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(Quasi-)Degeneration, Quantum Corrections and Neutrino Mixing*

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In the case of neutrinos having significant masses to be measured in direct search, there are sizeable corrections to the PMNS matrix. We show the universality of loop effects from any new physics sector and discuss the seesaw-extended MSSM as an example of a non-minimal flavour violating theory with non-decoupling effects.

I. INTRODUCTION

In the Standard Model of elementary particle physics, neutrinos play a special role among the fermions: neutrinos are exactly massless and neutrinos only appear as left-handed fermions. On the one hand, oscillation experiments suggest non-vanishing neutrino masses [1, 2] which at least have to be in the sub-eV regime. On the other hand, masses of righthanded neutrinos are unconstrained by the symmetries of the Standard Model and can in principle be much larger than the electroweak scale [3].

a. Neutrino masses and quasi-degeneration The absolute mass scale of the light neutrinos, however, is still unknown. Global fits on neutrino data give very precise measurements of the mass squared differences $\Delta m^2_{ij} = m^2_i - m^2_j$ [4] (similar results are obtained by other groups [5, 6]):

$$\begin{aligned} \Delta m^2_{21} &= 7.54^{+0.26}_{-0.22} \times 10^{-5}\,\mathrm{eV}^2, \\ \Delta m^2_{31} &= 2.45 \pm 0.06 \times 10^{-3}\,\mathrm{eV}^2. \end{aligned} \tag{I.1}$$

Unknown as well is the hierarchy of the spectrum: the sign of the larger Δm^2_{31} is yet undetermined and distinguishes between normal (1-2-3) or inverted (3-1-2) hierarchy.

If the lightest neutrino is sufficiently heavy, direct mass searches as done by the Karlsruhe Tritium Neutrino Experiment (KATRIN) [7] or will be able to measure it—or set a tight upper bound on the effective electron neutrino mass $m^2_{\nu_e} = \sum_i \left|U_{ei}\right|^2 m^2_i$. After three years of measurement a sensitivity $m_{\nu_e} \leq 0.2\,\mathrm{eV}$ is expected [8].

b. Quark vs. lepton mixing In the quark sector, flavour mixing is encoded in the Cabibbo-Kobayashi-Maskawa (CKM) matrix [9, 10] whose mixing angles are measured to be small leading to a CKM matrix being closely the unit matrix. On the other hand, neutrino mixing is encoded in the leptonic mixing matrix according to Pontecorvo [11], Maki, Nakagawa and Sakata [12] (PMNS) which is strongly non-hierarchical and shows rather arbitrary mixing. A schematical comparison between both is shown in Fig. 1.

The small mixings of the quark sector suggest to generate them as an effect of quantum corrections, where the mixing pattern of the PMNS matrix is usually addressed by postulating flavour symmetries. However, the possibility of radiative generation of neutrino mixing becomes important in a regime where neutrino masses are quasi-degenerate which will be proven by the direct mass searches.

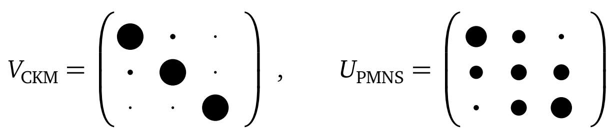

Figure 1: Sizes of the mixing matrix elements—CKM versus PMNS mixing.

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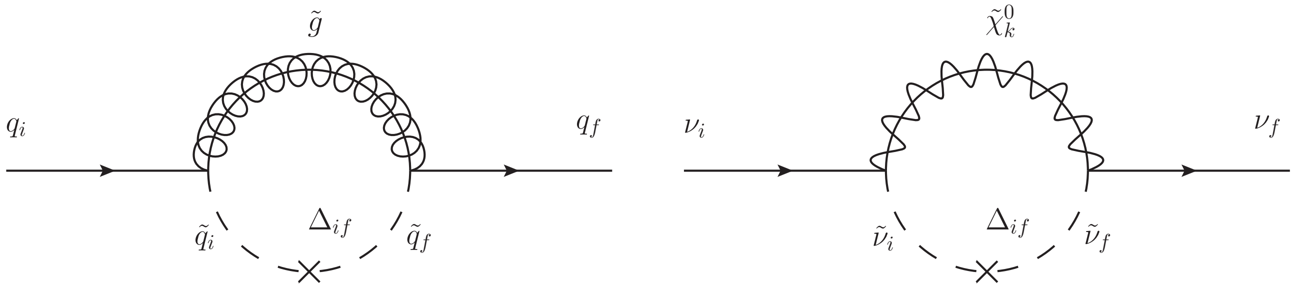

