

# Proceedings of the Workshop on High precision $\alpha_s$ measurements: from LHC to FCC-ee

CERN, Geneva  
October 12th–13th, 2015

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## **Abstract**

This document collects a summary of all contributions of the workshop on “*High precision measurements of  $\alpha_s$ : from LHC to FCC-ee*” held at CERN, October 12-13, 2015 (<http://indico.cern.ch/e/alphas2015>). The workshop explored in depth the latest developments on the determination of the QCD coupling  $\alpha_s$  from the key categories where high precision measurements are (or will be) available. Those include low-energy observables: (i) lattice QCD, (ii) pion decay factor, (iii) quarkonia and (iv)  $\tau$  decays, (v) soft parton-to-hadron fragmentation functions; as well as high-energy observables: (vi) global fits of parton distribution functions, (vii) hard parton-to-hadron fragmentation functions, (viii) jets in  $e^\pm$ -p DIS and photoproduction, (ix) event shapes and (x) jet cross sections in  $e^+e^-$  collisions, (xi) W boson and (xii) Z boson decays, and (xiii) top-quark and (xiv) jet cross sections in p-p collisions. The current status of the theoretical and experimental uncertainties associated to each extraction method, the improvements expected at the end of the LHC running, and future perspectives achievable in  $e^+e^-$  collisions at the Future Circular Collider (FCC-ee) with multi-inverse-attobarn integrated luminosities yielding  $10^{12}$  Z bosons and jets, and  $10^8$  W bosons and  $\tau$  leptons, are thoroughly reviewed and discussed.

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# 1 Introduction

The strong coupling  $\alpha_s$  is one of the fundamental parameters of the Standard Model (SM), setting the scale of the strength of the strong interaction theoretically described by Quantum Chromodynamics (QCD). At the reference Z pole mass scale, its measured value amounts to  $\alpha_s(m_Z^2) = 0.1185 \pm 0.0006$ . Given its current  $\delta\alpha_s \approx 0.6\%$  uncertainty—orders of magnitude larger than that of the gravitational ( $\delta G \approx 10^{-5}$ ), Fermi ( $\delta G_F \approx 10^{-8}$ ) and QED ( $\delta\alpha \approx 10^{-10}$ ) couplings—the strong coupling is the least precisely known of all fundamental constants in nature. Improving our knowledge of  $\alpha_s$  is a prerequisite to reduce the theoretical uncertainties in the calculations of all high-precision perturbative QCD (pQCD) processes whose cross sections or decay rates depend on higher-order powers of  $\alpha_s$ , as is the case for virtually all those measured at the LHC. In the Higgs sector, in particular, the uncertainty on  $\alpha_s$  is currently the second major contributor (after the bottom mass) to the parametric uncertainties of its dominant  $H \rightarrow b\bar{b}$  partial decay. The same applies for the extraction of the charm Yukawa coupling via future  $H \rightarrow c\bar{c}$  measurements.

The workshop “*High precision measurements of  $\alpha_s$ : from LHC to FCC-ee*” was held at CERN, October 12-13, 2015 (<http://indico.cern.ch/e/alphas2015>), as part of the FCC-ee *QCD and gamma-gamma physics* working group activities in the context of the preparation of the FCC-ee Yellow Report in 2016. The meeting brought together experts from several different fields to explore in depth the latest developments on the determination of the QCD coupling  $\alpha_s$  from the key categories where high precision measurements are (or will be) available, and put its emphasis on the following issues:

- What is the current state-of-the-art of each one of the  $\alpha_s$  determination methods, from the theoretical and experimental points of view?
- What is the current size of the theoretical (missing higher orders, electroweak corrections, power corrections, hadronization corrections,...) and experimental uncertainties associated to each measurement?
- What is the expected  $\alpha_s$  uncertainty in  $\sim 10$  years from now thanks to the ongoing (or expected) theoretical developments, plus  $\mathcal{O}(1 \text{ ab}^{-1})$  collected p-p data at 14 TeV at the LHC ?
- What are the improvements expected to be brought about by  $e^+e^-$  collisions at the FCC-ee ( $\sqrt{s} = 91, 160, 240$  and 350 GeV) with  $10^{12}$  Z bosons and jets, and  $10^8$  W bosons and  $\tau$  leptons collected ?
- What are the systematic errors that the FCC-ee detectors should target in order to match the expected statistical precision, or where that is not possible, what are the important theoretical targets that should be met or exceeded ?

