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$\Gamma(H \to b\bar{b})$ to order $\alpha\alpha_s$

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Abstract

We compute the decay rate of the Standard Model Higgs boson to bottom quarks to order $\alpha\alpha_s$. We apply the optical theorem and calculate the imaginary part of three-loop corrections to the Higgs boson propagator using asymptotic expansions in appropriately chosen mass ratios. The corrections of order $\alpha\alpha_s$ are of the same order of magnitude as the $\mathcal{O}(\alpha_s^3)$ QCD corrections but have the opposite sign.

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1 Introduction

After the discovery of a Higgs boson in run I of the CERN Large Hadron Collider it is one of the main tasks of run II to determine the properties of the new particle. Among them is the coupling to other particles. This is predominantly done by determining Higgs production cross sections and decay branching ratios, i.e. the ratio of the partial decay width of the Higgs boson to the considered particles normalized to the total decay rate. The latter is dominated by the partial decay rate to bottom quark, $\Gamma(H \to b\bar{b})$, which hence influences all branching ratios. Thus, $\Gamma(H \to b\bar{b})$ should be available as precisely as possible.

QCD corrections are known up to order α_s^4 (see, e.g., Refs. [1–7]) and first results of order α_s^5 induced by virtual top quarks have been obtained in Ref. [8]. Good convergence of the perturbative series is observed leading to a 0.1% contribution of the α_s^4 corrections to $\Gamma(H \to b\bar{b})$. As far as electroweak corrections are concerned only one-loop corrections are available which have been computed beginning of the nineties [9, 10]. At two- and three-loop order only the leading M_t^2 corrections are available [11–14]. In this work we compute QCD corrections to the full $\mathcal{O}(\alpha)$ result and thus obtain all contributions of order $\alpha\alpha_s$ to the partial decay rate of a Standard Model Higgs boson into bottom quarks. Analog corrections to the decay rates of the Z and W bosons have been computed in Refs. [15–17] and [18], respectively.

We parametrize the corrections to the decay rate as follows

$$\Gamma(H \to b\bar{b}) \;=\; \Gamma^{(0)} \left(1 + \Delta^{(\alpha_s)} + \Delta^{(\alpha)} + \Delta^{(\alpha\alpha_s)} + \dots\right) , \tag{1}$$

where the ellipses stand for higher order corrections in α and α_s. It is convenient to split the electroweak corrections into a weak and a QED contributions which to our order are separately finite and gauge invariant:

$$\begin{aligned} \Delta^{(\alpha)} &= \Delta^{(\mathrm{QED})} + \Delta^{(\mathrm{weak})} , \\ \Delta^{(\alpha\alpha_s)} &= \Delta^{(\mathrm{QED},\alpha_s)} + \Delta^{(\mathrm{weak},\alpha_s)} . \end{aligned} \tag{2}$$

The aim of this paper is the computation of the mixed corrections $\Delta^{(\alpha\alpha_s)}$. In Eq. (1) $\Gamma^{(0)}$ denotes the Born decay rate which is given by

$$\Gamma^{(0)} \;=\; \frac{N_c \alpha m_b^2 M_H}{8 s_W^2 M_W^2} \beta_0^3 , \tag{3}$$

where $N_c = 3$ is the number of colours and s_W is the sine of the weak mixing angle. $\beta_0 = \sqrt{1 - 4m_b^2/M_H^2}$ is the velocity of the produced bottom quarks which from now on we approximate to $\beta_0 = 1$. As an alternative to Eq. (3) one can replace the fine structure constant by Fermi's constant via

$$\frac{G_F}{\sqrt{2}} \;=\; \frac{\pi\alpha}{2 s_W^2 M_W^2} \frac{1}{1 - \Delta r} , \tag{4}$$

where the finite quantity Δr parametrizes the radiative corrections to the muon decay beyond QED corrections within the effective four-fermion theory [19]. This leads to

$$\Gamma^{(0)} = \frac{N_c G_F m_b^2 M_H}{4\sqrt{2}\pi}\,. \tag{5}$$

For later reference we provide the Born decay rate including higher order terms in $\epsilon = (4-d)/2$ which are useful in the renormalization procedure. For $d \neq 4$ both (3) and (5) have to be multiplied by the factor[1]

$$f(\epsilon) = \left(\frac{\mu^2}{M_H^2}\right)^{\epsilon}\left[1+\epsilon+\epsilon^2\left(4-\frac{\pi^2}{4}\right)+\mathcal{O}(\epsilon^3)\right]\,. \tag{6}$$

For later convenience we also list the one-loop QCD corrections including terms of order ϵ

$$\begin{aligned}\Delta^{(\alpha_s)} &= \frac{\alpha_s}{\pi}C_F\left\{\frac{17}{4}+\frac{3}{2}\ln\left(\frac{\mu^2}{M_H^2}\right)+\epsilon\left[\frac{179}{8}-\frac{7\pi^2}{8}-6\zeta(3)+\frac{23}{2}\ln\left(\frac{\mu^2}{M_H^2}\right)\right.\right.\\ &\quad\left.\left.+\frac{9}{4}\ln^2\left(\frac{\mu^2}{M_H^2}\right)\right]\right\}\,,\end{aligned} \tag{7}$$

where $C_F = 4/3$. To obtain this result the bottom quark mass has been renormalized in the $\overline{\rm MS}$ scheme. The one-loop QED corrections are obtained from $\Delta^{(\alpha_s)}$ with the help of

$$\Delta^{(\rm QED)} = \frac{\alpha Q_b^2}{C_F\alpha_s}\Delta^{(\alpha_s)}\,, \tag{8}$$

where $C_F = 4/3$ and $Q_b = -1/3$ is the charge of the bottom quark.

