Flavour Singlet $\mathcal{O}(\alpha_s^2 m_b^2/m_t^2)$ Corrections to the Partial Decay Rate $\Gamma(Z \to b\bar{b})$

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Abstract

We analytically compute the flavour singlet $\mathcal{O}(\alpha_s^2 m_b^2/m_t^2)$ radiative corrections to the partial decay rate $\Gamma(Z \to b\bar{b})$. These corrections arise from anomalous "double triangle" diagrams containing a single γ_5 matrix inside each of two closed fermion loops. They represent the next-to-leading term of the asymptotic expansion in the inverse large top mass. As a byproduct of the calculcation we confirm the results of B.Kniehl and J.H.Kühn for the flavour singlet massless corrections [**?**] as well as our previous $\mathcal{O}(\alpha_s^2 m_b^2/M_Z^2)$ result [**?**].

1 Introduction

The decay of the Z-boson into a bottom quark pair has become of increasing interest in the past. On the experimental side the measurement of the partial decay rate $\Gamma(Z \to b\bar{b})$ represents one of the high precision tests of the Standard Model, which are performed at LEP. The experimental uncertainty of this quantity $\Gamma(Z \to b\bar{b}) = (383 \pm 5.6)$ MeV is likely to be reduced even below 1% in the future [?].

In order to match this accuracy also on the theoretical side, a precise knowledge of radiative corrections to the partial Z-decay rate becomes mandatory. For this reason the influence of the finite bottom mass became a relevant topic in QCD calculations. Bottom mass effects have conveniently been studied by expanding radiative corrections in m_b^2/M_Z^2, which is certainly a small parameter. Several contributions of the quadratic mass corrections are known in the literature. Flavour non-singlet corrections were obtained to third order $\mathcal{O}(\alpha_s^3 m_b^2/M_Z^2)$ for the vector current induced rate [?] and to second order $\mathcal{O}(\alpha_s^2 m_b^2/M_Z^2)$ to the axial vector induced rate [?]. Flavour singlet contributions to the Z-width were first discussed for the massless case in [?]. It was pointed out that they arise at second order only for the axial induced rate, whereas the vector contribution vanishes at this order due to Furry's theorem. The corresponding mass corrections were obtained in [?].

In this paper we focus again on the flavour singlet corrections to the partial Z-width. The accurate measurement of the various free parameters of the Standard Model made it possible to derive constraints on the top mass. With $m_t = (158^{+30}_{-34} \pm 20 \pm 12)$ GeV [?] the top quark is expected to be roughly twice as heavy as the Z-boson. Therefore one can expect mass corrections of order $\mathcal{O}(m_b^2/m_t^2)$ to be of similar size as those of order $\mathcal{O}(m_b^2/M_Z^2)$. The inclusion of the $\mathcal{O}(m_b^2/m_t^2)$ contribution completes the discussion about flavour singlet bottom mass corrections.

Our approach for the actual calculation of the decay rate

$$\Gamma^A(Z \to b\bar{b}) = \frac{G_F M_Z^3}{24\sqrt{2}\pi} N_C a_b^2 R^A \tag{1}$$

is to employ its relation to the absorptive part of the corresponding axial current correlation function

$$R^A(s) = -\frac{4\pi}{s} \mathrm{Im}\Pi_1^A(s + i\epsilon). \tag{2}$$

Here Π_1^A arises from the Lorentz decomposition of the 2-point current correlator

$$\begin{aligned} \Pi_{\mu\nu}^A(q) &= i \int dx e^{iqx} \langle 0|\; T\; \Delta_\mu(x)\Delta_\nu(0)\; |0\rangle \\ &= g_{\mu\nu}\Pi_1^A(q^2) + q_\mu q_\nu \Pi_2^A(q^2) \end{aligned} \tag{3}$$

with $\Delta_\mu = A_\mu^t - A_\mu^b$, where we denote the axial vector current for the quark of flavour f with $A_\mu^f = \bar{\Psi}_f \gamma_\mu \gamma_5 \Psi_f$.