With those goals in mind, the workshop was organized along four broad sessions:

1. An introductory session, presenting the motivations of the workshop, the current status of the world average of the strong coupling, the impact of  $\alpha_s$  on Higgs cross sections and branching ratios, and on new physics constraints.

2. A session dedicated to  $\alpha_s$  determination at low energy including results from: lattice QCD, pion decay factor,  $\tau$  decay,  $Q\bar{Q}$  decays, and soft parton-to-hadron FFs.
3. A session dedicated to  $\alpha_s$  determination at higher energy scales including: global fits of parton distribution functions, hard parton-to-hadron fragmentation functions, jets in deep-inelastic scattering and photoproduction in  $e^\pm$ -p collisions,  $e^+e^-$  event shapes,  $e^+e^-$  jets, hadronic Z and W decays,  $\sigma(e^+e^- \rightarrow \text{hadrons})$ , electroweak fit,...
4. Recent experimental and theoretical results and plans for  $\alpha_s$  measurements at the LHC via top-quark pair and jets cross sections.

One important goal of the workshop was to facilitate discussion between the different groups, and in particular to give speakers the opportunity to explain details that one would normally not be able to present at a conference, but which have an important impact on the analyses. There were 40 physicists who took part in the workshop, and 24 talks were presented. Slides as well as background reference materials are available on the conference website

<http://indico.cern.ch/e/alphas2015>

The sessions and talks in the workshop program were:

- Introduction
  - “Introduction and goals of the workshop”, D. d’Enterria and P.Z. Skands
  - “World Summary of  $\alpha_s$  (2015)”, S. Bethke
  - “ $\alpha_s$  and physics beyond the Standard Model”, F. Sannino
  - “Impact of  $\alpha_s$  on Higgs production and decay uncertainties”, L. Mihaila
- Measurements of  $\alpha_s$  at low energy scales:
  - “ $\alpha_s$  from lattice QCD”, P. Mackenzie
  - “ $\alpha_s$  from the QCD static energy”, X. Garcia i Tormo
  - “ $\alpha_s$  from pion decay factor”, J.L. Kneur
  - “ $\alpha_s$  from hadronic tau decays”, A. Pich
  - “ $\alpha_s$  from hadronic quarkonia decays”, J. Soto i Riera
  - “ $\alpha_s$  from soft parton-to-hadron FFs”, R. Perez-Ramos and D. d’Enterria
- Measurements of  $\alpha_s$  at high energy scales:
  - “ $\alpha_s$  from global fits of parton distribution functions”, J. Blümlein
  - “ $\alpha_s$  from jets in DIS and photoproduction”, M. Klasen
  - “ $\alpha_s$  from scaling violations of hard parton-to-hadron FFs”, B. Kniehl
  - “ $\alpha_s$  from  $e^+e^-$  event shapes”, S. Kluth
  - “ $\alpha_s$  from  $e^+e^-$  C-parameter event shape”, A. Hoang
  - “ $\alpha_s$  from  $e^+e^-$  jet cross sections”, A. Banfi
  - “ $\alpha_s$  from hadronic Z decays and from the full electroweak fit”, K. Mönig
  - “ $\alpha_s$  from hadronic W decays”, M. Srebre
  - “ $\alpha_s$  from  $\sigma(e^+e^- \rightarrow \text{hadrons})$ ”, J. Kühn
- Measurements of  $\alpha_s$  at the LHC and conclusions:
  - “ $\alpha_s$  from top-pair cross sections at the LHC and beyond”, A. Mitov
  - “ $\alpha_s$  from top-pair cross sections”, G. Salam

- “Future prospects of  $\alpha_s$  from NNLO jets at the LHC and beyond”, J. Pires
- “ $\alpha_s$  determinations from ATLAS (status and plans)”, B. Malaescu
- “ $\alpha_s$  determinations from CMS (status and plans)”, K. Rabbertz
- Final discussion
- Conclusions (FCC-ee Yellow Report preparation)

This proceedings represent a collection of extended abstracts and references for the presentations, summarizing the most important results and issues. These written contributions will be incorporated into the FCC-ee Yellow Report under preparation.