The remainder of the paper is organized as follows: In the next Section we discuss the method we want to use for the three-loop diagrams of order $\alpha\alpha_s$ and apply it to the one-loop electroweak corrections. The $\mathcal{O}(\alpha\alpha_s)$ corrections are presented afterwards in Section 3. In Section 4 we discuss the numerical effect, compare with the known QCD corrections and conclude.

2 Corrections of order α

Before discussing the computation of the genuine diagrams of order α we briefly elaborate on the counterterm contribution. We follow Ref. [20] and introduce one-loop counterterms for the Higgs boson wave function (δZ_H), the vacuum expectation value (δv) and the bottom quark mass (δ_{m_b}). This leads to the following counterterm contribution of $\Delta^{(\rm weak)}$

$$\Delta_{\rm CT}^{(\rm weak)} = \Gamma^{(0)}\left(1-2\frac{\delta v}{v}+\delta Z_H-\Delta r+2\delta_{m_b}\right)\,, \tag{9}$$

[1]Throughout this paper we adopt a $\overline{\rm MS}$-like convention and set γ_E and $\log(4\pi)$ to zero.

where $\Gamma^{(0)}$ is given in Eq. (5). We do not include the on-shell wave function renormalization of the quarks in $\Delta_{\rm CT}^{\rm (weak)}$ since it is automatically taken into account when using the optical theorem (see below). In the on-shell scheme the mass counterterm is defined through

$$m_b^0 \quad = \quad M_b \left(1 + \delta_{m_b}^{\rm OS}\right) \,, \tag{10}$$

with M_b being the on-shell mass. We take $\delta_{m_b}^{\rm OS}$ from Ref. [21, 22] dropping all tadpole contributions. The divergent part of δ_{m_b} determines the $\overline{\rm MS}$ counterterm $\delta_{m_b}^{\overline{\rm MS}}$ and is in our approximation (i.e. at most m_b^2 terms in the decay rate) given by

$$\delta_{m_b}^{({\rm weak}),\overline{\rm MS}} \quad = \quad \delta_b^{\rm OS}\Big|_{1/\epsilon \text{ pole}} \quad = \quad \frac{\alpha}{\pi}\left(-\frac{3m_t^2}{32 s_W^2 M_W^2} + \frac{9 + 6s_W^2 - 8s_W^4}{96 c_W^2 s_W^2}\right)\frac{1}{\epsilon}\,. \tag{11}$$

The remaining terms on the right-hand side of Eq. (9) can be written in the form [20]

$$\begin{aligned}
v_r \quad &\equiv \quad -2\frac{\delta v}{v} + \delta Z_H - \Delta r \\
&= \quad -\frac{\Sigma^W(0)}{M_W^2} - \Sigma'^H(M_H^2) - \frac{2}{s_W c_W}\frac{\Sigma^{\gamma Z}(0)}{M_Z^2} - \frac{\alpha}{4\pi s_W^2}\left(6 + \frac{7 - 4s_W^2}{2s_W^2}\ln(c_W^2)\right) \\
&= \quad \frac{\alpha}{\pi s_W^2}\left\{ -\frac{1 + 2c_W^2}{8c_W^2}\frac{1}{\epsilon} - \frac{3(1+2c_W^2)}{32c_W^2} + \frac{3(1+2c_W^4)M_Z^2}{8c_W^2 M_H^2} + \frac{(13 - 2\sqrt{3}\pi)M_H^2}{32M_W^2} \right. \\
&\quad + \frac{(M_H^2 - 6M_W^2)(M_H^2 + 2M_W^2)}{4M_H^3\sqrt{-M_H^2 + 4M_W^2}}\arctan\frac{M_H}{\sqrt{-M_H^2 + 4M_W^2}} \\
&\quad + \frac{(M_H^2 - 6M_Z^2)(M_H^2 + 2M_Z^2)}{8c_W^2 M_H^3\sqrt{-M_H^2 + 4M_Z^2}}\arctan\frac{M_H}{\sqrt{-M_H^2 + 4M_Z^2}} \\
&\quad + \frac{(M_H^2 + 5M_W^2)}{48M_W^2}\ln\left(\frac{\mu^2}{M_H^2}\right) + \frac{(-1 + 11c_W^2 + 8c_W^4)}{48c_W^2}\ln\left(\frac{\mu^2}{M_Z^2}\right) \\
&\quad + \frac{M_H^2 c_W^2 - M_W^2(5 + 18c_W^2 + 8c_W^4)}{48M_W^2 c_W^2}\ln\left(\frac{\mu^2}{M_W^2}\right) \\
&\quad + \frac{(M_H^4 - 5M_H^2 M_W^2 - 5M_W^4)}{48M_W^2(M_H^2 - M_W^2)}\ln\left(\frac{M_W^2}{M_H^2}\right) + \frac{(-5 + 8c_W^2)(1 + c_W^2 + c_W^4)}{48c_W^2 s_W^2}\ln\left(\frac{M_W^2}{M_Z^2}\right) \\
&\quad \left. + N_c\left[\frac{m_t^2(8m_t^2 + M_H^2)}{16M_H^2 M_W^2} + \frac{m_t^2(2m_t^2 + M_H^2)\sqrt{4m_t^2 - M_H^2}}{4M_H^3 M_W^2}\arctan\frac{M_H}{\sqrt{4m_t^2 - M_H^2}}\right]\right\}\,,
\end{aligned} \tag{12}$$

