TTP93–1A
May 1994

ADDENDUM TO OUR PAPER: "RADIATIVE TAU DECAYS WITH ONE PSEUDOSCALAR MESON"

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Abstract

In a previous paper we have calculated the decay $\tau \to \pi\nu_\tau\gamma$. We used a given relative sign between the internal bremsstrahlung and the structure dependent radiation, which has been used by several other authors before. However, we believe that there are good arguments why the opposite sign is the physical one. We briefly discuss these arguments and then investigate how the predictions for the decay $\tau \to \pi\nu_\tau\gamma$ are affected by this sign.

In [1] we have calculated the radiative decay $\tau \to \pi(K)\nu_\tau\gamma$. In that paper we used structure dependent form factors F_V and F_A with negative values at $t = 0$, ie. $F_V(t) < 0$, $F_A(0) < 0$ (see Eqs. (39) and (59) in [1]). In other words, for the relative sign s between the structure dependent radiation SD and the internal bremsstrahlung IB which is defined by

$$s := \frac{F_V(0) f_\pi}{|F_V(0) f_\pi|} \tag{1}$$

we used $s = -1$. We took this sign from [2], where a detailed derivation of the amplitude for $\pi \to e\nu_e\gamma$ is given. This sign agrees with the one in [3, 4]. (In comparing with the literature, some care is needed because of varying conventions for the form factors and for ϵ^{0123}.) The sign $s = -1$ in [2], however, was chosen arbitraryly, since no explicit model predicting the phases of f_π and F_V was considered [5].

Note that the value of s affects the interference between SD and IB only. In the case of the radiative pion decay $\pi \to e\nu_e\gamma$, this interference is vanishingly small, so up to now a direct measurement of s was not possible.

However, the sign s can be determined by using specific models or symmetry relations between the phases of the form factors F_V or F_A and that of other observables. For the vector form factor, it is shown in [6] that it is related to the anomalous form factor H in K_{l4} decays and that the available data for H and $SU(3)$ symmetry relations require $s = +1$. In [7, 8], the processes $\pi(K) \to l\nu_l\gamma$ are calculated within the framework of chiral perturbation theory, also resulting in $s = +1$.

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Therefore we believe that most probably $s = +1$ is the physical choice. We will discuss below how the predictions for $\tau \to \pi\nu_\tau\gamma$ change under $s = -1 \to +1$. In fact it will turn out that a measurement of this decay yields s.

In [1] we compared our results to those of [9, 10]. We stated that we agreed with [9] if we used their particle data and their parametrization of the form factors, but that we did not reproduce the numerical results of [10]. Now it appears that this is due to the fact that the authors in [9] use $s = -1$ while those in [10] have $s = +1$. With the latter choice and the parametrization of [10] we fully reproduce their results.

Now we will discuss the changes which occur in [1] if we take $s = +1$. We have to replace $F_V(0)$ and $F_A(0)$ of [1] by

$$F_V(0) = +0.0270 \qquad F_A(0) = +0.0116 \tag{2}$$

Then the rates for the internal bremsstrahlung Γ_{IB} and for structure dependent radiation Γ_{SD} remain unaffected, while the interference term Γ_{INT} changes its sign. So in the numerical discussion in Sec. 5 of [1], the sign of all interference contributions (IB-V, IB-A and INT, cf. Eqns. (27)) have to be reversed. This concerns tables 1, 2, 4. The interference becomes destructive, such that the difference between the pure internal bremsstrahlung IB and the full answer Γ_{total} is always reduced. Similarly most of the numbers in table 3 are somewhat reduced, from about 2.8 down to values of about 1.9. Also in the photon spectrum (Fig. 4(a,b)), the interference INT (dotted) must be subtracted rather than added to IB and SD, if $s = +1$. This brings the total spectrum even closer to the pure internal bremsstrahlung (IB), especially for large x. While Fig. 4(c) is not affected at all, in Fig. 4(d) the numbers on the vertical axis change their sign. Fig. 5 does not change much, again the total rate comes closer to the internal bremsstrahlung.

