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The two-loop helicity amplitudes for $gg \to V_1V_2 \to 4$ leptons

Andreas von Manteuffel,[a] **Lorenzo Tancredi**[b]

[a] *PRISMA Cluster of Excence, Institute of Physics, Johannes Gutenberg University, 55099 Mainz, Germany*

[b] *Institut für Theoretische Teilchenphysik, Karlsruhe Institute of Technology, Engesserstrasse 7, 76128 Karlsruhe, Germany*

E-mail: manteuffel@uni-mainz.de, lorenzo.tancredi@kit.edu

Abstract: We compute the two-loop massless QCD corrections to the helicity amplitudes for the production of two electroweak gauge bosons in the gluon fusion channel, $gg \to V_1V_2$, keeping the virtuality of the vector bosons V_1 and V_2 arbitrary and taking their decays into leptons into account. The amplitudes are expressed in terms of master integrals, whose representation has been optimised for fast and reliable numerical evaluation. We provide analytical results and a public `C++` code for their numerical evaluation on `HepForge` at http://vvamp.hepforge.org.

Keywords: QCD, Collider Physics, NLO and NNLO Calculations

Contents

1 Introduction 1

2 Partonic current for $gg \to V_1V_2$ 3

3 Helicity amplitudes for $gg \to V_1V_2 \to 4$ leptons 8

4 Calculation of the form factors 11

5 Numerical C++ implementation and results 14

6 Conclusions 16

A Form factor relations 17

1 Introduction

Pair production processes for electroweak vector bosons provide a rich spectrum of observables, which are crucial to test in depth the $SU(2)_L \times U(1)_Y$ gauge sector of the Standard Model. In particular, the production of pairs of resonant vector bosons allows for precise studies of the electroweak triple gauge couplings, while considering off-shell vector boson pairs is required for precision Higgs phenomenology. Furthermore, diboson production processes are important backgrounds in direct new physics searches. The main production channel for pairs of vector bosons at hadron colliders is quark-antiquark annihilation and great progress has been achieved in the last years with the computation of the next-to-next-to-leading order (NNLO) QCD corrections to $q\bar{q} \to \gamma\gamma$ [1], $q\bar{q} \to Z\gamma$ [2], $q\bar{q} \to ZZ$ [3] and $q\bar{q} \to W^+W^-$ [4] production at the LHC. Furthermore, the fermionic NNLO corrections to $q\bar{q} \to \gamma^*\gamma^*$ were derived in [5].

The gluon fusion channel contributes to W^+W^-, ZZ, $Z\gamma$ and $\gamma\gamma$ production. As a quark-loop induced process, its leading order (LO) cross section is suppressed by two powers of the strong coupling constant with respect to that of the quark channel. This implies that it formally contributes only at NNLO in the perturbative expansion of the hadronic process, but numerical enhancements may be expected due to the large gluon luminosities at typical energies for diboson production at the LHC. For the gluon-induced processes, the one-loop amplitudes and the corresponding one-loop squared interference terms have been computed long ago [6–12]. Their impact on the total cross section was found to range approximately from 5% to more than 10% for different final states at the LHC, and to rise with increasing collider energy [1–4]. These values can substantially increase up to about 30% when particular sets of cuts, relevant for example for Higgs boson searches, are applied [13]. It is therefore clear that the inclusion of gluon channel

contributions can be important in order to achieve a description of the full process which matches the experimental precision. Beyond the actual size of the known leading order corrections in the gluon channel, it is unclear how large the associated theory uncertainty actually is. By comparison with Higgs production in gluon fusion [14–16], the conventional LO scale variation is not expected to allow for a reliable estimate of the size of neglected higher order corrections. In order to control the theory uncertainty to the level of the NNLO prediction for the quark induced process, it is therefore very desirable to compute the next-to-leading order (NLO) contributions for the gluon induced process. Currently this has been done only for $gg \to \gamma\gamma$ [17, 18], and the NLO corrections have been found to be not only sizeable but also important for stabilising the theoretical predictions [18]. Finally, precise theoretical predictions for $gg \to ZZ$ can be useful for constraining the total Higgs boson decay width at the LHC [19–21].

Technically, the computation of the NLO corrections to $gg \to V_1V_2$ requires two ingredients, the two-loop virtual corrections to $gg \to V_1V_2$ and the one-loop real-virtual corrections to the corresponding radiative processes with one more parton in the final state. By now the computation of the one-loop amplitudes with an extra gluon does not constitute any conceptual difficulty and can be pursued with standard techniques for one-loop multi-legs processes [22–28]. The two-loop amplitudes, on the other hand, are known only for $gg \to \gamma\gamma$ [17] and for $gg \to Z\gamma$ [29], in both cases for on-shell final state photons. In order to obtain physical predictions, both contributions need to be combined using a subtraction scheme to isolate and cancel unphysical IR divergences. In this case, a NLO scheme [30, 31] would be sufficient.

In this paper we calculate the missing two-loop massless QCD corrections to $gg \to V_1V_2$, with $V_1V_2 = W^+W^-, ZZ, Z\gamma^*, \gamma^*\gamma^*$. The calculation builds upon the master integrals for four-point functions with massless propagators and two massive external legs, which were computed recently in the case of equal masses in [32, 33], and in the case of different masses in [34–37]. The former were used for the first NNLO fully-inclusive calculations of ZZ [3] and W^+W^- [4] production at the LHC, while the latter allowed the computation of the two-loop corrections to $q\bar{q}' \to V_1V_2$ [37, 38]. A subset of these master integrals was also computed independently in [5, 39]. While the inclusion of massive top-loop mediated subprocesses would be of interest for some phenomenological applications [19, 40], the computation of the two-loop amplitudes requires knowledge of challenging new master integrals, which should be addressed in the future.

The paper is structured as follows. In Section 2 we describe the tensor decomposition of the partonic current for the process $gg \to V_1V_2$ and consider the possible electroweak coupling structures. We include the vector boson decays and describe the helicity amplitudes for the process $gg \to V_1V_2 \to 4$ leptons in terms of scalar form factors in Section 3. The actual calculation of the loop contributions to these form factors is described in Section 4, which includes a dicussion of UV renormalisation, IR subtraction and various checks we performed on our results. In Section 5 we present numerical results obtained with our `C++` implementation. Finally, we conclude in Section 6. In Appendix A we give explicit formulae for obtaining the physical form factors appearing in the helicity amplitudes from the original tensor coefficients computed in this paper. We provide computer readable files

for our analytical results and our `C++` code for the numerical evaluation of the amplitudes on our `VVamp` project page on `HepForge` at http://vvamp.hepforge.org.

2 Partonic current for $gg \to V_1V_2$

We consider the production of two massive off-shell vector bosons, V_1V_2, in the gluon fusion channel,

$$g(p_1) + g(p_2) \longrightarrow V_1(p_3) + V_2(p_4), \tag{2.1}$$

where $V_1V_2 = \gamma^*\gamma^*$, ZZ, $Z\gamma^*$, W^+W^-. The final states $W^\pm\gamma^*$ and $W^\pm Z$ instead are forbidden by charge conservation. Since the two vector bosons are off-shell we have in the general case

$$p_1^2 = p_2^2 = 0\,, \quad p_3^2 > 0, \quad p_4^2 > 0, \quad p_3^2 \neq p_4^2, \tag{2.2}$$

with the usual Mandelstam invariants defined as

$$s = (p_1 + p_2)^2\,, \qquad t = (p_1 - p_3)^2\,, \qquad u = (p_2 - p_3)^2\,, \tag{2.3}$$

and the relation

$$s + t + u = p_3^2 + p_4^2\,. \tag{2.4}$$

The physical region for the scattering kinematics has the boundary $t\,u = p_3^2\,p_4^2$ and fulfils

$$s \geq \left(\sqrt{p_3^2} + \sqrt{p_4^2}\right)^2, \qquad \frac{1}{2}\big(p_3^2 + p_4^2 - s - \kappa\big) \leq t \leq \frac{1}{2}\big(p_3^2 + p_4^2 - s + \kappa\big) \tag{2.5}$$

where κ is the Källén function

$$\kappa\left(s, p_3^2, p_4^2\right) \equiv \sqrt{s^2 + p_3^4 + p_4^4 - 2(s\,p_3^2 + p_3^2\,p_4^2 + p_4^2\,s)}\,. \tag{2.6}$$

We denote the scattering amplitude for the process (2.1) by

$$S(p_1, p_2, p_3) = S_{\mu\nu\rho\sigma}(p_1, p_2, p_3)\,\epsilon_1^\rho(p_1)\,\epsilon_2^\sigma(p_2)\,\epsilon_3^{*\,\mu}(p_3)\,\epsilon_4^{*\,\nu}(p_4)$$

where ϵ_1, ϵ_2 are the polarisation vectors of the incoming gluons, ϵ_3, ϵ_4 are the polarisation vectors of the outgoing massive vector bosons and $p_4 = p_1 + p_2 - p_3$. Since we will consider leptonic decays of the massive vector bosons we will be able to construct the full amplitude including the decays from the partonic current

$$S_{\mu\nu}(p_1, p_2, p_3) = S_{\mu\nu\rho\sigma}(p_1, p_2, p_3)\,\epsilon_1^\rho(p_1)\,\epsilon_2^\sigma(p_2)$$

for the $2 \to 2$ process. In particular, it is only the latter which receives (pure) QCD corrections at any order in perturbation theory.