where v is the vacuum expectation value and Σ^W, $\Sigma^{\gamma Z}$ and Σ^H denote the two-point functions of the corresponding bosons in the notation given in Ref. [20]. The prime in the case of the Higgs boson two-point function denotes the derivative w.r.t. the external momentum squared, q^2. Afterwards $q^2 = M_H^2$ is chosen.

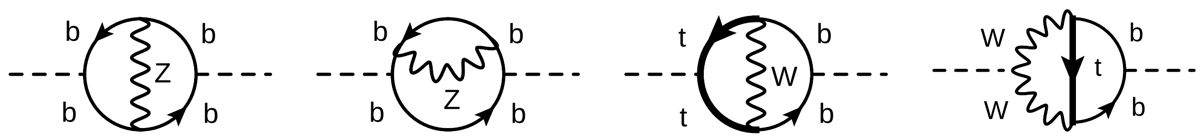

Figure 1: Sample Feynman diagram contributing to the $\mathcal{O}(\alpha)$ corrections of $\Gamma(H \to b\bar{b})$. External dashed lines denote the Higgs boson.

For the evaluation of the decay rate $\Gamma(H \to b\bar{b})$ we use the optical theorem which for our application has the form

$$\Gamma(H \to b\bar{b}) \quad = \quad \frac{1}{M_H}\mathrm{Im}\left[\Sigma_H(q^2 = M_H^2 + i\epsilon)\right] \,, \tag{13}$$

where $\Sigma_H(q^2)$ is the Higgs boson two-point function which is evaluated on the Higgs boson mass shell. As a consequence we have to consider Feynman diagrams as shown in Fig. 1 to evaluate the $\mathcal{O}(\alpha)$ corrections. In this approach we automatically take into account the on-shell wave function renormalization which is the reason why we have not considered it in Eq. (9). Note that we neglect m_b corrections except the leading m_b^2 factor and thus the contributing diagrams either contain Z bosons (possibly together with neutral Goldstone bosons) or W and/or charged Goldestone bosons and top quarks.

We express our final result in terms of the $\overline{\mathrm{MS}}$ bottom quark mass which, as is well known, leads to a better perturbative behaviour of the decay rate. In this context we briefly want to discuss the tadpole contributions to the bottom quark propagator (see also discussions in Refs. [21, 22]). In fact, besides the diagrams in Fig. 1 there are also contributions where a closed loop is connected via a Z or Higgs boson to the bottom quark line, so-called tadpoles. These contributions are exactly canceled by the on-shell counterterm contributions to the bottom quark mass. For this reason we drop the tadpoles in both parts from the very beginning. Note, however, that after dropping the tadpole contribution in the counterterm δ_{m_b}, it becomes dependent on the electroweak gauge parameters $\xi_{W/Z}$. The same is true for the contribution from the diagrams in Fig. 1. In the sum $\xi_{W/Z}$ drops out. In case the bottom quark mass is renormalized in the $\overline{\mathrm{MS}}$ scheme there is no cancellation and the final expression for $\Gamma(H \to b\bar{b})$ remains $\xi_{W/Z}$-dependent. Note, however, that also the numerical value of m_b in the $\overline{\mathrm{MS}}$ scheme (formally) depends on $\xi_{W/Z}$ since in the extraction of m_b from the comparison of theoretical calculations and experimental data (see, e.g., Ref. [23]) no electroweak tadpoles are included. The $\xi_{W/Z}$-dependence in $\Gamma(H \to b\bar{b})$ and m_b cancels.

In our calculation we adopt Feynman gauge in the electroweak sector but allow for general gauge parameter ξ_S in gluon propagator. In our final result ξ_S drops out which is a welcome check. Our Feynman integrals involve the mass scales M_H, M_t, M_W and M_Z.

Before presenting numerical results let us fix our input parameters which are given by [23–

25]

$$
\begin{aligned}
M_t &= 173.34 \text{ GeV}\,, \\
M_H &= 125.09 \text{ GeV}\,, \\
M_W &= 80.385 \text{ GeV}\,, \\
M_Z &= 91.1876 \text{ GeV}\,, \\
m_b(m_b) &= 4.163 \text{ GeV}\,, \\
G_F &= 1.1663788(6) \times 10^{-5} \text{ GeV}^{-2}\,, \\
\alpha_s(M_Z) &= 0.1185
\end{aligned}
\tag{14}
$$

where the four-loop QCD conversion [26] of the on-shell to the $\overline{\text{MS}}$ top quark mass leads to $m_t(m_t) = 163.47$ GeV and $m_t(M_H) = 166.97$ GeV.

