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$\mathcal{O}(\alpha_s^2)$ corrections to the running top-Yukawa coupling and the mass of the lightest Higgs boson in the MSSM

D. Kunz[(a)], L. Mihaila[(a)], N. Zerf[(b)]

(a) Institut für Theoretische Teilchenphysik, Karlsruhe Institute of Technology (KIT), 76128 Karlsruhe, Germany

(b) Department of Physics, University of Alberta, 4-181 CCIS, Edmonton AB T6G 2E1, Canada

E-mail: david.kunz@kit.edu, luminita-nicoletta.mihaila@kit.edu, zerf@ualberta.ca

Abstract

In this paper we propose a method to compute the running top-Yukawa coupling in supersymmetric models with heavy mass spectrum based on the "running" and "decoupling" procedure. In order to enable this approach we compute the two-loop SUSY-QCD radiative corrections required in the decoupling process. The method has the advantage that large logarithmic corrections are automatically resummed through the Renormalization Group Equations. As phenomenological application we study the effects of this approach on the SUSY-QCD corrections to the prediction of the lightest Higgs boson mass at three-loop accuracy. We observe a significant reduction of the renormalization scale dependence as compared to the direct method, that is based on the conversion relation between the running and pole mass for the top quark. The effect of resummation of large logarithmic contributions consists in an increased prediction for the Higgs boson mass, an observation in agreement with the previous analyses.
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1 Introduction

The recent discovery of the Higgs boson at the Large Hadron Collider (LHC) was a major milestone not just in particle physics but in the history of science. It bears the knowledge how the mass comes about at quantum level by the Higgs mechanism. With the Higgs boson discovery, particle physics entered a new era of tremendous intensity of detailed and careful study of its properties. Hopefully, accurate understanding of the Higgs phenomenology together with new information from experiments at the LHC will provide us a tool for exploring new physics.

Great interest is currently devoted to the study of the Higgs boson couplings to the

electroweak gauge bosons W and Z and to the top- and bottom-quarks or tau-leptons. Deviations in these couplings could possibly be observed once the currently large uncertainties will be improved as part of the LHC program and at a future Higgs factory. It has been shown that the Higgs couplings will be sensitive to new physics at the multi-TeV scale once percent level precision on coupling measurements will be achieved (for a recent review see [1]).

The aim of this paper is to propose a method for the computation of the top-Yukawa coupling within the Minimal Supersymmetric Standard Model (MSSM) taking into account radiative corrections at $\mathcal{O}(\alpha_s^2)$ accuracy. In general, the relationships between the running couplings and the physical observables like particle masses are affected by large radiative corrections [2, 3]. Within the SM, the relationship between the running top-Yukawa coupling and the physical top-quark pole mass receives three type of radiative corrections: i) higher order corrections to the running Yukawa coupling, ii) contribution to the fermion pole mass, and iii) corrections to the relation between the Fermi constant and the SM parameters. The first contribution is known at three-loop accuracy [4, 5] taking into account corrections from all sectors of the SM. The radiative corrections to the top-quark pole mass are known in QCD up to three-loop order [6–9] and in the electroweak sector up to two loops [10–15]. The third contribution is known in the SM with two- and three-loop accuracy for the genuine electroweak [10, 16, 17] and mixed QCD-electroweak [18] sectors, respectively. As was explicitly shown there are two sources for the large radiative corrections: the pure QCD contributions to the top-quark pole mass of about 10 GeV, 2 GeV and 0.5 GeV [6] at one-, two- and three-loop order, respectively; and the tadpole diagrams when they are taken into account for a gauge-independent definition of the running-mass [13]. Their magnitude is comparable with that from QCD sector at the one-loop order and amounts to about 0.5 GeV at two loops. As can be concluded from the numerical values cited above the radiative corrections are very important for the QCD sector, and even the third-order terms in the perturbative series are necessary in order to cope with the current experimental accuracy on the top-quark mass [19]. However, the situation is going to change if the International Linear Collider (ILC) is built, where a precision of $\mathcal{O}(100)$ MeV is expected.

When physics beyond the SM is considered, the radiative corrections to the top-quark pole mass might receive much larger contributions than in the SM and even diagrams beyond the three-loop order have to be taken into account to reach the current experimental accuracy. By now, the radiative corrections to the fermion pole mass are known at two-loop order for a general theory with massless gauge bosons [20]. The numerical evaluation of the two-loop self-energy diagrams has been implemented in the code `TSIL` [21]. For supersymmetric models with masses at the few TeV scale [1] the radiative corrections to the top-quark pole mass increase by about a factor of four as compared to the SM results as we show in section 3.2. Thus, the two-loop contributions might become one order of magnitude larger than the experimental uncertainties and the effects of higher order corrections have to be considered. The computation of on-shell self-energy diagrams

[1] In accordance with the lower bound from the direct SUSY searches at the LHC (for a review see [22]).

with several mass scales at the three-loop order is, for the time being, feasible only using asymptotic expansion techniques and is computationally very involved. In this paper, we propose an alternative method that can be applied as long as the top-quark mass is much smaller than the masses of supersymmetric particles. In the present paper, we focus on the dominant SUSY-QCD corrections to the running Yukawa coupling in the MSSM and postpone the study of the contributions originating from Yukawa and electroweak interactions for a later publication. The SUSY-QCD corrections to the relation between the running Yukawa coupling and the top-quark pole mass reduce to the corrections to the ratio between the pole and the running masses. Explicitly, the running top-quark mass is determined in the SM with the highest available precision from the experimentally measured pole mass [2]. Then, the running mass is evolved up to the SUSY scale using the Renormalization Group Equations (RGEs) of the SM. Afterwards, the running top-quark mass in the SM is converted to its value in the MSSM. In this step, threshold corrections at the SUSY-scale are required. In the last step, the running top-quark mass in the MSSM is evolved at the desired energy scale with the help of MSSM RGEs. As the RGEs in the SM [4, 5] and the MSSM [24, 25] are known to three-loop order, the threshold corrections are required at the two-loop order. They are known for light quark masses (e.g. bottom quark) in the SM to three-loop accuracy [26, 27] and in the MSSM to two loops [28, 29]. Great interest was devoted to the determination of the effective bottom-Yukawa coupling in SUSY-models with a large $\tan\beta$ parameter [30–32]. For these models the resummation of $\tan\beta$ enhanced contributions is considered on top of the two-loop order calculation.

One goal of this paper is to present the computation of the two-loop SUSY-QCD threshold corrections to the running top-quark mass. The calculation is similar with the ones performed in [28, 29]. The advantage of the method presented here as compared to the direct calculation of on-shell self-energy diagrams in the MSSM is that the occurring large logarithms of the form $\ln(M_{\rm top}/M_{\rm SUSY})$ are automatically resummed by the use of RGEs. The result is a much better convergence of the perturbative series as will be explicitly shown in the next sections.
The second aim is to study the phenomenological effects of the above calculation. Obviously, the top-Yukawa coupling is an essential ingredient in all processes involving interactions between the Higgs boson and top-quark and top-squarks. However, the most prominent example is probably the effects on the lightest Higgs boson mass, that receives radiative corrections enhanced by the fourth power in top-Yukawa coupling. A detailed numerical analysis of the effects of the proposed method for the determination of the running Yukawa coupling on the SUSY-QCD corrections to the lightest Higgs boson mass will be discussed in section 4.

The paper is structured as follows: in section 2 we present our computational framework; explicit analytical results are discussed in section 3 together with their numerical implementation; in section 4 we perform a phenomenological analysis of the effects of the two-loop SUSY-QCD corrections determined in the previous section on the SUSY-QCD prediction of the lightest Higgs boson mass with three-loop accuracy; in section 5 we

[2]For a detailed discussion about the distinction between the pole and the measured top-quark mass we refer to [23].

summarise our conclusions.

2 Framework

An elegant approach to get rid of unwanted large logarithms occurring in the predictions for observables in multi-scale processes is to formulate an effective theory (ET) (for a review see Ref. [33]). The parameters of the ET must be modified in order to take into account the effects of the heavy fields. The ET parameters are related to the parameters of the full theory by the so-called matching or decoupling relations. These take into account threshold corrections generated by the heavy degrees of freedom that are integrated out when the ET is constructed. In the following, we concentrate on the calculation of the decoupling coefficients for the strong coupling and the top-quark mass within SUSY-QCD. They are defined through the following relations between the bare quantities

$$\begin{aligned} \alpha_s^{0,\prime} &= \zeta_{\alpha_s}^0 \alpha_s^0 \\ m_t^{0,\prime} &= \zeta_{m_t}^0 m_t^0 \,, \end{aligned} \tag{1}$$

where the primes label the quantities in the effective theory. The decoupling coefficients have been computed in QCD including corrections up to the four-loop order for the strong coupling [34,35] and three-loop order for quark masses [26,27]. In the MSSM the two-loop SUSY-QCD [28,29,36] and SUSY-EW [28,32] expressions are known. Very recently, even the three-loop SUSY-QCD corrections to decoupling coefficient of the strong coupling were computed [37].

We consider SUSY-QCD with $n_f = 6$ active quark and squark flavours and $n_{\tilde{g}} = 1$ gluinos. Furthermore, we assume that all SUSY-particles including squarks and the gluino are much heavier than the SM particles. Integrating out the heavy fields from the full Lagrange density, we obtain the Lagrange density corresponding to the "effective" SM with n_f quarks plus non-renormalizable interactions. The latter are suppressed by negative powers of the heavy masses and will be neglected here.
Since the decoupling coefficients are universal quantities, they are independent of the momenta carried by the incoming and outgoing particles. The authors of Refs. [26] showed that the bare decoupling coefficients for the quark mass ζ_m^0 and for the strong coupling constant ζ_s^0 can be derived via the relations

$$\begin{aligned} \zeta_{\alpha_s}^0 &= \left[\frac{1 + \Gamma_{gcc}^{0,h}(0,0)}{(1 + \Pi_c^{0,h}(0))\sqrt{1 + \Pi_g^{0,h}(0)}} \right]^2 , \\ \zeta_{m_t}^0 &= \frac{1 - \Sigma_s(0)}{\sqrt{\zeta_L \zeta_R}} \quad \text{with} \\ \zeta_L &= 1 + \Sigma_v(0) - \Sigma_A(0) \quad \text{and} \quad \zeta_R = 1 + \Sigma_v(0) + \Sigma_A(0) \,, \end{aligned} \tag{2}$$

