On the NNLO QCD corrections to single-top production at the LHC

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We present a fully-differential calculation of the NNLO QCD corrections to the t-channel mechanism for producing single top quarks at the LHC. We work in the structure function approximation, computing QCD corrections to the light- and heavy-quark lines separately and neglecting the dynamical cross-talk between the two. The neglected contribution, which appears at NNLO for the first time, is color-suppressed and is expected to be sub-dominant. Within this approximation, we find that, for the total cross section, NNLO QCD corrections are in the few percent range and, therefore, are comparable to NLO QCD corrections. We also find that the scale independence of the theoretical prediction for single-top production improves significantly once NNLO QCD corrections are included. Furthermore, we show how these results change if a cut on the transverse momentum of the top quark is applied and derive the NNLO QCD prediction for the ratio of single top and single anti-top production cross sections at the 8 TeV LHC.

I. INTRODUCTION

Studies of top quarks produced in hadron collisions are important for understanding many properties of these heavy particles, including their masses, their couplings to electroweak gauge bosons, their Cabbibo-Kobayashi-Maskawa matrix element V_{tb} etc. In many cases, the precision reached in measuring these quantities is already close to a few a percent, thanks to the successful top quark physics programs at the Tevatron and the LHC. Further high-statistics data samples, that will become available during a forthcoming 13 TeV run of the LHC, will remove statistical uncertainties as a limiting factor for these measurements (see e.g. [1]). As the result, theoretical uncertainties related to imprecise knowledge of production cross sections and kinematic distributions will become an important limiting factor in pushing precision measurements forward.

There are two main mechanisms for producing top quarks in hadron collisions. Both at the Tevatron and the LHC, the dominant one occurs due to strong interactions and, through such processes as $q\bar{q} \to t\bar{t}$ or $gg \to t\bar{t}$, leads to the production of $t\bar{t}$ pairs. The theoretical description of this production mechanism is very advanced; it includes NLO QCD and electroweak corrections, soft gluon resummations and, since recently, complete NNLO QCD corrections [2–7]. The second mechanism is governed by weak interactions and relies on the flavor changing transitions $W^* \to tb$, $b \to tW$ or $W^*b \to t$ to produce single top (or anti-top) quarks. Although sub-dominant relative to $t\bar{t}$ pair production, this mechanism yields a sizable fraction of top quark events both at the Tevatron and the LHC. Experimental conditions for studying single-top production at the two colliders are however, very different. Indeed, when top quarks decay, they produce leptons, missing energy and b-jets. Correspondingly, the main background for observing single-top production at a hadron collider is the direct production of W bosons in association with jets in general and with b-jets in particular. The severity of this background and the relative smallness $\mathcal{O}(1\ \text{pb})$ of the single-top production cross section made detailed studies of this process at the Tevatron very difficult. Nevertheless, the CDF and D0 collaborations confirmed the existence of the electroweak production mechanism for top quarks and measured the cross section for this process with, approximately, twenty percent precision [8–11]. Since the single-top production cross section is proportional to the electroweak coupling of a top quark to a W-boson, a $\mathcal{O}(20\%)$ measurement of the production cross section can be interpreted as a $\mathcal{O}(10\%)$ measurement of the CKM matrix element $|V_{tb}|$ or a $\mathcal{O}(20\%)$ measurement of the top quark width.

Experimental conditions improve dramatically at the LHC where the single top quark production cross section is significantly higher, approximately 60 pb at the 8 TeV LHC and 160 pb at the 14 TeV LHC. Given that expected integrated LHC luminosities are in the range of a few hundreds inverse femtobarns, millions of top quarks will be produced at the LHC by virtue of electroweak interactions alone, making high-precision studies of this production mechanism an important part of the experimental program. Indeed, already in the first run of the LHC, ATLAS and CMS collaborations improved significantly on the CDF and D0 results, by measuring the single-top -production cross sections with a ten percent accuracy [12–16]. Similar to what we discussed in the context of the Tevatron, such a measurement can be interpreted as a $\mathcal{O}(5\%)$ measurement of $|V_{tb}|$ and a $\mathcal{O}(10\%)$ measurement of the top quark width. This is the highest experimental precision available for these quantities currently.