In order to compute the partonic current it is useful to consider its tensor decomposition. Based on Lorentz invariance only, there are 138 independent tensor structures which

can contribute

$$
\begin{aligned}
S^{\mu\nu\rho\sigma}(p_1,p_2,p_3) = {} & a_1 g^{\mu\nu} g^{\rho\sigma} + a_2 g^{\mu\rho} g^{\nu\sigma} + a_3 g^{\mu\sigma} g^{\nu\rho} \\
& + \sum_{j_1,j_2=1}^{3} \Big(b^{(1)}_{j_1 j_2}\, g^{\mu\nu}\, p^{\rho}_{j_1}\, p^{\sigma}_{j_2} + b^{(2)}_{j_1 j_2}\, g^{\mu\rho}\, p^{\nu}_{j_1}\, p^{\sigma}_{j_2} + b^{(3)}_{j_1 j_2}\, g^{\mu\sigma}\, p^{\nu}_{j_1}\, p^{\rho}_{j_2} \\
& \qquad + b^{(4)}_{j_1 j_2}\, g^{\nu\rho}\, p^{\mu}_{j_1}\, p^{\sigma}_{j_2} + b^{(5)}_{j_1 j_2}\, g^{\nu\sigma}\, p^{\mu}_{j_1}\, p^{\rho}_{j_2} + b^{(6)}_{j_1 j_2}\, g^{\rho\sigma}\, p^{\mu}_{j_1}\, p^{\nu}_{j_2} \Big) \\
& + \sum_{j_1,j_2,j_3,j_4=1}^{3} c_{j_1 j_2 j_3 j_4} p^{\mu}_{j_1}\, p^{\nu}_{j_2} p^{\rho}_{j_3}\, p^{\sigma}_{j_4},
\end{aligned}
\tag{2.7}
$$

where the coefficients a_j, b^k_{ij} and c_{ijkl} are scalar functions of the kinematic invariants s, t, p_3^2, p_4^2 and of the space-time dimension d. Not all structures are relevant for our calculation. Many of them simply drop due to the transversality of the gluons' polarisation vectors

$$
\epsilon_1 \cdot p_1 = \epsilon_2 \cdot p_2 = 0\,. \tag{2.8}
$$

Moreover the tensor structure can be further simplified by fixing explicitly the gauge for the incoming gluons. A particularly simple choice is given by the symmetrical condition

$$
\epsilon_1 \cdot p_2 = \epsilon_2 \cdot p_1 = 0\,, \tag{2.9}
$$

which corresponds to the following rules for the polarisation sums

$$
\begin{aligned}
\sum_{\lambda_1} \epsilon^{\mu *}_{1\,\lambda_1}(p_1)\epsilon^{\nu}_{1\,\lambda_1}(p_1) &= -g^{\mu\nu} + \frac{p_1^\mu p_2^\nu + p_1^\nu p_2^\mu}{p_1\cdot p_2}\,, \\
\sum_{\lambda_2} \epsilon^{\mu *}_{2\,\lambda_2}(p_2)\epsilon^{\nu}_{2\,\lambda_2}(p_2) &= -g^{\mu\nu} + \frac{p_1^\mu p_2^\nu + p_1^\nu p_2^\mu}{p_1\cdot p_2}\,.
\end{aligned}
\tag{2.10}
$$

Further conditions can be applied on the polarisation vectors of the massive vector bosons V_1V_2. Since we consider their tree-level decays into massless leptons, the transversality of the leptonic decay currents can be rephrased as a transversality condition for the polarisation vectors,

$$
\epsilon_3 \cdot p_3 = \epsilon_4 \cdot p_4 = 0\,, \tag{2.11}
$$

with the corresponding polarisation sums

$$
\begin{aligned}
\sum_{\lambda_3} \epsilon^{\mu *}_{3\,\lambda_3}(p_3)\epsilon^{\nu}_{3\,\lambda_3}(p_3) &= -g^{\mu\nu} + \frac{p_3^\mu p_3^\nu}{p_3^2}\,, \\
\sum_{\lambda_4} \epsilon^{\mu *}_{4\,\lambda_4}(p_4)\epsilon^{\nu}_{4\,\lambda_4}(p_4) &= -g^{\mu\nu} + \frac{p_4^\mu p_4^\nu}{p_4^2}\,.
\end{aligned}
\tag{2.12}
$$

Imposing the constraints (2.8), (2.9) and (2.11) one is left with only 20 independent tensor structures and we can write the partonic current according to

$$
S^{\mu\nu}(p_1,p_2,p_3) = \sum_{j=1}^{20} A_j(s,t,p_3^2,p_4^2)\, T_j^{\mu\nu}\,, \tag{2.13}
$$

where the A_j are scalar functions of s, t, p_3^2, p_4^2 and d. The tensors $T_j^{\mu\nu}$ are defined as

$$
\begin{aligned}
T_1^{\mu\nu} &= \epsilon_1\cdot\epsilon_2\, g^{\mu\nu}\,, & T_2^{\mu\nu} &= \epsilon_1^\mu\,\epsilon_2^\nu\,, & T_3^{\mu\nu} &= \epsilon_1^\nu\,\epsilon_2^\mu\,, & T_4^{\mu\nu} &= \epsilon_1\cdot\epsilon_2\, p_1^\mu\, p_1^\nu\,,\\
T_5^{\mu\nu} &= \epsilon_1\cdot\epsilon_2\, p_1^\mu\, p_2^\nu\,, & T_6^{\mu\nu} &= \epsilon_1\cdot\epsilon_2\, p_2^\mu\, p_1^\nu\,, & T_7^{\mu\nu} &= \epsilon_1\cdot\epsilon_2\, p_2^\mu\, p_2^\nu\,, & T_8^{\mu\nu} &= \epsilon_2\cdot p_3\,\epsilon_1^\mu\, p_1^\nu\,,\\
T_9^{\mu\nu} &= \epsilon_2\cdot p_3\,\epsilon_1^\mu\, p_2^\nu\,, & T_{10}^{\mu\nu} &= \epsilon_2\cdot p_3\,\epsilon_1^\nu\, p_1^\mu\,, & T_{11}^{\mu\nu} &= \epsilon_2\cdot p_3\,\epsilon_1^\nu\, p_2^\mu\,, & T_{12}^{\mu\nu} &= \epsilon_1\cdot p_3\,\epsilon_2^\mu\, p_1^\nu\,,\\
T_{13}^{\mu\nu} &= \epsilon_1\cdot p_3\,\epsilon_2^\mu\, p_2^\nu\,, & T_{14}^{\mu\nu} &= \epsilon_1\cdot p_3\,\epsilon_2^\nu\, p_1^\mu\,, & T_{15}^{\mu\nu} &= \epsilon_1\cdot p_3\,\epsilon_2^\nu\, p_2^\mu\,, & T_{16}^{\mu\nu} &= \epsilon_1\cdot p_3\,\epsilon_2\cdot p_3\, g^{\mu\nu}\,,\\
T_{17}^{\mu\nu} &= \epsilon_1\cdot p_3\,\epsilon_2\cdot p_3\, p_1^\mu\, p_1^\nu\,, & T_{18}^{\mu\nu} &= \epsilon_1\cdot p_3\,\epsilon_2\cdot p_3\, p_1^\mu\, p_2^\nu\,,\\
T_{19}^{\mu\nu} &= \epsilon_1\cdot p_3\,\epsilon_2\cdot p_3\, p_2^\mu\, p_1^\nu\,, & T_{20}^{\mu\nu} &= \epsilon_1\cdot p_3\,\epsilon_2\cdot p_3\, p_2^\mu\, p_2^\nu\,.
\end{aligned}
\tag{2.14}
$$

We stress that the tensor decomposition (2.13) is based only on Lorentz symmetry, gauge invariance and the properties of the boson decays and holds therefore at every order in perturbative QCD. Moreover, no assumption has been made on the dimensionality of space-time and the result is valid for any values of the parameter d.

The scalar form factors A_j can be extracted from the amplitude (2.13) by applying suitable projecting operators. The projectors themselves can be decomposed in the same 20 tensors as

$$
P_j^{\mu\nu} = \sum_{i=1}^{20} B_{ji}\,(T_i^{\mu\nu})^\dagger \qquad \text{for } j = 1,\ldots,20, \tag{2.15}
$$

where also B_{ji} are functions of the external invariants and d. Their explicit form can be determined imposing

$$
\sum_{pol} P_j^{\mu'\nu'}\left[\epsilon_{3\mu'}\epsilon_{4\nu'}\epsilon_{3\mu}^*\epsilon_{4\nu}^*\right] S^{\mu\nu} = A_j \quad \text{for } j = 1,...,20, \tag{2.16}
$$

where the polarisation sums are evaluated in d dimensions according to (2.10) and (2.12). The explicit results for the coefficients B_{ji} are rather lengthy and we prefer not to write them here explicitly. Computer readable files for the latter are given on our project page at `HepForge`.

