,b,c) \equiv a^2 + b^2 + c^2 - 2(ab + bc + ca)$ we have for $S(a,b,c) > 0$ and $a > b > c$

$$F(a,b,c) = -4\sqrt{S(a,b,c)}\left[M\left(\coth^{-1}\left(\frac{a+b-c}{\sqrt{S(a,b,c)}}\right)\right) + M\left(\coth^{-1}\left(\frac{c+a-b}{\sqrt{S(a,b,c)}}\right)\right)\right.$$
$$\left. -M\left(\coth^{-1}\left(\frac{a-b-c}{\sqrt{S(a,b,c)}}\right)\right)\right]\,, \quad \text{(B.2)}$$

and for $S(a,b,c) < 0$ and $a > b > c$

$$F(a,b,c) = -4\sqrt{-S(a,b,c)}\left[L\left(\arctan\left(\frac{c-a-b}{\sqrt{-S(a,b,c)}}\right)\right) + L\left(\arctan\left(\frac{a-b-c}{\sqrt{-S(a,b,c)}}\right)\right)\right.$$
$$\left. +L\left(\arctan\left(\frac{b-c-a}{\sqrt{-S(a,b,c)}}\right)\right) - \frac{\pi}{2}\ln 2\right]\,. \quad \text{(B.3)}$$

We also present the special cases

$$\kappa^2 I(m_1^2, m_2^2, 0) = \sum_{i=1,2} m_i^2\left[-\frac{1}{2\epsilon^2} - \frac{3 - 2\overline{\ln}\, m_i^2}{2\epsilon} - \tfrac{7}{2} - \frac{\pi^2}{12} + 3\overline{\ln}\, m_i^2\right] - \tfrac{1}{4}(m_1^2 + m_2^2)\left(\overline{\ln}\, m_1^2 + \overline{\ln}\, m_2^2\right)^2$$
$$-(m_1^2 - m_2^2)(\overline{\ln}\,(m_1^2 - m_2^2) - \ln 2)(\overline{\ln}\, m_1^2 - \overline{\ln}\, m_2^2) - 2(m_1^2 - m_2^2)M\left(\coth^{-1}\left(\frac{m_1^2 + m_2^2}{m_1^2 - m_2^2}\right)\right)\,,$$

$$\kappa^2 I(m^2, 0, 0) = m^2\left[-\frac{1}{2\epsilon^2} - \frac{3 - 2\overline{\ln}\, m^2}{2\epsilon} - \tfrac{7}{2} - \frac{\pi^2}{4} + 3\overline{\ln}\, m^2 - \overline{\ln}^2 m^2\right]\,,$$

$$\kappa^2 I(m^2, m^2, 0) = m^2 \left[-\frac{1}{\epsilon^2} - \frac{3 - 2\overline{\ln}\, m^2}{\epsilon} - 7 - \frac{\pi^2}{6} + 6\overline{\ln}\, m^2 - 2\overline{\ln}^2 m^2 \right] , \tag{B.4}$$

where $m_1^2 > m_2^2$. The function M, which is related to the Spence function, and the Lobaschevski function, L, are given by

$$M(x) \equiv -\int_0^x \mathrm{d}y \ln \sinh y \,, \qquad \text{and} \qquad L(x) \equiv -\int_0^x \mathrm{d}y \ln \cos y \,. \tag{B.5}$$

The one-loop effective potential is

$$V^{(1)} = N_c \left[2K(u) - K(\tilde{u}_L) - K(\tilde{u}_R) \right] , \tag{B.6}$$

The relevant top Yukawa and QCD contributions to the two-loop effective potential are

$$\begin{aligned} V^{(2)} &= g_s^2 (N_c^2 - 1) \{ [-2uI(u,u,0) + J(u,u)] \\ &+ \left[\tilde{u}_L I(\tilde{u}_L, \tilde{u}_L, 0) + \tfrac{1}{4} J(\tilde{u}_L, \tilde{u}_L) \right. \\ &\quad \left. + \tilde{u}_R I(\tilde{u}_R, \tilde{u}_R, 0) + \tfrac{1}{4} J(\tilde{u}_R, \tilde{u}_R) \right] \\ &+ [L(\tilde{u}_R, \tilde{g}, u) + L(\tilde{u}_L, \tilde{g}, u)] \\ &+ \tfrac{1}{4} [J(\tilde{u}_L, \tilde{u}_L) + J(\tilde{u}_R, \tilde{u}_R)] \Big\} \\ &+ N_c g_t^2 \Big\{ \Big[L(0,u,u) + L(0,0,u) + L(\tilde{u}_L, \tilde{h}, u) \\ &\quad + L(\tilde{u}_R, \tilde{h}, u) + L(\tilde{d}_L, \tilde{h}, u) + L(\tilde{u}_R, \tilde{h}, 0) \Big] \\ &- \Big[uI(\tilde{u}_L, \tilde{u}_L, 0) + uI(\tilde{d}_L, \tilde{u}_L, 0) + uI(\tilde{u}_R, \tilde{u}_R, 0) \\ &\quad + \tilde{h} I(\tilde{u}_L, \tilde{u}_R, H) + \tilde{h} I(\tilde{d}_L, \tilde{u}_R, H) \Big] \\ &+ \Big[J(\tilde{u}_L, \tilde{u}_R) + J(\tilde{d}_L, \tilde{u}_R) \Big] \Big\} \,. \end{aligned} \tag{B.7}$$

For convenience we have defined

$$\begin{aligned} L(a,b,c) &\equiv J(c,b) - J(a,b) - J(a,c) \\ &- (a-b-c) I(a,b,c) \,. \end{aligned} \tag{B.8}$$

The derivative of $V^{(2)}$ in eq. (11) is obtained by using the chain rule and

$$\frac{\mathrm{d}\tilde{u}_L}{\mathrm{d}u} = \frac{\mathrm{d}\tilde{u}_R}{\mathrm{d}u} = 1 \,, \quad \text{and} \qquad \frac{\mathrm{d}\tilde{d}_L}{\mathrm{d}u} = \frac{\mathrm{d}\tilde{g}}{\mathrm{d}u} = 0 \,. \tag{B.9}$$

We have computed the second derivative of $V^{(2)}$ numerically and analytically. Furthermore, we have checked both analytically and numerically that the right hand side of eq. (11) is independent of ϵ and the renormalization scale.

Remark : We have done the two-loop calculation in dimensional reduction. Operationally this was done using dimensional regularization plus the contributions from the so-called ϵ-scalars [14]. The latter contribution is divergent due to the $1/\epsilon^2$ pole from the two-loop integral. Without it eq. (11) is not finite.

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