It is important to emphasize that there are several experimentally distinguishable ways to produce single top quarks through electroweak interactions. Indeed, writing the primary electroweak tbW-vertex in three different ways, $W^*b \to t$, $W^* \to tb$, $b \to tW$, we obtain dis-

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tinct mechanisms for single top quark production that are usually referred to as the t-channel ($W^*b \to t$) process, the s-channel process ($W^* \to tb$) and the tW production ($b \to Wt$). Among these three mechanisms, the t-channel process has the largest cross section both at the 8 TeV LHC and at the Tevatron contributing, respectively, 82% and 65% to the total cross section σ_t. The s-channel process is 33% of σ_t at the Tevatron and is about 5% at the 8 TeV LHC. The tW production is negligible at the Tevatron and contributes $\mathcal{O}(15\%)$ to σ_t at the 8 TeV LHC. However, since tW production can be distinguished from the other two mechanisms, it is usually treated separately in experimental analyses. Also, we note that for the higher-energy LHC, the t-, s- and tW production channels contribute in similar proportions as for the 8 TeV LHC.

Theoretical results for single top quark production are available at an ever increasing level of sophistication. These include NLO QCD and electroweak predictions in four- or five-flavor scheme for both stable [17–21] and decaying [22–30] top quarks, resummations [31–34] and fixed order computations matched to parton showers [35–38]. Focusing on NLO QCD corrections, we note that they are small, of the order of a few percent, for the t-channel single-top production. On the other hand, NLO QCD corrections for the s-channel single-top production are large, of the order of fifty percent, both at the Tevatron and the LHC. Corrections to the associated tW production are known to be moderate at both colliders [23].

We note that the smallness of the NLO QCD corrections to the t-channel single-top production cross section is the result of strong cancellations between different sources of such corrections, e.g. different partonic channels. It is unclear if these cancellations are accidental or generic and if the smallness of NLO QCD corrections implies that NNLO QCD corrections are, in fact, even smaller as should be the case for convergent perturbative series. The most obvious reflection of this fact is the strong sensitivity of the NLO QCD prediction for t-channel single-top production to choices of factorization and renormalization scales. In fact, the sensitivity of the NLO QCD corrections to these scales is comparable to the size of corrections themselves. Therefore, if the scale variation is an indication of the size of missing higher-order QCD corrections, it is tempting to conclude that NLO QCD corrections to t-channel single-top production cross section are *accidentally* small and that the natural size of NNLO QCD corrections to this process is at a few percent level. Therefore, we expect that NLO and NNLO QCD corrections to the single-top production cross section are of a similar size. This implies that NNLO QCD corrections to t-channel single-top production cross section must be computed to enable studies of electroweak production of top quarks at the LHC with a percent accuracy.

The goal of this paper is to make the first step towards a high-precision prediction for the single-top production at the LHC, by providing fully-differential NNLO QCD corrections to the t-channel single top quark production in the approximation where corrections to light quark $q \to q'W^*$ and heavy quark $W^*b \to t$ weak transitions are treated (almost) independently from each other. More precisely, we neglect all dynamical cross-talk between corrections to the light and heavy quark lines, which then depend on each other only through kinematic phase-space constraints. At NLO, this approximation is exact due to color conservation. At NNLO however, the exchange of two (real or virtual) gluons in a color-singlet state between light and heavy quark lines, shown in Fig. 1d, leads to a non-vanishing contribution to the cross section. We expect this contribution to be small since it is suppressed by at least two powers of the number of colors $N_c = 3$ relative to the "factorizable" contributions shown in Fig. 1a-c. Therefore, we neglect the non-factorizable contributions in the rest of the paper.

The paper is organized as follows. In Section II we briefly discuss the technical details of the calculation. In Section III we show some results for NNLO QCD corrections to single-top and single anti-top production at the 8 TeV LHC. We conclude in Section IV.