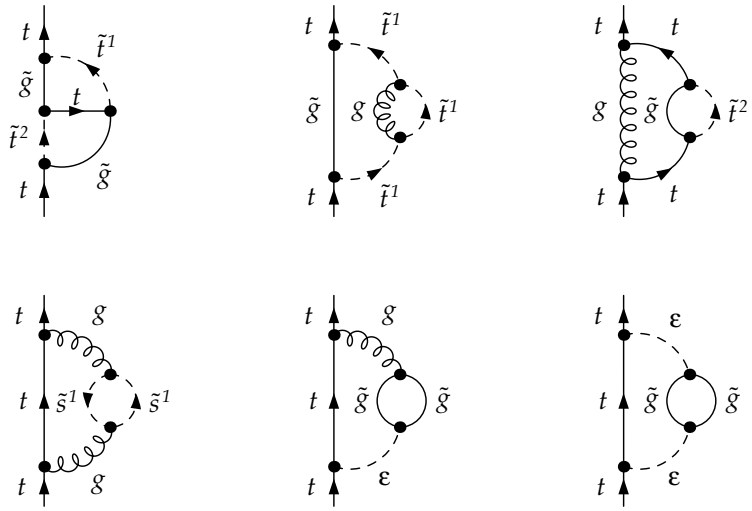

Figure 1: Sample Feynman diagrams that contribute to the two-loop corrections to ζ_{m_t}.

where $\Sigma_s(p^2)$, $\Sigma_v(p^2)$ and $\Sigma_A(p^2)$ are the scalar, the vector and the axial-vector components of the top-quark self-energy defined through

$$\Sigma(p^2) = \not{p}(\Sigma_v(p^2) + \gamma_5\Sigma_A(p^2)) + m_t\Sigma_s(p^2)\,. \tag{3}$$

Note that the axial-vector component starts contributing at the two-loop order. $\Pi_c^{0,h}(p^2)$ and $\Pi_g^{0,h}(p^2)$ are the ghost and gluon vacuum polarizations and $\Gamma_{gcc}^{0,h}(p,q)$ denotes the amputated Green function contributing to the gluon-ghost-ghost vertex. The superscript h indicates that in the framework of Dimensional Regularization (DREG) or Dimensional Reduction (DRED) only diagrams containing at least one heavy particle inside the loops contribute and that only the hard regions in the asymptotic expansion of the diagrams are taken into account.
In Fig. 1 some sample Feynman diagrams contributing to the decoupling coefficients for the top-quark mass are shown.

The Feynman diagrams were generated with `QGRAF` [39] and further processed with `q2e` and `exp` [40, 41]. The reduction of various vacuum integrals to the master integral was performed with a self written `FORM` [42] routine [38]. The reduction of topologies with two different massive and one massless lines requires a careful treatment. The related master integral can be easily derived from its general expression valid for massive lines, given in Ref. [43].

The finite decoupling coefficients are obtained upon the renormalization of the bare parameters. They are given by

$$\zeta_s = \frac{Z_s}{Z_s'}\zeta_s^0\,, \quad \zeta_m = \frac{Z_m}{Z_m'}\zeta_m^0\,, \tag{4}$$

where Z_s' and Z_m' correspond to the renormalization constants in the effective theory, and Z_s and Z_m denote the same quantities in the full theory. Since we are interested in the

two-loop results for ζ_i , $i = s, m$, the corresponding renormalization constants for SUSY-QCD and QCD have to be implemented with the same accuracy. Analytical results for the latter up to the three-loop order can be found in *e.g.* Refs. [33, 44, 45].

Apart from the renormalization constants of the external fields, also parameter renormalization is required. For the renormalization of the gluino and squark masses and the squark mixing angle we choose the on-shell scheme. This scheme allows us to directly use the physical parameters in the running analyses making the implementation very simple. The explicit formulae at the one-loop order can be found in Refs. [46, 47]. The two-loop counterterms are known analytically only for specific mass hierarchies [48] and numerically for arbitrary masses [21].
For the calculation of ζ_{α_s} the simultaneous renormalization of up- and down-type squarks is required. We follow the prescription proposed in Ref. [49] and fixed the counterterms for the up-squarks and the heavier down-squark to be on-shell and derive the counterterm for the lighter down-squark accordingly. In our limit of neglecting the top-quark mass as compared to the SUSY mass scale it holds

$$\delta m_{\tilde{d}_1} = \delta m^{\rm os}_{\tilde{u}_1} \,, \quad \text{with} \quad u = u, s, t \quad \text{and} \quad d = d, c, b \,, \tag{5}$$

where $\delta m_{\tilde{d}_1}$ stands for the mass counterterm of the light down-squark and $\delta m^{\rm os}_{\tilde{u}_1}$ denotes the on-shell mass counterterm of the light up-squark.

For the computation of the decoupling coefficient of the top-quark mass at order $\mathcal{O}(\alpha_s^2)$ one needs to renormalize in addition the top-quark mass and the ϵ-scalar mass. As the top-quark mass is neglected w.r.t. SUSY particle masses, an explicit dependence of the radiative corrections on m_t can occur only through top-Yukawa coupling. In order to avoid the occurrence of large logarithms of the form $\alpha_s^2 \log(\mu^2/m_t^2)$ with $\mu \simeq \tilde{M}$, where $\tilde{M}$ stands for the SUSY scale, one has to renormalize the top-Yukawa coupling in the $\overline{\rm DR}$ scheme. In this way, the large logarithms are absorbed into the running mass and the higher order corrections are maintained small. As a consequence, the top-squark mixing parameter $X_t = A_t - \mu_{\rm SUSY}/\tan\beta$ will be renormalized in a mixed scheme. Its counterterm is derived from the relation between the top-quark and squark masses and the mixing angle and mixing parameter.

Finally, the last parameter to be renormalized is the ϵ-scalar mass [3]. To obtain decoupling coefficients independent of the unphysical parameter m_ϵ, one has to modify the top squark masses by finite quantities [51, 52]. We adopt in the present calculation the method proposed in Ref. [28] to choose the ϵ-scalars massive and integrate them out together with the SUSY particles. In this way we achieve conversion from $\overline{\rm MS}$ to $\overline{\rm DR}$ schemes and decoupling of the heavy masses in a single step.

[3] For a review on their role in multi-loop calculation see [50].

3 Two-loop threshold corrections

The two-loop results for the decoupling coefficient for the strong coupling is very compact and we reproduce them below.

$$
\begin{aligned}
\zeta_{\alpha_s} = & -\frac{\alpha_s}{4\pi}\frac{1}{3}\left[C_A\left(1+2L_{\tilde{g}}\right)+T\sum_{\tilde{q}}\sum_{i=1,2}L_{\tilde{q}_i}\right]+\left(\frac{\alpha_s}{4\pi}\right)^2\left\{C_A^2\left(-\frac{125}{18}-\frac{44}{9}L_{\tilde{g}}+\frac{4}{9}L_{\tilde{g}}^2\right)\right. \\
& + C_A T\frac{2}{9}\left[30+\sum_{\tilde{q}}\sum_{i=1,2}\left(6\frac{M_{\tilde{q}_i}^2}{M_{\tilde{g}}^2}+6\frac{M_{\tilde{g}}^2-M_{\tilde{q}_i}^2}{M_{\tilde{g}}^2}B_{0,\mathrm{fin}}(M_{\tilde{g}}^2,M_{\tilde{q}_i},0)+2L_{\tilde{q}_i}L_{\tilde{g}}\right.\right. \\
& \left.\left. -\frac{2M_{\tilde{g}}^4-5M_{\tilde{g}}^2M_{\tilde{q}_i}^2+6M_{\tilde{q}_i}^4}{M_{\tilde{g}}^2(M_{\tilde{g}}^2-M_{\tilde{q}_i}^2)}L_{\tilde{q}_i}+3\frac{M_{\tilde{g}}^2}{M_{\tilde{g}}^2-M_{\tilde{q}_i}^2}L_{\tilde{g}}\right)\right]+T^2\left(\frac{1}{3}\sum_{\tilde{q}}\sum_{i=1,2}L_{\tilde{q}_i}\right)^2 \\
& + C_F T\frac{2}{3}\left[+\sum_{\tilde{q}}\sum_{i=1,2}\left(1+\frac{M_{\tilde{g}}^2}{M_{\tilde{q}_i}^2}-\frac{M_{\tilde{g}}^2-M_{\tilde{q}_i}^2}{M_{\tilde{q}_i}^2}B_{0,\mathrm{fin}}(M_{\tilde{q}_i}^2,M_{\tilde{g}},0)-2\frac{3M_{\tilde{g}}^2-2M_{\tilde{q}_i}^2}{M_{\tilde{g}}^2-M_{\tilde{q}_i}^2}L_{\tilde{q}_i}\right.\right. \\
& \left.+\left(4+\frac{M_{\tilde{g}}^2}{M_{\tilde{q}_i}^2}+\frac{2M_{\tilde{q}_i}^2}{M_{\tilde{g}}^2-M_{\tilde{q}_i}^2}\right)L_{\tilde{g}}\right)+\sum_{\mathrm{gen}}\left(-3\frac{M_{\tilde{q}_{u1}}^2}{M_{\tilde{q}_{d1}}^2}-\frac{M_{\tilde{g}}^2-M_{\tilde{q}_{u1}}^2}{M_{\tilde{q}_{d1}}^2}B_{0,\mathrm{fin}}(M_{\tilde{q}_{u1}}^2,M_{\tilde{g}},0)\right. \\
& \left.\left.\left.+\frac{M_{\tilde{g}}^2-M_{\tilde{q}_{d1}}^2}{M_{\tilde{q}_{d1}}^2}B_{0,\mathrm{fin}}(M_{\tilde{q}_{d1}}^2,M_{\tilde{g}},0)+L_{\tilde{q}_{d1}}-\frac{M_{\tilde{q}_{u1}}^2}{M_{\tilde{q}_{d1}}^2}L_{\tilde{q}_{u1}}+\frac{1}{2}\frac{M_{\tilde{q}_{d1}}^2}{M_{\tilde{g}}^2-M_{\tilde{q}_i}^2}L_{\tilde{g}}\right)\right]\right\} \qquad (6)
\end{aligned}
$$

In the formula above the sum $\sum_q$ runs over all quark flavours and $\sum_{\mathrm{gen}}$ over the number of generations. C_A, C_F are the quadratic Casimir invariants for the adjoint and fundamental representations, $T = 1/2$ is the Dynkin index and $L_x = \ln(\mu^2/m_x^2)$, with $x = \tilde{g}, \tilde{q}$. $B_{0,\mathrm{fin}}(p^2, m_1, m_2)$ denotes the fine part of the B_0-function [53]. The asymmetry w.r.t. up- and down-type quarks originate from the special renormalization scheme of down-type squarks relative to the up-type squarks. Here, $\alpha_s = \alpha_s^{(\mathrm{SQCD})}$ denotes the strong coupling constant in the full theory.

3.1 Limits

The final results for two-loop threshold corrections for ζ_{m_t} are too lengthy to be displayed here. They are available in `Mathematica` and `Fortran` format from http://www.ttp.kit.edu/Progdata/ttp14/ttp14-025. Instead, we present them for two special mass hierarchies.