Figure 2: Flavour changing self energy by the virtue of SUSY corrections for quarks (left) and neutrinos (right).

II. RADIATIVE FLAVOUR VIOLATION

In the Standard Model, flavour violation is a property of the Yukawa couplings of fermions to the Higgs sector. Any theory beyond the Standard Model either provides *minimal flavour violation*, where all sources of flavour violation are in the Yukawa couplings, or additional sources of flavour mixing that are tightly constrained by flavour observables.

The general minimal supersymmetric Standard Model (MSSM) carries in principle arbitrary flavour structures in the terms that softly break supersymmetry. Soft breaking scalar masses as well as trilinear couplings of scalar superpartners to the Higgs scalars can mix flavour in a way that is not accounted for in the Yukawa couplings. Those contributions enter via the sfermionic mass matrix and induce flavour changes in self-energies by supersymmetric (SUSY) quantum corrections as shown in Fig. 2.

Appending those self-energies to charged current interactions (see Fig. 3 for the leptonic case), the flavour mixing matrix associated with that interaction befalls renormalization according to [13]. It was shown that in the MSSM such corrections are sufficient to generate quark mixing radiatively [14, 15]. Moreover, finite supersymmetric threshold corrections have the power to generate fermion masses radiatively [16–19]. Large effects from supersymmetry are also known in lepton flavour violation [20].

The renormalized electron-neutrino-W vertex can then be written as

$$i\frac{g}{\sqrt{2}}\gamma^{\mu}P_L U^{\dagger}_{\mathrm{PMNS}} \to i\frac{g}{\sqrt{2}}\gamma^{\mu}P_L\left(U^{(0)\dagger} + U^{(0)\dagger}\Delta U^{e} + \Delta U^{\nu}U^{(0)\dagger}\right), \tag{II.1}$$

with $U^{(0)}$ being the unrenormalized mixing matrix. The correction from the neutrino leg ΔU^{ν} is sensitive to the mass spectrum and the flavour-changing self-energy Σ^{ν}_{ij}

$$\Delta U^{\nu}_{fi} \sim \frac{m_{\nu_f}\Sigma^{\nu}_{fi}}{\Delta m^2_{fi}} \tag{II.2}$$

