The Strong CP Problem and Discrete Symmetries

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We discuss a possible solution to the strong CP problem which is based on spontaneous CP violation and discrete symmetries. At the same time we predict in a simple way the almost right-angled quark unitarity triangle angle ($\alpha \simeq 90°$) by making the entries of the quark mass matrices either real or imaginary. To prove the viability of our strategy we present a toy flavour model for the quark sector.

Keywords: Strong CP Problem; Discrete Symmetries; CP Violation; Flavour Models.

1. Motivation

It is fair to say that quantum chromodynamics (QCD) has emerged as the well-established theory of strong interactions. However, there are still puzzles about the strong interactions. One of them is the smallness of CP violation. Already in the 1970s it was realised that the QCD Lagrangian can violate CP due to instanton effects[1,2] which is described by the strong phase

$$\bar{\theta} = \theta + \arg\det(M_u M_d) \,, \tag{1}$$

where θ is the coefficient of $\alpha_s/(8\pi)\tilde{G}_{\mu\nu}G^{\mu\nu}$, $G_{\mu\nu}$ is the field strength tensor of QCD, $\tilde{G}_{\mu\nu}$ its dual, and $\arg\det(M_u M_d)$ is the anomalous contribution from the quark masses. While θ and $\arg\det(M_u M_d)$ are transformed into each other via a chiral transformation, the combination $\bar{\theta}$ stays invariant. Experiments put stringent bounds on $\bar{\theta} \lesssim 10^{-11}$, see Ref. 3,4, which is much smaller than the Jarlskog invariant, $J = \left(2.96^{+0.20}_{-0.16}\right) \times 10^{-5}$, see Ref. 3. Therefore, the essence of the strong CP problem is the question why the two contributions to $\bar{\theta}$ sum up to such a small number.

There are three main ideas put forward to explain the smallness of $\bar{\theta}$. The first and simplest solution is that one of the quarks is massless[2]. In this case the strong CP phase $\bar{\theta}$ is unphysical. However, recent data strongly suggests that all quarks are massive[3].

The second popular solution is the so-called axion[5] where $\bar{\theta}$ is promoted to a dynamical degree of freedom which is set to small values by a potential. This solution is very elegant but albeit there have been extensive searches for axions there have been no convincing experimental hints for their existence so far[3].

The third approach solves the strong CP problem by breaking parity (or CP) spontaneously. Then on the fundamental level the term $\alpha_s/(8\pi)\tilde{G}_{\mu\nu}G^{\mu\nu}$, which violates parity as well as CP, is forbidden by either parity and/or CP, which are assumed to be fundamental symmetries. For a short overview and more references, see Ref. 6, where we introduce the class of models discussed here.

As we discuss in the next section, where we outline our strategy, our class of models is based on a sum rule for the phases in the CKM matrix[7] suggesting a simple structure for quark mass matrices with either real or purely imaginary elements[8]. For an alternative class of textures models see, for instance, Refs. 9–13. As we will see our structure is realised in a simple manner in flavour models based on discrete symmetries where the CP symmetry is spontaneously broken using a method dubbed discrete vacuum alignment[14]. This method was used as well in various flavour models[15,16] which nevertheless usually put a stronger focus on the lepton sector.

2. The Strategy

If CP is a fundamental symmetry of the Lagrangian, the strong CP phase $\bar{\theta}$ vanishes on the fundamental level. However, in order to explain CP violation in weak interactions, CP has to be broken spontaneously. And this has to be done in a controlled way to keep the strong CP phase $\bar{\theta}$ at least tiny enough to be in agreement with experimental data.

In our class of models[6] we have quark mass matrices with $\arg\det(M_u M_d) = 0$ but still the value for the CKM phase is realistic. Furthermore, we disfavour unnatural cancellations between the phases in the up-type and the down-type quark sector. Hence, $\det M_u$ and $\det M_d$ should be real (and positive) by itself already.

One possible choice is, for instance, that M_u is completely real and has a negligible 1-3 mixing (1-3 element), and that

$$M_d = \begin{pmatrix} 0 & * & 0 \\ * & \mathrm{i} * & * \\ 0 & 0 & * \end{pmatrix} \ , \tag{2}$$

where $'*'$ are arbitrary but real entries. The only non-trivial complex phase appears in the purely imaginary 2-2 element of M_d. Then the determinants of both mass matrices are real.

