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MSSM Higgs Self-Couplings: Two-Loop $\mathcal{O}(\alpha_t\alpha_s)$ Corrections

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We analyze the minimal supersymmetric Higgs self-couplings at $\mathcal{O}(\alpha_t\alpha_s)$ within the effective potential approach. The two-loop corrections turn out to be of moderate size in the $\overline{\rm DR}$ scheme if the central scale is chosen as half the SUSY scale. The inclusion of the two-loop corrections reduces the renormalization scale dependence to the per-cent level. These results have a significant impact on measurements of the trilinear Higgs self-couplings at the LHC and a future e^+e^- collider.

The Higgs mechanism [1] is a cornerstone of the Standard Model (SM) and its supersymmetric extensions. The masses of the fundamental particles, i.e. electroweak gauge bosons, leptons and quarks, are generated by interactions with Higgs fields. The recently discovered particle with a mass of ~125 GeV at the LHC [2] seems to be the SM Higgs boson, i.e. all tested properties such as its couplings to the other SM particles, its spin and $\mathcal{CP}$ quantum numbers agree with the SM predictions [3]. However, the errors of the measured couplings to fermions and vector bosons leave room for deviations from the SM values, which naturally arise in SM extensions as e.g. the minimal supersymmetric extension (MSSM).

The MSSM requires the introduction of two Higgs doublets. After electroweak symmetry breaking there are five elementary Higgs particles, two $\mathcal{CP}$-even (h, H), one $\mathcal{CP}$-odd (A) and two charged $(H^\pm)$. At lowest order all couplings and masses of the MSSM Higgs sector are fixed by two independent input parameters, which are generally chosen as $\mathrm{tg}\beta = v_2/v_1$, the ratio of the two vacuum expectation values (vevs) $v_{1,2}$, and the pseudoscalar Higgs mass M_A. Including the one-loop and dominant two- and three-loop corrections the upper bound on the light scalar Higgs mass is $M_h \lesssim 135$ GeV [4]. The Higgs boson couplings to fermions and gauge bosons depend on mixing angles α and β, which are defined by diagonalizing the neutral and charged Higgs mass matrices.

One of the most important tests of the Higgs sector in the future is the measurement of the Higgs potential, i.e. the self-interactions of the Higgs particles. It is possible that the trilinear Higgs self-coupling could be measured at the LHC after the high-luminosity upgrade [5], while a measurement of the quartic Higgs coupling will be out of reach at any foreseen collider due to the tiny signal rates [6]. Within the MSSM the prospects for the trilinear Higgs couplings can be better due to the possible appearance of resonant Higgs decays into lighter Higgs pairs as e.g. the heavy scalar Higgs decay in $gg \to H \to hh$ for values of $\mathrm{tg}\beta \lesssim 10$ [7]. The proper treatment of the signal rates within the MSSM requires the determination of the radiative corrections to the effective trilinear Higgs couplings supplemented by moderate process-dependent corrections [8]. Many years ago the one-loop corrections to the effective trilinear Higgs couplings have been shown to be large [9, 10]. However, sizable residual uncertainties of these effective couplings, arising by integrating out the heavy SUSY particles and the top quark, are left over. In order to reduce these uncertainties a two-loop calculation of the trilinear Higgs couplings is required. The one-loop corrections are dominated entirely by top and stop loop contributions. Only for large values of $\mathrm{tg}\beta$ can the bottom/sbottom loop contributions become relevant thanks to the large enhancement of the bottom Yukawa couplings in this regime [9]. In this work we will describe the two-loop SUSY–QCD corrections to the top/stop-loop induced corrections [11].

We will parametrize the two MSSM Higgs doublets as

$$H_1 = \begin{pmatrix} H_1^0 \\ -H_1^- \end{pmatrix}, \qquad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}, \tag{1}$$

where $H_{1,2}^\pm$ denote the four charged Higgs fields in the current eigenstate basis. The neutral Higgs fields decompose into $v_{1,2}$ and scalar/pseudoscalar components as

$$H_j^0 \;=\; \frac{1}{\sqrt{2}}(v_j + S_j + iP_j) \quad (j = 1, 2)\,. \tag{2}$$

The neutral physical Higgs and would-be Goldstone fields emerge from rotations by the mixing angles α and β,

$$\begin{aligned} S_1 &= Hc_\alpha - hs_\alpha, \qquad & P_1 = G^0 c_\beta - As_\beta, \\ S_2 &= Hs_\alpha + hc_\alpha, \qquad & P_2 = G^0 s_\beta + Ac_\beta\,. \end{aligned} \tag{3}$$