II. TECHNICAL DETAILS OF THE CALCULATION

Our goal is to compute NNLO QCD corrections to t-channel single top quark production. The top quarks are considered stable. In the approximation where only factorizable corrections are retained, the calculation can be divided into three different parts. We need to compute *i)* NNLO QCD corrections to the weak transition on a heavy quark line $W^*b \to t$, c.f. Fig.1a; *ii)* NNLO QCD corrections to the weak transition on a light quark line $u \to W^*d$, c.f. Fig.1b; and *iii)* a product of NLO QCD corrections to weak transitions on both heavy and light quark lines, c.f. Fig.1c. These three contributions are individually infra-red and collinear finite, gauge invariant, and can be considered separately. We now briefly illustrate some of the technical details of our computation, starting from corrections to the heavy quark line.

As a preliminary remark, we note that the computation of NNLO QCD corrections to a process X requires three ingredients: 1) two-loop QCD corrections to X; 2) one-loop QCD corrections to X+jet and 3) tree-level matrix element for the $X + 2$ jet process. All of the required ingredients to compute NNLO QCD corrections to the $W^*b \to t$ transition, for arbitrary invariant mass of the W-boson, can be obtained by crossing the two-loop, one-loop and tree amplitudes used by us recently in the computation of NNLO QCD corrections to top quark decay $t \to W^*b$ [39]. This crossing is straightforward for one- and two-loop virtual amplitudes to the $0 \to tW\bar{b}$ vertex [40–44] (since they depend on a very small number of kinematic invariants) and for tree-level amplitudes $t \to bW^*gg$ and $t \to W^*bq\bar{q}$. The crossing

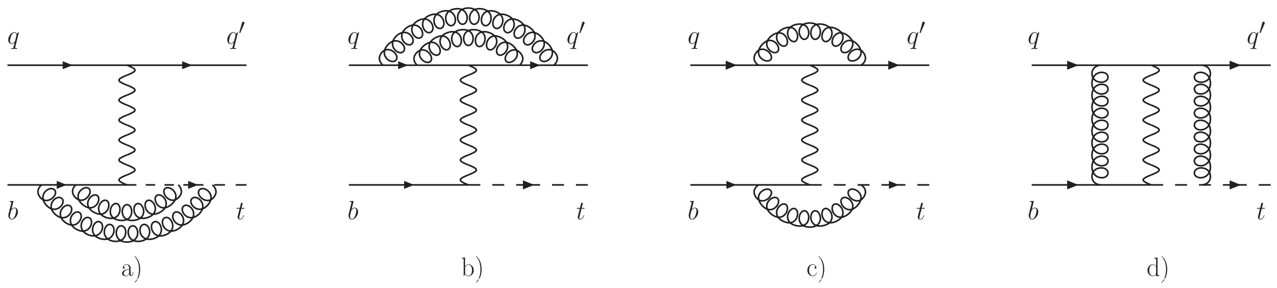

FIG. 1: Schematic representation of different contributions to the NNLO QCD corrections for the t-channel single-top production. From left to right: a) NNLO corrections the the heavy quark line, b) NNLO corrections to the light quark line, c) product of NLO corrections to the heavy and the light quark lines, and d) non-factorizable contributions that are neglected in this paper. Corresponding real emission diagrams, as well as crossed channels, are not shown.

is potentially more challenging for one-loop corrections to $t \to bgW^*$ [23], that we borrow from the MCFM program [45] since, in this case, the number of kinematic invariants is larger. To ensure that this analytic continuation is correct, we computed $0 \to t\bar{b}gW^*$ amplitudes in physical kinematics, where an off-shell W^* boson and a b-quark collide to produce a gluon and a top quark, using our own implementation of the Passarino-Veltman reduction procedure for one-loop tensor integrals [46], and found complete agreement with the result obtained by crossing one-loop MCFM amplitudes.