CERN, October 2015

David d’Enterria  
Peter Skands

## 2 Proceedings Contributions

# $\alpha_s$ from $\sigma(e^+e^- \rightarrow \text{hadrons})$

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## 3 Introduction

The cross section for electron-positron annihilation into hadrons is one of the cleanest ways for measuring the strong coupling constant. In principle a wide range of energies exists which can be exploited for this purpose, ranging from the region below the charm threshold, say around 3 GeV, up to the highest energies available at electron-positron colliders.

Apart from the regions very close to the charm-, bottom- and top-threshold the cross section can be evaluated by taking the limit of massless quarks. The leading term for QCD corrections is thus given by the extremely simple form  $(1 + a_s)$  with  $a_s \equiv \alpha_s/\pi$ . Higher order terms, however, exhibit a non-trivial dependence on the number of active flavours. Also the difference between vector- and axial vector current induced reactions starts (in the massless limit) in order  $\alpha_s^2$ . Closely related to this quantity are QCD-corrections to  $\tau$ -lepton decays. Although in principle extremely clean and simple, the predictions are, in this case, at the limit of applicability of perturbative QCD. This fact is well visible from the difference between the predictions based on “Fixed Order” and “Contour Improved” (see [1] and references therein) and will be discussed in more detail in [2].

The following discussion will, therefore, be limited to decays of the virtual photon into hadrons, i.e. the famous  $R$ -ratio, or the corresponding object at higher energies, in particular the decay rate of the  $Z$ -boson into hadrons,  $\Gamma(Z \rightarrow \text{hadrons})$ , and the closely related quantities for the  $W$  or the Higgs-boson decay.

## 4 The $R$ -ratio below the bottom threshold

The  $R$ -ratio, given by  $\sigma(e^+e^- \rightarrow \text{hadrons})/\sigma_{\text{pt}}$ , can be measured at vastly different energies. In the low energy region the annihilation proceeds through the virtual photon only. A combined fit [3] to CLEO results [4], based on four-loop accuracy, gives

$$\alpha_s^{(4)}(9^2 \text{ GeV}^2) = 0.160 \pm 0.024 \pm 0.024 \quad (1)$$

which, at the scale of  $M_Z$ , corresponds to

$$\alpha_s^{(5)}(M_Z^2) = 0.110 \begin{pmatrix} +0.014 \\ -0.017 \end{pmatrix}. \quad (2)$$

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\*Presented by Johann H. Kühn

This CLEO result, in turn, can be combined with earlier measurements by BESS [5], MD-1 [6] and CLEO [7], and leads to [3]

$$\alpha_s^{(4)}(9^2 \text{ GeV}^2) = 0.182 \begin{pmatrix} +0.022 \\ -0.025 \end{pmatrix} \quad (3)$$

corresponding to

$$\alpha_s^{(5)}(M_Z^2) = 0.119 \begin{pmatrix} +0.009 \\ -0.011 \end{pmatrix}. \quad (4)$$

This result is in nice agreement with a more recent measurement of the BESS collaboration [8]

$$R_{uds} = 2.224 \pm 0.019 \pm 0.089 \quad (5)$$

taken at energy values below the charm threshold.

Let us emphasize again that more precise measurements in this region, based on the enormous statistic of the BESS and BELLE experiment, would be highly welcome.

## 5 Status and perspectives for $e^+e^- \rightarrow \text{hadrons}$ at the $Z$ resonance

### a.) QCD corrections

At present the most precise determination of  $R$  is based on the measurement of  $Z$  decays, more precisely on the measurement of  $\Gamma_{had}$  and  $\Gamma_{had}/\Gamma_{lept}$ , where corrections have been calculated to  $\mathcal{O}(\alpha_s^4)$  [??]. Effects from non-vanishing charm and bottom quark masses will be mentioned below.

Even in the massless limit there are three different types of contributions

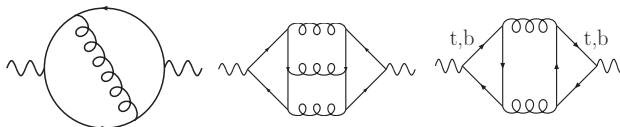


Figure 1: Non-singlet & singlet, vector & axial correlators.