3.1.1 Scenario A

We consider first the case of all supersymmetric particles having masses of the same order of magnitude and being much heavier than the top-quark.

$$
\begin{aligned}
& m_{\tilde{u}} = \ldots = m_{\tilde{b}} = m_{\tilde{t}} = m_{\tilde{g}} = \tilde{M} \quad \gg \quad m_t \\
& \alpha_s^{(6)} = \zeta_s^{\tilde{M}}\,\alpha_s^{(\mathrm{SQCD})}\,, \qquad m_t^{(6)} = \zeta_{m_t}^{\tilde{M}}\,m_t^{(\mathrm{SQCD})}\,.
\end{aligned}
$$

$\zeta_s^{\tilde{M}}$, $\zeta_{m_t}^{\tilde{M}}$ are functions of the supersymmetric mass $\tilde{M}$, the soft SUSY breaking parameters $X_t = A_t - \mu_{\rm SUSY}/\tan\beta$, the strong coupling constant in the full theory $\alpha_s^{(\rm SQCD)}$, and the decoupling scale μ. The superscript (6) indicates that the parameters are defined in QCD with $n_f = 6$ quarks.

$$
\begin{aligned}
\zeta_{m_t}^{\tilde{M}} \;=\; & -\frac{\alpha_s}{4\pi} C_F \left(-1 + \frac{X_t}{\tilde{M}} + L_{\tilde{M}}\right) \\
& + \left(\frac{\alpha_s}{4\pi}\right)^2 \Bigg\{ C_F^2 \Bigg[-\frac{71}{8} - \frac{13}{2} L_{\tilde{M}} + \frac{1}{2} L_{\tilde{M}}^2 + \frac{X_t}{\tilde{M}} \left(-5 + 3 L_{\tilde{M}}\right) \Bigg] \\
& + C_F T \Bigg[\frac{109}{3} - 16 L_{\tilde{M}} + 12 L_{\tilde{M}}^2 + 12 \frac{X_t}{\tilde{M}} \left(-1 + L_{\tilde{M}}\right) \Bigg] \\
& + C_F C_A \Bigg[-\frac{23}{72} - \frac{37}{6} L_{\tilde{M}} - \frac{1}{2} L_{\tilde{M}}^2 - \frac{X_t}{\tilde{M}} \left(1 + 3 L_{\tilde{M}}\right) \Bigg] \Bigg\}
\end{aligned}
\tag{7}
$$

3.1.2 Scenario B

The second scenario we consider is the so called "split-SUSY" one, with squarks much heavier than all the other particles:

$$
\begin{aligned}
& m_{\tilde{u}} = \ldots = m_{\tilde{b}} = m_{\tilde{t}} = \tilde{M} \quad \gg \quad m_{\tilde{g}} \quad \gg \quad m_t \\
& \alpha_s^{(6)} = \zeta_s^{\tilde{q}}\, \alpha_s^{(\rm SQCD)}, \qquad m_t^{(6)} = \zeta_{m_t}^{\tilde{q}}\, m_t^{(\rm SQCD)}.
\end{aligned}
$$

The result reads:

$$
\begin{aligned}
\zeta_{m_t}^{\tilde{q}} =& \frac{\alpha_s}{4\pi} C_F \Bigg\{ \frac{1}{2} - L_{\tilde{M}} + \frac{1}{\tilde{M}^2} (M_{\tilde{g}}^2 - 2 M_{\tilde{g}} X_t) \\
& + \frac{1}{\tilde{M}^4} \Big[M_{\tilde{g}}^4 (1 + L_{\tilde{M}} - L_{\tilde{g}}) + 2 X_t M_{\tilde{g}}^3 (-1 + L_{\tilde{g}} - L_{\tilde{M}}) \Big] \Bigg\} \\
& + \left(\frac{\alpha_s}{4\pi}\right)^2 C_F \Bigg\{ -\frac{C_A}{72 \tilde{M}^4} \Big[\tilde{M}^4 \big(-481 + 432 L_{\tilde{M}} + 108 L_{\tilde{M}}^2 \\
& + 120 L_{\tilde{g}} - 72 L_{\tilde{g}}^2 + 576 \zeta(2) \big) - 36 M_{\tilde{g}}^3 \big(M_{\tilde{g}} (38 + 4 L_{\tilde{M}} \\
& + 13 L_{\tilde{M}}^2 + 2 L_{\tilde{g}} - 20 L_{\tilde{M}} L_{\tilde{g}} + 7 L_{\tilde{g}}^2 - 20 \zeta(2)) \\
& - 4 X_t (5 + 19 L_{\tilde{M}} + 3 L_{\tilde{M}}^2 - 16 L_{\tilde{g}} - 3 L_{\tilde{M}} L_{\tilde{g}} + 2 \zeta(2)) \big) \\
& + 72 \tilde{M}^2 M_{\tilde{g}} \big(2 X_t (7 + 6 L_{\tilde{M}} - 3 L_{\tilde{g}} + 2 \zeta(2)) \\
& + M_{\tilde{g}} (-15 + 3 L_{\tilde{M}} - 6 L_{\tilde{g}} + 10 \zeta(2)) \big) \Big]
\end{aligned}
$$

$$
\begin{aligned}
&+ \frac{C_F}{8\tilde{M}^4}\Big[- 8\tilde{M}^2 M_{\tilde{g}}\big(M_{\tilde{g}}(21 + L_{\tilde{M}} - 20\zeta(2)) \\
&+ X_t(5 - 6L_{\tilde{M}} - 8\zeta(2))\big) - 2M_{\tilde{g}}^3\big(M_{\tilde{g}}(175 + 60L_{\tilde{M}}^2 \\
&+ 90L_{\tilde{g}} + 56L_{\tilde{g}}^2 - 2L_{\tilde{M}}(43 + 58L_{\tilde{g}}) - 104\zeta(2)) \\
&+ 4X_t(15 - 6L_{\tilde{M}}^2 + 11L_{\tilde{g}} + L_{\tilde{M}}(-17 + 6L_{\tilde{g}}) - 8\zeta(2))\big) \\
&+ \tilde{M}^4\big(- 189 - 48L_{\tilde{M}} + 4L_{\tilde{M}}^2 + 120\zeta(2)\big)\Big] \\
&+ \frac{T}{3\tilde{M}^4}\Big[- 36\tilde{M}^2 M_{\tilde{g}}\big((-1 + L_{\tilde{M}})M_{\tilde{g}} + X_t - 2L_{\tilde{M}}X_t\big) \\
&+ 3M_{\tilde{g}}^3\big((-13 - 12L_{\tilde{M}}^2 + 6L_{\tilde{g}} + 6L_{\tilde{M}}(-3 + 2L_{\tilde{g}}))M_{\tilde{g}} \\
&+ 4(7 + 6L_{\tilde{M}}^2 + L_{\tilde{M}}(9 - 6L_{\tilde{g}}) - 3L_{\tilde{g}})X_t\big) \\
&+ \tilde{M}^4\big(127 - 30L_{\tilde{M}} + 36L_{\tilde{M}}^2 - 36\zeta(2)\big)\Big]\Big\} \qquad (8)
\end{aligned}
$$

We have checked the formulae above against the exact calculation both analytically and numerically. First, we have computed the two-loop decoupling coefficient ζ_{m_t} for scenarios A and B making use of asymptotic expansion method (explicitly Large Mass expansion) that is available in the code `exp` and compared with the expansion of the exact result. For scenario A we obtain agreement for the dominant term (*i.e.* neglecting corrections proportional with the mass differences between the SUSY particles.) For scenario B, we verified the agreement for the first three terms of the expansion in mass ratio $M_{\tilde{g}}^2/\tilde{M}^2$. Also, the direct numerical comparison of the exact and asymptotically expanded results gives very good agreement.

3.2 Numerical results

In this section we present the phenomenological effects of the two-loop SUSY-QCD threshold corrections on the prediction of the running top-quark mass at the SUSY mass scale. We also present the comparison with the direct prediction obtained from the ratio between the running and the pole mass within SUSY-QCD as described in the code `TSIL` [21]. Our method can be summarised in the following sequence:

$$
M_t^{\rm OS} \overset{(i)}{\to} m_t^{\overline{\rm MS}}(M_t) \overset{(ii)}{\to} m_t^{\overline{\rm MS}}(\mu_{\rm dec}) \overset{(iii)}{\to} m_t^{\rm SQCD,\overline{DR}}(\mu_{\rm dec}) \overset{(iv)}{\to} m_t^{\rm SQCD,\overline{DR}}(\mu)\,, \qquad (9)
$$

where $M_t^{\rm OS}$ denotes the top-quark pole mass and $m_t^{\overline{\rm MS}}$ and $m_t^{\rm SQCD,\overline{DR}}$ stand for the running top-quark mass in the SM and SUSY-QCD in the $\overline{\rm MS}$ and $\overline{\rm DR}$ schemes, respectively. $\mu_{\rm dec}$ is the scale at which the decoupling is performed and it is usually chosen comparable with SUSY masses. If not stated otherwise, we fix it to be the arithmetic average over the squarks and gluino on-shell masses:

$$
\mu_{\rm dec} = \frac{1}{13}\big[M_{\tilde{g}} + \sum_{\tilde{q}} M_{\tilde{q}}\big]\,. \qquad (10)
$$

But of course, $\mu_{\rm dec}$ can be chosen arbitrarily and the dependence of the running top-quark on it is a measure of the theoretical uncertainties (for details, see Fig. 2 and its discussion). Also in the numerical setup, we implemented it as a free parameter that can be varied. In the step (i) the relation between the top-quark pole and running masses in the SM is required. We implemented the three-loop results [6–9]. The RGEs for the SM and the MSSM that are necessary in the steps (ii) and (iv) are known to three-loop accuracy [4,5,24,25] as well. Let us mention, that in QCD the quark anomalous dimension was recently computed even at the five loop order [54]. For consistency, the threshold corrections evaluated in the step (iii) are necessary at two loops.