This structure of the mass matrices can be realised from the spontaneous breaking of CP and we indeed have a solution for the strong CP problem as we will show in the following. And furthermore this very simple structure can also correctly reproduce the right quark unitarity triangle, as it was demonstrated in Ref. 7, since it satisfies the phase sum rule

$$\alpha \approx \delta_{12}^d - \delta_{12}^u \approx 90^\circ \ , \tag{3}$$

where α is the angle of the CKM unitarity triangle measured to be close to 90°[3] and $\delta_{12}^{d/u}$ are the phases of the complex 1-2 mixing angles diagonalising the quark mass

matrices (for the conventions used, see Ref. 7). Now any model, which generates such a structure could do the trick, and in the following we will discuss one possible example.

Suppose we have a (discrete, non-Abelian) family symmetry G_F with triplet representations (we use as an example A_4, but S_4, T', $\Delta(27)$, etc. would work equally well). See Ref. 17 for a recent review on family symmetries. In our toy model we assume the right-handed down-type quarks to transform as triplets under G_F while all other quarks are singlets. Then the rows of M_d are proportional to the vacuum expectation values (vevs) of family symmetry breaking Higgs fields, so-called flavon fields, which are triplets under G_F as well. M_u is generated by vevs of singlet flavon fields.

We introduce four flavon triplets with the following alignments in flavour space

$$\langle\phi_1\rangle \sim \begin{pmatrix}1\\0\\0\end{pmatrix} \ , \quad \langle\phi_2\rangle \sim \begin{pmatrix}0\\1\\0\end{pmatrix} \ , \quad \langle\phi_3\rangle \sim \begin{pmatrix}0\\0\\1\end{pmatrix} \ , \quad \langle\tilde{\phi}_2\rangle \sim \mathrm{i} \begin{pmatrix}0\\1\\0\end{pmatrix} \ , \tag{4}$$

where we have explicitly shown the phases and which can be achieved by standard vacuum alignment techniques. Note that only $\tilde{\phi}_2$ has a complex (imaginary) vev.

To fix the phases of these vevs we use the method described in Ref. 14, which we want to sketch here for a singlet flavon field ξ. Suppose ξ is charged under a discrete Z_n symmetry and apart from that neutral then we can write down a superpotential for ξ

$$\mathcal{W} = P\left(\frac{\xi^n}{\Lambda^{n-2}} \mp M^2\right) \ , \tag{5}$$

where P is a total singlet and M and Λ mass parameters. We have dropped couplings for brevity and since we assume fundamental CP symmetry the couplings and the mass parameters are real.[a] For the scalar potential for ξ we find

$$V = |F_P|^2 = \left|\frac{\xi^n}{\Lambda^{n-2}} \mp M^2\right|^2 . \tag{6}$$

and since $|F_P| \stackrel{!}{=} 0$ the vev of ξ has to satisfy

$$\langle\xi^n\rangle = \pm\,\Lambda^{n-2} M^2 \ . \tag{7}$$

This means

$$\arg(\langle\xi\rangle) = \begin{cases} \frac{2\pi}{n} q \ , \quad q = 1, \ldots, n & \text{for ``}-\text{'' in Eq. (6)}, \\ \frac{2\pi}{n} q + \frac{\pi}{n} \ , \quad q = 1, \ldots, n & \text{for ``}+\text{'' in Eq. (6)}. \end{cases} \tag{8}$$

Here the phases of the vevs do not depend on potential parameters, a situation which has been dubbed 'calculable phases' in the literature[20]. In Ref. 19 this was understood as the result of an accidental CP symmetry of the potential.

[a] Note that we use the generalised CP transformation, which is trivial with respect to A_4. It agrees with the ordinary CP transformation for real representations of A_4. See Refs. 18,19 for a recent discussion of generalised CP in the context of non-Abelian discrete symmetries.

Due to the stringent constraints on $\bar{\theta}$, special care needs to be taken with possible corrections to this parameter. The most important corrections are:

(1) Higher dimensional operators in the superpotential that could spoil the structure of the mass matrices and hence generate a non-vanishing $\arg\det(M_u M_d)$.
(2) Corrections which are induced from the soft SUSY breaking terms.

Here we are only going to touch the first point. For the second point we refer to the discussion in Ref. 6.