- i.) The non-singlet vector and axial vector correlators  $r_{NS}$  [??].
- ii.) The singlet correlator  $r_S^V$ , which starts in  $\mathcal{O}(\alpha_s^3)$  and is present for the vector current only [??].
- iii.) A contribution to the axial correlator,  $r_S^A$ , resulting from the top-bottom doublet with  $4m_t^2 \gg s \gg 4m_b^2$ , which starts in order  $\alpha_s^2$  [??].

The corresponding predictions are available to order  $\alpha_s^4$ . For an extremely conservative error estimate based on the variation of the scale parameter  $\mu$  between  $M_Z/3$  and  $3M_Z$  this corresponds to the variation

$$\delta\Gamma_{NS} = 101 \text{ KeV}, \quad \delta\Gamma_S^V = 2.7 \text{ KeV}, \quad \delta\Gamma_S^A = 42 \text{ KeV} \quad (6)$$

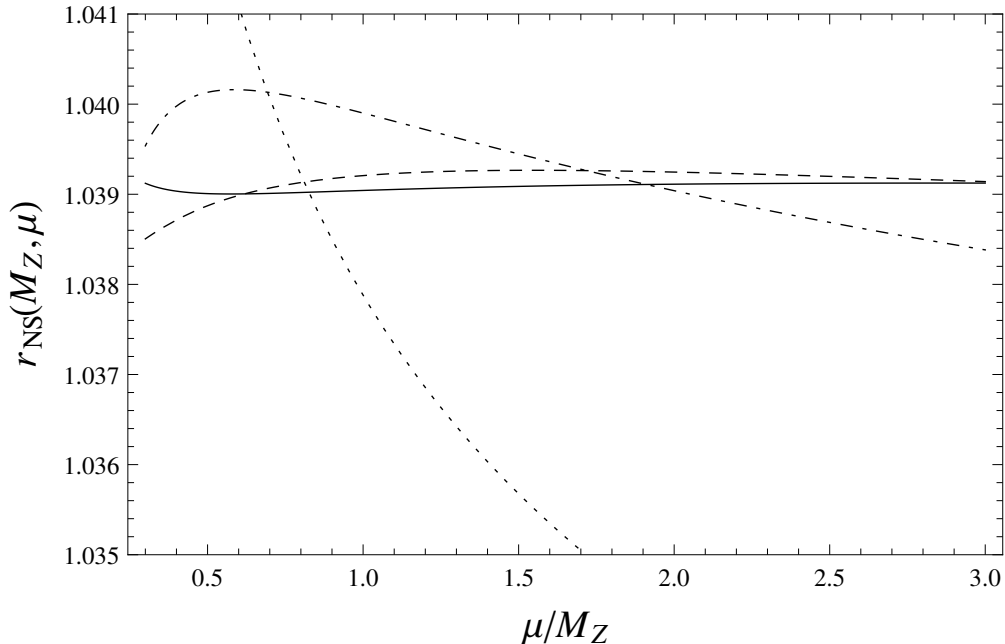


Figure 2: Predictions for  $r_{NS}$  in increasing orders of  $\alpha_s$  [10].

and, correspondingly, to a variation of the strong coupling around  $3 \cdot 10^{-4}$ . This theory error is completely negligible compared to the present experimental uncertainty around 0.026. In fact, it is even comparable to the error of  $\delta\Gamma_{had} \approx 100$  KeV expected for an analysis at the FCC-ee. Various additional small corrections must be applied, which, however, have been calculated some time ago: quark mass corrections up to order  $m_b^2\alpha_s^4$  and  $m_b^4\alpha_s^3$  [??] and mixed QCD and electroweak corrections that will be discussed in the following.

### b.) Mixed electroweak and QCD corrections

For light quarks terms of order  $\alpha\alpha_s$  have been evaluated some time ago [16]. The difference between the correct two-loop terms and the product of the corresponding two one-loop terms

$$\Delta\Gamma \equiv \Gamma(\text{two loop (EW} \star \text{QCD)}) - \Gamma_{\text{Born}}\delta_{\text{EW}}^{\text{NLO}}\delta_{\text{QCD}}^{\text{NLO}} = -0.59(3) \quad (7)$$

is again of high importance, if a precision of  $\delta\Gamma \approx 0.1$  MeV should be reached at some point in the future. The next, presently unknown three-loop term is smaller by another factor of order  $\alpha_s/\pi$ , times an unknown coefficient which should not exceed 5 in order to keep this term under control.