The partonic current is the only one which receives contributions from QCD radiative corrections and, for two gluons of helicities λ_1 and λ_2, can be written as

$$
S_{\mu\nu}(p_1^{\lambda_1}, p_2^{\lambda_2}, p_3) = \delta^{a_1 a_2} \sum_j \mathcal{C}_{V_1V_2}^{[j]}\, S_{\mu\nu\rho\sigma}^{[j]}(p_1,p_2,p_3)\epsilon_{1\lambda_1}^\rho(p_1)\epsilon_{2\lambda_2}^\sigma(p_2)\,, \tag{2.17}
$$

where $\delta^{a_1a_2}$ is the overall colour structure and the index j runs over different possible classes of diagrams discussed below, see also Fig. 1, which are characterised by different electroweak couplings $\mathcal{C}_{V_1V_2}^{[j]}$.

Before proceeding, it is convenient to introduce some notations needed in the following. As long as we work in QCD, we only need to consider the coupling of electroweak vector bosons V to fermions. We parametrise this as

$$
\mathcal{V}_\mu^{V,f_1,f_2} = -i\,e\,\Gamma_\mu^{Vf_1f_2}\,, \qquad \text{where} \quad e = \sqrt{4\,\pi\,\alpha} \quad \text{is the positron charge}\,, \tag{2.18}
$$

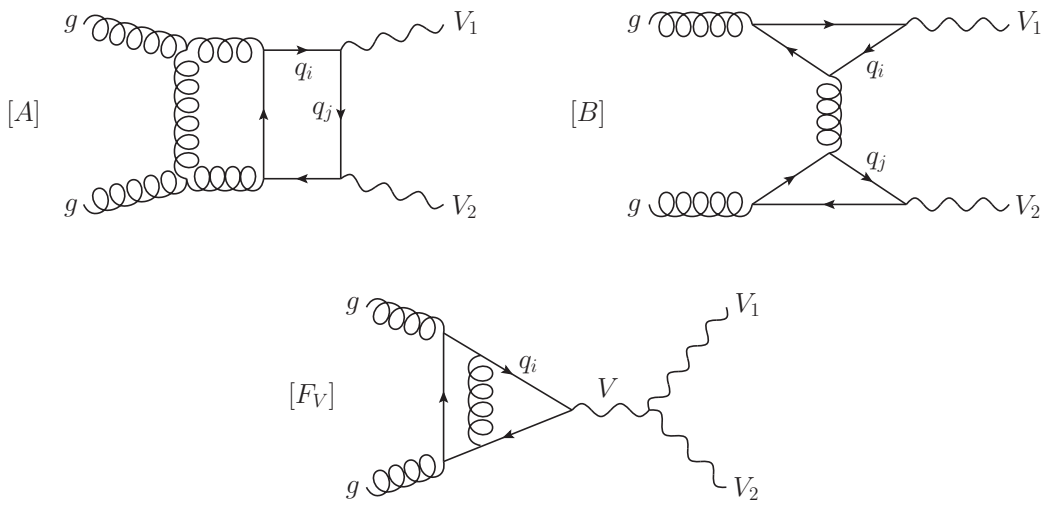

Figure 1. Example Feynman diagrams for the process $gg \to V_1V_2$ at the two-loop level, where the vector bosons couple to the same fermion loop, $[A]$, to different fermion loops, $[B]$ or to an intermediate vector boson, $[F_V]$. The sum of all type $[B]$ contribution and the sum of all type $[F_V]$ contributions vanish, respectively.

in such a way that all fermion charges are expressed in units of e and

$$\Gamma_\mu = L^V_{f_1f_2}\,\gamma_\mu\left(\frac{1-\gamma_5}{2}\right) + R^V_{f_1f_2}\,\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)\,, \tag{2.19}$$

with

$$L^\gamma_{f_1f_2} = -e_{f_1}\,\delta_{f_1f_2} \qquad R^\gamma_{f_1f_2} = -e_{f_1}\,\delta_{f_1f_2}\,, \tag{2.20}$$

$$L^Z_{f_1f_2} = \frac{I_3^{f_1} - \sin^2\theta_w e_{f_1}}{\sin\theta_w\cos\theta_w}\,\delta_{f_1f_2}\,, \qquad R^Z_{f_1f_2} = -\frac{\sin\theta_w e_{f_1}}{\cos\theta_w}\,\delta_{f_1f_2}\,, \tag{2.21}$$

$$L^W_{f_1f_2} = \frac{1}{\sqrt{2}\,\sin\theta_w}\,\epsilon_{f_1f_2}\,, \qquad R^W_{f_1f_2} = 0\,, \tag{2.22}$$

where $\epsilon_{f_1f_2}$ is unity for $f_1 \neq f_2$, but belonging to the same isospin doublet, and zero otherwise.

Let us consider the different electroweak coupling structures in detail. It is clear that, since we do not take any electroweak radiative corrections into account, at least one of the two vector bosons must be coupled to an internal fermion loop. In order to compute the one- and two-loop QCD corrections we need to consider the following three possibilities, see Fig. 1.

Class A Both vector bosons V_1V_2 are attached to the same fermion loop. In this case the diagrams are proportional to the charge weighted sum of the quark flavours, which we denote as $\mathcal{C}^{[A]}_{V_1V_2} = N_{V_1V_2}$. These diagrams could in principle yield two different contributions. One, proportional to the sum of the vector-vector and the axial-axial couplings, in which all dependence on γ_5 cancels out. The second, instead, contains the vector-axial coupling and is linear in γ_5. Due to charge parity conservation this

last contribution is expected to always vanish identically for massless quarks running in the loops, for any choice of V_1 and V_2 [8, 9, 40]. One then easily finds that

$$N_{\gamma\gamma} = \sum_i e_{q_i}^2 \,, \qquad N_{ZZ} = \frac{1}{2}\sum_i \left[\left(L^Z_{q_i q_i}\right)^2 + \left(R^Z_{q_i q_i}\right)^2\right] \,,$$
$$N_{WW} = \frac{1}{2}\sum_{i\,j} \left(L^W_{q_i q_j} L^W_{q_j q_i}\right) \,, \qquad N_{Z\gamma} = -\frac{1}{2}\sum_i \left(L^Z_{q_i q_i} + R^Z_{q_i q_i}\right) e_{q_i} \,, \tag{2.23}$$

where the indices i, j run over the flavours of the quarks in the loop.

Class B The two vector bosons are attached to two different fermion loops. This configuration is of course possible only starting from two loops on. Each fermion loop contains both a vector and an axial piece. For the case of two-loop massless QCD corrections relevant here, both contributions can be shown to vanish. The axial contribution cancels out for degenerate isospin doublets, while the vector piece must sum up to zero due to Furry's theorem.

Class $\mathbf{F_V}$ Only for the case of $V_1V_2 = W^+W^-$, one should also take into account the s-channel production diagrams, where the incoming gluons produce an intermediate electroweak gauge boson $V = \gamma^*/Z^*$, which then decays into the outgoing W-pair, see Fig. 1. Charge-parity invariance ensures that the vector part of these diagrams must sum up to zero. Again, the axial part cancels out for degenerate isospin doublets, and therefore also in the case of massless quarks running in the loops.

For the case of the one- and two-loop contributions considered here, we can therefore simplify (2.17) to

$$S_{\mu\nu}(p_1^{\lambda_1}, p_2^{\lambda_2}, p_3) = \delta^{a_1 a_2}\, N_{V_1V_2}\, S^{[A]}_{\mu\nu\rho\sigma}(p_1, p_2, p_3)\epsilon^{\rho}_{1\lambda_1}(p_1)\epsilon^{\sigma}_{2\lambda_2}(p_2) \,, \tag{2.24}$$

with $N_{V_1V_2}$ given in (2.23) and consider the coefficients $A_j^{[A]}$ defined by

$$A_j(s, t, p_3^2, p_4^2) = \delta_{a_1 a_2} N_{V_1V_2} A_j^{[A]}(s, t, p_3^2, p_4^2). \tag{2.25}$$

It is instructive to study the transformations of the partonic current (2.24) under permutations of the external legs. We define the following two permutations

$$\pi_{12} := p_1 \leftrightarrow p_2 \Rightarrow \{\, t \leftrightarrow u \,\},$$
$$\pi_{34} := p_3 \leftrightarrow p_4 \Rightarrow \{\, t \leftrightarrow u \,,\;\; p_3^2 \leftrightarrow p_4^2 \,\}. \tag{2.26}$$