The vevs are defined as $v_1 = vc_\beta, v_2 = vs_\beta$ with $v \approx 246$ GeV. The tree-level Higgs potential is given by

$$\begin{aligned} V_0 &= m_1^2|H_1|^2 + m_2^2|H_2|^2 - B\mu\epsilon_{ij}(H_1^i H_2^j + h.c.) \\ &+ \frac{g^2+g'^2}{8}(|H_1|^2 - |H_2|^2)^2 + \frac{g^2}{2}|H_1^\dagger H_2|^2\,, \end{aligned} \tag{4}$$

where $m_{1,2}^2 = m_{H_{1,2}}^2 + \mu^2$ with $m_{H_{1,2}}, \mu$ denoting the soft SUSY-breaking Higgs and Higgsino mass parameters respectively. The object ϵ_{ij} is the antisymmetric two-dimensional tensor, while g, g' are the isospin and hypercharge gauge couplings. The parameters $m_{1,2}$ are eliminated by the minimization condition of the effective potential, while the parameter $B\mu$ is traded for the pseudoscalar mass M_A. Taking the second derivatives of V_0 with respect to the Higgs fields yields the mass matrices, while the third and fourth derivatives define the trilinear and quartic Higgs couplings, respectively. After rotation to the physical mass eigenstates, the neutral trilinear Higgs couplings at leading order are given by

$$\begin{aligned}
\lambda_{hhh} &= 3\frac{M_Z^2}{v} c_{2\alpha} s_{\alpha+\beta}, \quad \lambda_{HHH} = 3\frac{M_Z^2}{v} c_{2\alpha} c_{\alpha+\beta}, \\
\lambda_{Hhh} &= \frac{M_Z^2}{v} [2 s_{2\alpha} s_{\alpha+\beta} - c_{2\alpha} c_{\alpha+\beta}], \\
\lambda_{HHh} &= -\frac{M_Z^2}{v} [2 s_{2\alpha} c_{\alpha+\beta} + c_{2\alpha} s_{\alpha+\beta}], \\
\lambda_{hAA} &= \frac{M_Z^2}{v} c_{2\beta} s_{\alpha+\beta}, \quad \lambda_{HAA} = -\frac{M_Z^2}{v} c_{2\beta} c_{\alpha+\beta} \,. \quad (5)
\end{aligned}$$

One-Loop Corrections. These couplings are subject to radiative corrections. Using dimensional reduction in $n = 4 - 2\epsilon$ dimensions, the leading top/stop-induced corrections of $\mathcal{O}(\alpha_t)$ to the effective potential in Landau gauge can be cast into the form [12]

$$\begin{aligned}
V_1 &= \frac{3}{16\pi^2} \Bigg\{ \bar{m}_t^4 \left[\frac{1}{\epsilon} + \frac{3}{2} - \log \frac{\bar{m}_t^2}{Q^2} \right. \\
&\qquad \left. + \epsilon \left(\frac{7}{4} - \frac{3}{2} \log \frac{\bar{m}_t^2}{Q^2} + \frac{1}{2} \log^2 \frac{\bar{m}_t^2}{Q^2} + \frac{1}{2} \zeta_2 \right) \right] \\
&\qquad - \frac{1}{2} [(\bar{m}_t \leftrightarrow \bar{m}_{\tilde{t}_1}) + (\bar{m}_t \leftrightarrow \bar{m}_{\tilde{t}_2})] \Bigg\} \qquad (6)
\end{aligned}$$

with the field-dependent mass parameters defined as

$$\begin{aligned}
\bar{m}_t^2 &= |X|^2 \,, \\
\bar{m}_{\tilde{t}_{1,2}}^2 &= \frac{1}{2} \Big(\tilde{M}_{\tilde{t}_R}^2 + \tilde{M}_{\tilde{t}_L}^2 + 2\bar{m}_t^2 \\
&\qquad \mp \sqrt{(\tilde{M}_{\tilde{t}_L}^2 - \tilde{M}_{\tilde{t}_R}^2)^2 + 4|\tilde{X}|^2} \Big) \,, \\
X &= h_t H_2^0, \qquad \tilde{X} = h_t \left[A_t H_2^0 - \mu H_1^{0*} \right] \,, \quad (7)
\end{aligned}$$

where the parameters $\tilde{M}_{\tilde{t}_{L/R}}$ include the D-terms,

$$\begin{aligned}
&\tilde{M}_{\tilde{t}_{L/R}}^2 = M_{\tilde{t}_{L/R}}^2 + D_{\tilde{t}_{L/R}} \,, \\
&D_{\tilde{t}_L} = M_Z^2 \left(\frac{1}{2} - \frac{2}{3} s_W^2 \right) c_{2\beta}, \; D_{\tilde{t}_R} = M_Z^2 \, \frac{2}{3} s_W^2 c_{2\beta} \,. \, (8)
\end{aligned}$$