Similar comments apply to  $\Gamma(Z \rightarrow b\bar{b})$ . The present result,  $\Gamma(Z \rightarrow b\bar{b})/\Gamma_Z \equiv R_b = 0.1512 \pm 0.0005$ , obtained at LEP corresponds to an error of about 1.3 MeV. Pushing the relative error  $\delta\Gamma(Z \rightarrow b\bar{b})/\Gamma_Z$  down to  $2 \cdot 10^{-5}$ , would imply a precision of 0.05 MeV for the partial width  $\Gamma(Z \rightarrow b\bar{b})$ . This should be compared to the present result of order  $\alpha\alpha_s$ , which is given by

$$\Gamma(Z \rightarrow b\bar{b}) - \Gamma(Z \rightarrow d\bar{d}) = (-5.69 - 0.79)_{\mathcal{O}(\alpha)} + (+0.50 + 0.06)_{\mathcal{O}(\alpha\alpha_s)} \text{ MeV} \quad (8)$$

separated into  $m_t^2$  enhanced terms [17] and the rest [18].

The size of these terms has even motivated the evaluation of the  $m_t^2$ -enhanced corrections [19] of order  $G_F m_t^2 \alpha_s^2$  with the result  $\delta\Gamma_b \approx 0.1$  MeV for the non-singlet contributions, a result



drastically below the current precision and comparable in size to the planned precision of the FCC-ee.

Let us mention that many corrections seem to be significantly smaller if expressed in terms of the  $\overline{\text{MS}}$ -mass [??]

$$\bar{m}_t(\bar{m}_t) = m_{pole}(1 - 1.33 a_s - 6.46 a_s^2 - 60.27 a_s^3 - 704.28 a_s^4). \quad (9)$$

Note that the  $\overline{\text{MS}}$ -mass may well be directly accessible at an  $e^+e^-$  collider with high precision, e.g. through a measurement of the potential subtracted mass with a precision of 20 – 30 MeV.

## 6 $M_W$ from $G_F$ , $M_Z$ , $\alpha$ and the rest

The precision of the present  $M_W$ -measurement amounts to 23 MeV. For the FCC-ee option a precision of 0.5 to 1 MeV is foreseen. In the Standard Model  $M_W$  can be considered a derived quantity that can be calculated from  $G_F$ ,  $M_Z$ ,  $M_t$ ,  $\Delta\alpha$  and  $m_H$ . Let us, for the moment, focus on the top mass dependence, which is obtained from the following chain of equations

$$M_W^2 = f(G_F, M_Z, m_t, \Delta\alpha, \dots) \quad (10)$$

$$= \frac{M_Z^2}{2(1 - \delta\rho)} \left( 1 + \sqrt{1 - \frac{4\pi\alpha(1 - \delta\rho)}{\sqrt{2}G_F M_Z^2} \left( \frac{1}{1 - \Delta\alpha} + \dots \right)} \right), \quad (11)$$

$$\delta M_W \approx M_W \frac{1}{2} \frac{\cos^2 \theta_w}{\cos^2 \theta_w - \sin^2 \theta_w} \delta\rho \approx 5.7 \times 10^4 \delta\rho [MeV], \quad (12)$$

$$\delta\rho_t = 3X_t \left( 1 - 2.8599 \left( \frac{\alpha_s}{\pi} \right) - 14.594 \left( \frac{\alpha_s}{\pi} \right)^2 - 93.1 \left( \frac{\alpha_s}{\pi} \right)^3 \right). \quad (13)$$