Because of Bose symmetry these two permutations must leave the partonic amplitude unchanged. This enforces a well defined behaviour of the coefficients $A_j(s, t, p_3^2, p_4^2)$ under the action of π_{12} and π_{34}. From direct inspection of (2.13) one finds that the following

relations must be fulfilled:

$$\begin{aligned}
\pi_{12}: \quad & A_1^{[A]}(s,u,p_3^2,p_4^2) = A_1^{[A]}(s,t,p_3^2,p_4^2)\,, \qquad A_2^{[A]}(s,u,p_3^2,p_4^2) = A_3^{[A]}(s,t,p_3^2,p_4^2)\,, \\
& A_4^{[A]}(s,u,p_3^2,p_4^2) = A_7^{[A]}(s,t,p_3^2,p_4^2)\,, \qquad A_5^{[A]}(s,u,p_3^2,p_4^2) = A_6^{[A]}(s,t,p_3^2,p_4^2)\,, \\
& A_8^{[A]}(s,u,p_3^2,p_4^2) = A_{13}^{[A]}(s,t,p_3^2,p_4^2)\,, \qquad A_9^{[A]}(s,u,p_3^2,p_4^2) = A_{12}^{[A]}(s,t,p_3^2,p_4^2)\,, \\
& A_{10}^{[A]}(s,u,p_3^2,p_4^2) = A_{15}^{[A]}(s,t,p_3^2,p_4^2)\,, \qquad A_{11}^{[A]}(s,u,p_3^2,p_4^2) = A_{14}^{[A]}(s,t,p_3^2,p_4^2)\,, \\
& A_{16}^{[A]}(s,u,p_3^2,p_4^2) = A_{16}^{[A]}(s,t,p_3^2,p_4^2)\,, \qquad A_{17}^{[A]}(s,u,p_3^2,p_4^2) = A_{20}^{[A]}(s,t,p_3^2,p_4^2)\,, \\
& A_{18}^{[A]}(s,u,p_3^2,p_4^2) = A_{19}^{[A]}(s,t,p_3^2,p_4^2)\,,
\end{aligned} \tag{2.27}$$

$$\begin{aligned}
\pi_{34}: \quad & A_1^{[A]}(s,u,p_4^2,p_3^2) = A_1^{[A]}(s,t,p_3^2,p_4^2)\,, \qquad A_2^{[A]}(s,u,p_4^2,p_3^2) = A_3^{[A]}(s,t,p_3^2,p_4^2)\,, \\
& A_4^{[A]}(s,u,p_4^2,p_3^2) = A_4^{[A]}(s,t,p_3^2,p_4^2)\,, \qquad A_5^{[A]}(s,u,p_4^2,p_3^2) = A_6^{[A]}(s,t,p_3^2,p_4^2)\,, \\
& A_7^{[A]}(s,u,p_4^2,p_3^2) = A_7^{[A]}(s,t,p_3^2,p_4^2)\,, \qquad A_8^{[A]}(s,u,p_4^2,p_3^2) = -A_{10}^{[A]}(s,t,p_3^2,p_4^2)\,, \\
& A_9^{[A]}(s,u,p_4^2,p_3^2) = -A_{11}^{[A]}(s,t,p_3^2,p_4^2)\,, \qquad A_{12}^{[A]}(s,u,p_4^2,p_3^2) = -A_{14}^{[A]}(s,t,p_3^2,p_4^2)\,, \\
& A_{13}^{[A]}(s,u,p_4^2,p_3^2) = -A_{15}^{[A]}(s,t,p_3^2,p_4^2)\,, \qquad A_{16}^{[A]}(s,u,p_4^2,p_3^2) = A_{16}^{[A]}(s,t,p_3^2,p_4^2)\,, \\
& A_{17}^{[A]}(s,u,p_4^2,p_3^2) = A_{17}^{[A]}(s,t,p_3^2,p_4^2)\,, \qquad A_{18}^{[A]}(s,u,p_4^2,p_3^2) = A_{19}^{[A]}(s,t,p_3^2,p_4^2)\,, \\
& A_{20}^{[A]}(s,u,p_4^2,p_3^2) = A_{20}^{[A]}(s,t,p_3^2,p_4^2)\,,
\end{aligned} \tag{2.28}$$

It is interesting to notice that, upon exploiting all of these crossing relations, only 9 out of the 20 coefficients $A_j^{[A]}$ turn out to be effectively independent, while the other 11 coefficients can be obtained by crossing of the external legs.

3 Helicity amplitudes for $gg \to V_1V_2 \to 4$ leptons

We consider physical processes, where the two off-shell vector bosons decay into lepton pairs

$$g(p_1) + g(p_2) \to V_1(p_3) + V_2(p_4) \to l_5(p_5) + \bar{l}_6(p_6) + l_7(p_7) + \bar{l}_8(p_8) \tag{3.1}$$

such that $p_3 = p_5 + p_6\,,\ p_4 = p_7 + p_8$ and $p_5^2 = p_6^2 = p_7^2 = p_8^2 = 0$. As long as we consider QCD radiative corrections the amplitudes $\mathcal{M}^{V_1V_2}_{\lambda_1\lambda_2\lambda_3\lambda_4}$ can be written, at any order in perturbation theory, as the product of the partonic current for $gg \to V_1V_2$ with the two leptonic currents for the decay products, $V_1 \to l_5\bar{l}_6$ and $V_2 \to l_7\bar{l}_8$, mediated by the propagators of the two off-shell vector bosons V_1V_2. We write the propagator for an off-shell vector boson in the R_ξ gauge as

$$P^V_{\mu\nu}(q) = \frac{i\,\Delta^V_{\mu\nu}(q,\xi)}{D_V(q)}\,, \tag{3.2}$$

with

$$\Delta^V_{\mu\nu}(q,\xi) = \left(-g_{\mu\nu} + (1-\xi)\frac{q_\mu q_\nu}{q^2 - \xi m_V^2}\right)\,, \tag{3.3}$$

$$D_{\gamma^*}(q) = q^2\,, \qquad D_{Z,W}(q) = (q^2 - m_V^2 + i\,\Gamma_V m_V)\,, \tag{3.4}$$

where m_V is its mass and Γ_V is its decay width. In our case the massive vector bosons decay to massless fermions such that the term proportional to $(1-\xi)$ can be dropped.

In the following we will consider fixed helicities of the external particles and compute the amplitudes for the different helicity configurations. Since the decay leptons are massless, helicity is conserved along the leptonic decay currents and the amplitude can be written as

$$\mathcal{M}^{V_1V_2}_{\lambda_1\lambda_2\lambda_3\lambda_4}(p_1,p_2;p_5,p_6,p_7,p_8), \tag{3.5}$$

where λ_1 and λ_2 are the helicities of the incoming gluons, while λ_3 and λ_4 are the helicities of the two leptonic currents. It is clear that there are 16 different helicity configurations, depending on the different possibilities for the initial and final states. Each gluon has two possible helicity states, which we denote by L (-) and R (+), and similarly each leptonic current occurs in either left- or right-handed configuration, again denoted by L and R, respectively, such that $\lambda_j = L, R$, for $j = 1, ..., 4$. As we will show explicitly later on, all 16 helicity configurations can be obtained from only two independent ones, by simple permutations of the external legs and complex conjugation. We choose as independent configurations the following two

$$\mathcal{M}^{V_1V_2}_{LLLL}(p_1,p_2;p_5,p_6,p_7,p_8)\,, \qquad \mathcal{M}^{V_1V_2}_{LRLL}(p_1,p_2;p_5,p_6,p_7,p_8)\,. \tag{3.6}$$

With the notations introduced above we write the two independent helicity amplitudes (3.6), up to two loops, as:

$$\mathcal{M}^{V_1V_2}_{\lambda_1\lambda_2LL}(p_1,p_2;p_5,p_6,p_7,p_8) = i\,(4\pi\alpha)^2\,\frac{L^{V_1}_{f_5f_6}\,L^{V_2}_{f_7f_8}}{D_{V_1}(p_3)D_{V_2}(p_4)}\,\mathrm{M}_{\lambda_1\lambda_2LL}(p_1,p_2;p_5,p_6,p_7,p_8)\,, \tag{3.7}$$

where the *basic amplitudes* $\mathrm{M}_{\lambda_1\lambda_2LL}(p_1,p_2;p_5,p_6,p_7,p_8)$ are constructed from the partonic current (2.17) and the leptonic currents (3.9) according to

$$\mathrm{M}_{\lambda_1\lambda_2LL}(p_1,p_2;p_5,p_6,p_7,p_8) = \epsilon^{\rho}_{1\lambda_1}(p_1)\epsilon^{\sigma}_{2\lambda_2}(p_2)\,S_{\mu\nu\rho\sigma}(p_1,p_2,p_3)\,L^{\mu}_L(p_5^-,p_6^+)\,L^{\nu}_L(p_7^-,p_8^+)\,. \tag{3.8}$$

The leptonic decay currents do not receive any QCD corrections and are simple tree-level objects. They can be easily expressed in the usual spinor-helicity notation [41, 42] as

$$L^{\mu}_L(p_5^-,p_6^+) = \bar{u}_-(p_5)\,\gamma^{\mu}\,v_+(p_6) = [6\,|\gamma^{\mu}|\,5\,\rangle = \langle 5\,|\gamma^{\mu}|\,6\,]\,, \tag{3.9}$$

$$L^{\mu}_R(p_5^+,p_6^-) = \bar{u}_+(p_5)\,\gamma^{\mu}\,v_-(p_6) = [5\,|\gamma^{\mu}|\,6\,\rangle = \big(L^{\mu}_L(p_5^-,p_6^+)\big)^* = L^{\mu}_L(p_6^-,p_5^+)\,. \tag{3.10}$$

Note that, in this case, a permutation of the external momenta is equivalent to a complex conjugation of the current and it corresponds to a flip of the helicity $L \leftrightarrow R$.