The scale Q^2 is related to the 't Hooft mass scale μ_0 as $Q^2 = 4\pi\mu_0 e^{-\gamma_E}$ with the Euler-constant γ_E. The top Yukawa coupling $h_t = \sqrt{2} m_t/(v s_\beta)$ defines the corresponding coupling $\alpha_t = h_t^2/(4\pi)$. Including the loop-corrected minimization condition and the loop-corrected pseudoscalar Higgs mass the third derivative of this effective potential reproduces the results of Ref. [9] for the trilinear Higgs couplings. Analogously the quartic Higgs couplings can be derived from the fourth derivatives.

Two-Loop Corrections. The two-loop SUSY–QCD corrections to the effective potential are given by [13]

$$\begin{aligned}
V_2 = &\; \frac{\alpha_s}{8\pi^3} \Big\{ J(\bar{m}_t^2, \bar{m}_t^2) - 2\bar{m}_t^2 I(\bar{m}_t^2, \bar{m}_t^2, 0) \\
&+ \Bigg[\frac{1}{4}(2 - s_{2\bar{\theta}}^2) J(\bar{m}_{\tilde{t}_1}^2, \bar{m}_{\tilde{t}_1}^2) + \frac{s_{2\bar{\theta}}^2}{4} J(\bar{m}_{\tilde{t}_1}^2, \bar{m}_{\tilde{t}_2}^2) \\
&+ \bar{m}_{\tilde{t}_1}^2 I(\bar{m}_{\tilde{t}_1}^2, \bar{m}_{\tilde{t}_1}^2, 0) + J(m_{\tilde{g}}^2, \bar{m}_t^2) - J(\bar{m}_{\tilde{t}_1}^2, m_{\tilde{g}}^2) \\
&- J(\bar{m}_{\tilde{t}_1}^2, \bar{m}_t^2) - (\bar{m}_{\tilde{t}_1}^2 - m_{\tilde{g}}^2 - \bar{m}_t^2) I(\bar{m}_{\tilde{t}_1}^2, m_{\tilde{g}}^2, \bar{m}_t^2) \\
&- 2 m_{\tilde{g}} \bar{\xi} I(\bar{m}_{\tilde{t}_1}^2, m_{\tilde{g}}^2, \bar{m}_t^2) + (\bar{m}_{\tilde{t}_1}^2 \leftrightarrow \bar{m}_{\tilde{t}_2}^2, \bar{\xi} \to -\bar{\xi}) \Bigg] \Big\} (9)
\end{aligned}$$

with the additional field-dependent parameters

$$\begin{aligned}
s_{2\bar{\theta}}^2 &= \frac{4|\tilde{X}|^2}{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^2} \,, \\
\bar{\xi} &= 2 \frac{\Re e(X) \Re e(\tilde{X}) + \Im m(X) \Im m(\tilde{X})}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} \,. \qquad (10)
\end{aligned}$$

The gluino mass is denoted by $m_{\tilde{g}}$ and the two-loop integrals I, J are defined and calculated in [14]. We have calculated the derivatives of the two-loop corrected Higgs potential up to fourth order in the Higgs fields. After implementing the minimization condition and the pseudoscalar Higgs mass at the two-loop level, we have renormalized the top mass, stop masses, stop mixing angle and A_t parameters of the one-loop corrected Higgs potential within the $\overline{\mathrm{DR}}$ scheme. This scheme choice ensures the relation between the stop mixing angle and A_t,

$$s_{2\theta} = \frac{2 m_t (A_t - \mu/\mathrm{tg}\beta)}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} \,. \qquad (11)$$

The $\overline{\mathrm{DR}}$ counter terms are given by

$$\begin{aligned}
\delta m_t &= -C_F \frac{\alpha_s}{2\pi} m_t \left[\frac{1}{\epsilon} + \log \frac{Q^2}{\mu_R^2} \right] \,, \\
\delta m_{\tilde{t}_{1/2}}^2 &= C_F \frac{\alpha_s}{4\pi} \Big[\mp (m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2) s_{2\theta}^2 \\
&\qquad - 4(m_{\tilde{g}}^2 + m_t^2 \mp m_{\tilde{g}} m_t s_{2\theta}) \Big] \left[\frac{1}{\epsilon} + \log \frac{Q^2}{\mu_R^2} \right] \,, \\
\delta A_t &= C_F \frac{\alpha_s}{\pi} m_{\tilde{g}} \left[\frac{1}{\epsilon} + \log \frac{Q^2}{\mu_R^2} \right] \,. \qquad (12)
\end{aligned}$$

