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Leading Top Mass Corrections of Order $\mathcal{O}(\alpha\alpha_s m_t^2/M_W^2)$ to the Partial Decay Rate $\Gamma(Z \to b\bar{b})$

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Abstract

The order $\mathcal{O}(\alpha\alpha_s m_t^2/M_W^2)$ corrections to the partial decay rate of $\Gamma(Z \to b\bar{b})$ are computed using the heavy top mass expansion. They are given by the absorptive part of the corresponding three loop Z-self energy diagrams. Our method provides an independent way to calculate the QCD corrections to the electroweak one loop result for $\Gamma(Z \to b\bar{b})$ and our result represents another confirmation of the one recently published by Fleischer et al.

1 Introduction

Experiments at LEP have limited the top mass m_t to be larger than the mass of the Z boson. Many observables measured at LEP nevertheless are affected by m_t due to the appearance of the top quark as a virtual state in higher order radiative corrections.

Of particular interest has been the investigation of the partial decay rate $\Gamma(Z \to b\bar{b})$ of the Z boson into a b quark pair. On the one hand the present experimental uncertainty of this quantity $\Gamma(Z \to b\bar{b}) = (383 \pm 5.6)$ MeV is very likely to be even further reduced below 1% in the future [1]. On the other hand the m_t dependence enters $\Gamma(Z \to b\bar{b})$ through non-oblique corrections, which helps to disentangle top effects from possible new physics effects.

The advanced status of the analysis of $\Gamma(Z \to b\bar{b})$ is reflected by the number of groups being involved in the calculation of radiative corrections to this quantity. The one loop electroweak corrections to the partial Z decay rate were obtained in [2, 3, 4]. On top of it the knowledge of QCD corrections to this result became desirable in view of the high experimental precision of $\Gamma(Z \to b\bar{b})$. This calculation was first performed to order $\mathcal{O}(\alpha\alpha_s m_t^2/M_W^2)$ in the heavy top mass limit by [5]. It involved the computation of two loop vertex corrections as well as gluon bremsstrahlung corrections to the one loop electroweak vertex correction and found a similar screening of the leading top mass effect as is known from the universal ρ-parameter. The result was confirmed recently by two other calculations [6, 7], where in the latter a current algebra approach has been employed.

In this letter we present another independent confirmation of the $\mathcal{O}(\alpha\alpha_s m_t^2/M_W^2)$ radiative corrections to $\Gamma(Z \to b\bar{b})$. Our method is based on the calculation of the three loop propagator type (p-)diagrams

$$\Pi_{\mu\nu}(q^2) = g_{\mu\nu}\Pi_1(q^2) + q_\mu q_\nu \Pi_2(q^2) \tag{1}$$

which are obtained by attaching to the electroweak diagrams of Figure 1 an internal gluon line in all possible ways. After summing all diagrams the decay rate is given by the absorptive part of the spectral function Π_1

$$\Gamma(Z \to b\bar{b}) = \frac{1}{M_Z}\mathrm{Im}\Pi_1(M_Z^2). \tag{2}$$

There are several advantages of our approach. The number of diagrams is rather small (17 topologically different diagrams). Each p-diagram is infrared (IR) finite, since it comprises just those contributions of vertex corrections and gluon bremsstrahlung for which IR cancellation takes place. The attribution of the problem to propagator type diagrams is also convenient as there exist elaborated techniques to compute just p-integrals (for a recent review see [8]).

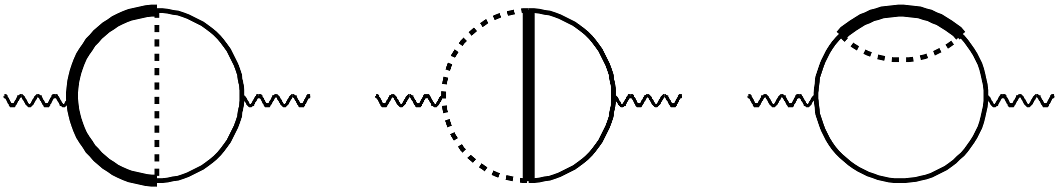

Figure 1: The electroweak diagrams contributing to the self energy of the Z boson. Dashed lines: Higgs ghost, thin lines: bottom quark, thick lines: top quark.