Let us in a first step discuss the contribution from the Feynman diagrams which do not involve top quarks. As massive particles they only contain Z bosons or neutral Goldstone bosons and thus they depend on q^2/M_Z^2 where $q^2 = M_H^2$ is the square of the external momentum. At $\mathcal{O}(\alpha)$ an exact calculation is possible, however, at $\mathcal{O}(\alpha\alpha_s)$ the occurring integrals become complicated. Thus we evaluate this class of Feynman diagrams in the limit $q^2 \ll M_Z^2$ and apply a Padé approximation to construct an approximation for the physical limit $q^2 = M_H^2$. In principle one could also imagine to consider $q^2 \gg M_Z^2$. However, this limit contains decays of the form $H \to ZZ$ which are kinematically forbidden. On the other hand, for $q^2 \ll M_Z^2$ we neglect contributions from $H \to Zb\bar{b}$, which are, however, strongly phase-space suppressed. Furthermore, it is possible to experimentally distinguish this final state from $H \to b\bar{b}$. Note that the decay $H \to Zb\bar{b}$ is not included in the result of Ref. [10].

In the limit $q^2 \ll M_Z^2$ we obtain for $\Delta^{(\text{weak,Z})}$ the expansion

$$
\Delta^{(\text{weak,Z})} = \sum_{j\geq 0} d_{2j} \left(\frac{M_H^2}{M_Z^2}\right)^j = \sum_{j\geq 0} D_{2j}\,. \tag{15}
$$

where the coefficients D_k are given in Table 1. $\Delta^{(\text{weak,Z})}$ includes all relevant contributions from $\Delta_{\text{CT}}^{(\text{weak})}$ and is thus finite in the limit $\epsilon \to 0$. The bottom quark is renormalized in the $\overline{\text{MS}}$ scheme. The counterterm contribution is not expanded in M_H^2/M_Z^2 and is contained in the coefficient D_0. Furthermore, we choose $\mu^2 = M_H^2$ for the renormalization scale.

In a next step we use the results in Table 1 and construct various Padé approximations. The results are shown in Table 2 where also the exact result for $\Delta^{(\text{weak,Z})}$ from Ref. [10] is displayed. The deviation of the numerical approximation based on the [4/4] Padé expression and the exact result [10] is about 0.01% which justifies the use of this method at order $\alpha\alpha_s$.

Let us next turn to the contribution involving top quarks and a W and/or charged Goldstone bosons. To simplify the integrals and to obtain simple final expressions we assume

k	D_k
0	-0.008768
2	-0.000847
4	-0.000196
6	$+0.000269$
8	-0.000534
10	$+0.000894$
12	-0.001515
14	$+0.002568$
16	-0.004383

Table 1: Coefficients D_k from Eq. (15) for $\mu^2 = M_H$.

Padé approximant	$\Delta^{(\text{weak,Z})}$
[3/3]	-0.009744
[3/4]	-0.009746
[4/3]	-0.009747
[4/4]	-0.009746
exact	-0.009747

Table 2: Numerical results for $\Delta^{(\text{weak,Z})}$ obtained from the construction of Padé approximations using the coefficients in Table 1. The last row contains the exact result from Ref. [10].

one of the following hierarchies

$$\begin{aligned} &(A) \qquad M_H^2 \ll 4M_W^2 \ll 4M_t^2\,, \\ &(B) \qquad M_H^2 \ll 4M_W^2 \approx 4M_t^2\,. \end{aligned} \tag{16}$$

We stress that the $\mathcal{O}(\alpha)$ corrections can be computed without any assumptions on the relative size of the involved mass scales. However, at order $\alpha\alpha_s$ the hierarchies in Eq. (16) significantly simplify the calculation.

In Eqs. (16) the strong hierarchy ("$\ll$") means that we apply an asymptotic expansion [28] in the corresponding mass ratio. In the case of an approximation sign we Taylor-expand the integrand in the mass difference. As a result we obtain $\Delta^{(\text{weak})}$ in the form

$$\begin{aligned} \Delta^{(\text{weak})} &= \sum_{i\geq -1} c_{2i}^{(A)} \left(\frac{M_W^2}{M_t^2}\right)^i = \sum_{i\geq -1} C_{2i}^{(A)}\,, \\ &= \sum_{i\geq -1} c_{2i}^{(B)} \left(\frac{M_t^2 - M_W^2}{M_t^2}\right)^i = \sum_{i\geq -1} C_{2i}^{(B)}\,, \end{aligned} \tag{17}$$

k	$C_k^{(A)}$ ($\overline{\text{MS}}$)	$\Delta^{(\text{weak})}$
-2	$+0.005146$	$+0.005146$
0	$-0.004506 - 0.009746\vert_Z$	-0.009106
2	-0.000166	-0.009272
4	-0.000100	-0.009372
6	-0.000105	-0.009477
8	-0.000088	-0.009565
10	$+0.000048$	-0.009517
12	-0.000029	-0.009546
14	$+0.000001$	-0.009545
exact		-0.009549
k	$C_k^{(A)}$ (on-shell)	$\Delta^{(\text{weak})}$
-2	$+0.004842$	$+0.004842$
0	$-0.004754 - 0.009746\vert_Z$	-0.009659
2	-0.000145	-0.009804
4	-0.000095	-0.009899
6	-0.000078	-0.009977
8	-0.000065	-0.010042
10	$+0.000029$	-0.010012
12	-0.000018	-0.010031
14	$+0.000000$	-0.010030
exact		-0.010034