Let us now discuss the most important changes, which occur in the pion-photon invariant mass spectrum. Using $s = +1$ we obtain Figs. 1 and 2, which are to be compared with Figs. 6 and 7 of [1]. The destructive interference substantially diminishes the total rate above the ρ resonance peak at $t = m_\rho^2$ (Fig. 1(a)). The destructive interference contribution decreases strongly if t approaches $m_{a_1}^2$ from below, such that with $s = +1$ a a_1 resonance peak becomes visible (Fig. 1(b)). As is shown in Fig. 2, the width Γ_{a_1} can be measured quite well in this spectrum if $s = +1$.

In order to remain brief, we will not give all the numbers for the decay rates and plots of the spectra for $s = +1$ in this addendum. They can, however, be found in [11].

To conclude let us repeat our main results. First we find that the decay $\tau \to \pi\nu_\tau\gamma$ allows for direct measurement of the relative sign s between the internal bremsstrahlung and the structure dependent radiation. Second, if $s = +1$, which is most probably the physical choice, then the difference between the pure internal bremsstrahlung and the physical spectrum is very small in most of the phase space, because of the destructive interference between the internal bremsstrahlung and the structure dependent radiation. Third, if $s = +1$, a measurement of the pion-photon invariant mass spectrum allows for a measurement of Γ_{a_1}.

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Figure 1: Pion-photon invariant mass spectrum of the decay $\tau \to \nu\pi\gamma$ (using the standard parameter set)