Our calulation is performed with dimensional regularization in the $\overline{\text{MS}}$-scheme. We also transform our final result to the on-shell scheme to compare it with the one of [5, 6, 7]. For dimensions $D \neq 4$ we use the definition of the hermitean, anticommuting γ_5 $(\gamma_5^2 = 1, \{\gamma_5, \gamma_\mu\} = 0)$. We are working in the 't Hooft gauge with an arbitrary QCD gauge parameter. Explicit verification of QCD gauge invariance serves as an internal check of the result.

In this work the calculation is restricted to the leading m_t^2 dependence of the strong-electroweak corrections. We isolate this contribution by using a large top mass expansion and keeping only the leading term. The second order term of this series would yield the next to leading top mass effect of order $\mathcal{O}(\alpha\alpha_s)$ and will be investigated in the future.

A short description of the heavy mass expansion to be used and further details of the calculation are given in Section 2. Section 3 contains the discussion of the results and conclusions.

2 Calculation of the Order $\mathcal{O}(\alpha\alpha_s m_t^2/M_W^2)$ Corrections to $\Gamma(Z \to b\bar{b})$

The top mass dependence of the electroweak one-loop result for $\Gamma(Z \to b\bar{b})$ is due to the appearance of the top quark as a virtual particle. Intermediate top states originate from the $t-b$ transitions, which in general may be induced by the exchange of a charged Higgs ghost $\Phi^\pm$ or a W boson. In this paper we are interested in the QCD corrections to the leading m_t^2 term of the $\mathcal{O}(\alpha)$ corrected partial Z width. To this order only diagrams with a Higgs ghost exchange contribute, because the Higgs ghost-fermion coupling is proportional to the fermion mass, which is not the case for the coupling of the W boson to fermions.

The method of our calculation is to consider the relevant Z boson two point functions and compute their absorptive parts. Note that only the function Π_1 is relevant for the Z decay rate (see eq.(2)). We may therefore simplify the loop integrals by contracting $\Pi_{\mu\nu}$ with the transversal Lorentz tensor ($D = 4 - 2\epsilon$):

$$\Pi_1(q^2) = \frac{1}{D-1}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\Pi^{\mu\nu}(q^2) \tag{3}$$

In order to derive the $\mathcal{O}(\alpha\alpha_s m_t^2/M_W^2)$ corrections to $\Gamma(Z \to b\bar{b})$ we have to calculate all three loop diagrams which can be constructed from those of Figure 1 by inserting an additional gluon line in all possible ways. However, we are working in the limit of large top masses in order to derive the dominant top mass contribution. In this limit no explicit computation of genuine three loop integrals has to be performed and the calculation may be drastically simplified in the following manner.

We employ an effective field theory approach by integrating out the heavy top quark from the theory. This results in a power expansion with respect to the inverse heavy mass $1/m_t$, from which we obtain the leading contribution $\propto m_t^2$. This large mass expansion is constructed as follows[1]. Let Γ be a graph contributing to the polarization operator Π_1. The corresponding Feynman integral $< \Gamma > (q^2)$ in the limit of the heavy top quark may be represented in the form of the following asymptotic expansion:

$$< \Gamma > (q^2) \underset{m_t\to\infty}{=\!=} \sum_\gamma C_\gamma^{(t)} * < \Gamma/\gamma >^{\text{eff}} \tag{4}$$