Once the tensor decomposition of the partonic current is fixed, it is straight-forward to express the two basic helicity amplitudes M_{LLLL} and M_{LRLL} in (3.8) in the usual spinor-helicity notation [41, 42]. We replace the gluon polarisation vectors according to

$$\epsilon^{\mu}_{1L}(p_1) = \frac{[2|\gamma^{\mu}|1\rangle}{\sqrt{2}[12]}\,, \quad \epsilon^{\mu}_{1R}(p_1) = \frac{\langle 2|\gamma^{\mu}|1]}{\sqrt{2}\langle 21\rangle}\,, \quad \epsilon^{\mu}_{2L}(p_2) = \frac{[1|\gamma^{\mu}|2\rangle}{\sqrt{2}[21]}\,, \quad \epsilon^{\mu}_{2R}(p_2) = \frac{\langle 1|\gamma^{\mu}|2]}{\sqrt{2}\langle 12\rangle}\,, \tag{3.11}$$

which is of course compatible with the polarisation sums (2.10) and (2.12). In doing this, we assume that the external states can be treated as 4-dimensional and this allows to reduce considerably the number of independent structures that are required for parametrising a specific helicity configuration. Using (3.11) we find that both basic amplitudes can be written in terms of 9 independent spinor structures as

$$\begin{aligned}\mathrm{M}_{\lambda_1\lambda_2 LL}(p_1,p_2;p_5,p_6,p_7,p_8) = C_{\lambda_1\lambda_2}\Big\{ & [2\,\not{p}_3\,1\rangle\,\Big(E_1^{\lambda_1\lambda_2}\langle 57\rangle[68] \\ & + E_2^{\lambda_1\lambda_2}\,\langle 15\rangle\langle 17\rangle[16][18] + E_3^{\lambda_1\lambda_2}\,\langle 15\rangle\langle 27\rangle[16][28] \\ & + E_4^{\lambda_1\lambda_2}\,\langle 25\rangle\langle 17\rangle[26][18] + E_5^{\lambda_1\lambda_2}\,\langle 25\rangle\langle 27\rangle[26][28]\Big) \\ & + E_6^{\lambda_1\lambda_2}\,\langle 15\rangle\langle 17\rangle[16][28] + E_7^{\lambda_1\lambda_2}\,\langle 15\rangle\langle 17\rangle[26][18] \\ & + E_8^{\lambda_1\lambda_2}\,\langle 15\rangle\langle 27\rangle[26][28] + E_9^{\lambda_1\lambda_2}\,\langle 25\rangle\langle 17\rangle[26][28]\Big\}\,, \end{aligned} \tag{3.12}$$

where the 18 newly introduced form factors $E_j^{\lambda_1\lambda_2}$ are simple linear combinations of the scalar coefficients A_j. The spinor structure of the amplitudes for the configurations $LLLL$ and $LRLL$ differs only by an overall factor which reads in the two cases

$$C_{LL} = [1\,\not{p}_3\,2\rangle\frac{\langle 12\rangle}{[12]}\,, \qquad C_{LR} = [2\,\not{p}_3\,1\rangle\,, \tag{3.13}$$

but the form factors E_j^{LL} and E_j^{LR} are different. We also note, in passing, that the spinor structure of (3.12) exhibits also a *formal* similarity to that of the RLL amplitude for $q\bar{q}' \to V_1V_2 \to l_5\bar{l}_6l_7\bar{l}_8$ [37, 38], again up to an overall factor and with, of course, completely unrelated form factors. Similar as before, we also define the functions $E_j^{\lambda_1\lambda_2\,[A]}$

$$E_j^{\lambda_1\lambda_2}(s,t,p_3^2,p_4^2) = \delta_{a_1a_2}N_{V_1V_2}E_j^{\lambda_1\lambda_2\,[A]}(s,t,p_3^2,p_4^2). \tag{3.14}$$

The explicit expressions for the form factors $E_j^{\lambda_1\lambda_2}$ in terms of the coefficients A_j are given in Appendix A.

In order to obtain all 16 helicity amplitudes from (3.12), one should recall that complex conjugation has the effect of reversing the helicity of the external gluons,

$$\big(\epsilon_{1L}^{\mu}(p_1)\big)^* = \epsilon_{1R}^{\mu}(p_1)\,, \qquad \big(\epsilon_{2L}^{\mu}(p_2)\big)^* = \epsilon_{2R}^{\mu}(p_2)\,, \tag{3.15}$$

and similarly for the leptonic currents, see (3.10) (3.9). We define with the symbol $[...]^C$ a complex-conjugation operation which, when applied on the amplitudes $\mathrm{M}_{\lambda_1\lambda_2 LL}$, acts *only on the spinor structures*, i.e. leaves invariant the form factors $E_j^{\lambda_1\lambda_2}$. Given the explicit form of (3.12), it is easy to see that this corresponds to simply exchanging angle brackets with squared bracket and vice versa

$$[\mathrm{M}_{\lambda_1\lambda_2 LL}]^C \equiv \mathrm{M}_{\lambda_1\lambda_2 LL}\left(\langle ij\rangle \leftrightarrow [ij]\right)\,. \tag{3.16}$$

Hence, we can derive the missing helicity amplitudes for left-handed leptonic currents from the two basic amplitudes as

$$\begin{aligned}
\mathrm{M}_{RLLL}(p_1,p_2;p_5,p_6,p_7,p_8) &= [\mathrm{M}_{LRLL}(p_1,p_2;p_6,p_5,p_8,p_7)]^C\,,\\
\mathrm{M}_{RRLL}(p_1,p_2;p_5,p_6,p_7,p_8) &= [\mathrm{M}_{LLLL}(p_1,p_2;p_6,p_5,p_8,p_7)]^C\,,
\end{aligned} \tag{3.17}$$

where one should note that the lepton and anti-lepton momenta are exchanged in the r.h.s. in order to have a left-handed leptonic currents on the l.h.s. The corresponding formulae for the basic amplitudes for right-handed leptonic currents can be obtained from the ones above by simple permutations of the lepton and anti-lepton momenta

$$\begin{aligned}
\mathrm{M}_{\lambda_1\lambda_2 RL}(p_1,p_2;p_5,p_6,p_7,p_8) &= \mathrm{M}_{\lambda_1\lambda_2 LL}(p_1,p_2;p_6,p_5,p_7,p_8)\,,\\
\mathrm{M}_{\lambda_1\lambda_2 LR}(p_1,p_2;p_5,p_6,p_7,p_8) &= \mathrm{M}_{\lambda_1\lambda_2 LL}(p_1,p_2;p_5,p_6,p_8,p_7)\,,\\
\mathrm{M}_{\lambda_1\lambda_2 RR}(p_1,p_2;p_5,p_6,p_7,p_8) &= \mathrm{M}_{\lambda_1\lambda_2 LL}(p_1,p_2;p_6,p_5,p_8,p_7)\,.
\end{aligned} \tag{3.18}$$

With these formulae also all the 16 physical amplitudes $\mathcal{M}^{V_1V_2}_{\lambda_1\lambda_2\lambda_3\lambda_4}$ in (3.7) can be easily obtained, recalling that in the case of right-handed leptonic currents one should, of course, exchange the corresponding couplings $L^V_{f_if_j} \leftrightarrow R^V_{f_if_j}$.

As we already stated above, the partonic current receives contributions form QCD radiative corrections and it can be expanded as

$$S_{\mu\nu\rho\sigma}(p_1,p_2,p_3) = \left(\frac{\alpha_s}{2\pi}\right) S^{(1)}_{\mu\nu\rho\sigma}(p_1,p_2,p_3) + \left(\frac{\alpha_s}{2\pi}\right)^2 S^{(2)}_{\mu\nu\rho\sigma}(p_1,p_2,p_3) + \mathcal{O}(\alpha_s^3)\,, \tag{3.19}$$

where obviously the perturbative expansion starts only at one-loop order. Of course also the coefficients A_j, and equivalently the form factors $E_j^{\lambda_1\lambda_2}$, have the same expansion

$$\begin{aligned}
A_j &= \left(\frac{\alpha_s}{2\pi}\right) A_j^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 A_j^{(2)} + \mathcal{O}(\alpha_s^3)\,,\\
E_j^{\lambda_1\lambda_2} &= \left(\frac{\alpha_s}{2\pi}\right) E_j^{(1),\lambda_1\lambda_2} + \left(\frac{\alpha_s}{2\pi}\right)^2 E_j^{(2),\lambda_1\lambda_2} + \mathcal{O}(\alpha_s^3)\,.
\end{aligned} \tag{3.20}$$