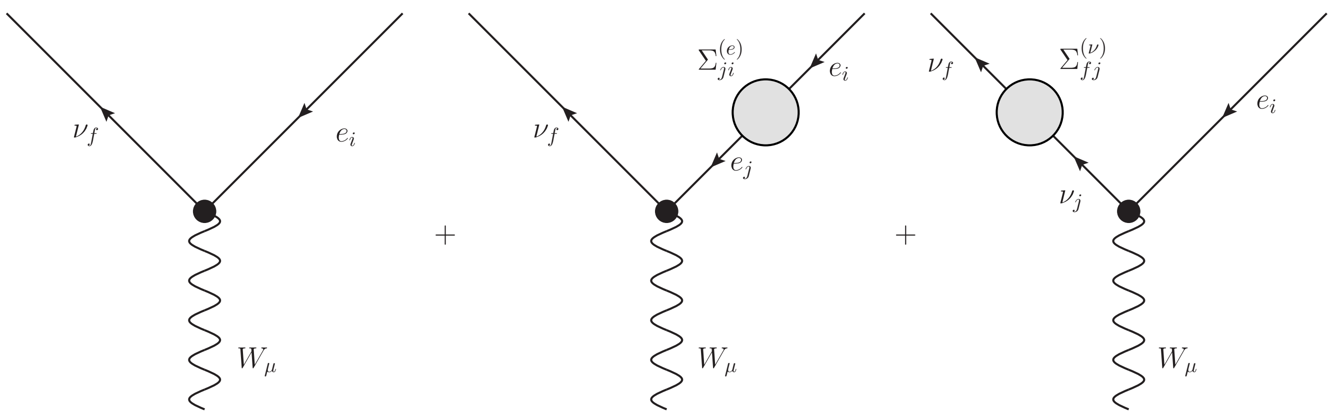

Figure 3: Renormalization of the leptonic mixing matrix by flavour changing self-energies at external legs.

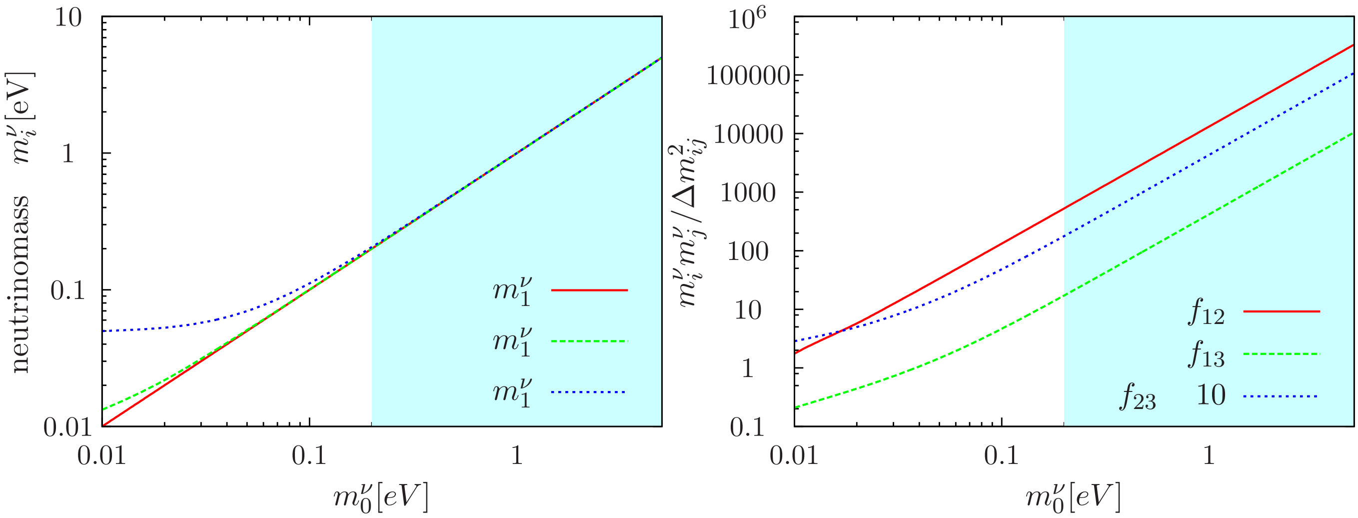

Figure 4: Enhancement factors of radiative corrections to neutrino mixing (right). The shaded area shows the KATRIN discovery regime.

and the charged lepton contribution ΔU^e small in contrast [21].

In the quasi-degenerate regime of neutrino masses, there occurs a strong enhancement of this type of corrections that can significantly supersede loop suppression factors if the absolute neutrino mass is around the KATRIN discovery limit $m_\nu^{(0)} \approx 0.35\,\mathrm{eV}$, Fig. 4:

$$f_{ij} = \frac{m_{\nu_f} m_{\nu_i}}{\Delta m_{fi}^2} \lesssim 5 \times 10^5.$$

III. THE MSSM WITH RIGHTHANDED NEUTRINOS

The standard mass generation mechanism for fermions would force neutrino Yukawa couplings being orders of magnitude smaller than the others. In contrast, the seesaw mechanism provides an effective and elegant way to cope with light neutrinos by keeping their Dirac masses m_D at the electroweak scale. A rough estimate of scales sets the mass of righthanded neutrinos m_R around $10^{13\ldots14}$ GeV for $m_D = \mathcal{O}(m_t)$:

$$m_{\nu_\ell} \sim m_D^2/m_R \approx \mathcal{O}(0.1\,\mathrm{eV}).$$

The implementation of this simplest type of seesaw mechanism in the MSSM involves the addition of righthanded neutrino superfields: chiral superfields that are gauge singlets under the Standard Model group, coupled to left-handed $SU(2)_L$ doublets via a Yukawa-type interaction and that have Majorana-type masses in the superpotential:

$$\mathcal{W}^\ell = \mu H_d \cdot H_u - Y_\ell^{IJ} H_d \cdot L_L^I E_R^J + Y_\nu^{IJ} H_u \cdot L_L^I N_R^J + \frac{1}{2} m_R^{IJ} N_R^I N_R^J, \tag{III.1}$$

with $L_L = (\ell_L, \tilde{\ell}_L) \in SU(2)_L$ and $E_R = (e_L^c, \tilde{e}_R^*)$, $N_R = (\nu_L^c, \tilde{\nu}_R^*)$. The dot-product denotes the SU(2)-invariant multiplication. Summation over I and J is understood, for symmetry reasons we take the same number of N_R fields as L_L.

To softly break supersymmetry soft breaking terms have to be included in the sneutrino sector as well:

$$\begin{aligned}\mathcal{V}_{\text{soft}} = &(\mathcal{M}_{\tilde{\ell}}^2)^{IJ} \tilde{L}_L^{I*} \tilde{L}_L^J + (\mathcal{M}_{\tilde{e}}^2)^{IJ} \tilde{e}_R^I \tilde{e}_R^{J*} + (\mathcal{M}_{\tilde{\nu}}^2)^{IJ} \tilde{\nu}_R^I \tilde{\nu}_R^{J*} \\ &- \left[(B_\nu)^{IJ} \tilde{\nu}_R^{I*} \tilde{\nu}_R^{J*} + A_e^{IJ} H_1 \cdot \tilde{L}_L^I \tilde{e}_R^{J*} - A_\nu^{IJ} H_2 \cdot \tilde{L}_L^I \tilde{\nu}_R^{J*} + \text{h.c.}\right].\end{aligned} \tag{III.2}$$