Table 3: Coefficients $C_k^{(A)}$ as defined in Eq. (17) (middle column). In the n^{th} row of the right column the sum including the first n terms is shown. On the top part we adopt the $\overline{\text{MS}}$ and below the on-shell definition for the top quark mass. The Z boson contribution is marked by $|_Z$.

where the coefficients $c_k^{(A)}$ and $c_k^{(B)}$ are expansions in $M_H^2/(4M_W^2)$. Note that by definition $C_0^{(A)}$ and $C_0^{(B)}$ contain the contribution from $\Delta^{(\text{weak,Z})}$.

We show our results for hierarchy (A) in Table 3 adopting again $\mu^2 = M_H^2$ and the $\overline{\text{MS}}$ definition for the bottom quark mass. For the top quark mass both the $\overline{\text{MS}}$ and on-shell mass value is used.

Note that $\Delta^{(\text{weak})} \sim 1/x$ as $x \to 0$. For this reason we show in Fig. 2 the quantity $x\Delta^{(\text{weak})}$ as a function of $x = M_W^2/M_t^2$ and compare the expansion obtained for the hierarchies (A) and (B) with the exact result [10].[2] For the plot we use the on-shell definition of the top quark mass and set $\mu^2 = M_H^2$. The numerical values are obtained by keeping M_W fixed and varying M_t. We take into account expansion terms up to $k = 12$ (see Table 3) which corresponds to the expansion depth which is available at order $\alpha\alpha_s$ (cf. Section 3).

[2]There is a typo in the quantity ΔT_{10+11} in Eq.(A.2) of [10]: the minus sign in front of $m_{f'}^2$ should be a plus sign.

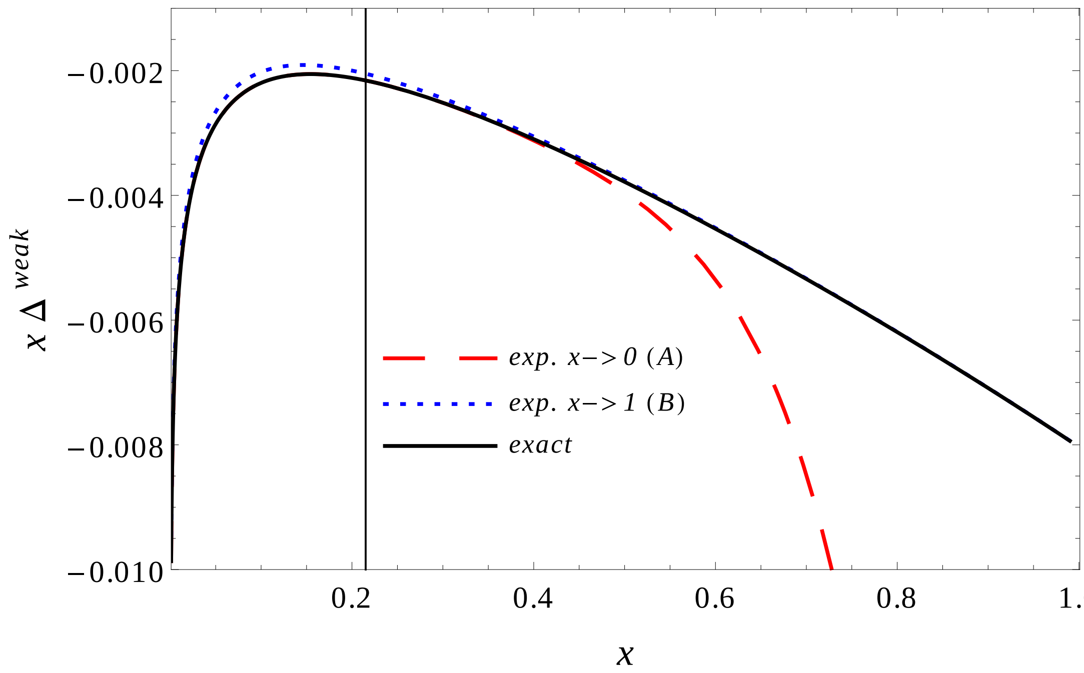

Figure 2: Comparison of $x\Delta^{(\text{weak})}$ as obtained for the hierarchies (A) and (B) with the exact result as a function of $x = M_W^2/M_t^2$. The black (solid) curve shows the exact result, the (red) dashed curve the expansion for $x \to 0$ and the (blue) dotted curve the expansion around $x = 1$. The vertical line indicates the experimental result for $x \approx 0.215$. For the renormalization scale of the bottom quark $\mu^2 = M_H^2$ has been chosen. Note that $x\Delta^{(\text{weak})}$ behaves as $\log(x)$ for $x \to 0$.