For the explicit numerical evaluation we use for the SM parameters their values cited in Ref. [55]. For the MSSM parameters we employ two scenarios that we call "heavy Higgs" and "heavy sfermions", respectively. We obtain the numerical values with the help of the spectrum generator `SOFTSUSY` [56]. The input parameters for the spectrum generator are as follows:

i) In the "heavy sfermions" scenario we define all $\overline{\rm DR}$ breaking parameters at the input scale following the Supersymmetry Les Houches Accord (SLHA) [57, 58] `EXTPAR 0` $Q_{\rm in} = \tilde{m}_t$. Where $\tilde{m}_t$ is an alias for the right handed top squark mass breaking parameter `EXTPAR 46` and kept as free parameter. Further we identify the third generation doublet mass breaking $\tilde{m}_{Q_3}$ alias `EXTPAR 43` with $\tilde{m}_t$. All other sfermion mass breaking parameters have a common value $\tilde{m}_f = \tilde{m}_t + 1$TeV. The trilinear couplings (`EXTPAR 11-13`) are given by $A_t = 20$GeV and $A_\tau = A_b = 4$TeV. The gaugino mass parameters (`EXTPAR 1-3`) are set to $M_1 = M_2 = M_3 = 1.5$TeV. For the bilinear coupling of H_u and H_d (`EXTPAR 23`) we chose $\mu_{\rm SUSY} = 200$GeV. The mass of the pseudo scalar Higgs boson (`EXTPAR 23`) is set to $M_A = 1$TeV. Finally, the ratio of the two vacuum expectation values (`MINPAR 3`) is set to $\tan\beta = 10$. The given parameters choice results in very weakly mixing top squarks, which are about 1TeV lighter than the other sfermions and thus have largest impact in the decoupling process. By increasing the value of $\tilde{m}_t$ one automatically pushes the squark mass spectrum to higher values. However, in order to successfully describe the currently measured mass for the lightest Higgs boson, one is forced to use multi TeV range values for $\tilde{m}_t$, because of the very weak mixing between the top squarks which have nearly equal masses.

ii) In the "heavy Higgs" scenario we define all $\overline{\rm DR}$ breaking parameters at the input scale $Q_{\rm in} = \sqrt{M_{\rm SUSY}^2 + M_t^{\rm OS\,2}}$. Here $M_{\rm SUSY} = 1$TeV is a common breaking mass parameter for all sfermions except $\tilde{m}_t$ (`EXTPAR 46`), which we keep as free parameter. The remaining input parameters are given by:

$$\begin{aligned} A_\tau = A_b &= 2469.49\text{GeV}, & A_t &= 1.5\text{TeV}, & \mu_{\rm SUSY} &= 200\text{GeV}, \\ M_1 &= 5s_w^2/(3c_w^2)M_2, & M_2 &= 200\text{GeV}, & M_3 &= 800\text{GeV}, \\ M_A &= 1\text{TeV}, & \tan\beta &= 20\,. \end{aligned} \tag{11}$$

Here s_w and c_w are the sine and cosine of the Weinberg mixing angle θ_W. In contrast to the first scenario we do have light Higgs masses for sub TeV values of $\tilde{m}_t$ due to the stop mixing. Moreover one can have a very light stop of order 300GeV for $\tilde{m}_t$ values having nearly same size.

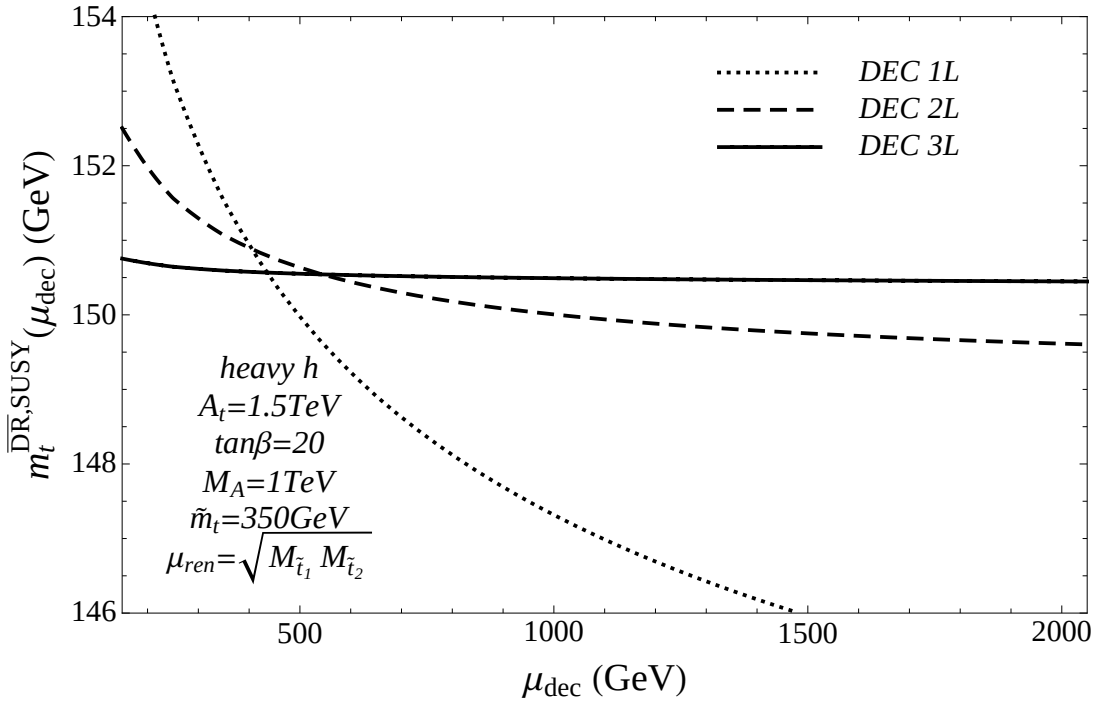

Figure 2: Decoupling scale dependence of the running top quark mass for the "heavy Higgs" scenario. The curves show the results obtained within the decoupling method at one (dotted line), two (dashed line) and three (full line) loops. The renormalization scale was fixed to $\mu_{\rm ren} = \sqrt{M_{\tilde{t}_1} M_{\tilde{t}_2}}$.

Please note that for the pure SUSY-QCD analysis done in this paper, the effect of changing breaking parameters of particles transforming as QCD singlets is very weak with respect to SUSY-QCD decoupling effects. We provide the SLHA input files for the two scenarios considered here in electronic format on the web page http://www.ttp.kit.edu/Progdata/ttp14/ttp14-025.

In the following, we denote as leading order (LO) value for the running top-quark mass in the decoupling method (DEC), the value obtained with one-loop RGEs and without threshold corrections. The next-to-leading (NLO) and the next-to-next-to-leading order (NNLO) values in the decoupling method are calculated employing two- and three-loop RGEs and one- and two-loop threshold corrections, respectively. The NLO top-quark mass computed in the direct method (with the code `TSIL`) takes into account the one-loop relation between the running- and pole-quark mass, whereas the NNLO prediction is based on the two-loop relation.

In a first step, we study the dependence of the running top-quark mass on the unphysical parameter $\mu_{\rm dec}$. As this parameter is not fixed by the theory, the dependence of the physical quantities on it provides us a measure of the accuracy of the method itself. We display in Fig. 2 such a dependence for the "heavy Higgs" scenario. It is particularly

important to perform the study in this scenario as one of the supersymmetric particles (the light top-squark) has the mass at an intermediate scale of about 300 GeV, whereas the rest of the particles have masses around 1 TeV. The natural question to be addressed in this case is whether the one-step decoupling approach, where all supersymmetric particles are integrated out at once, is still appropriate or a multi-step procedure is required. In the Fig. 2 the dotted, dashed and full lines depict the running top quark mass evaluated at scale $\mu_{\rm ren} = \sqrt{M_{\tilde{t}_1} M_{\tilde{t}_2}}$ at LO, NLO and NNLO and the decoupling scale is varied in the range from $M_{\rm top}$ to $2M_{\rm SUSY}$. As expected from theoretical consideration and clearly illustrated in the plot, a substantial improvement of the stability of the predictions w.r.t. the variation of $\mu_{\rm dec}$ is obtained when going from one- to three-loop accuracy. While at the two-loop level a variation of the top-quark mass of about 2 GeV is still present, at three loops the variation amounts to less than 100 MeV. Since the expected experimental accuracy for the top-quark mass measurement even at the future ILC does not go below 100 MeV, we can conclude that the method proposed here is well suited also for scenarios with lighter supersymmetric particles with masses around 300 GeV.

Furthermore, in Fig. 3 we present the running top-quark mass in the full theory (in the "heavy sfermion" scenario) as a function of the scale $\mu_{\rm ren}$ at which it is evaluated. The black (middle) curves display the results obtained with the method proposed in this paper (that we refer at as the decoupling method and is denoted as "DEC" in the legend of the plot) at one (dotted line) two (dashed line) and three (full line) loops. The blue curves show the predictions obtained directly via the ratio between the top-quark pole and the running masses in the SUSY-QCD, using the code `TSIL`. The dashed line corresponds to the one-loop results and the full line displays the two-loop contributions. As can be seen from the figure, the radiative corrections for the decoupling method are very small (tenth of MeV between one- and two-loop order contributions and negligible at the three loops) as compared with the current experimental uncertainty on the top quark pole mass of about 1 GeV. The perturbative series is very well converging and the contributions from the unknown higher order corrections are negligible for all renormalization scales. The radiative corrections obtained via the direct method are much larger than the experimental uncertainty. Even at the two-loop order, they amount to 10 GeV at the electro-weak scale and increase further at renormalization scales comparable with the squark masses. In this case higher order contributions are necessary to bring the theoretical precision at the same level as the experimental one. One observes also that the predictions obtained in the two methods at the two-loop order agree very well for small renormalization scales below 400 GeV. This can be explained by the fact that in this case the logarithmic contributions (of the form $\ln(M_{\rm top}/\mu)$) are small and the resummation gives only minor corrections. When the running top-quark mass is evaluated at high energy scales the resummation of the large logarithms becomes important and the difference between the two predictions can reach about 10 GeV. Let us also point out that in the domain where the resummation is expected to bring only small effects the differences between the two methods decrease considerably when going from one to two loops as is expected in perturbation theory.