3. The Model

In this section we briefly sketch the toy model presented in Ref. 6 which serves as a proof that the strategy outlined before can be realized in an explicit model controlling higher dimensional operators in the superpotential.

As gauge symmetry we stick to the Standard Model gauge group and impose CP to be a fundamental symmetry. We choose here as non-Abelian discrete family symmetry A_4 which is frequently used in flavour model building, since it allows to readily realise the observed large lepton mixing (which we will not consider here) and since it is the smallest discrete group with triplet representations. To avoid unwanted operators and to implement the discrete vacuum alignment mechanism we have additionally the shaping symmetry $Z_4^5 \times Z_2 \times U(1)_R$. The family symmetry is broken by the ϕ_i, $i = 1, 2, 3$, and $\tilde{\phi}_2$ which are triplets under A_4, cf. Eq. (4). Additionally there are five singlet flavons ξ_i, $i = u, c, t, d, s$, which all receive real vevs.

To arrange for the flavon vev configuration to be dynamically realised along the lines outlined in Sec. 2, additional symmetries and fields have to be introduced. This discussion is somewhat lengthy and technical such that we will skip the detailed discussion of this nevertheless important ingredient. The interested reader can find the full superpotential to align the flavon vevs in Ref. 6.

Instead we want to discuss in somewhat more detail the couplings of the flavons to the matter sector and the corrections from higher-dimensional effective operators. After symmetry breaking, the mass matrices will be generated by the superpotential (remember that the right-handed down-type quarks form A_4 triplets while all other matter fields are A_4 singlets)

$$\mathcal{W}_d = Q_1 \bar{d} H_d \frac{\phi_2 \xi_d}{\Lambda^2} + Q_2 \bar{d} H_d \frac{\phi_1 \xi_d + \tilde{\phi}_2 \xi_s + \phi_3 \xi_t}{\Lambda^2} + Q_3 \bar{d} H_d \frac{\phi_3}{\Lambda} \,, \tag{9}$$

$$\mathcal{W}_u = Q_1 \bar{u}_1 H_u \frac{\xi_u^2}{\Lambda^2} + Q_1 \bar{u}_2 H_u \frac{\xi_u \xi_c}{\Lambda^2} + Q_2 \bar{u}_2 H_u \frac{\xi_c}{\Lambda} + (Q_2 \bar{u}_3 + Q_3 \bar{u}_2) H_u \frac{\xi_t}{\Lambda} + Q_3 \bar{u}_3 H_u \,, \tag{10}$$

which results from integrating out the heavy messenger fields and where we dropped couplings for the sake of brevity. Trivial A_4 contractions are not explicitly shown[b]

[b]The only non-trivial contraction is between $\bar{d}$ and the ϕ_i, which form a singlet contracted by the

and Λ denotes a generic messenger scale which is larger than the family symmetry breaking scale M_F.

Replacing Higgs and flavon fields with their respective vevs we find the following quark mass matrices

$$M_d = \begin{pmatrix} 0 & b_d & 0 \\ b'_d \,\mathrm{i}\, c_d & d_d \\ 0 & 0 & e_d \end{pmatrix} \qquad \text{and} \qquad M_u = \begin{pmatrix} a_u & b_u & 0 \\ 0 & c_u & d_u \\ 0 & d'_u & e_u \end{pmatrix} \ . \tag{11}$$

where we use the left-right convention $-\mathcal{L} = \overline{u^i_L}(M_u)_{ij}u^j_R + \overline{d^i_L}(M_d)_{ij}d^j_R +$ H.c.. Note that due to the fundamental CP symmetry and its peculiar breaking pattern, all entries are real apart from the 2-2 element of M_d. As discussed before in the strategy section, it predicts the right quark unitarity triangle[7] in terms of a phase sum rule

$$\alpha \approx \delta^d_{12} - \delta^u_{12} \approx 90^\circ \ , \tag{12}$$

where the angle α of the CKM unitarity triangle is close to 90°[3].

In this toy model, we concentrate on the explanation of CP violation in strong and weak interactions. Therefore, we are content with the prediction of the smallness of the strong CP phase and the correct CP phase in the CKM matrix. We are able to fit all masses and mixing angles, cf. Ref. 7. A more realistic model should obviously aim at predicting the masses and mixing angles as well, which happens quite naturally in a GUT context, for instance. In fact, a similar texture has been obtained in a GUT based model in Ref. 16, which could solve the strong CP problem as well.