One observes that for $x \lesssim 0.4$ a perfect description is obtained from hierarchy (A) (red, dashed curve) and above $x \approx 0.4$ the result from hierarchy (B) (blue, dotted curve) agrees perfectly with the exact result (black line). For the physical value $x \approx 0.215$ one obtains

$$\Delta_B^{(\text{weak})} \approx -0.009525\,, \tag{18}$$

which has to be compared with the results in Table 3 in the lower panel. There is a notable deviation of about 5% to the exact result which has its origin in the divergent behaviour proportional to $1/x$ for $x \to 0$. For this reason we concentrate in Section 3 on hierarchy (A).

3 Corrections of order $\alpha\alpha_s$

In this Section we consider the quantity $\Delta^{(\alpha\alpha_s)}$ of Eq. (1). An analytic expression for $\Delta^{(\text{QED},\alpha_s)}$ can easily be obtained from the $\mathcal{O}(\alpha_s^2)$ QCD corrections (see, e.g., Ref. [29])

after adopting the colour factors. It reads

$$\Delta^{(\mathrm{QED},\alpha_s)} = Q_b^2 \frac{\alpha\alpha_s}{\pi^2} C_F \left[\frac{691}{32} - \frac{3}{4}\pi^2 - \frac{9}{2}\zeta(3) + \frac{105}{8} \ln\left(\frac{\mu^2}{M_H^2}\right) + \frac{9}{4}\ln^2\left(\frac{\mu^2}{M_H^2}\right)\right] . \quad (19)$$

To obtain $\Delta^{(\mathrm{weak},\alpha_s)}$ we proceed as follows:

- We consider the imaginary part of the three-loop propagator-type diagrams which are obtained by dressing the $\mathcal{O}(\alpha)$ diagrams (cf. Fig. 1 for examples) in all possible ways with one gluon. This part can be split, in analogy to the $\mathcal{O}(\alpha)$ corrections, into a contribution involving Z or Goldstone bosons and into a contribution involving W and/or charged Goldstone bosons and top quarks.

- The bare bottom quark mass in the Born result has to be replaced by the $\overline{\mathrm{MS}}$ renormalized counterpart using corrections of order $\alpha\alpha_s$. The corresponding counterterm is available from Ref. [22,30] which we have checked by an independent calculation. It is given by

$$\delta_{m_b}^{(\mathrm{weak},\alpha_s),\overline{\mathrm{MS}}} = C_F \frac{\alpha\alpha_s}{\pi^2} \left[\frac{1}{\epsilon^2} \left(-\frac{1}{16} - \frac{7}{128 c_W^2} - \frac{9}{128 s_W^2} + \frac{9 m_t^2}{64 M_W^2 s_W^2} \right) + \frac{1}{\epsilon} \left(\frac{1}{96} + \frac{31}{768 c_W^2} + \frac{27}{256 s_W^2} - \frac{3 m_t^2}{32 M_W^2 s_W^2} \right) \right] . \quad (20)$$

  Note that $\delta_{m_b}^{(\mathrm{weak},\alpha_s),\overline{\mathrm{MS}}}$ contains poles up to order $1/\epsilon^2$ and thus the Born result is needed up to order ϵ^2 terms.

- $\Gamma^{(0)}\Delta^{(\alpha)}$ has to be available up to order ϵ and the bottom and top quark masses have to be renormalized using one-loop counterterms of $\mathcal{O}(\alpha_s)$ which are given by

$$\delta_{m_q}^{(\alpha_s),\mathrm{OS}} = \frac{m_q^{\mathrm{bare}}}{M_q} - 1 = -\frac{\alpha_s}{\pi} C_F \left(\frac{3}{4\epsilon} + 1 + \frac{3}{4} \ln \frac{\mu^2}{M_q^2} \right) , \quad (21)$$

  with $q = b, t$. The corresponding $\overline{\mathrm{MS}}$ counterterm is obtained by dropping the finite part on the right-hand side of the above equation.

- $\Gamma^{(0)}\Delta^{(\alpha_s)}$ has to be available up to order ϵ and the bottom quark mass has to be renormalized using one-loop counterterms of $\mathcal{O}(\alpha)$ which is given in Eq. (11) .

- There is a contribution where v_r from Eq. (12) multiplies $\Gamma^{(0)}\Delta^{(\alpha_s)}$. Since the latter is finite we do not need the $\mathcal{O}(\epsilon)$ part of v_r. On the other hand, since v_r contains $1/\epsilon$ poles $\Gamma^{(0)}\Delta^{(\alpha_s)}$ is needed including $\mathcal{O}(\epsilon)$ terms.