4 Calculation of the form factors

The calculation of the coefficients $E_j^{\lambda_1\lambda_2}$ proceeds as follows. We produce all one- and two-loop Feynman diagrams relevant for $gg \to V_1V_2$ using `Qgraf` [43]. In particular we focus only on diagrams in classes A and B, for which we find 8 diagrams at one loop and 138 diagrams at two loops. Diagrams in class F_V, in fact, are simple three-point functions, which sum up to zero due to charge-parity invariance. The coefficients A_j are then calculated by applying the projectors defined in (2.15) on the different Feynman diagrams. We insert the Feynman rules in our diagrams, where we employ the Feynman-'t Hooft gauge ($\xi = 1$) for internal gluons. After evaluation of Dirac traces and contraction of Lorentz indices every Feynman diagram is expressed as linear combination of a large number of scalar integrals. The latter belong to the family of the massless four-point functions with two off-shell legs of different virtualities and can be reduced to a small set of master integrals using integration-by-parts identities [44–47]. We employ `Reduze 2` [48–51] to map all

scalar integrals to the three integral families given in [33] and their crossed versions, and subsequently to reduce them to master integrals. In this way, we obtain analytical expressions for the coefficients A_j as linear combinations of the latter. For the master integrals we employ the solutions presented in [37]. With the explicit expressions for the coefficients A_j at the different perturbative orders, it is easy to obtain the corresponding results for the $E_j^{\lambda_1\lambda_2}$ using the formulae given in Appendix A. `Form` [52] was used extensively for all intermediate algebraic manipulations.

Because of the lack of any tree-level contribution to the process $gg \to V_1V_2$, the UV and IR pole-structure of the one- and two-loop amplitudes is very simple. Clearly, the one-loop amplitude must be both UV- and IR-finite, and therefore the pole structure of the two-loop amplitude will be, *effectively*, what one usually encounters for a one-loop QCD amplitude. Since all considerations described in this section hold identically for any A_j and $E_i^{\lambda_1\lambda_2}$, in what follows we will use the symbol Ω for any of the scalar coefficients,

$$\Omega \in \left\{ A_j \,, E_i^{\lambda_1\lambda_2} \right\} \,, \qquad \text{for any} \quad j = 1, ..., 20, \quad i = 1, ..., 9, \quad \lambda_1\lambda_2 = LL, LR \,.$$

We start by performing UV-renormalisation in the $\overline{\text{MS}}$ scheme. In massless QCD this amounts to replacing the bare coupling, α_0, with the renormalised one, $\alpha_s = \alpha_s(\mu^2)$, where μ is the renormalisation scale. Here we only need the one-loop relation

$$\alpha_0 \, \mu_0^{2\epsilon} \, S_\epsilon = \alpha_s \, \mu^{2\epsilon} \left[1 - \frac{\beta_0}{\epsilon} \left(\frac{\alpha_s}{2\pi} \right) + \mathcal{O}(\alpha_s^2) \right] \,, \tag{4.1}$$

where

$$S_\epsilon = (4\pi)^\epsilon \, \mathrm{e}^{-\epsilon\gamma} \,, \qquad \text{with the Euler-Mascheroni constant} \quad \gamma = 0.5772... \,, \tag{4.2}$$

$\epsilon = (4-d)/2$, μ_0 is the mass-parameter introduced in dimensional regularisation to maintain a dimensionless coupling in the bare QCD Lagrangian density, and finally β_0 is the first order of the QCD β-function

$$\beta_0 = \frac{11\, C_A - 4\, T_F \, N_f}{6} \,, \quad \text{with} \quad C_A = N \,, \quad C_F = \frac{N^2-1}{2\,N} \,, \quad T_F = \frac{1}{2} \,. \tag{4.3}$$

The renormalisation is performed at $\mu^2 = s$, the invariant mass squared of the vector-boson pair. The renormalised form factors read then, in terms of the un-renormalised ones,

$$\begin{aligned} \Omega^{(1)} &= S_\epsilon^{-1} \Omega^{(1),\text{un}}, \\ \Omega^{(2)} &= S_\epsilon^{-2} \Omega^{(2),\text{un}} - \frac{\beta_0}{\epsilon} S_\epsilon^{-1} \Omega^{(1),\text{un}} \,. \end{aligned} \tag{4.4}$$

After UV renormalisation, the two-loop coefficients $\Omega^{(2)}$ contain still residual IR singularities. In any IR-safe observable these divergences are cancelled by the corresponding ones produced in one-loop radiative processes with one more external parton. In the present case of $gg \to V_1V_2$, as discussed already above, the IR-poles at two loops are of NLO type and their structure has been know for a long time. Here, we choose to follow the conventions

used for the NNLO corrections to $q\bar{q} \to V_1 V_2$ in [37], which required a NNLO subtraction scheme. The exact structure of the IR poles up to NNLO in QCD was predicted first by Catani [53]. We present our results in a slightly modified scheme described in [54], which is well suited for the q_T-subtraction formalism.

We define the IR finite amplitudes at renormalisation scale μ in terms of the UV renormalised ones as follows

$$\begin{aligned}\Omega_{q_T}^{(1),\text{finite}} &= \Omega^{(1)}\,,\\ \Omega_{q_T}^{(2),\text{finite}} &= \Omega^{(2)} - I_1(\epsilon)\,\Omega^{(1)}\,,\end{aligned} \tag{4.5}$$

where for the gluon-fusion channel we have

$$I_1(\epsilon) = I_1^{soft}(\epsilon) + I_1^{coll}(\epsilon)\,, \tag{4.6}$$

$$I_1^{soft}(\epsilon) = -\frac{\mathrm{e}^{\epsilon\gamma}}{\Gamma(1-\epsilon)}\left(\frac{\mu^2}{s}\right)^{\epsilon}\left(\frac{1}{\epsilon^2} + \frac{i\pi}{\epsilon} + \delta_{q_T}^{(0)}\right) C_A\,, \tag{4.7}$$

$$I_1^{coll}(\epsilon) = -\frac{1}{\epsilon}\beta_0\left(\frac{\mu^2}{s}\right)^{\epsilon}. \tag{4.8}$$

Following [54] we then put $\delta_{q_T}^{(0)} = 0$. We provide the explicit analytical results for the finite remainders of the coefficients A_j in this scheme, obtained for $\mu^2 = s$, on our project page at `HepForge`.

Finally, it is straight-forward to convert these finite remainders into the Catani's original subtraction scheme [53], as extensively described in [37]. For the present case we obtain the conversion formulae

$$\begin{aligned}\Omega_{\text{Catani}}^{(1),\text{finite}} &= \Omega_{q_T}^{(1),\text{finite}},\\ \Omega_{\text{Catani}}^{(2),\text{finite}} &= \Omega_{q_T}^{(2),\text{finite}} + \Delta I_1\,\Omega_{q_T}^{(1),\text{finite}},\end{aligned} \tag{4.9}$$

with ΔI_1, in the case of a gg initial state, is given by

$$\Delta I_1 = -\frac{1}{2}\pi^2 C_A + i\pi\beta_0\,. \tag{4.10}$$

In order to test the correctness of our results we have performed a number of checks, which we list in the following.

1. First of all, we computed explicitly all one- and two-loop diagrams relevant for $gg \to V_1 V_2$, including those diagrams in class B which are expected not to give any contribution due to Furry's theorem, see Section 4. We have verified that, after reduction to master integrals, all diagrams in class B sum up to zero.

2. We have verified explicitly that the coefficients A_j respects the expected symmetry relations derived in (2.27) and (2.28).

3. We have verified explicitly that the IR poles of the two-loop amplitude have the structure predicted by Catani's formula, see Section 4. This provides a strong check of the correctness of the result.