In Fig. 4 the running top-quark mass is shown as a function of the squark mass breaking parameter $\tilde{m}_t$, that can be interpreted in the "heavy sfermion" scenario as a

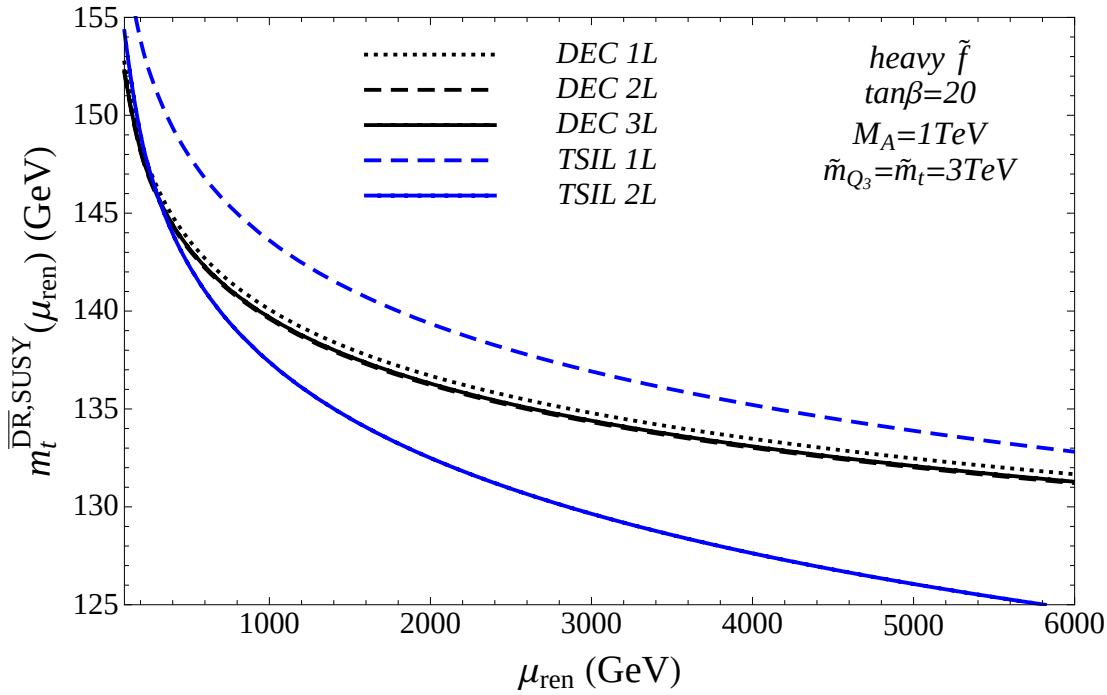

Figure 3: Renormalization scale dependence of the running top quark mass for the "heavy sfermion" scenario. The black curves show the results obtained within the decoupling method at one (dotted line), two (dashed line) and three (full line) loops. The blue lines display the results obtained with the code `TSIL` at one (dashed line) and two (full line) loops.

scale for the SUSY mass spectrum. In this case the renormalization scale is chosen as geometric mean value of the top-squark masses. One notices that the radiative corrections calculated with the decoupling approach are very small for all SUSY scales. The direct computation deliver even at the two-loop order radiative corrections of about $\mathcal{O}(10)$ GeV. As expected, the two methods provide results in very good agreement at the two-loop order for low SUSY mass scales, where no large logarithmic corrections are present.

Fig. 5 shows the running top-quark mass as a function of the renormalization scale in the "heavy Higgs" scenario. For the chosen input parameters, the predictions of the two methods differ at the one-loop level by more than 10 GeV. At the two-loop level the predictions of the two approaches agree well within the present experimental uncertainty on the top-quark pole mass for low-energy scales, whereas for large renormalization scales the difference amounts to few GeV. Let us also point out that the differences between one- and two-loop contributions within the decoupling method amounts to about 4 GeV, whereas the genuine three-loop contributions are very small, below 100 MeV. This observation proves the good convergence of the perturbative methods in the decoupling approach. The direct calculation based on the code `TSIL` provides similarly large radiative corrections at the two-loop level. However, in order to reduce the theoretical uncertainty

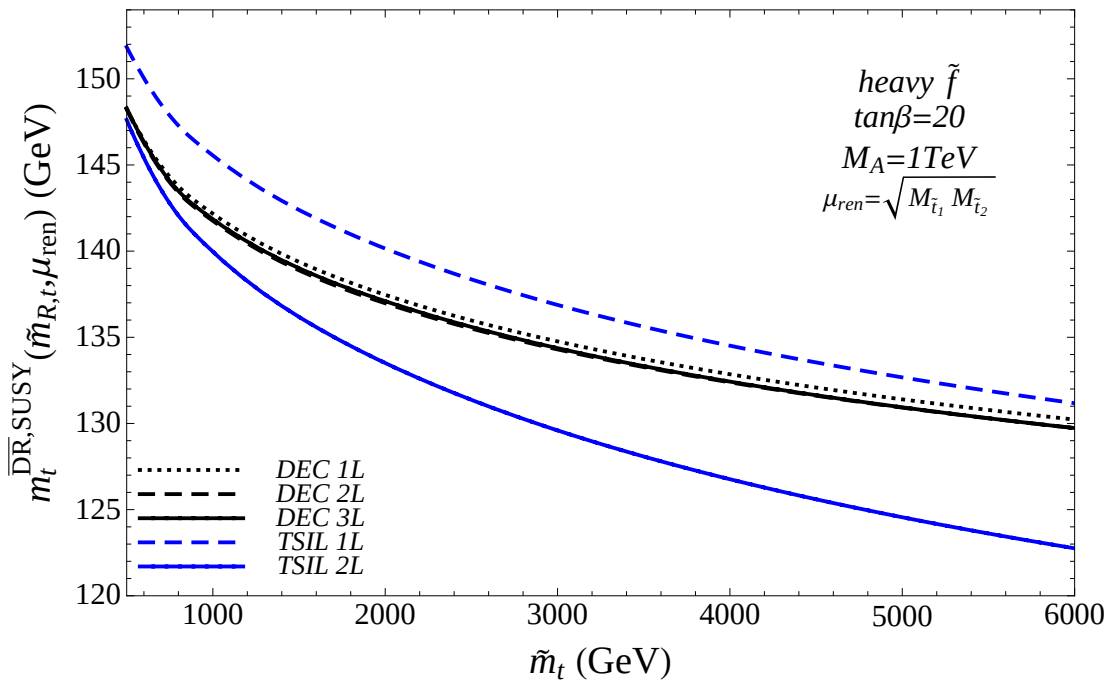

Figure 4: Dependence of the running top-quark mass on the SUSY scale $\tilde{m}_t$ for the "heavy sfermion" scenario. The convention for the curves is the same as in Fig. 3.

at a similar level with the experimental one, we need higher order radiative corrections that are currently not available.

In summary, we conclude that the two methods provide results in good agreement for low SUSY mass scales or renormalization scales, but they differ significantly when the SUSY particles become heavy, in the multi TeV range. Also, the rapid convergence of the perturbative series for the decoupling method allows us to reduce the genuine theoretical uncertainties due to unknown higher order corrections well below the present experimental uncertainty on the top-quark pole mass. The discrepancies between the predictions obtained within the two methods can have important phenomenological implications, depending on the process and observables under consideration.

4 The mass of the lightest Higgs boson

The Higgs boson mass measurement by ATLAS $125.5 \pm 0.2 \pm 0.6$ GeV [59] and CMS $125.7 \pm 0.3 \pm 0.3$ GeV [60] already reached an amazing precision. In the MSSM, the lightest Higgs boson mass is predicted. Beyond the tree-level approximation, it is a function of the top-squark masses and mixing parameters. It grows logarithmically with the top-squark masses and can be used to determine an upper bound for the supersymmetric (SUSY) mass scale from the measured Higgs boson mass, once the mixing parameters are fixed. This approach has received considerable attention recently [61–63], partially because the

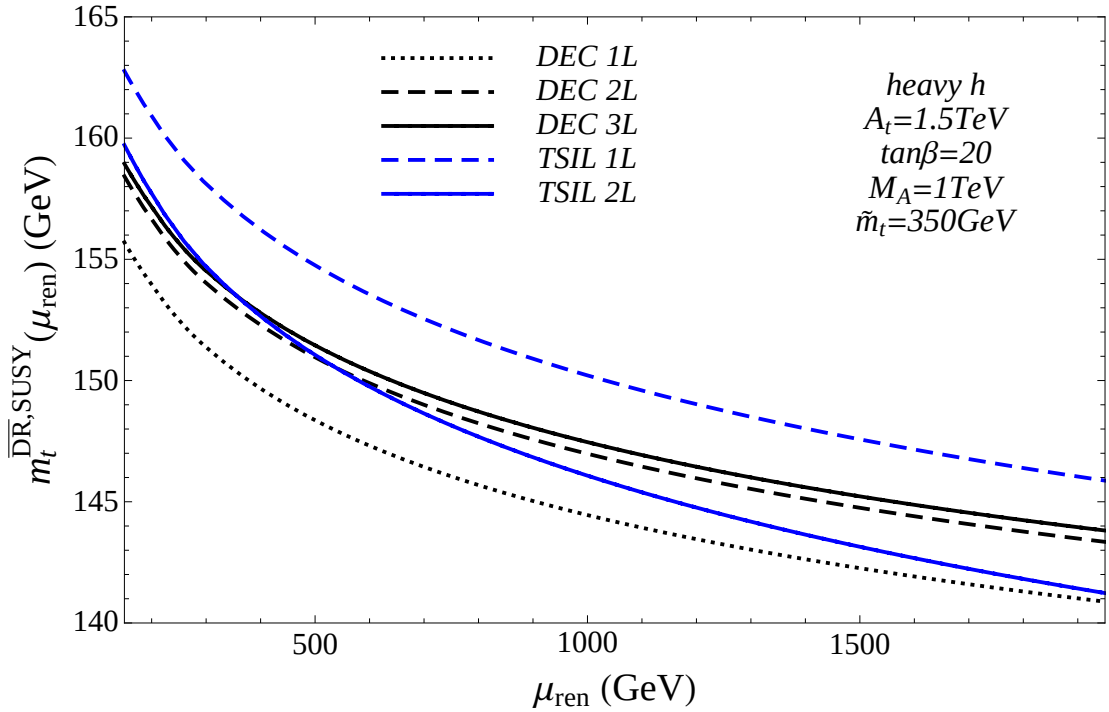

Figure 5: Dependence of the running top quark mass on the renormalization scale within the "heavy Higgs" scenario. The convention for the curves is the same as in Fig. 3.

direct searches for SUSY particles at the LHC remained unsuccessful, indicating a possible lower bound for their mass scale in the TeV range.
Since the dependence of the Higgs boson mass on the SUSY masses is logarithmic, high-precision measurements and theoretical predictions are required. The genuine theoretical uncertainties due to unknown higher order corrections are expected to grow with the SUSY mass scale. For top-squark masses in the multi TeV range they were estimated to be of about $5-7$ GeV [62,63]. By now, the complete one-loop [64,65] and dominant two-loop [66, 67] corrections are implemented in the numerical programs FeynHiggs [68] and CPsuperH [69] using on-shell particle masses, and in SOFTSUSY [56] and SPheno [70] using running parameters. The dominant SUSY-QCD three-loop corrections are taken into account in the code `H3m` [48], for which a mixed renormalization scheme was employed. However, the three-loop contributions are known only for specific mass hierarchies between the SUSY particles. The dominant contributions to the leading (LL) and next-to-leading (NLL) logarithmic terms in the ratio between the top quark mass and the typical scale of SUSY particle masses $\ln(M_{\rm top}/M_{\rm SUSY})$ have been obtained in Ref [71]. Very recently, the generalization of the LL and NLL approximation has been derived [62] and implemented in the code `FeynHiggs`, up to the seventh loop-order. Furthermore, in Ref. [63] the recent calculations of the three-loop beta-functions for the SM coupling and the two-loop corrections to the Higgs boson mass in the SM have been used to derive (presumably) the dominant NNLL corrections at the four-loop order.