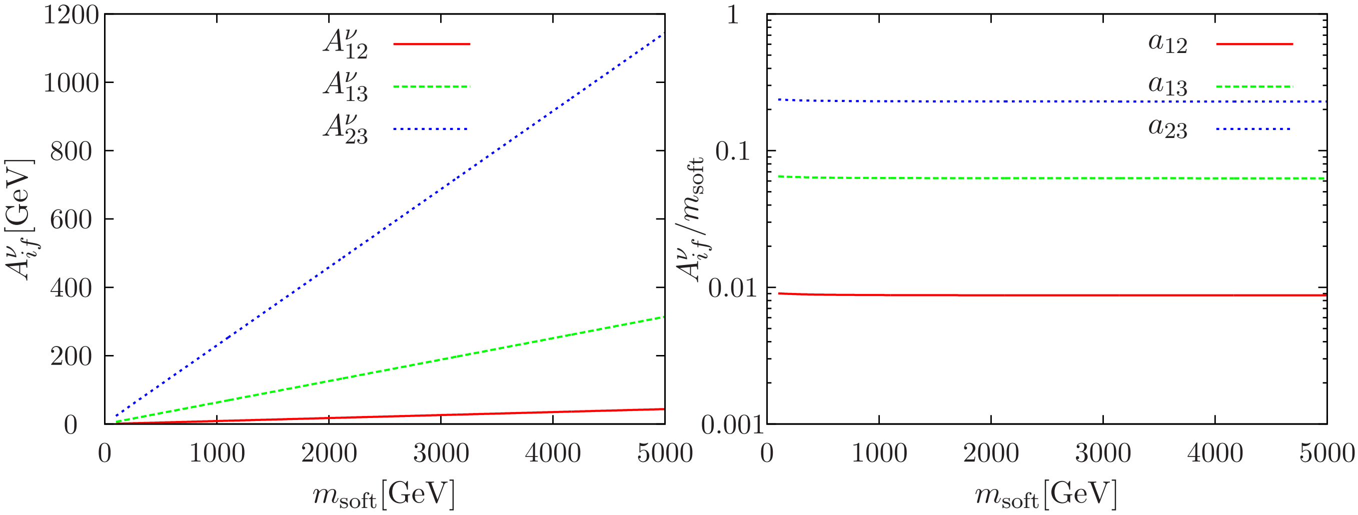

Figure 5: Off-diagonal neutrino A terms to radiatively generate PMNS mixing. As can be seen from the right-hand side, this effect does not decouple with increasing SUSY masses.

IV. SOME RESULTS

The SUSY one-loop corrections according to the flavour changing self-energies in Fig. 3 are dependent on the soft breaking parameters introduced in Eq. III.2, besides the traditional MSSM parameters: $\Delta U^{\nu} = \Delta U^{\nu}(\mathcal{M}^2_{\tilde\ell}, \mathcal{M}^2_{\tilde\nu}, A_\nu)$.

Radiative lepton decays $\ell_j \to \ell_i \gamma$ severely constrain the off-diagonal elements of the soft breaking mass $\mathcal{M}^2_{\tilde\ell}$ as well as the left-right mixing of charged sleptons, basically A_e. To comply with those flavour observables, we set them to zero in the numerical example below: $A_e \equiv 0$ and $\mathcal{M}^2_{\tilde\ell} = \mathcal{M}^2_{\tilde e} = \mathcal{M}^2_{\tilde\nu} \equiv m^2_{\rm soft}\mathbb{1}$. The absolute mass scale for the light neutrinos lies at the KATRIN discovery limit: $m^{(0)}_\nu = 0.35\,\text{eV}$.

To show the power of the method, we artificially suppress potential flavour mixing in the neutrino Yukawa coupling by alignment to the charged lepton Yukawa coupling which always can be rotated into a flavour diagonal basis without affecting the charged current mixing (in the superpotential of Eq. (III.1)). The only flavour violation which is left sits in the neutrino A terms—Fig. 5 shows the size of the off-diagonal A^{ν}_{ij} needed to produce the corresponding PMNS mixing

$$|U_{12}| \approx 0.52, \qquad |U_{13}| \approx 0.15, \qquad |U_{23}| \approx 0.58 \tag{IV.1}$$

correctly.

V. CONCLUSIONS

As has been shown in this talk, there are radiative corrections to the leptonic mixing matrix which get sizeable if neutrinos are nearly degenerate in masses. We have not included in the analysis the influence of those corrections to the masses themselves. However, since the generic structure is not limited to supersymmetric corrections but only depends on the size of the flavour changing neutrino self-energies Σ^{ν}_{ij} the description can be generalized to any theory with new flavour structures allowing for non-vanishing Σ^{ν}_{ij} whith $i \neq j$. In the case of MSSM extensions, the corrections do not decouple with increasing size of the SUSY scale. To reduce the influence on flavour changing branching ratios $\ell_j \to \ell_i \gamma$ a specific restriction on the SUSY parameter was taken. However, for the numerical example given in this talk $\text{BR}(\mu \to e\gamma) \lesssim 10^{-28}$ complying with data and similarly other Lepton Flavour Violation constraints are obeyed as well. The evaluation of the branching ratios was performed according to the results of [22].