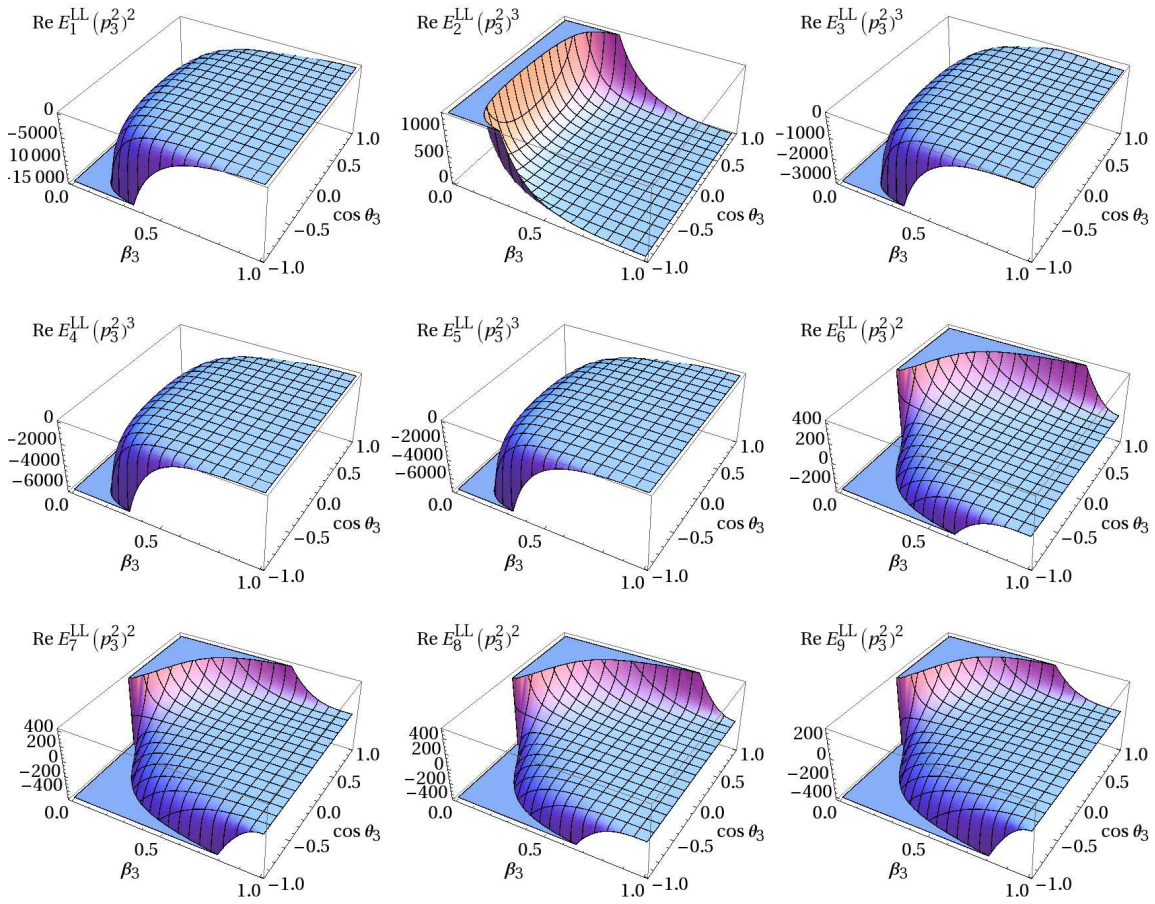

Figure 2. Real parts of the two loop form factors $E_j^{(2),LL\,[A]}$ for the process $gg \to V_1V_2$. The plots illustrate their dependence on the velocity, β_3, and the cosine of the scattering angle, $\cos\theta_3$, of the vector boson V_1, where $p_4^2 = 2p_3^2$ is chosen for the vector boson virtualities.

4. We have performed a thorough comparison of our results with an independent calculation of the same process [55]. Specifically, we compared our results prior to UV renormalisation and IR subtraction. While the representation of the amplitudes in terms of spinor structures in [55] has a different form than our decomposition (3.12), we found that both are equivalent. For the full helicity amplitudes we have found perfect numerical agreement at one- and two-loop order. Moreover, expressing the form factors defined in [55] as linear combinations of our form factors $E_j^{\lambda_1\lambda_2}$, we have verified that for each of them independently we have perfect numerical agreement at one- and two-loop order.

5 Numerical C++ implementation and results

For the numerical evaluation of the helicity amplitudes for $gg \to V_1V_2 \to 4$ leptons, we implemented our results for the form factors $E_j^{\lambda_1\lambda_2\,[A]}$ and $A_j^{[A]}$ at one- and two-loop order in a dedicated C++ code. The implementation is based on the solutions for the master inte-

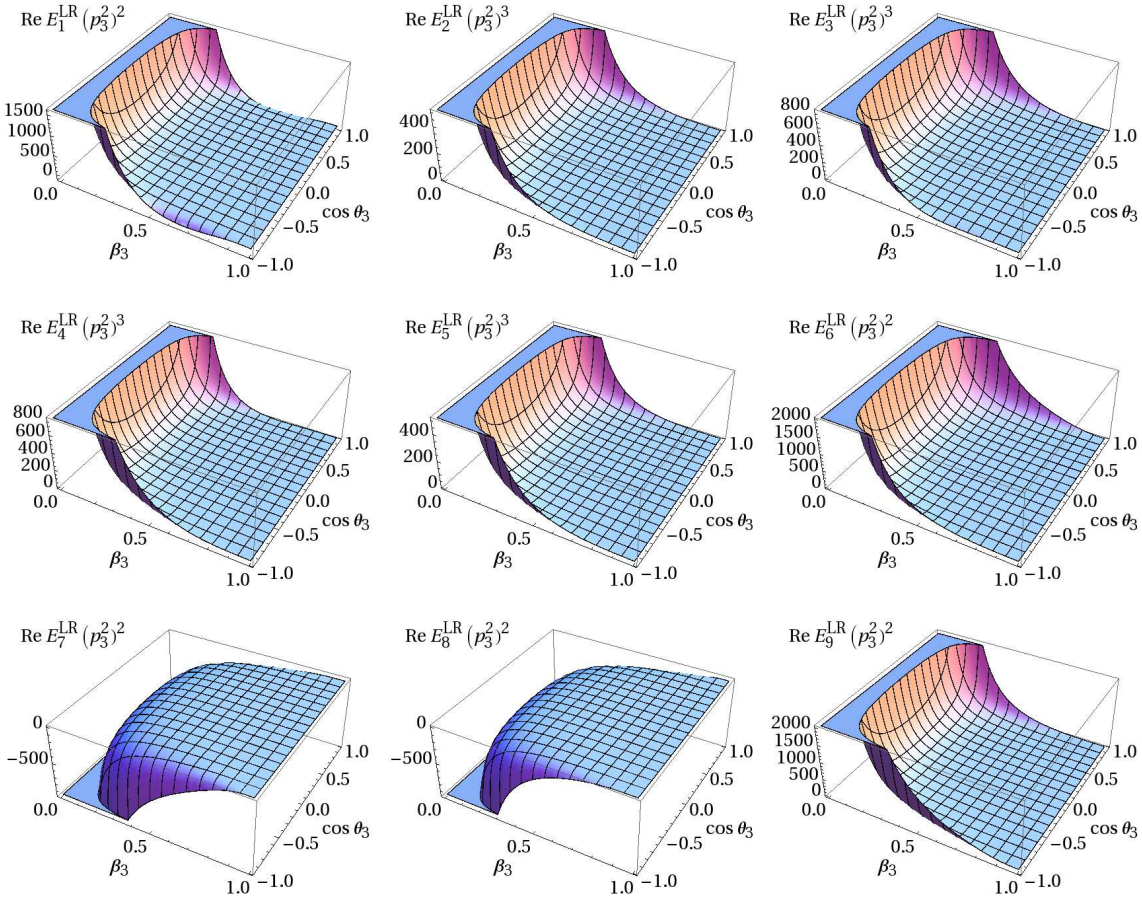

Figure 3. Real parts of the two loop form factors $E_j^{(2),LR\,[A]}$ for the process $gg \to V_1V_2$. The plots illustrate their dependence on the velocity, β_3, and the cosine of the scattering angle, $\cos\theta_3$, of the vector boson V_1, where $p_4^2 = 2p_3^2$ is chosen for the vector boson virtualities.

grals presented in [37], which were specifically constructed for fast and reliable numerical evaluations. We organised our form factor implementation in form of a library, which is supplemented by a simple command line interface. We provide the software package for public download on `HepForge` at `http://vvamp.hepforge.org`.

For the numerical evaluation of the multiple polylogarithms encountered in the solutions for the master integrals, we employ their implementation [56] in the `GiNaC` [50] library. To identify and account for possible numerical instabilities of the form factors in collinear or other potentially problematic regions of phase space, the code compares numerical evaluations, which are obtained using different floating point data types, similar to the setup used in [37]. If the results obtained with different precision settings differ beyond a user-defined tolerance, the code successively increases the precision until the target precision is met. For the rather central benchmark point of [38], the double precision mode of our code takes roughly $600ms$ on a single computer core and results in at least 11 significant digits for all of the $E_j^{\lambda_1\lambda_2\,[A]}$. In order to estimate the actual precision, the default behaviour of our code is to reevaluate the algebraic expressions in quad precision, which results in a

total run-time of roughly $3s$ for this phase space point. The described version of our code implements a minimal set of 9 coefficients A_j and employs four evaluations of them with different kinematics in order to derive the remaining form factors using crossing relations. If required, it is straight-forward to further improve the evaluation speed, either by proper caching of multiple polylogarithms or, at the price of an increased code size, by an explicit implementation of all form factors, as we did for the process $q\bar{q}' \to V_1V_2$ in [37].

In order to illustrate the form factors and the reliability of the code, we used the latter to plot the real part of the two-loop form factors for the case $p_4^2 = 2p_3^2$ in Figures 2 and 3. In the plots, we vary the relativistic velocity β_3 and the cosine of the scattering angle $\cos\theta_3$ of the vector boson V_1, where $\beta_3 = \kappa/(s+p_3^2-p_4^2)$ and $\cos\theta_3 = (2t+s-p_3^2-p_4^2)/\kappa$. Compared to the results for the form factors E_j in the process $q\bar{q}' \to V_1V_2$ in [37], we observe strong enhancements for the forward, backward and production threshold regions for the form factors in the present case. However, for the physical helicity amplitudes (3.12) we wish to point out that an additional dampening (very) close to the aforementioned phase space boundaries should be taken into account due to the additional overall factors C_{LL} and C_{LR} (3.13).