In this section we focus on the numerical effects that the new prescription for the determination of the running top-quark mass and top-Yukawa coupling has on the predictions of the lightest Higgs boson mass taking into account SUSY-QCD radiative corrections. We implemented the resummation of the large logarithms of the form $\ln(M_{\rm top}/M_{\rm SUSY})$ contained in the running of the top-Yukawa coupling (as discussed in the previous section) on top of the three-loop SUSY-QCD corrections to the lightest Higgs boson mass encoded in the program `H3m`. As will be shown, this type of resummation is necessary for SUSY masses in the multi TeV range and enable us to reduce the effects of the unknown higher order contributions. In the following, we evaluate the running top-quark mass and couplings with the highest accuracy both for the decoupling and the direct methods and use them further for the calculation of the lightest Higgs boson mass at one-, two- and three-loop accuracy.

In order to allow a convenient evaluation of general MSSM scenarios including the stated low scale ones, the `Mathematica` package `SLAM` [72] has been implemented in `H3m`. It provides an easy to use interface for calling and reading SUSY spectrum generator output full automatically using the SLHA. Moreover it enables the ability to save and recall SUSY spectra in and from a data base. Besides reading in user provided SLHA spectrum generator output files, it is now possible to enter the SLHA input file for the spectrum generators defining the SUSY scenario directly in `Mathematica` relaxing the restriction to predefined scenarios in earlier versions of `H3m`.
The MSSM parameters derived in this way are stored and further used to compute the running top-quark mass, the running top-Yukawa coupling and the strong coupling constant in the MSSM using the decoupling method described in section 2. This step is

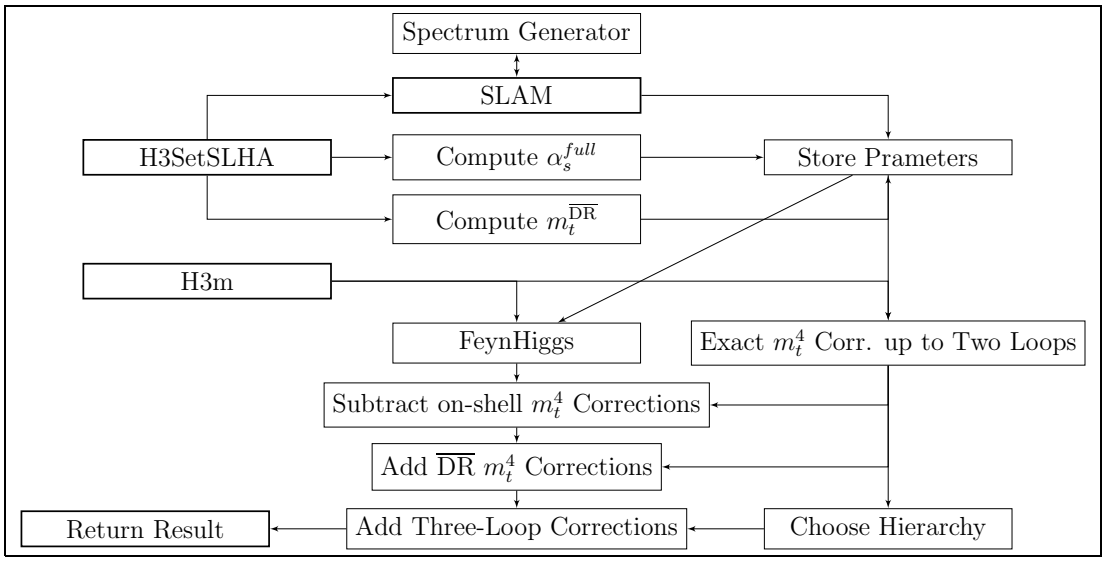

Figure 6: Flowchart of the new version of `H3m`. First, the user calls the code `SLAM` to set the MSSM parameters. A subsequent call of `H3m` computes M_h.

realized through stand-alone routines as is also explained in the flowchart plot in Fig. 6. Afterwards, the stored values for the input MSSM parameters together with the $\overline{\rm DR}$ couplings just derived are delivered to the code `H3m`. The computation of the lightest Higgs boson mass follows then the steps described in Ref. [48]. The user has also the possibility to choose the way the running top-quark mass is computed. The command `SetOptions[H3GetSLHA, calcmt->{``MtTSIL''}]` allows to use at this stage the code `TSIL`, whereas the decoupling method is implemented as default option. A direct comparison between the predictions obtained with the two methods will be presented below. The new version of the code `H3m` together with few simple example programs are available from `https://www.ttp.kit.edu/Progdata/ttp14/ttp14-025/`. Apart from the implementation of the interface program `SLAM` and the routine for the computation of the running top-quark mass and the strong coupling constant through the decoupling method, we improved on the determination of the mixing angle and reduced oscillations in the Higgs mass by setting the W-boson mass fixed to is on-shell value as given by the PDG [55].

In Fig. 7 it is shown the dependence of the lightest Higgs boson mass M_h in the "heavy sfermion" scenario on the renormalization scale, taking into account beyond one-loop only SUSY-QCD radiative corrections. The black curves display the one-(dotted line), two-(dashed line) and three-loop (full line) contributions obtained with the running top-quark mass in the decoupling method. The blue curves present the same predictions using the code `TSIL` for the calculation of the running top-quark mass. The explicit value of the running-top quark mass can be read from Fig. 3. It is known that the renormalization scale dependence of an observable gives an estimation for the magnitude of unknown

higher order corrections. This enables us to use it for the determination of the theoretical uncertainty. As expected the renormalization scale dependence is reduced when going from one- to two- and to three-loop order corrections in both schemes. However, the direct determination of the running top-quark mass (blue curves) is affected by large logarithmic corrections that in turn induces large radiative corrections to the Higgs boson mass. Even at the three-loop order, the scale variation of M_h amounts to about 5 GeV, more than an order of magnitude larger than the current experimental accuracy on M_h and few times bigger than the parametric uncertainties. In contrast, the resummation of the logarithmic corrections to the running top-Yukawa coupling through the decoupling method renders the scale dependence of M_h at three-loop order very mild about tens of MeV. One can also observe, that low values for the renormalization scale around the top-quark pole mass are not well suited for the present scenario, especially when the three-loop order contributions are not taken into account. In this case, radiative corrections even beyond the three-loop order are required in order to cope with the experimental precision. The difference between the predictions for M_h obtained with the two methods is sizeable and can amount to few GeV for large values of the renormalization scale.

The dependence of M_h on the SUSY breaking parameter $\tilde{m}_t$ is shown in Fig. 8. As described in section 3.2, $\tilde{m}_t$ can be interpreted as an estimation of the SUSY mass parameter. The convention for the line style is the same as in the previous figure. The renormalization scale is fixed as the geometric mean value of the top-squark masses, thus in the TeV range. The radiative corrections to the Higgs boson mass increase with the SUSY mass scale as expected. The predictions obtained using the two methods for the derivation of the top-Yukawa coupling are in good agreement for low SUSY scales of about 500 GeV, but they differ significantly for heavy SUSY spectrum in the multi TeV range. The Higgs boson mass predicted through the decoupling method is always heavier and has a steeper dependence on the SUSY spectrum as compared to the one obtained through the direct method. This difference can be explained by the effects of resumming large logarithms within the first approach. Since we use the RGEs at the three-loop order the next-to-next-to-leading-logarithms are resummed. A similar behaviour of predictions for M_h based on resummation was observed in the previous works [62, 63]. Let us point out the big impact of the resummation procedure for constraining the SUSY parameter space. While the prediction of M_h through the decoupling method allows SUSY mass scales of about 4 TeV, the present scenario is already excluded when the direct method is employed. This observation explains also the need for very precise theoretical predictions for the Higgs boson mass.

In Fig. 9 the dependence of M_h on the renormalization scale is shown for the "heavy Higgs" scenario. As can be read from the figure, the scale dependence is reduced when higher order radiative corrections are taken into account. However, even at the three-loop order the variation of M_h with the renormalization scale can amount to few GeV for both methods. It is also important to notice that low values of the renormalization scale are characterized by large radiative contributions. A better alternative is the choice of the renormalization scales above 500 GeV. In the decoupling method this choice will reduce the scale variation to about 200 MeV. As for the previous scenario, we observe a milder

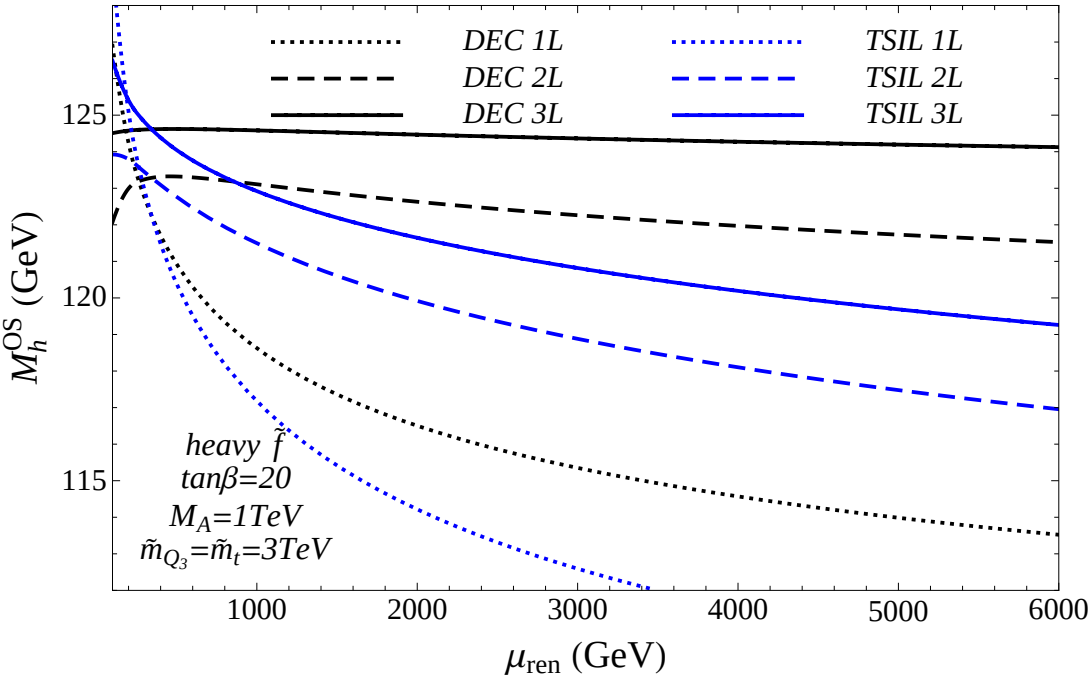

Figure 7: Dependence of the Higgs boson mass on the renormalization scale within the "heavy sfermion" scenario. The black curves display the one-(dotted line), two-(dashed line) and three-loop (full line) contributions to the lightest Higgs boson mass, when the running top-quark mass is determined using the decoupling method. For all three curves the running top-quark mass is evaluated with three-loop accuracy. The blue curves present the same predictions using for the calculation of the running top-quark mass the code `TSIL`. Beyond one-loop, only the SUSY-QCD contributions are taken into consideration.