The diagrams $< \Gamma/\gamma >^{\text{eff}}$ of the effective theory are by definition composed of the fields of the light particles (namely the b-quark, the Higgs ghosts and the gluon), whereas the whole top dependence is contained in the "coefficient functions" $C_\gamma^{(t)}$. Diagrammatically the latter are represented by all possible *hard* subgraphs of Γ. By definition, a subgraph γ of Γ is hard if, first, it comprises all the *heavy* lines (i.e. the top quark propagators) and, second, it becomes one-particle-irreducible after contracting its heavy lines. An example is illustrated in Figure 2. Our notation $\gamma * < \Gamma/\gamma >^{\text{eff}}$ means that in the original diagram Γ the hard subgraph γ is reduced to an effective blob. The corresponding Feynman subintegral $C_\gamma^{(t)}$ is then expanded in a (in general multidimensional) formal Taylor series with respect to its (small) external momenta and to light masses eventually contained in $C_n^{(t)}$. This Taylor expansion is then inserted in the effective blob and the whole expression is evaluated.

There are different mass scales involved in our problem. We neglect the bottom mass throughout in the calculation. The W boson mass M_W enters through the

[1]In performing concrete calculations we make use of the so-called method of projectors [9] in a diagram-wise approach of refs. [10]. The reader interested in the mathematical proofs will find them in [11, 12].

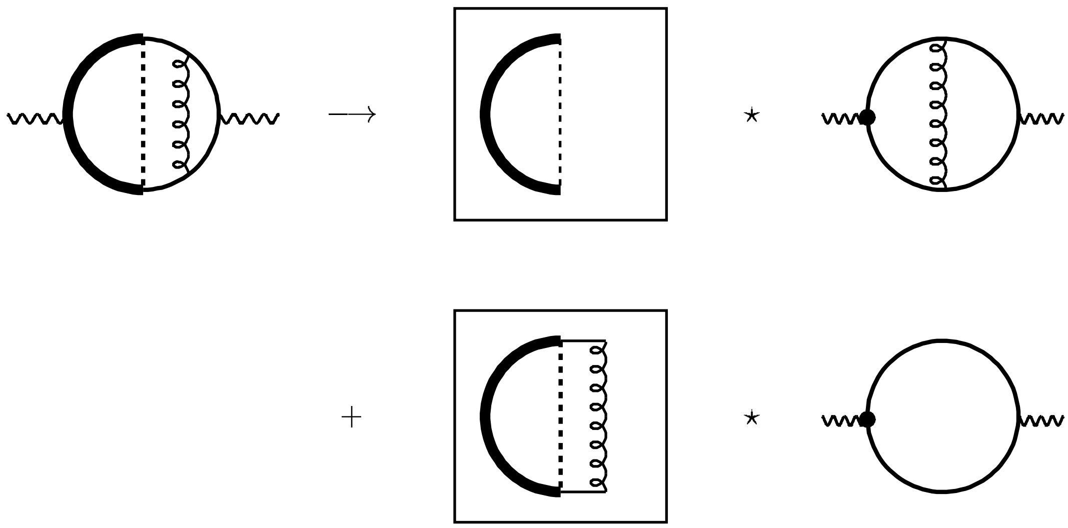

Figure 2: One example for the large mass expansion of a diagram with QCD corrections. Only those hard subgraphs (denoted by the boxes) which contribute to order $\mathcal{O}(\alpha\alpha_s m_t^2/M_W^2)$ are depicted.

propagator of the Higgs ghost

$$D(p) = \frac{1}{i}\frac{1}{\xi_W M_W^2 - p^2 - i\epsilon} \tag{5}$$

and is of the same order as the energy scale of the process, i.e. the mass of the Z boson. Finally in our approximation the top mass is assumed to be very large ($m_t \to \infty$). Therefore we use M_W as an expansion parameter within the hard subgraphs $C_n^{(t)}$, whereas it should be kept finite within the effective theory. As a consequence of the evaluation of $C_n^{(t)}$ at $M_W = 0$ the electroweak gauge parameter ξ_W vanishes trivially in the leading top mass correction.