- The fermion-loop contributions to v_r [see Eq. (12)] receive two-loop QCD corrections which are multiplied by the Born decay rate. Since the fermionic contribution to

$\Sigma^{\gamma Z}(0)$ vanishes only $\Sigma^W(0)$ and $\Sigma'^H(M_H^2)$ get correction terms of order $\alpha\alpha_s$. We compute them in the limit of a heavy top quark and obtain in the $\overline{\mathrm{MS}}$ scheme

$$
\begin{aligned}
& -\frac{\Sigma^W(0)}{M_W^2} - \Sigma'^H(M_H^2)\Bigg|_{\mathcal{O}(\alpha\alpha_s)} \\
&= \frac{N_c\alpha\alpha_s}{\pi^2 s_W^2}\Bigg\{\frac{m_t^2}{M_W^2}\left[\frac{19}{72}+\frac{7}{24}\ln\left(\frac{\mu^2}{m_t^2}\right)-\frac{1}{12}\zeta(2)\right]-\frac{M_H^2}{M_W^2}\frac{61}{3240} \\
&\quad +\frac{M_H^4}{m_t^2M_W^2}\left[-\frac{503}{100800}+\frac{3}{560}\ln\left(\frac{\mu^2}{m_t^2}\right)\right] \\
&\quad +\frac{M_H^6}{m_t^4M_W^2}\left[-\frac{9523}{15876000}+\frac{1}{630}\ln\left(\frac{\mu^2}{m_t^2}\right)\right] \\
&\quad +\frac{M_H^8}{m_t^6M_W^2}\left[-\frac{100687}{2514758400}+\frac{1}{2464}\ln\left(\frac{\mu^2}{m_t^2}\right)\right] \\
&\quad +\frac{M_H^{10}}{m_t^8M_W^2}\left[\frac{154559}{19423404000}+\frac{1}{10010}\ln\left(\frac{\mu^2}{m_t^2}\right)\right]+\mathcal{O}\left(\frac{M_H^{12}}{m_t^{10}M_W^2}\right)\Bigg\}\,, \qquad (22)
\end{aligned}
$$

where $m_t = m_t(\mu)$.

The individual terms develop poles up to order $1/\epsilon^2$, which cancel in the sum.

We are now in the position to present results for the order $\alpha\alpha_s$ corrections. In analogy to Eqs. (15) and (17) we introduce

$$
\begin{aligned}
\Delta^{(\mathrm{weak},\alpha_s,\mathrm{Z})} &= \sum_{j\geq 0} d_{2j}\left(\frac{M_H^2}{M_Z^2}\right)^j = \sum_{j\geq 0} D_{2j}\,, \\
\Delta^{(\mathrm{weak},\alpha_s)} &= \sum_{i\geq -1} c_{2i}^{(A)}\left(\frac{M_W^2}{M_t^2}\right)^i = \sum_{i\geq -1} C_{2i}^{(A)} \qquad (23)
\end{aligned}
$$

where for convenience $\Delta^{(\mathrm{weak},\alpha_s,\mathrm{Z})}$ is added to the coefficient $C_0^{(A)}$.

In the case of $\Delta^{(\mathrm{weak},\alpha_s,\mathrm{Z})}$ we proceed as at order α: we compute nine expansion terms for the (formal) limit $q^2 \ll M_Z^2$ and set $q^2 = M_H^2$. After including the corresponding counterterm contributions we obtain the expansion coefficients listed in Table 4. Afterwards we construct several Padé approximants and obtain the results in Table 5. We observe a similar stability as at $\mathcal{O}(\alpha)$ and estimate the final result as

$$
\Delta^{(\mathrm{weak},\alpha_s,\mathrm{Z})} = -0.00195(1)\,, \qquad (24)
$$

which has an uncertainty of about 0.5%, an accuracy sufficient for all foreseeable applications.

In Table 6 we present the results for the coefficients $C_k^{(A)}$. We observe a continuous decrease of the magnitude leading to a stable result with two significant digits after

k	D_k
0	-0.001692
2	-0.000032
4	-0.000547
6	$+0.000836$
8	-0.001347
10	$+0.002183$
12	-0.003604
14	$+0.006033$
16	-0.010223

Table 4: Coefficients D_k from Eq. (23) for $\mu^2 = M_H^2$.

Padé approximant	$\Delta^{(\mathrm{weak},\alpha_\mathrm{s},\mathrm{Z})}$
[3/3]	-0.001955
[3/4]	-0.001954
[4/3]	-0.001960
[4/4]	-0.001953

Table 5: Numerical results for $\Delta^{(\mathrm{weak},\alpha_\mathrm{s},\mathrm{Z})}$ obtained from the construction of Padé approximations using the coefficients in Table 4.

including six expansion terms, a similar behaviour as at order α. The seventh and eighth terms confirm this approximation. It is also interesting to note that the contribution from the Z boson diagrams amounts to about 65% of the total result. Furthermore, the leading m_t^2 contribution amounts to less than 20% of $\Delta^{(\mathrm{weak},\alpha_\mathrm{s})}$ but to more than 50% of the W boson diagrams, i.e., $\Delta^{(\mathrm{weak},\alpha_\mathrm{s})} - \Delta^{(\mathrm{weak},\alpha_\mathrm{s},\mathrm{Z})}$.