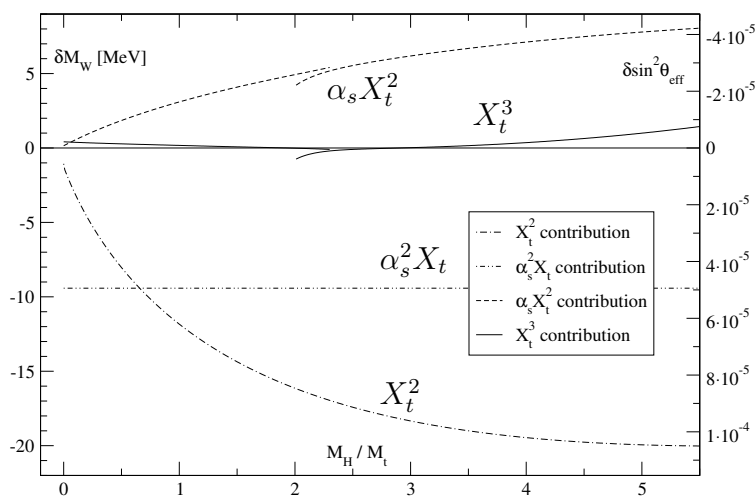


Fig. 3: Shifts in  $M_W$  and in  $\sin^2 \Theta_W$  from various corrections to  $\Delta\rho$  [23].

In this case the three-loop term amounts to 9.5 MeV, the four-loop term to 2.1 MeV [??]. In three loop approximation also purely weak ( $\sim X_t^3$ ) and mixed (QCD\*electroweak)

of order  $\alpha_s X_t^2$  three-loop terms are available, corresponding to  $\delta M_W = 0.2$  MeV and 2.5 MeV respectively. These are shown in Fig. 3 together with the terms of order  $\alpha_s X_t^2$  and  $X_t^2$ .

Let us emphasize that there are still various uncalculated terms of order 0.1 MeV – 0.5 MeV, for example the four-loop term of order  $\alpha_s^2 X_t^2$  or higher order terms relating pole and  $\overline{\text{MS}}$ -mass.

## 7 Perspectives for $e^+e^- \rightarrow$ hadrons above the $Z$ -resonance

No detailed analysis of the cross section exists in the moment for the energy region above the  $Z$ -resonance. The situation is significantly more complicated than the one around the  $Z$ -boson or at low energies:

As a consequence of Furry’s theorem the interference between Born and one-loop correction vanishes, as long as only vector currents are involved. This is no longer true for a mixture of vector and axial currents, such that the interference (relative to the Born term) is proportional to order  $\alpha$ . In combination with the large radiative tail this enhances the radiative corrections drastically and no prediction for the  $R$ -ratio at a level required for a precise determination of the  $Zf\bar{f}$  coupling is available at present.

## 8 Perspectives for $e^+e^- \rightarrow Z + H(\rightarrow$ hadrons)

### a.) $H \rightarrow b\bar{b}$

Up to this point only QCD corrections for vector and axial currents have been considered, where the first terms are equal for both cases and given by  $(1 + \alpha_s/\pi)$ . In contrast, for the case of Higgs boson decay into a quark-antiquark pair, the leading QCD correction is of the form  $(1 + 17/3 \cdot \alpha_s/\pi)$ . The full correction for the decay rate into  $b$  quarks is given by

$$\Gamma(H \rightarrow b\bar{b}) = \frac{G_F M_H}{4\sqrt{2}\pi} m_b^2(M_H) R^S(s = M_H^2, \mu^2 = M_H^2) \quad (14)$$

$$R^S(M_H) = 1 + 1 + 5.667 a_s + 29.147 a_s^2 + 41.758 a_s^3 - 825.7 a_s^4 = 1.2298 \quad (15)$$

for  $\alpha_s(M_H) = 0.108$ , corresponding to  $\alpha_s(M_Z) = 0.118$ .

The theory uncertainty of the correction factor has been reduced from 5 (four loop) to 1.5 (five loop) permille, where a variation of  $\mu$  between  $M_H/3$  and  $3M_H$  has been assumed and  $\alpha_s(M_H)$  has been fixed to 0.108.

The parametric uncertainties can be traced to the value of  $\alpha_s$ , of  $m_b$  at low energies, say 10 GeV, and to the running of  $m_b$  from 10 GeV to  $M_H$ . For the values of  $\alpha_s$  and  $m_b$  we adopt  $\alpha_s(M_Z) = 0.1189 \pm 0.002$  and  $m_b(10 \text{ GeV}) = 3610 - \frac{\alpha_s - 0.1189}{0.002}$ . Running  $m_b$  to the scale of  $M_H$  one obtains  $m_b(M_H) = 2759 \pm 8 |_{m_b} \pm 27 |_{\alpha_s}$  MeV.