In general, the computation of NNLO QCD corrections to *any* process is made complicated by the fact that all three ingredients for NNLO computations that we listed above are separately infra-red and collinear divergent. We regularize and extract these divergences by constructing subtraction terms, following the approach described in Refs. [47–49][1]. The subtraction terms involve products of lower-multiplicity matrix elements with appropriate splitting functions or eikonal currents that need to be computed for both initial and final state partons. The initial state singularities were absent in the computation of NNLO QCD corrections to top decay [39] but they are an important part in the computation of single-top production. The corresponding splitting functions, both tree and one-loop, are well-known [52–59] and the one-loop eikonal current for the $bW^* \to t$ transition [60] is again obtained by crossing the current that we employed in the top quark decays computation [39].

In computing radiative corrections to top quark decay we were able to argue that, due to simple kinematics of that process, we do not need to consider a true extension of momenta of any particle to d-dimensional space. This is so because, when the initial top quark is at rest, for all sub-processes it is possible to choose a reference frame where momenta of all particles can be parametrized by four-dimensional vectors [39]. However, in the case of single-top production, the kinematics is richer and we are forced to extend the parametrization of momenta of final state particles in such a way that explicit $(d-4)$-dimensional momenta components appear. We note that such an extension is non-trivial if we want to *i*) keep the *phase-space locality* of subtraction terms and *ii*) represent $(d-4)$-dimensional momenta components in such a way that the numerical integration of amplitudes remains possible. To deal with this issue, we closely follow the implementation described in Ref. [61].

Even after all the relevant ingredients for the computation of NNLO QCD radiative corrections to weak transition on a heavy quark line are put together, the result still contains collinear singularities. These singularities are removed by the renormalization of parton distribution functions. Since parton distribution functions mix under collinear renormalization, we are forced to consider single top quark production in such partonic channels as $gW^* \to t\bar{b}$ that appears first at next-to-leading order and $qW^* \to t\bar{b}q$ that appears first at NNLO. The calculation of radiative corrections to those channels proceeds along the same lines as for $W^*b \to t$; the only difference is that virtual corrections, either two- or one-loop, do not necessarily contribute to those channels.

Although in our approximation NNLO QCD corrections to the heavy quark line are treated as independent from corrections to the light quark line, the heavy and the light quark lines do influence each other due to kinematic constraints. Indeed, for computing radiative corrections, it is convenient to treat the t-channel single-top production as a whole process and parametrize kinematics of the full $ub \to dt$ scattering, rather than the kinematics of the $W^*b \to t$ transition only. Therefore, when considering corrections to the heavy quark line we would like to parametrize the kinematics of a scattering process where a massive particle and a massless particle are produced in the collision of two massless particles, and where no singularities are associated with the massless outgoing particle. It is easy to realize that the phase-space parametrization for this case can be directly borrowed from the calculation of the Higgs boson production in association with a jet [61]. The corresponding formu-

[1] For other approaches to NNLO computations, see [50]. Some phenomenological applications can be found in [7, 39, 51, 61].

$p_\perp$	$\sigma_{\rm LO}$, pb	$\sigma_{\rm NLO}$, pb	$\delta_{\rm NLO}$	$\sigma_{\rm NNLO}$, pb	$\delta_{\rm NNLO}$
0 GeV	$53.8^{+3.0}_{-4.3}$	$55.1^{+1.6}_{-0.9}$	$+2.4\%$	$54.2^{+0.5}_{-0.2}$	-1.6%
20 GeV	$46.6^{+2.5}_{-3.7}$	$48.9^{+1.2}_{-0.5}$	$+4.9\%$	$48.3^{+0.3}_{-0.02}$	-1.2%
40 GeV	$33.4^{+1.7}_{-2.5}$	$36.5^{+0.6}_{-0.03}$	$+9.3\%$	$36.5^{+0.1}_{+0.1}$	-0.1%
60 GeV	$22.0^{+1.0}_{-1.5}$	$25.0^{+0.2}_{+0.3}$	$+13.6\%$	$25.4^{-0.1}_{+0.2}$	$+1.6\%$