We sketch now the UV completion of our toy model which justifies completely the effective operators we have given before. We will furthermore discuss all higher-dimensional operators which give corrections to the mass matrices and to the flavon alignment. They will not alter the structure of the mass matrices and hence our conclusions remain unchanged.

We will not go through the details of the full renormalisable superpotential here, which can be found in Ref. 6. The relevant point is that the symmetries and the chosen field content allow only for certain higher-dimensional operators depicted by their respective supergraphs in Fig. 1. After the heavy messenger fields are integrated out we end up first of all with the leading operators which we needed to get the right flavon alignment and the right quark mass matrices.

Beyond those operators we did not find any higher-dimensional operators produced at tree-level that would contribute to the down-type quark sector. In contrast, for the up-type quarks there are some additional operators allowed which give (real)

$SO(3)$-type inner product '$\cdot$'.

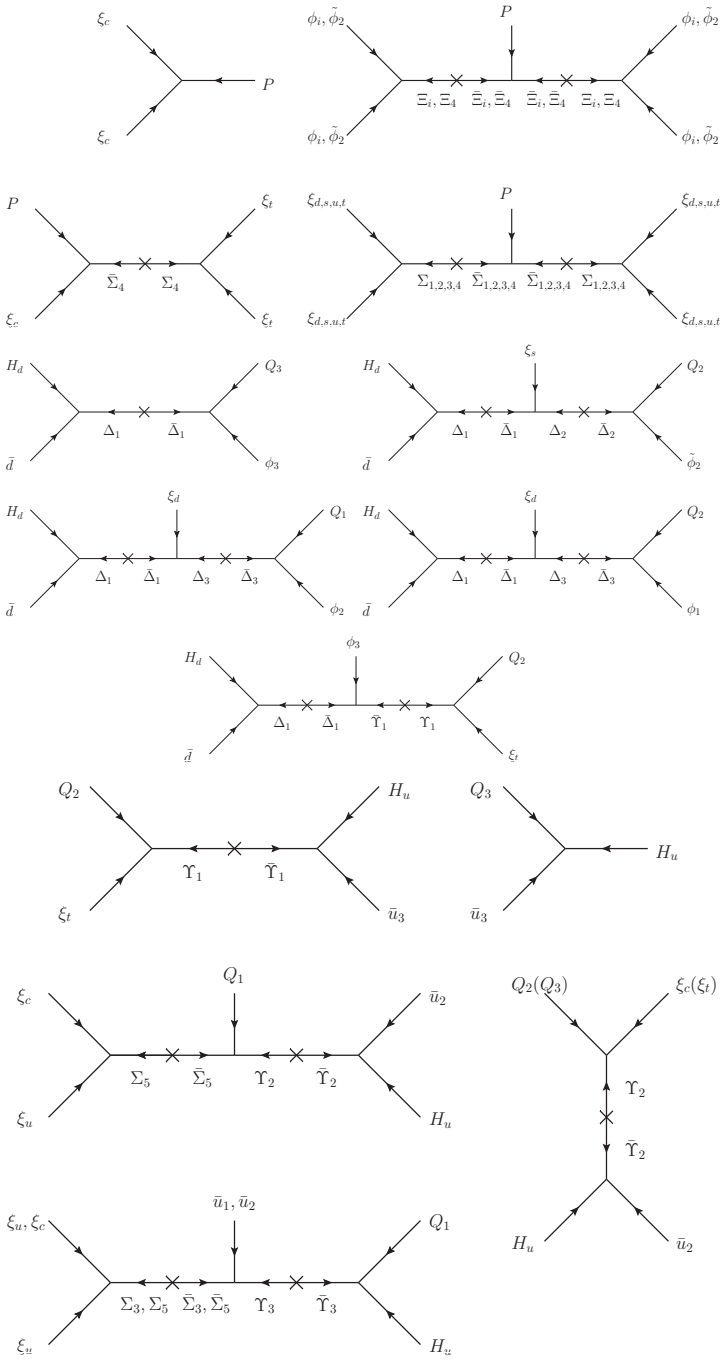

Fig. 1. The supergraphs before integrating out the messengers in our model. For the flavon sector only the diagrams are shown which fix the phases of the flavon vevs. For more details, see Ref. 6.