This is different in the case of the QCD gauge. The QCD gauge parameter ξ is introduced through the gluon propagator

$$D^{\mu\nu}_{\alpha\beta}(k) = \frac{1}{i}\delta_{\alpha\beta}\frac{-g^{\mu\nu} + (\xi - 1)\dfrac{k^\mu k^\nu}{-k^2 - i\epsilon}}{-k^2 - i\epsilon} \tag{6}$$

and is kept arbitrary for an explicit check of gauge invariance. The calculation shows that those diagrams which originate from the same electroweak two point function of Figure 1 (characterized by the vertices $Zt\bar{t}$, $Z\Phi^+\Phi^-$ and the quark self energy) already form a gauge invariant subset of diagrams.

The isolation of the leading m_t^2 term in the large mass expansion allows to discard contributions at three different levels. First, only Higgs ghost exchange graphs need to be considered. As was noted before, W boson exchange diagrams contribute at most to next to leading order (namely $\sim \ln(m_t^2/M_W^2)$. Second, only subgraphs with the highest dimension in m_t are taken into account. Finally, higher order contributions of the Taylor expansion of the hard subgraphs lead to higher orders in $1/m_t$ and may be discarded.

The heavy mass expansion described above is a powerful method for the calculation of radiative corrections. (As an exercise we have also used this method to reproduce the $\mathcal{O}(\alpha \ln(m_t^2/M_W^2))$ term in the electroweak result for $\Gamma(Z \to b\bar{b})$ by [2].) Due to the application of this procedure the original three loop integrals simplify a lot. They factorize and may be reduced to one loop massive tadpoles and two loop massless integrals or to one loop massive tadpoles and two loop massless integrals. The tadpole integrals are given by the following analytical expressions [13]:

$$
\begin{aligned}
T_1 = & \int_E \frac{d^D p}{(M^2+p^2)^a p^{2b}} \\
& \left(M^2\right)^{2-\epsilon-a-b-c} \left(\frac{\mu^2}{M^2}\right)^{\epsilon} \frac{\Gamma(2-\epsilon-b)\Gamma(a+b+\epsilon-2)}{\Gamma(2-\epsilon)\Gamma(a)} \\
T_2 = & \int_E \frac{d^D p d^D l}{(M^2+p^2)^a (M^2+l^2)^b (l+p)^{2c}} \\
= & \left(M^2\right)^{4-2\epsilon-a-b} \left(\frac{\mu^2}{M^2}\right)^{2\epsilon} \cdot \\
& \frac{\Gamma(a+c-2+\epsilon)\Gamma(b+c-2+\epsilon)\Gamma(a+b+c-4+2\epsilon)\Gamma(2-\epsilon-c)}{\Gamma(2-\epsilon)\Gamma(a)\Gamma(b)\Gamma(a+b+2c-4+2\epsilon)},
\end{aligned} \tag{7}
$$

where μ^2 is the renormalization scale of dimensional regularization. For the calculation of the massless integrals we have used the program LOOPS [14], which employes the algorithm of Ref. [15] to do massless p-integrals.

Before we describe the renormalization of the strong-electroweak three loop diagrams, let us briefly recall the situation of the purely electroweak case. Each two loop graph of Figure 1 develops quadratic $1/\epsilon^2$-poles due to its UV divergent subgraphs and its overall divergence. In the sum of all diagrams the subdivergences compensate each other and an overall $1/\epsilon$-pole remains. In other words, all counterterms introduced to subtract the divergences of the various subgraphs cancel each other. This is known as the generalization of the QED Ward identities and manifests the fact that the fermion-Higgs ghost interaction does not contribute to the renormalization of the weak coupling constant.

In the case of the QCD corrected three loop diagrams a similar cancellation of subdivergences takes place and all $1/\epsilon^3$ poles drop out. Similarly, all $1/\epsilon^2$-poles cancel if the $\mathcal{O}(\alpha\alpha_s m_t^2/M_W^2)$ contribution is taken into account, which is induced by the order $\mathcal{O}(\alpha m_t^2/M_W^2)$ through replacing the bare top mass by the renormalized one:

$$m_t^{bare} = \bar{m}_t \left(1 - \frac{\alpha_s}{\pi}\frac{1}{\epsilon}\right) \tag{8}$$

Thus after subtraction of all subdivergences, we are left with a simple overall divergence in the real part of the result, whereas its imaginary part is finite and transforms to the corrected decay rate $\Gamma(Z \to b\bar{b})$ according to eq.(2).