4 Numerical results and conclusions

In Table 7 we summarize our results for the $\mathcal{O}(\alpha)$ and $\mathcal{O}(\alpha\alpha_s)$ corrections where the electroweak part is split into QED and weak corrections. The contribution from the Z boson diagrams is listed for completeness; their contribution is contained in $\Delta^{(\mathrm{weak})}$ and $\Delta^{(\mathrm{weak},\alpha_\mathrm{s})}$, respectively. For comparison also the QCD corrections up to $\mathcal{O}(\alpha_s^4)$ [6] based on computations of the imaginary part of the massless Higgs correlators are shown in Table 7. Top quark induced QCD corrections due to an effective $Hb\bar{b}$ coupling, which are in general small (see, e.g. Eq. (14) of Ref. [8]), are not shown.

Both at one- and two-loop order the weak corrections are negative whereas the QED corrections are positive. One furthermore observes that the weak corrections are about an order of magnitude larger than the QED terms. For $\mu^2 = M_H^2$ the weak corrections

k	$C_k^{(A)}$ ($\overline{\mathrm{MS}}$)	$\Delta^{(\mathrm{weak},\alpha_\mathrm{s})}$
-2	-0.000479	-0.000479
0	$-0.000514 - 0.001953\|_Z$	-0.002946
2	-0.000044	-0.002990
4	$+0.000018$	-0.002972
6	$+0.000003$	-0.002970
8	$+0.000005$	-0.002964
10	$+0.000004$	-0.002960
12	$+0.000002$	-0.002959
k	$C_k^{(A)}$ (on-shell)	$\Delta^{(\mathrm{weak},\alpha_\mathrm{s})}$
-2	-0.000481	-0.000481
0	$-0.000382 - 0.001953\|_Z$	-0.002816
2	-0.000032	-0.002848
4	-0.000006	-0.002854
6	-0.000008	-0.002862
8	$+0.000011$	-0.002851
10	-0.000010	-0.002861
12	$+0.000002$	-0.002860

Table 6: Coefficients $C_k^{(A)}$ at order $\alpha\alpha_s$ as defined in Eq. (23). In the n^{th} row of the right column the sum including the first n terms is shown. On the top part we adopt the $\overline{\mathrm{MS}}$ and below the on-shell definition for the top quark mass.

	$\Delta^{(\alpha_\mathrm{s})}$	$\Delta^{(\alpha_\mathrm{s}^2)}$	$\Delta^{(\alpha_\mathrm{s}^3)}$	$\Delta^{(\alpha_\mathrm{s}^4)}$
QCD	0.2040	0.0378	0.0020	-0.0014
	$\Delta^{(\mathrm{QED})}$	$\Delta^{(\mathrm{QED},\alpha_\mathrm{s})}$		
QED/QCD	0.0011	0.0001		
	$\Delta^{(\mathrm{weak})}$	$\Delta^{(\mathrm{weak},\alpha_\mathrm{s})}$	$\Delta^{(\mathrm{weak,Z})}$	$\Delta^{(\mathrm{weak},\alpha_\mathrm{s},\mathrm{Z})}$
weak/QCD	-0.0100	-0.0029	-0.0097	-0.0020

Table 7: Numerical result for the QCD, QED and weak one-loop and mixed two-loop corrections for $\mu^2 = M_H^2$. Note that $\Delta^{(\mathrm{weak})}$ and $\Delta^{(\mathrm{weak},\alpha_\mathrm{s})}$ contain the contributions from $\Delta^{(\mathrm{weak,Z})}$ and $\Delta^{(\mathrm{weak},\alpha_\mathrm{s},\mathrm{Z})}$, respectively.

amount to about -1% which is significantly smaller than the one-loop QCD correction ($+20\%$), however, it is of the same order of magnitude as the two-loop $\mathcal{O}(\alpha_s^2)$ corrections obtained from the massless Higgs correlator, which amount to $+3.8\%$ (see, e.g., Ref. [6]). At the same value of μ the correction term $\Delta^{(\mathrm{weak},\alpha_\mathrm{s})}$ amounts to about -0.3% which is a factor three larger than the one-loop QED corrections and which is of the same order of magnitude, but with the opposite sign, as the three-loop QCD corrections. It is interesting to note that the four-loop QCD corrections are -0.1%.

Finally, it is interesting to comment on the assumption the QED and QCD corrections factorize, an approach often chosen in case $\mathcal{O}(\alpha\alpha_s)$ terms are missing. To do this we define

$$\Delta^{(\alpha\alpha_s,\mathrm{non-fact.})} \;=\; \Delta^{(\alpha\alpha_s)} - \Delta^{(\alpha)}\Delta^{(\alpha_s)}\,, \tag{25}$$

which shall be small in case the factorization approach works. From the numbers in Table 7 we obtain

$$\Delta^{(\alpha\alpha_s,\mathrm{non-fact.})} \;=\; -0.000831\,, \tag{26}$$

which corresponds to about 30% of $\Delta^{(\alpha\alpha_s)}$.

To summarize, in this letter we have computed the complete $\mathcal{O}(\alpha\alpha_s)$ mixed corections to the decay rate $\Gamma(H \to b\bar{b})$. They provide a negative shift of about -0.3% to $\Gamma(H \to b\bar{b})$ which corresponds to about 30% of the one-loop electroweak corrections and which is of the same order of magnitude as the three-loop QCD corrections.

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