TABLE I: QCD corrections to t-channel single top quark production cross sections at 8 TeV LHC with a cut on the transverse momentum of the top quark $p_\perp$. Cross sections are shown at leading, next-to-leading and next-to-next-to-leading order in dependence of the factorization and renormalization scale $\mu = m_t$ (central value), $\mu = 2m_t$ (upper value) and $\mu = m_t/2$ (lower value). Corrections at NLO and at NNLO (relative to the NLO) are shown in percent for $\mu = m_t$.

las for the phase-space parametrization relevant for the $ub \to dt$, $ub \to dtg$ and $ub \to dtgg$ sub-processes, as well as a discussion of an appropriate choices of variables relevant for the extraction of singularities can be found in that reference. Using the language of that paper, we only need to consider "initial-state" sectors since there are no collinear singularities associated with final state particles due to the fact that top quarks are massive. All calculations required for initial-state sectors are documented in Ref. [61] except that here we need soft and collinear limits for incoming quarks, rather than gluons, and the soft current for a massive particle. This, however, is a minor difference that does not affect the principal features of the computational method.

The above discussion of the NNLO QCD corrections to the heavy quark line can be applied almost verbatim to corrections to the light quark line. The two-loop corrections for the $0 \to q\bar{q}'W^*$ vertex are known since long ago [62–64]. One-loop corrections to $0 \to q\bar{q}'gW^*$ scattering are also well-known; we implemented the result presented in [65] and again checked the implementation against an independent computation based on the Passarino-Veltman reduction. Apart from different amplitudes, the only minor difference with respect to corrections to the heavy quark line is that in this case there are collinear singularities associated with both, the incoming and the outgoing quark lines. We deal with this problem splitting the real-emission contribution into sectors, see Ref. [61]. In the language of that paper, we have to consider "initial-initial", "final-final" and mixed "initial-final" sectors. Finally, we briefly comment on the contribution shown in Fig.1c. We note that, although formally NNLO, it is effectively the product of NLO corrections to the heavy and the light quark lines, so that it can be dealt with using techniques familiar from NLO computations.

We will now comment on our treatment of γ_5. For perturbative calculations at higher orders the presence of the Dirac matrix γ_5 is a nuisance since it can not be continued to d-dimensions in a straightforward way. While computationally-efficient ways to deal with γ_5 in computations, that employ dimensional regularization, exist (see e.g. Ref. [66]), they are typically complex and untransparent. Fortunately, there is a simple way to solve the γ_5 problem in our case. Indeed, in the calculation of virtual corrections to the tWb weak vertex, γ_5 is taken to be anti-commuting [40–43]. This enforces the left-handed polarization of the b-quark and removes the issue of γ_5 altogether. Indeed, if we imagine that the weak $b \to t$ transition is facilitated by the vector current but we select the b-quark with left-handed polarization only, we will obtain the same result as when the calculation is performed with the anti-commuting γ_5. Since the cancellation of infra-red and collinear divergences occurs for each polarization of the incoming b-quark separately, this approach completely eliminates the need to specify the scheme for dealing with γ_5 and automatically enforces simultaneous conservation of vector and axial currents – a must-have feature if quantum anomalies are neglected. Of course, this requires that we deal with the γ_5 appearing in real emission diagrams in the same way as in the virtual correction and this is, indeed, what we do by using helicity amplitudes, as described in [39].

We have performed several checks to ensure that our calculation of NNLO QCD corrections to single top quark production is correct. For example, we have compared all the tree-level matrix elements that are used in this computation, e.g. $ub \to dt + ng$, with $0 \le n \le 2$, $ub \to dt + q\bar{q}$, $ug \to d\bar{b}t + mg$, $0 \le m \le 1$, against MadGraph [67] and found complete agreement. We have extracted one-loop amplitudes for $0 \to Wt\bar{b}g$ from MCFM [45] and checked them against our own implementation of the Passarino-Veltman reduction, for both the $W^*b \to tg$ and the $W^*g \to t\bar{b}$ processes. We have cross-checked one-loop amplitudes for $W^*u \to dg$ and related channels against MadLoop [68]. In the intermediate stages of the computation, we also require reduced tree and one-loop amplitudes computed to higher orders in ϵ, as explained e.g. in Ref. [61]. We checked that their contributions drop out from the final results, in accord with the general conclusion of Ref. [69].