corrections to the entries of the up-type quark mass matrix:

$$\mathcal{W}_{\rm u}^{\rm corr} = Q_1 \bar{u}_1 H_u \left(\frac{\xi_c^2 \xi_u^2 + \phi_1^2 \phi_2^2}{\Lambda^4} + \frac{\xi_c \xi_t^2 \xi_u^2}{\Lambda^5} + \frac{\xi_t^4 \xi_u^2}{\Lambda^6} \right) + Q_1 \bar{u}_2 H_u \frac{\xi_t^2 \xi_u}{\Lambda^3} + Q_2 \bar{u}_2 H_u \frac{\xi_t^2}{\Lambda^2} \, . \tag{13}$$

These corrections are subleading real corrections to real entries of the Yukawa matrix and hence do not alter the fact that $\bar{\theta} = 0$.

Comparatively complicated are the additional effective operators for the flavon alignment

$$\begin{aligned}
\mathcal{W}_{\rm flavon}^{\rm corr} = & \frac{P}{\Lambda^4}(\phi_1^2 \phi_2^2 \xi_u^2 + \xi_u^4 \xi_c^2) + \frac{P}{\Lambda^5} \xi_c \xi_t^2 \xi_u^4 \\
& + \frac{P}{\Lambda^6}(\xi_t^4 \xi_u^4 + \xi_c^2 \xi_u^2 \phi_1^2 \phi_2^2 + \phi_1^4 \phi_2^4 + \xi_u^4 (\phi_1^4 + \phi_2^4)) \\
& + \frac{P}{\Lambda^7} \xi_c \xi_t^2 \xi_u^2 \phi_1^2 \phi_2^2 + \frac{P}{\Lambda^8} \xi_u^2 (\xi_c^2 \xi_u^2 (\phi_1^4 + \phi_2^4) + \phi_1^2 \phi_2^2 (\xi_t^4 + \phi_1^4 + \phi_2^4)) \\
& + \frac{P}{\Lambda^9} \xi_c \xi_t^2 \xi_u^4 (\phi_1^4 + \phi_2^4) + \frac{P}{\Lambda^{10}} (\phi_1^4 + \phi_2^4)(\xi_t^4 \xi_u^4 + \xi_c^2 \xi_u^2 \phi_1^2 \phi_2^2 + \phi_1^4 \phi_2^4) \\
& + \frac{P}{\Lambda^{11}} \xi_c \xi_t^2 \xi_u^2 \phi_1^2 \phi_2^2 (\phi_1^4 + \phi_2^4) + \frac{P}{\Lambda^{12}} \xi_t^4 \xi_u^2 \phi_1^2 \phi_2^2 (\phi_1^4 + \phi_2^4) \, .
\end{aligned} \tag{14}$$

Nevertheless, a close inspection reveals that our alignment including the phases of the flavon vevs is not altered by these additional operators which can be supported by symmetry arguments[6].

Finally, let us briefly comment on the effects anomalies might have on our results. The gauge symmetries remain anomaly free (after adding the leptons), because we do not add new chiral fermions, which are charged under the Standard Model gauge group. In addition, as we do not introduce non-trivial singlet representations of A_4, the A_4 group is anomaly free, but some of the auxiliary Z_n symmetries appear to be anomalous.[c] However, since we do not specify here a complete model (including leptons, a SUSY breaking sector etc.), we cannot make definite statements about anomalies but we assume that the effects of anomalies are either cancelled in the complete theory or sufficiently small.

4. Relation to Other Models

In this section we want to discuss briefly how our class of models is related to other models explaining the smallness of the strong CP phase by a spontaneous breaking of CP. We will especially focus on the Nelson-Barr models of spontaneous CP violation[23,24] being the first and most studied models. Although there are certain similarities, our model, for instance, does not fulfil the Barr criteria[24].

[c]For a general discussion of anomalies of discrete symmetry groups the reader is referred to Refs. 21,22.

We do not want to repeat the whole discussion. Instead, we just give the mass matrices in both setups. In the Nelson-Barr setup the mass matrix for the down-type quarks including heavy vector-like quarks (what we call messenger fields) would read

$$M_D \sim \begin{pmatrix} Y v_d & 0 \\ \langle \phi \rangle & M_\Upsilon \end{pmatrix} , \tag{15}$$

where they assume $Y v_d$ and M_Υ to be real by CP symmetry and only the vev of some symmetry breaking fields which governs the couplings of the light to the heavy fields induces CP violation. In such a setup one could get weak CP violation while $\bar{\theta} \sim \arg\det M_D$ still vanishes.