While the quark mass anomalous dimension  $\gamma_4$  has been obtained in five loop approximation already some time ago [26] the corresponding result for the QCD beta-function is still missing at present. This corresponds to an uncertainty

$$\frac{\delta m_b^2(M_H)}{m_b^2(M_H)} = -1.4 \times 10^{-4} \left( \frac{\beta_4}{\beta_0} = 0 \right) \quad | \quad -4.3 \times 10^{-4} \left( \frac{\beta_4}{\beta_0} = 100 \right) \quad | \quad -7.3 \times 10^{-4} \left( \frac{\beta_4}{\beta_0} = 200 \right) \quad (16)$$

which should be compared to  $\delta\Gamma(H \rightarrow b\bar{b})/\Gamma(H \rightarrow b\bar{b}) = 2.0 \times 10^{-4}$  expected for the FCC-ee. Assuming an improvement of our knowledge of the strong coupling by a factor 10, corresponding to  $\delta\alpha_s = 2 \times 10^{-4}$  and for  $m_b$  by a factor 4 ( $\delta m_b(M_H)/m(M_H) = 10^{-3}$ ) one finds

$$\frac{\delta\Gamma_{H \rightarrow b\bar{b}}}{\Gamma_{H \rightarrow b\bar{b}}} = \pm 2 \times 10^{-3}|_{m_b} \pm 1.3 \times 10^{-3}|_{\alpha_s, \text{running}} \pm 1 \times 10^{-3}|_{\text{theory}}. \quad (17)$$

### b.) $H \rightarrow gg$

Another hadronic decay mode of the Higgs boson with a large branching ratio is the one into two gluons, which starts in order  $\alpha_s^2$  and is presently known [27] to order  $\alpha_s^5$ :

$$\begin{aligned} \Gamma(H \rightarrow gg) &= K \cdot \Gamma_{\text{Born}}(H \rightarrow gg) & (18) \\ K &= 1 + 17.9167 a'_s + (156.81 - 5.7083 \ln \frac{M_t^2}{M_H^2})(a'_s)^2 \\ &+ (467.68 - 122.44 \ln \frac{M_t^2}{M_H^2} + 10.94 \ln^2 \frac{M_t^2}{M_H^2})(a'_s)^3. \\ K &= 1 + 17.9167 a'_s + 152.5(a'_s)^2 + 381.5(a'_s)^3 \\ &= 1 + 0.65038 + 0.20095 + 0.01825. \end{aligned}$$

where  $M_t = 175 \text{ GeV}$ ,  $m_H = 120 \text{ GeV}$  and  $a'_s = \alpha_s^{(5)}(M_H)/\pi$  had been assumed. Note that an experimental precision for  $\sigma(HZ) \times Br(H \rightarrow gg)$  of 1.4% has been claimed in [28]. Mass corrections to the Higgs-boson decay rate result can be found in [29].

## 9 Summary and Conclusions

Improved measurements of the total cross section for electron-positron annihilation just below the charm and the bottom threshold would lead to model independent tests of the running of  $\alpha_s$  over a wide energy range. At high energies the precise control of QCD corrections is crucial for the detailed and precise comparison of theory predictions with experimental results at a future circular or linear electron-positron collider.

A significant improvement in the determination of the  $Z$  decay rate would immediately lead to a correspondingly improved value of the strong coupling constant. In combination with an improved determination of the fine structure constant at the scale of the  $Z$ -boson this would lead to a significantly improved prediction for  $M_W$  at the level of 1 MeV, corresponding to the anticipated precision of a future measurement.

A collider with an energy around 250 GeV would be ideally suited for the measurement of  $e^+e^- \rightarrow ZH$ , leading to the precise determination of the branching ratios of the Higgs boson into its various decay channels. Examples are presented which involve corrections up to order  $\alpha_s^4$  for the decay into  $b\bar{b}$  and order  $\alpha_s^5$  for decays into  $gg$ . These demonstrate that predictions with a precision around one percent are well within reach for the dominant decay modes.

## References

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