One of the most important checks is provided by the cancellation of infra-red and collinear divergences. Indeed, the technique for NNLO QCD computations described in Refs. [47–49] leads to a Laurent expansion of different contributions to differential cross sections in the dimensional regularization parameter ϵ; coefficients of this expansion are computed by numerical integration. Independence of physical cross sections on the regularization parameter is therefore achieved numerically, when different contributions to such cross sections (two-loop virtual corrections, one-loop corrections to single

$p_\perp$	$\sigma_{\rm LO}$, pb	$\sigma_{\rm NLO}$, pb	$\delta_{\rm NLO}$	$\sigma_{\rm NNLO}$, pb	$\delta_{\rm NNLO}$
0 GeV	$29.1^{+1.7}_{-2.4}$	$30.1^{+0.9}_{-0.5}$	+3.4%	$29.7^{+0.3}_{-0.1}$	−1.3%
20 GeV	$24.8^{+1.4}_{-2.0}$	$26.3^{+0.7}_{-0.3}$	+6.0%	$26.2^{-0.01}_{-0.1}$	−0.4%
40 GeV	$17.1^{+0.9}_{-1.3}$	$19.1^{+0.3}_{+0.1}$	+11.7%	$19.3^{-0.2}_{+0.1}$	+1.0%
60 GeV	$10.8^{+0.5}_{-0.7}$	$12.7^{+0.03}_{+0.2}$	+17.6%	$12.9^{-0.2}_{+0.2}$	+1.6%

TABLE II: QCD corrections to the t-channel single anti-top production cross sections at 8 TeV LHC with a cut on the transverse momentum of the anti-top quark $p_\perp$. Cross sections are shown at leading, next-to-leading and next-to-next-to-leading order in dependence of the factorization and renormalization scale $\mu = m_t$ (central value), $\mu = 2m_t$ (upper value) and $\mu = m_t/2$ (lower value). Corrections at NLO and at NNLO (relative to the NLO) are shown in percent for $\mu = m_t$.

real emission contributions, double real emission contributions, renormalization, collinear subtractions of parton distribution functions, etc.) are combined. The numerical cancellation of the $\mathcal{O}(\epsilon^i)$ contributions, $-4 \le i \le -1$ is an important check of the calculation. We computed partonic cross sections for the t-channel single-top production at three different center of mass energies and observed cancellation of $1/\epsilon^4, 1/\epsilon^3, 1/\epsilon^2$ and $1/\epsilon$ singularities. For the $1/\epsilon$ contributions to the cross section, we find that the cancellation is at the per mill level, independent of the center-of-mass collision energy. For higher poles, cancellations improve by, roughly, an order of magnitude per power of $1/\epsilon$. We have also checked that similar degree of cancellations is achieved for hadronic cross sections, which are computed by integrating partonic cross sections with parton distribution functions.

III. RESULTS

We are now in position to present the results of our calculation. We have chosen to consider the 8 TeV LHC. We use the MSTW2008 set for parton distribution functions and α_s; when results for N^{k}LO cross sections are reported, the relevant PDF set and α_s value are used. We also set the CKM matrix to the identity matrix, the top quark mass to $m_t = 173.2$ GeV, the Fermi constant to $G_F = 1.16639 \times 10^{-5}$ GeV^{-2} and the mass of the W boson to 80.398 GeV. The factorization and renormalization scales are by default set to the value of the top quark mass m_t and varied by a factor two to probe sensitivity of the results to these unphysical scales.[2] We account for all partonic channels. At LO, this means that the light quark transition is initiated either by an up-type quark or by a down-type anti-quark, while the heavy quark transition can only be initiated by a b-quark. At NLO, the gluon channel opens up, both for the light and the heavy quark transitions. At NNLO, in addition to that, we also have to take into account pure singlet contributions, for example $W^*b \to b\bar{u}d$ for the light quark line and $W^*u \to u\bar{b}t$ for the heavy quark line. Although we include all partonic channels in our calculation, it turns out that their contributions to single-top production differ significantly. Indeed, we find that it is important to include $bu \to dt$, $gu \to dt\bar{b}$, $qu \to dqt\bar{b}$ and $gb \to q\bar{q}'t$ in the computation of NLO and NNLO QCD corrections while other channels can, in principle, be neglected.