In our toy model we can explicitly write down the corresponding mass matrix

$$M_D \sim \begin{pmatrix} 0 & 0 & 0 & \langle \phi_2 \rangle^T & 0 & 0 \\ 0 & 0 & \langle \tilde{\phi}_2 \rangle^T & \langle \phi_1 \rangle^T & \langle \xi_t \rangle & \langle \xi_c \rangle \\ 0 & \langle \phi_3 \rangle^T & 0 & 0 & 0 & \langle \xi_t \rangle \\ \langle H_d \rangle & M_{\Delta_1} & 0 & 0 & 0 & 0 \\ 0 & \langle \xi_s \rangle & M_{\Delta_2} & 0 & 0 & 0 \\ 0 & \langle \xi_d \rangle & 0 & M_{\Delta_3} & 0 & 0 \\ 0 & \langle \phi_3 \rangle^T & 0 & 0 & M_{\Upsilon_1} & \langle \xi_t \rangle \\ 0 & 0 & 0 & 0 & 0 & M_{\Upsilon_2} \end{pmatrix} , \tag{16}$$

which has the determinant

$$\det M_D \sim \langle H_d \rangle^3 M_{\Delta_2}^3 M_{\Delta_3}^3 M_{\Upsilon_1} M_{\Upsilon_2} \langle \xi_d^2 \rangle \langle \phi_1 \rangle \langle \phi_2 \rangle \langle \phi_3 \rangle , \tag{17}$$

which is real because $\langle \tilde{\phi}_2 \rangle$ does not appear. This is only due to our alignment.

5. Summary and Conclusions

In this proceedings we have discussed a recently proposed novel approach to solve the strong CP problem in the context of spontaneous CP violation without the need for an axion. We assume CP to be a fundamental symmetry of nature and use discrete, Abelian and non-Abelian (family) symmetries to break it in such a way that the anomalous contribution to the CP violating QCD parameter $\bar{\theta}$ from the quark mass matrices vanishes at tree-level. Simultaneously the CKM phase is predicted to have its observed large value in a simple and transparent way.

An essential ingredient of this approach is that the phases of the symmetry breaking vevs are fixed to certain discrete values with either being real or purely imaginary in the simplest possible setup which is governed in our example by the discrete vacuum alignment method[14]. Nevertheless, other models reproducing the texture from eq. (2) could do the same trick.

Our toy model is supersymmetric, which helps to fix the flavon vev phases and forbids via the non-renormalisation theorem the appearance of new, unwanted operators in the superpotential from loop corrections, which could spoil our solution

for the strong CP problem. Furthermore, the model is based on the family symmetry A_4 with an $U(1)_R$ symmetry and the shaping symmetry $Z_2 \times Z_4^5$ forbidding unwanted operators and providing a mechanism to fix the phases of the flavon vevs via the discrete vacuum alignment method. We discussed an UV completion of the model in that sense that we give a list of heavy messenger fields which generate the desired effective operators after being integrated out. This enables us to show explicitly that our solution for the strong CP problem is not affected by higher order corrections (ignoring non-perturbative and SUSY breaking effects).

Finally, we discussed the relation between our novel class of models to the well known Nelson-Barr models[23,24]. In the Nelson-Barr models direct couplings between the light sector and the heavy sector are partially forbidden in such a way that the total mass matrix exhibits a special block structure. This is different in our class of models, where all light fields can couple to all heavy messenger fields in principle. The determinant of the total mass matrix in their case is real due to the mentioned block structure, while in our case it is real due to our vacuum alignment (including phases).

The class of models presented here casts new light on an old problem, the strong CP problem. There have been several previous attempts to solve it in terms of spontaneous CP violation in combination with flavour symmetries but our strategy differs significantly from these previous approaches. Most notably, we simultaneously have large CP violation in the CKM matrix with a right-angled unitarity triangle in a simple way, without any contribution to $\bar{\theta}$ from the quark mass matrices. Furthermore, the techniques to handle the symmetry breaking of discrete non-Abelian family symmetries, like in our example model A_4, was first developed in the context of the large leptonic mixing angles and finds here an unexpected new application. Also the method to fix the flavon vev phases was developed to give a dynamical explanation for the phase sum rule but was then in succeeding papers used in the lepton sector as well.

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