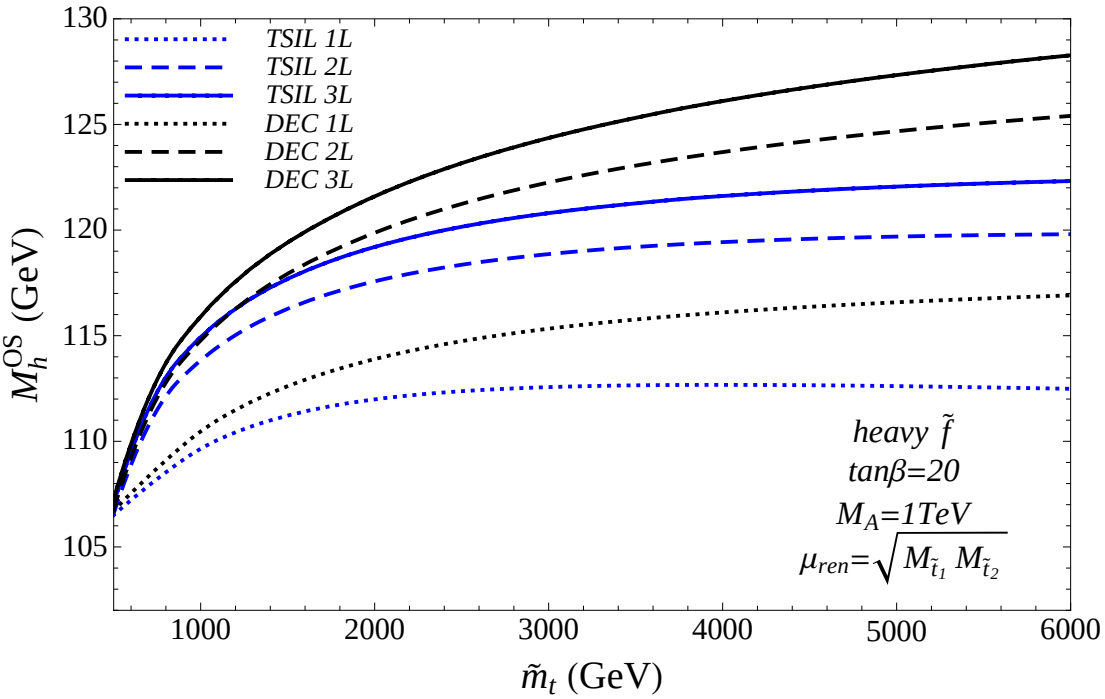

Figure 8: Dependence of the lightest Higgs boson mass on the SUSY scale $\tilde{m}_t$ for the "heavy sfermion" scenario. The same convention for the line style is used as for the Fig. 7.

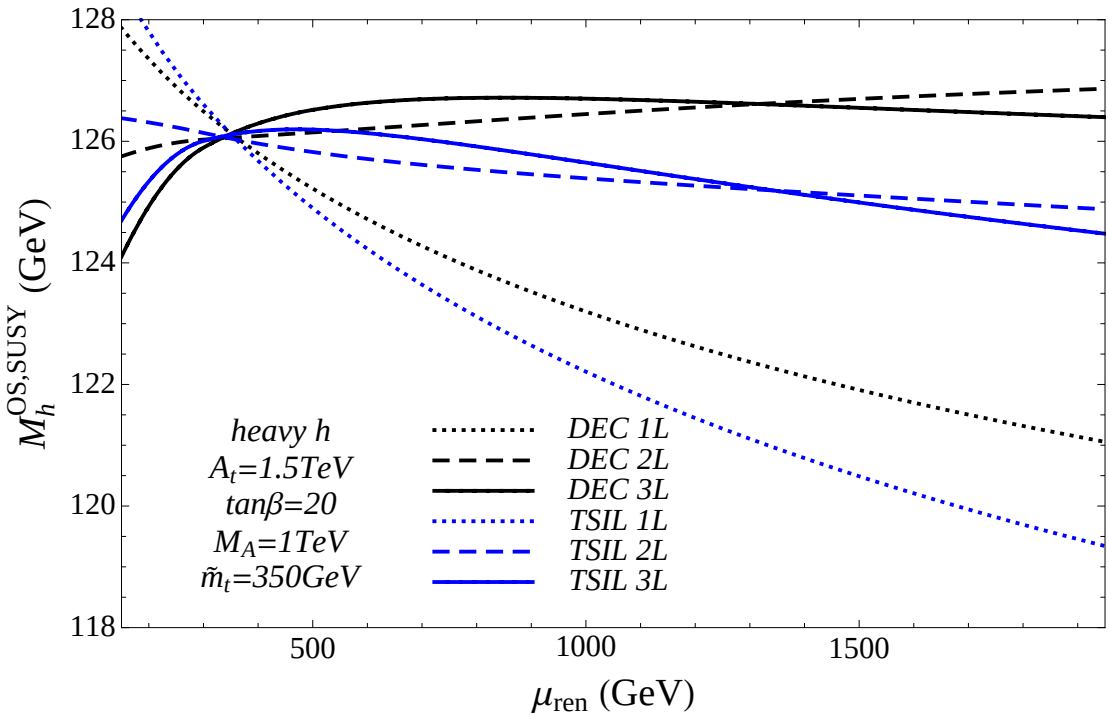

Figure 9: Dependence of the lightest Higgs boson mass on the renormalization scale within the "heavy Higgs" scenario. The same convention for the line style is used as in Fig. 7.

scale variation when the decoupling method is employed, that can be associated with smaller higher order corrections. The difference between the predictions obtained in the two frameworks amounts to few GeV for renormalization scales in the TeV range. The two methods provide same values for M_h for renormalization scale of about 400 MeV, for which also the predictions for the running top-quark mass coincide.

5 Conclusions

In this paper, we consider the calculation of $\mathcal{O}(\alpha_s^2)$ radiative corrections to the running top-quark mass and Yukawa coupling with the MSSM. Our method is based on the "running and decoupling" technique, that has the advantage to resumm the large logarithms by the use of RGEs. Our numerical analysis performed in section 3.2 showed that the method is very stable upon higher order radiative corrections. The remaining theoretical uncertainty is estimated by half of the magnitude of genuine three-loop order contributions and amounts to about 100 MeV. The method proposed here provides results for the running top-quark mass in good agreement with the predictions of the code `TSIL` for light SUSY spectra and for low-energy scales at which the top-quark mass is computed. For heavy SUSY particles and for high renormalization scales the differences between the two methods can easily reach few GeV. However, the relation between the running and the

on-shell top-quark mass in the MSSM, on which the code TSIL rely, is affected by large radiative corrections in the range of GeV. One also observes that the two methods agree better when going from LO to NLO and NNLO.
The exact analytical results of our method are available in electronic format. In addition, we provide in the paper the results for two specific mass hierarchies.

The second part of the paper presents the effects that the new determination of the running top-Yukawa coupling has on the prediction of the lightest Higgs boson mass in the SUSY-QCD. We observe a much milder dependence on the renormalization scale of the Higgs boson mass predicted with three-loop accuracy. The renormalization scale dependence is usually interpreted as a measure for the missing higher order corrections. Thus, this improvement is very welcomed given the present difficulty to achieve radiative corrections to the Higgs boson mass beyond the two-loop level. We also notice that the predictions obtained through the decoupling method are higher than the one derived in the direct method. The difference can amount to several GeV for SUSY masses in the TeV range. This behaviour can be explained by the effect of resumming the large logarithms through the use of RGEs in the determination of the running top-Yukawa coupling.
Furthermore, we implemented the decoupling method described above together with the code SLAM, that provides an interface to spectrum generators, in the existing code H3m. In this way, the code H3m computes the three-loop SUSY-QCD corrections to the Higgs boson mass, taking into account the resummation of the large logarithms of the form $\ln(M_{\rm top}/M_{\rm SUSY})$.

Finally, we want to stress that for the final prediction of the lightest Higgs boson mass within the MSSM, in addition to the SUSY-QCD corrections discussed in this paper also higher order corrections induced by the top- and bottom-Yukawa couplings have to be considered. A similar analysis for the Yukawa sector is not yet feasible because the two-loop contributions to the decoupling coefficient of the Yukawa couplings induced by mixed QCD and top- or bottom-Yukawa corrections are not known in the literature. This analysis is beyond the scope of the present paper and we postpone it to a future project. However, it is important to emphasize that the contributions that we do not consider here (including two-loop top- and bottom-Yukawa and electro-weak corrections) can range from 0.5 GeV to 2.5 GeV without resummation effects, depending on the parameters of the two scenarios analysed. These numbers have to be compared with the sum of the one- and two- and three-loop SUSY-QCD contributions that can reach from 20 GeV up to 35 GeV when no resummation effects are included. Thus, the effects of resummation discussed above that account only for the SUSY-QCD sector, although not complete, provide an essential contribution to the prediction of the lightest Higgs boson mass for supersymmetric models with heavy particles.

Acknowledgments

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References

[1] C. Englert, A. Freitas, M. Muhlleitner, T. Plehn, M. Rauch, M. Spira and K. Walz, arXiv:1403.7191 [hep-ph].

[2] A. Sirlin and R. Zucchini, Nucl. Phys. B **266** (1986) 389.

[3] R. Hempfling and B. A. Kniehl, Phys. Rev. D **51** (1995) 1386 [hep-ph/9408313].

[4] K. G. Chetyrkin and M. F. Zoller, JHEP **1206** (2012) 033 [arXiv:1205.2892 [hep-ph]].

[5] A. V. Bednyakov, A. F. Pikelner and V. N. Velizhanin, Phys. Lett. B **722** (2013) 336 [arXiv:1212.6829].

[6] K. G. Chetyrkin and M. Steinhauser, Phys. Rev. Lett. **83** (1999) 4001 [hep-ph/9907509].

[7] K. G. Chetyrkin and M. Steinhauser, Nucl. Phys. B **573** (2000) 617 [hep-ph/9911434].

[8] K. Melnikov and T. v. Ritbergen, Phys. Lett. B **482** (2000) 99 [hep-ph/9912391].