3 Results and Discussion

In order to present the result of our calculation let us write the partial decay rate $\Gamma(Z \to b\bar{b})$ as the sum of the Born decay rate Γ_{Born} and the correction $\Delta\Gamma$

$$\Gamma(Z \to b\bar{b}) = \Gamma_{Born} + \Delta\Gamma, \tag{9}$$

for which we obtain

$$\begin{aligned}
\Gamma_{Born} &= \frac{G_F M_Z^3}{24\sqrt{2}\pi} N_C (v_b^2 + a_b^2) \\
\Delta\Gamma &= -\frac{G_F^2 M_Z^3}{48\pi^3}\left(1 - \frac{2}{3}s_W^2\right) N_C \bar{m}_t^2(\mu^2)\left[1 - \left(\frac{\pi^2 - 11}{3} - 2\ln\frac{\mu^2}{\bar{m}_t^2}\right)\frac{\alpha_s(\mu^2)}{\pi}\right]
\end{aligned} \tag{10}$$

Here $\bar{m}_t(\mu^2)$ and $\alpha_s(\mu^2)$ are the renormalized mass and the strong coupling constant respectively at the renormalization scale μ^2. Furthermore the vector and axial couplings read $v_b = -1 + \frac{4}{3}s_W^2$, $a_b = -1$. We transform this result from the $\overline{\text{MS}}$-scheme to the on-shell scheme via

$$\bar{m}(\mu^2) = m_{OS}\left(1 - \frac{\alpha_s}{\pi}\left[\frac{4}{3} + \ln\frac{\mu^2}{m_{OS}^2}\right]\right) \tag{11}$$

Inserting into eq.(10) with $\mu^2 = M_Z^2$ gives

$$\Delta\Gamma = -\frac{G_F^2 M_Z^3}{48\pi^3}\left(1 - \frac{2}{3}s_W^2\right) N_C m_t^2 \left[1 - \frac{\pi^2 - 3}{3}\frac{\alpha_s(M_Z^2)}{\pi}\right] \tag{12}$$

As was shown in [2] one can account for electroweak corrections to the decay rate by introducing two form factors ρ, κ and replacing the Fermi constant and the weak mixing angle by corresponding effective parameters ρG_F and $\kappa \sin^2\theta_W$ in Γ_{Born}. The order $\mathcal{O}(\alpha\alpha_s m_t^2/M_W^2)$ corrections to the decay rate may be organized in a similar way. The partial Z decay rate is expressed in the form

$$\Gamma(Z \to b\bar{b}) = \frac{G_F M_Z^3}{12\sqrt{2}\pi} N_C \rho \left[1 - \frac{4}{3}\kappa s_W^2 + \frac{8}{9}\kappa^2 s_W^4\right] \tag{13}$$

with

$$
\begin{aligned}
\rho &= 1 + \Delta\rho \\
&= 1 - \frac{G_F}{2\sqrt{2}\pi^2} m_t^2 \left(1 - \frac{\pi^2 - 3}{3} \frac{\alpha_s}{\pi}\right) \\
\kappa &= 1 + \Delta\kappa \\
&= 1 - \frac{1}{2}\Delta\rho
\end{aligned}
\tag{14}
$$

This result is in agreement with that of Fleischer et al. [5].

To conclude, we have calculated the QCD corrections to the leading top mass contribution of the electroweak one loop result for the partial Z width $\Gamma(Z \to b\bar{b})$. For this purpose we have computed corresponding propagator type diagrams and extracted their imaginary part. The calulation was performed in the large mass limit. The leading m_t^2 contribution was obtained through a heavy mass expansion in the inverse top mass. This work represents an independent way to determine the $\mathcal{O}(\alpha\alpha_s m_t^2/M_W^2)$ corrections and confirms the recent results of other groups.

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