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[1] **Super-Kamiokande Collaboration** Collaboration, Y. Fukuda *et al.*, "Evidence for oscillation of atmospheric neutrinos," *Phys.Rev.Lett.* **81** (1998) 1562–1567, arXiv:hep-ex/9807003 [hep-ex].
[2] **SNO Collaboration** Collaboration, Q. Ahmad *et al.*, "Direct evidence for neutrino flavor transformation from neutral current interactions in the Sudbury Neutrino Observatory," *Phys.Rev.Lett.* **89** (2002) 011301, arXiv:nucl-ex/0204008 [nucl-ex].
[3] P. Minkowski, "$\mu \to e\gamma$ at a Rate of One Out of 1-Billion Muon Decays?," *Phys.Lett.* **B67** (1977) 421.
[4] F. Capozzi, G. Fogli, E. Lisi, A. Marrone, D. Montanino, *et al.*, "Status of three-neutrino oscillation parameters, circa 2013," *Phys.Rev.* **D89** (2014) 093018, arXiv:1312.2878 [hep-ph].
[5] D. Forero, M. Tortola, and J. Valle, "Neutrino oscillations refitted," arXiv:1405.7540 [hep-ph].
[6] M. Gonzalez-Garcia, M. Maltoni, and T. Schwetz, "Updated fit to three neutrino mixing: status of leptonic CP violation," arXiv:1409.5439 [hep-ph].
[7] **KATRIN Collaboration** Collaboration, A. Osipowicz *et al.*, "KATRIN: A Next generation tritium beta decay experiment with sub-eV sensitivity for the electron neutrino mass. Letter of intent," arXiv:hep-ex/0109033 [hep-ex].
[8] **KATRIN Collaboration** Collaboration, J. Angrik *et al.*, "KATRIN design report 2004," *FZKA-7090* (2005) .
[9] N. Cabibbo, "Unitary Symmetry and Leptonic Decays," *Phys.Rev.Lett.* **10** (1963) 531–533.
[10] M. Kobayashi and T. Maskawa, "CP Violation in the Renormalizable Theory of Weak Interaction," *Prog.Theor.Phys.* **49** (1973) 652–657.
[11] B. Pontecorvo, "Inverse beta processes and nonconservation of lepton charge," *Sov.Phys.JETP* **7** (1958) 172–173.
[12] Z. Maki, M. Nakagawa, and S. Sakata, "Remarks on the unified model of elementary particles," *Prog.Theor.Phys.* **28** (1962) 870–880.
[13] A. Denner and T. Sack, "Renormalization of the Quark Mixing Matrix," *Nucl.Phys.* **B347** (1990) 203–216.
[14] J. Ferrandis and N. Haba, "Supersymmetry breaking as the origin of flavor," *Phys.Rev.* **D70** (2004) 055003, arXiv:hep-ph/0404077 [hep-ph].
[15] A. Crivellin and U. Nierste, "Supersymmetric renormalisation of the CKM matrix and new constraints on the squark mass matrices," *Phys.Rev.* **D79** (2009) 035018, arXiv:0810.1613 [hep-ph].
[16] A. Lahanas and D. Wyler, "Radiative Fermion Masses and Supersymmetry," *Phys.Lett.* **B122** (1983) 258.
[17] A. Masiero, D. V. Nanopoulos, and K. Tamvakis, "Radiative Fermion Masses in Supersymmetric Theories," *Phys.Lett.* **B126** (1983) 337.
[18] T. Banks, "Supersymmetry and the Quark Mass Matrix," *Nucl.Phys.* **B303** (1988) 172.
[19] F. Borzumati, G. R. Farrar, N. Polonsky, and S. D. Thomas, "Soft Yukawa couplings in supersymmetric theories," *Nucl.Phys.* **B555** (1999) 53–115, arXiv:hep-ph/9902443 [hep-ph].
[20] F. Borzumati and A. Masiero, "Large Muon and electron Number Violations in Supergravity Theories," *Phys.Rev.Lett.* **57** (1986) 961.
[21] J. Girrbach, S. Mertens, U. Nierste, and S. Wiesenfeldt, "Lepton flavour violation in the MSSM," *JHEP* **1005** (2010) 026, arXiv:0910.2663 [hep-ph].
[22] J. Hisano, T. Moroi, K. Tobe, and M. Yamaguchi, "Lepton flavor violation via right-handed neutrino Yukawa couplings in supersymmetric standard model," *Phys.Rev.* **D53** (1996) 2442–2459, arXiv:hep-ph/9510309 [hep-ph].