[9] P. Marquard, L. Mihaila, J. H. Piclum and M. Steinhauser, Nucl. Phys. B **773** (2007) 1 [hep-ph/0702185].

[10] M. Faisst, J. H. Kühn, T. Seidensticker and O. Veretin, Nucl. Phys. B **665** (2003) 649 [hep-ph/0302275].

[11] F. Jegerlehner and M. Y. Kalmykov, Acta Phys. Polon. B **34** (2003) 5335 [hep-ph/0310361].

[12] F. Jegerlehner and M. Y. .Kalmykov, Nucl. Phys. B **676** (2004) 365 [hep-ph/0308216].
J. Fleischer, M. Y. .Kalmykov and A. V. Kotikov, Phys. Lett. B **462** (1999) 169 [hep-ph/9905249].

[13] M. Faisst, J. H. Kühn and O. Veretin, Phys. Lett. B **589** (2004) 35 [hep-ph/0403026].

[14] B. A. Kniehl and O. L. Veretin, Nucl. Phys. B **885** (2014) 459 [arXiv:1401.1844 [hep-ph]].

[15] D. Eiras and M. Steinhauser, JHEP **0602** (2006) 010 [hep-ph/0512099].

[16] S. Fanchiotti, B. A. Kniehl and A. Sirlin, Phys. Rev. D **48** (1993) 307 [hep-ph/9212285].
A. Djouadi and P. Gambino, Phys. Rev. D **49** (1994) 3499 [Erratum-ibid. D **53** (1996) 4111] [hep-ph/9309298].

[17] M. Awramik and M. Czakon, Phys. Rev. Lett. **89** (2002) 241801 [hep-ph/0208113].
A. Onishchenko and O. Veretin, Phys. Lett. B **551** (2003) 111 [hep-ph/0209010].

[18] K. G. Chetyrkin, J. H. Kühn and M. Steinhauser, Phys. Rev. Lett. **75** (1995) 3394 [hep-ph/9504413].
L. Avdeev, J. Fleischer, S. Mikhailov and O. Tarasov, Phys. Lett. B **336** (1994) 560 [Erratum-ibid. B **349** (1995) 597] [hep-ph/9406363].

[19] [ATLAS and CDF and CMS and D0 Collaborations], arXiv:1403.4427 [hep-ex].

[20] S. P. Martin, Phys. Rev. D **72** (2005) 096008 [hep-ph/0509115].

[21] S. P. Martin and D. G. Robertson, Comput. Phys. Commun. **174** (2006) 133 [hep-ph/0501132]. S. P. Martin, Phys. Rev. D **68** (2003) 075002 [hep-ph/0307101].

[22] N. Craig, arXiv:1309.0528 [hep-ph].

[23] S. Moch, S. Weinzierl, S. Alekhin, J. Blumlein, L. de la Cruz, S. Dittmaier, M. Dowling and J. Erler *et al.*, arXiv:1405.4781 [hep-ph].

[24] P. M. Ferreira, I. Jack and D. R. T. Jones, Phys. Lett. B **387** (1996) 80 [hep-ph/9605440].

[25] R. V. Harlander, L. Mihaila and M. Steinhauser, Eur. Phys. J. C **63** (2009) 383 [arXiv:0905.4807 [hep-ph]].

[26] K. G. Chetyrkin, B. A. Kniehl and M. Steinhauser, Nucl. Phys. B **510** (1998) 61 [hep-ph/9708255].

[27] M. Steinhauser, Phys. Rev. D **59** (1999) 054005 [hep-ph/9809507].

[28] A. V. Bednyakov, Int. J. Mod. Phys. A **22** (2007) 5245 [arXiv:0707.0650 [hep-ph]].
A. V. Bednyakov, Int. J. Mod. Phys. A **25** (2010) 2437 [arXiv:0912.4652 [hep-ph]].

[29] A. Bauer, L. Mihaila and J. Salomon, JHEP **0902** (2009) 037 [arXiv:0810.5101 [hep-ph]].

[30] M. S. Carena, D. Garcia, U. Nierste and C. E. M. Wagner, Nucl. Phys. B **577** (2000) 88 [hep-ph/9912516].

[31] L. Mihaila and C. Reisser, JHEP **1008** (2010) 021 [arXiv:1007.0693 [hep-ph]].

[32] D. Noth and M. Spira, JHEP **1106** (2011) 084 [arXiv:1001.1935 [hep-ph]].

[33] M. Steinhauser, Phys. Rept. **364** (2002) 247 [hep-ph/0201075].

[34] Y. Schröder and M. Steinhauser, JHEP **0601** (2006) 051 [hep-ph/0512058].

[35] K. G. Chetyrkin, J. H. Kuhn and C. Sturm, Nucl. Phys. B **744** (2006) 121 [hep-ph/0512060].

[36] R. Harlander, L. Mihaila and M. Steinhauser, Phys. Rev. D **72** (2005) 095009 [hep-ph/0509048].

[37] A. Kurz, M. Steinhauser and N. Zerf, JHEP **1207** (2012) 138 [arXiv:1206.6675 [hep-ph]].

[38] J. Salomon, "Das Laufen und die Vereinigung der Eichkopplungen des Standardmodells zur Drei-Schleifen-Ordnung", PhD Thesis, Karlsruhe Institute of Technology, 2012.

[39] P. Nogueira, J. Comput. Phys. **105** (1993) 279.

[40] R. Harlander, T. Seidensticker and M. Steinhauser, Phys. Lett. B **426** (1998) 125 [hep-ph/9712228].

[41] T. Seidensticker, [hep-ph/99].

[42] J. A. M. Vermaseren, arXiv:math-ph/0010025.

[43] A. I. Davydychev and J. B. Tausk, Nucl. Phys. B **397**, 123 (1993).

[44] I. Jack and D. R. T. Jones, Phys. Lett. B **333** (1994) 372 [arXiv:hep-ph/9405233].

[45] A. Bednyakov, A. Onishchenko, V. Velizhanin and O. Veretin, Eur. Phys. J. C **29** (2003) 87 [hep-ph/0210258].

[46] D. M. Pierce, J. A. Bagger, K. T. Matchev and R. j. Zhang, Nucl. Phys. B **491**, 3 (1997) [arXiv:hep-ph/9606211].

[47] R. V. Harlander and M. Steinhauser, JHEP **0409** (2004) 066 [hep-ph/0409010].

[48] P. Kant, R. V. Harlander, L. Mihaila and M. Steinhauser, JHEP **1008** (2010) 104 [arXiv:1005.5709 [hep-ph]].

[49] S. Heinemeyer, W. Hollik, H. Rzehak and G. Weiglein, Eur. Phys. J. C **39** (2005) 465 [hep-ph/0411114].

[50] L. Mihaila, Adv. High Energy Phys. **2013** (2013) 607807 [arXiv:1310.6178 [hep-ph]].

[51] I. Jack, D. R. T. Jones, S. P. Martin, M. T. Vaughn and Y. Yamada, Phys. Rev. D **50** (1994) 5481 [arXiv:hep-ph/9407291].

[52] S. P. Martin, Phys. Rev. D **65** (2002) 116003 [arXiv:hep-ph/0111209].

[53] A. Denner, Fortsch. Phys. **41** (1993) 307 [arXiv:0709.1075 [hep-ph]].

[54] P. A. Baikov, K. G. Chetyrkin and J. H. Khn, arXiv:1402.6611 [hep-ph].

[55] J. Beringer et al. (Particle Data Group), Phys. Rev. D **86** (2012) 010001

[56] B. C. Allanach, Comput. Phys. Commun. **143** (2002) 305 [hep-ph/0104145].

[57] P. Z. Skands, B. C. Allanach, H. Baer, C. Balazs, G. Belanger, F. Boudjema, A. Djouadi and R. Godbole *et al.*, JHEP **0407** (2004) 036 [hep-ph/0311123].

[58] B. C. Allanach, C. Balazs, G. Belanger, M. Bernhardt, F. Boudjema, D. Choudhury, K. Desch and U. Ellwanger *et al.*, Comput. Phys. Commun. **180** (2009) 8 [arXiv:0801.0045 [hep-ph]].

[59] [ATLAS Collaboration], ATLAS-CONF-2013-014.

[60] [CMS Collaboration], CMS-PAS-HIG-13-005.

[61] J. L. Feng, P. Kant, S. Profumo and D. Sanford, Phys. Rev. Lett. **111** (2013) 131802 [arXiv:1306.2318 [hep-ph]].

[62] T. Hahn, S. Heinemeyer, W. Hollik, H. Rzehak and G. Weiglein, Phys. Rev. Lett. **112** (2014) 141801 [arXiv:1312.4937 [hep-ph]].

[63] P. Draper, G. Lee and C. E. M. Wagner, Phys. Rev. D **89** (2014) 055023 [arXiv:1312.5743 [hep-ph]].

[64] J. R. Ellis, G. Ridolfi and F. Zwirner, Phys. Lett. B **257** (1991) 83.
Y. Okada, M. Yamaguchi and T. Yanagida, Prog. Theor. Phys. **85** (1991) 1.
H. E. Haber and R. Hempfling, Phys. Rev. Lett. **66** (1991) 1815.

[65] A. Brignole, Phys. Lett. B **281** (1992) 284.
P. H. Chankowski, S. Pokorski and J. Rosiek, Nucl. Phys. B **423** (1994) 437 [hep-ph/9303309].
A. Dabelstein, Z. Phys. C **67** (1995) 495 [hep-ph/9409375].

[66] S. Heinemeyer, W. Hollik and G. Weiglein, Eur. Phys. J. C **9** (1999) 343 [hep-ph/9812472].

[67] G. Degrassi, P. Slavich and F. Zwirner, Nucl. Phys. B **611** (2001) 403 [hep-ph/0105096].
A. Brignole, G. Degrassi, P. Slavich and F. Zwirner, Nucl. Phys. B **631** (2002) 195 [hep-ph/0112177].

[68] S. Heinemeyer, W. Hollik and G. Weiglein, Comput. Phys. Commun. **124** (2000) 76, T. Hahn et al., Comput. Phys. Commun. **180** (2009) 1426; www.feynhiggs.de.

[69] J. S. Lee, A. Pilaftsis, M. S. Carena, S. Y. Choi, M. Drees, J. R. Ellis and C. E. M. Wagner, Comput. Phys. Commun. **156** (2004) 283; *ibid.* **180** (2009) 312; *ibid.* **184** (2013) 1220.

[70] W. Porod, Comput. Phys. Commun. **153** (2003) 275 [hep-ph/0301101].

[71] S. P. Martin, Phys. Rev. D **75** (2007) 055005 [hep-ph/0701051].

[72] P. Marquard and N. Zerf, Comput. Phys. Commun. **185** (2014) 1153 [arXiv:1309.1731 [hep-ph]].