The simplest observable to discuss is the total cross section. Using the input parameters given in the previous paragraph, we find the leading order cross section for single-top production at 8 TeV LHC to be $\sigma_t^{\rm LO} = 53.8$ pb, if we set the renormalization and factorization scales to $\mu = m_t$. The next-to-leading order QCD cross section at $\mu = m_t$ is $\sigma_t^{\rm NLO} = 55.1$ pb, corresponding to an increase of the leading order cross section by 2.5 percent. It is important to realize that this small increase is the result of significant cancellations between various sources of QCD corrections. For example, NLO QCD corrections in the bq partonic channel increase the leading order cross section by 10%, which is more in line with the expected size of NLO QCD corrections. However, this positive correction is largely canceled by the quark-gluon channel that appears at next-to-leading order for the first time. The gluon-initiated channels have large and negative cross sections. Indeed, the $qg \to t\bar{b}q'$ and $gb \to q\bar{q}'t$ partonic processes change the leading order cross section by −14%. When the leading order cross section is computed with NLO PDFs, it increases by 8%. Finally, when all the different contributions are combined, a small positive change in the single-top production cross section at NLO is observed. The scale dependence of leading and next-to-leading order cross sections is shown in Table I. For the total single-top production cross section, we observe that the residual scale dependence at NLO is at a few percent level. For $\mu = m_t$, the NNLO QCD cross section is $\sigma_t^{\rm NNLO} = 54.2$ pb, corresponding to a decrease of the NLO cross section by −1.5%. The magnitude of NNLO corrections is similar to the NLO corrections, illustrating the accidental smallness of the latter. As can be seen from Table I, the residual scale dependence of the NNLO result is very small, of the order of one percent.

The simplest observable, beyond the total cross section that one can study, is the cross section with a cut on the transverse momentum of the top quark. The corre-

[2] We note that by comparing NLO QCD corrections to single-top production in four- and five-flavor schemes, it was suggested [25] that choosing $m_t/2$ as a central value is more appropriate. Given reduced dependence on the renormalization/factorization scales at NNLO, this issue is less relevant for our computation.

sponding cross sections and QCD corrections are shown in Table I. It follows from there that the QCD corrections strongly depend on the minimal value of the top quark transverse momentum. As we already mentioned, the total cross section, corresponding to $p_\perp = 0$ exhibits very small NLO QCD corrections that are, in fact, comparable to the residual scale uncertainty. At higher values of the cut on the top transverse momentum, the corrections become larger and reach $+14\%$ at $p_\perp = 60$ GeV. However, the magnitude of the scale uncertainty of the NLO QCD prediction is nearly $p_\perp$-independent and remains at a few percent level for both small and large values of $p_\perp$. This suggests that, once the NNLO QCD corrections are taken into account, the single-top production cross section becomes known with a few percent precision for all values of $p_\perp$. This is indeed what happens, as one can see from Table I. Indeed, regardless of the size of NLO QCD corrections – that are very different for different values of $p_\perp$ – the NNLO QCD corrections are always in the range of just a few percent and the residual scale dependence is also in the one percent range. Therefore, availability of the NNLO QCD corrections enables very accurate predictions for single-top production for all values of the top quark transverse momentum.

We can also study the production of single anti-top quarks in proton-proton collisions. The corresponding results for the total cross section are shown in Table II. The magnitude of QCD corrections for anti-top are similar to that of the top, although they are somewhat larger. Nevertheless, also for the $\bar{t}$ case one can see an impressive stabilization of the NNLO QCD cross sections and only marginal residual dependence on the factorization and renormalization scales.