6 Conclusions

In this paper we computed the two-loop massless QCD corrections to the helicity amplitudes for the production of pairs of electroweak gauge bosons, V_1V_2, in the gluon fusion channel. For the calculation we employed the solutions for the master integrals presented in [37]. Contracting the diboson amplitude with the leptonic decay currents we have constructed the helicity amplitudes for $gg \to V_1V_2 \to 4$ leptons. We have compared our results to an independent calculation [55] and find perfect agreement. Our results for these amplitudes provide the fundamental ingredient required to compute the NLO corrections to diboson production processes in gluon fusion. These corrections would contribute formally at N^{3}LO to the processes $pp \to V_1V_2 + X$, but their inclusion may be important to match the expected experimental accuracy due to the large gluon luminosity at the LHC. In particular studying their impact is required to obtain a more reliable estimate of the theory uncertainty and to establish more precise constraints on the total Higgs decay width [19–21]. We provide both analytical results and a `C++` code for the numerical evaluation of the amplitudes on `HepForge` at `http://vvamp.hepforge.org`.

Acknowledgements

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A Form factor relations

In this Appendix we present the explicit formulae needed in order to compute the 18 form factors $E_j^{\lambda_1\lambda_2}$ defined for the amplitude (3.12), starting from the 20 form factors A_j defined in (2.13). For the M_{LLLL} amplitude we find

$$E_1^{LL} = \frac{2A_1 + A_2 + A_3}{t\,u - p_3^2\,p_4^2} - \frac{A_{16}}{s}\,,$$

$$E_2^{LL} = \frac{A_{14}(t-p_4^2) - A_{12}(u-p_3^2) - s\,A_4}{s(t\,u - p_3^2\,p_4^2)} + \frac{A_{17}}{2\,s}\,,$$

$$E_3^{LL} = \frac{A_{14}(u-p_4^2) - A_{13}(u-p_3^2) + A_2 + A_3 - s\,A_5}{s(t\,u - p_3^2\,p_4^2)} + \frac{A_{18}}{2\,s}\,,$$

$$E_4^{LL} = \frac{A_{15}(t-p_4^2) - A_{12}(t-p_3^2) + A_2 + A_3 - s\,A_6}{s(t\,u - p_3^2\,p_4^2)} + \frac{A_{19}}{2\,s}\,,$$

$$E_5^{LL} = \frac{A_{15}(u-p_4^2) - A_{13}(t-p_3^2) - s\,A_7}{s(t\,u - p_3^2\,p_4^2)} + \frac{A_{20}}{2\,s}\,,$$

$$E_6^{LL} = \frac{(u-p_3^2)(A_2-A_3)}{s(t\,u - p_3^2\,p_4^2)} + \frac{A_{10}-A_{14}}{s}\,, \qquad E_7^{LL} = \frac{(t-p_4^2)(A_2-A_3)}{s(t\,u - p_3^2\,p_4^2)} + \frac{A_8-A_{12}}{s}\,,$$

$$E_8^{LL} = \frac{(u-p_4^2)(A_2-A_3)}{s(t\,u - p_3^2\,p_4^2)} + \frac{A_9-A_{13}}{s}\,, \qquad E_9^{LL} = \frac{(t-p_3^2)(A_2-A_3)}{s(t\,u - p_3^2\,p_4^2)} + \frac{A_{11}-A_{15}}{s}\,. \tag{A.1}$$

For the M_{LRLL} amplitude we have instead

$$E_1^{LR} = \frac{A_2 + A_3}{t\,u - p_3^2\,p_4^2} + \frac{A_{16}}{s}\,, \qquad E_2^{LR} = -\frac{A_{17}}{2\,s}\,,$$

$$E_3^{LR} = \frac{A_2 + A_3}{s(t\,u - p_3^2\,p_4^2)} - \frac{A_{18}}{2\,s}\,, \qquad E_4^{LR} = \frac{A_2 + A_3}{s(t\,u - p_3^2\,p_4^2)} - \frac{A_{19}}{2\,s}\,,$$

$$E_5^{LR} = -\frac{A_{20}}{2\,s}\,, \qquad E_6^{LR} = \frac{(u-p_3^2)(A_2+A_3)}{s(t\,u - p_3^2\,p_4^2)} - \frac{A_{10}+A_{14}}{s}\,,$$

$$E_7^{LR} = -\frac{(t-p_4^2)(A_2+A_3)}{s(t\,u - p_3^2\,p_4^2)} - \frac{A_8+A_{12}}{s}\,, \qquad E_8^{LR} = -\frac{(u-p_4^2)(A_2+A_3)}{s(t\,u - p_3^2\,p_4^2)} - \frac{A_9+A_{13}}{s}\,,$$

$$E_9^{LR} = \frac{(t-p_3^2)(A_2+A_3)}{s(t\,u-p_3^2\,p_4^2)} - \frac{A_{11}+A_{15}}{s}\,. \tag{A.2}$$

Let us consider the behaviour of the form factors $E_j^{\lambda_1\lambda_2}$ under the two permutations π_{12} and π_{34} defined in (2.26). Using the crossing relations for the form factors $A_j^{[A]}$ (2.27) and (2.28) one easily finds the corresponding ones for the form factors $E_j^{\lambda_1\lambda_2\,[A]}$. To simplify our notation, we drop the superscript $[A]$ in the following. Under permutation π_{12} we obtain

$$\begin{aligned}
&E_1^{LL}(s,u,p_3^2,p_4^2) = E_1^{LL}(s,t,p_3^2,p_4^2)\,,\\
&E_2^{LL}(s,u,p_3^2,p_4^2) = E_5^{LL}(s,t,p_3^2,p_4^2) + \frac{(u-p_4^2)E_9^{LL}(s,t,p_3^2,p_4^2) - (t-p_3^2)E_8^{LL}(s,t,p_3^2,p_4^2)}{tu-p_3^2\,p_4^2}\,,\\
&E_3^{LL}(s,u,p_3^2,p_4^2) = E_4^{LL}(s,t,p_3^2,p_4^2) + \frac{(t-p_4^2)E_9^{LL}(s,t,p_3^2,p_4^2) - (t-p_3^2)E_7^{LL}(s,t,p_3^2,p_4^2)}{tu-p_3^2\,p_4^2}\,,\\
&E_6^{LL}(s,u,p_3^2,p_4^2) = -E_9^{LL}(s,t,p_3^2,p_4^2)\,, \qquad E_7^{LL}(s,u,p_3^2,p_4^2) = -E_8^{LL}(s,t,p_3^2,p_4^2)
\end{aligned} \tag{A.3}$$

and

$$\begin{aligned}
&E_1^{LR}(s,u,p_3^2,p_4^2) = E_1^{LR}(s,t,p_3^2,p_4^2)\,, \qquad E_2^{LR}(s,u,p_3^2,p_4^2) = E_5^{LR}(s,t,p_3^2,p_4^2)\,,\\
&E_3^{LR}(s,u,p_3^2,p_4^2) = E_4^{LR}(s,t,p_3^2,p_4^2)\,, \qquad E_6^{LR}(s,u,p_3^2,p_4^2) = E_9^{LR}(s,t,p_3^2,p_4^2)\,,\\
&E_7^{LR}(s,u,p_3^2,p_4^2) = E_8^{LR}(s,t,p_3^2,p_4^2)\,.
\end{aligned} \tag{A.4}$$

Under permutation π_{34}, instead, the form factors for both the LL and LR helicity configurations transform in the same way

$$\begin{aligned}
&E_1^{\lambda_1\lambda_2}(s,u,p_4^2,p_3^2) = E_1^{\lambda_1\lambda_2}(s,t,p_3^2,p_4^2)\,, \qquad E_2^{\lambda_1\lambda_2}(s,u,p_4^2,p_3^2) = E_2^{\lambda_1\lambda_2}(s,t,p_3^2,p_4^2)\,,\\
&E_3^{\lambda_1\lambda_2}(s,u,p_4^2,p_3^2) = E_4^{\lambda_1\lambda_2}(s,t,p_3^2,p_4^2)\,, \qquad E_5^{\lambda_1\lambda_2}(s,u,p_4^2,p_3^2) = E_5^{\lambda_1\lambda_2}(s,t,p_3^2,p_4^2)\,,\\
&E_6^{\lambda_1\lambda_2}(s,u,p_4^2,p_3^2) = -E_7^{\lambda_1\lambda_2}(s,t,p_3^2,p_4^2)\,, \quad E_8^{\lambda_1\lambda_2}(s,u,p_4^2,p_3^2) = -E_9^{\lambda_1\lambda_2}(s,t,p_3^2,p_4^2)\,.
\end{aligned} \tag{A.5}$$

Exploiting all of these crossing relations we find that only 9 out of the 18 form factors $E_j^{\lambda_1\lambda_2}$ are effectively independent, while the other 9 can be obtained by the crossing rules above. The number of independent form factors $E_j^{\lambda_1\lambda_2}$ coincides with the number of independent form factors A_j found in Section 2.

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