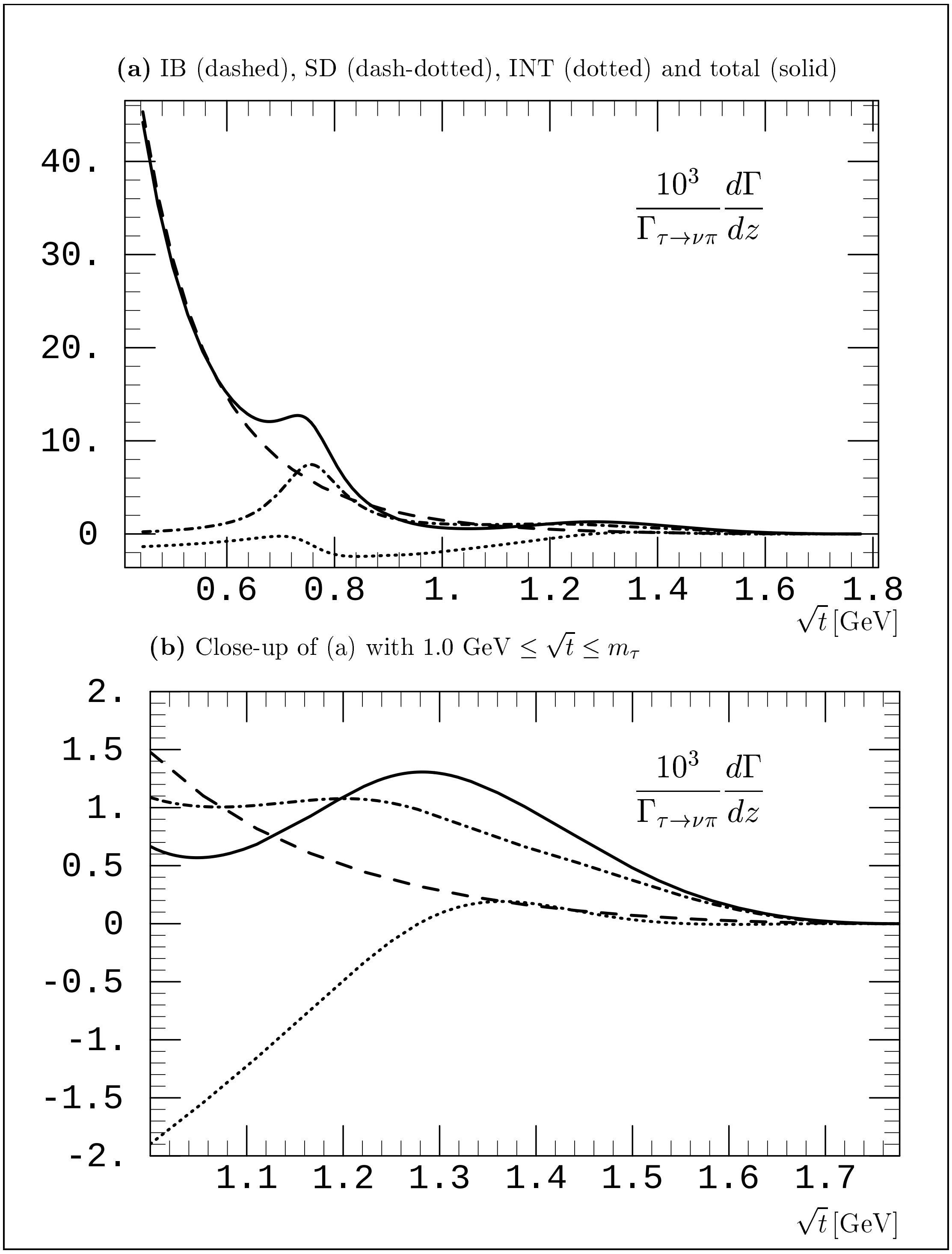
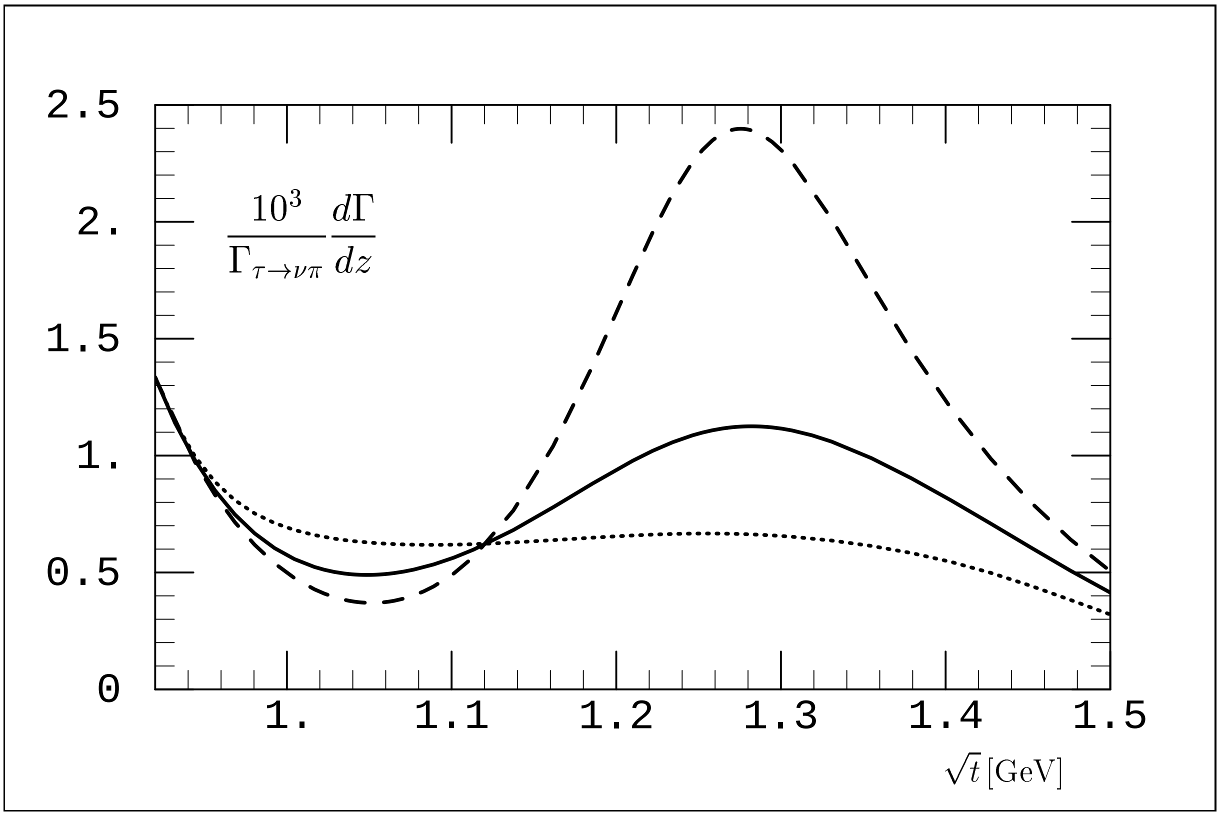


Figure 2: Pion-photon invariant mass spectrum in the a_1 mass region, using Γ_{a_1} = 250 (dashed), 400 (solid) and 650 MeV (dotted)



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