It is common in experimental analyses to measure and quote the sum of top and anti-top production cross sections. The combination of ATLAS and CMS t-channel single-t and single-$\bar{t}$ measurements at the 8 TeV LHC was recently given in Ref. [70]. They find $\sigma^{\rm exp}_{t\,\&\,\bar{t}} = 85 \pm 12$ pb. Our NNLO result follows from the sum of relevant entries at first rows in Tables I and II. We obtain $\sigma^{\rm NNLO}_{t\,\&\,\bar{t}} = 83.9^{+0.8}_{-0.3}$ pb, in good agreement with the measured value. For comparison, the NLO result $\sigma^{\rm NLO}_{t\,\&\,\bar{t}} = 85.2^{+2.5}_{-1.4}$ pb is similar, but significantly less precise than the NNLO result.

Another interesting observable [71, 72] is the ratio of the single-t and single-$\bar{t}$ cross sections, since this ratio is sensitive to the relative size of parton distribution functions for up and down quarks at moderate values of the Bjorken variable x. This ratio also depends on the top (or anti-top) $p_\perp$ cut. For $p_\perp = 0$ we find $\sigma_t/\sigma_{\bar{t}} = 1.849 \pm 0.005, \quad 1.831 \pm 0.001, \quad 1.825 \pm 0.001$ at leading, next-to-leading and next-to-next-to-leading order; the recent experimental result [72] is $1.95 \pm 0.1(\text{stat}) \pm 0.19(\text{syst})$. As it often happens with ratios, changes caused by the scale variation are probably not good indicators of uncertainty of theoretical predictions, especially so since the QCD corrections are small. For higher values of $p_\perp$, the ratio of top and anti-top cross sections increases. For example, taking $p_\perp = 60$ GeV, we find $\sigma_t/\sigma_{\bar{t}} = 2.037 \pm 0.007, \quad 1.969 \pm 0.01, \quad 1.969 \pm 0.02$ at leading, next-to-leading and next-to-next-to-leading order, respectively. We stress that in all cases the errors on the ratio do not include the PDF uncertainty which should be significant given the smallness of scale-variation errors. Turning this argument around, we note that, since the perturbative uncertainty on the ratio of top and anti-top production cross sections is very small, the precisely measured ratio of single-top and single-anti-top cross sections can be used to provide additional stringent constraints on parton distribution function.

IV. CONCLUSIONS

In this paper we described the calculation of NNLO QCD corrections to t-channel single-top production cross section at the LHC. We found that the NNLO QCD corrections are small, of the order of a few percent. The residual scale dependence of the single-top production cross section is below one percent, indicating that uncalculated higher order corrections are very small. Similar conclusions are reached also for the single-top production cross sections with a cut on the top transverse momentum although in that case the residual scale uncertainty can be larger. It is interesting to note that, with a cut on the top quark transverse momentum, the NLO QCD corrections are very different for different values of $p_\perp$ but the NNLO QCD corrections and residual scale dependences are small, independent of it. Therefore, it appears that, similar to the Drell-Yan processes $pp \to Z, W$ etc. and the $t\bar{t}$ pair production $pp \to t\bar{t}$, the single-top production cross section at the LHC is predicted with a percent-level accuracy at NNLO QCD. In principle, this should allow very precise measurements of the top quark electroweak couplings and ensuing indirect determinations of the top quark width.

A natural extension of our calculation is to include decays of the top quark in the narrow width approximation. This can be done in a relatively straightforward way by calculating all the scattering amplitudes, needed to describe single-top production, for a particular top quark spinor, that effectively accounts for the complete decay chain of the top quark, see e.g. Ref. [73]. Once this is done, computation of fiducial volume cross sections for realistically selected final states in single-top production as well as realistic kinematic distributions of top quark decay products becomes possible. We hope that such an extension of the present work will contribute towards improved measurements of the top quark electroweak coupling, the top quark width and the CKM matrix element $|V_{tb}|$ at the LHC.

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