The $\overline{\mathrm{DR}}$ counter term $\delta\theta$ can be derived from the relation (11). After renormalization we reproduce the known two-loop results for the Higgs masses [13, 15] and arrive at finite expressions for the trilinear and quartic Higgs couplings. These are finally rotated to the physical Higgs mass eigenstates by the radiatively corrected mixing angles α, β, which diagonalize the two-loop corrected scalar and pseudoscalar Higgs mass matrices. At two-loop order we have checked explicitly that the trilinear and quartic Higgs couplings $\lambda_{hhh}, \lambda_{hhhh}$ approach their two-loop SM limits for large M_A and SUSY masses in analogy with the one-loop analysis of Ref. [10].

In order to obtain consistent results for the Higgs self-couplings we used the following expressions for the running $\overline{\rm DR}$ parameters at the renormalization scale μ_R [16],

$$
\begin{aligned}
m_t(\mu_R) &= m_t(M_t)\left(\frac{\alpha_s(\mu_R)}{\alpha_s(M_t)}\right)^{\frac{8}{9}} \frac{9+10\alpha_s(\mu_R)/\pi}{9+10\alpha_s(M_t)/\pi}\,, \\
m_{\tilde g}(\mu_R) &= m_{\tilde g}(M_{\tilde g})\,\frac{\alpha_s(\mu_R)\,[6-7\alpha_s(\mu_R)/\pi]}{\alpha_s(M_{\tilde g})\,[6-7\alpha_s(M_{\tilde g})/\pi]}\,, \\
A_t(\mu_R) &= A_t(Q_0) + m_{\tilde g}(Q_0)\left\{-\frac{16}{9}\left[\frac{\alpha_s(\mu_R)}{\alpha_s(Q_0)}-1\right]\times\right. \\
&\quad \left.\left[1+\frac{7}{6}\frac{\alpha_s(Q_0)}{\pi}\right] - \frac{4}{27}\frac{\alpha_s(Q_0)}{\pi}\left[\frac{\alpha_s^2(\mu_R)}{\alpha_s^2(Q_0)}-1\right]\right\}\,, \\
M^2_{\tilde t_{L/R}}(\mu_R) &= M^2_{\tilde t_{L/R}}(Q_0) + m^2_{\tilde g}(Q_0)\left\{\frac{8}{9}\left[\frac{\alpha_s^2(\mu_R)}{\alpha_s^2(Q_0)}-1\right]\times\right. \\
&\quad \left.\left[1+\frac{7}{3}\frac{\alpha_s(Q_0)}{\pi}\right] - \frac{8}{81}\frac{\alpha_s(Q_0)}{\pi}\left[\frac{\alpha_s^3(\mu_R)}{\alpha_s^3(Q_0)}-1\right]\right\}\,, \\
\alpha_s(\mu_R) &= \frac{4\pi}{3\log(\mu_R^2/\Lambda^2)}\left\{1+\frac{14}{9}\frac{\log\log(\mu_R^2/\Lambda^2)}{\log(\mu_R^2/\Lambda^2)}\right\}, \quad (13)
\end{aligned}
$$

where Q_0 denotes the input scale for these parameters and Λ the QCD scale of the strong coupling α_s [20]. These expressions are valid up to the next-to-leading-log level of the renormalization group equations [17]. The $\overline{\rm DR}$ stop masses $m_{\tilde t_i}(\mu_R)$ are obtained from the running SUSY-breaking parameters as $[\tilde M_{\tilde t_{L/R}} = \tilde M_{\tilde t_{L/R}}(\mu_R)]$

$$
\begin{aligned}
m^2_{\tilde t_{1/2}}(\mu_R) &= m_t^2(\mu_R) + \frac{1}{2}\left[\tilde M^2_{\tilde t_L} + \tilde M^2_{\tilde t_R}\right. \\
&\left.\mp\sqrt{[\tilde M^2_{\tilde t_L} - \tilde M^2_{\tilde t_R}]^2 + 4m_t^2(\mu_R)[A_t(\mu_R)-\mu/{\rm tg}\beta]^2}\right]. \quad (14)
\end{aligned}
$$

Our running $\overline{\rm DR}$ parameters include the contributions of all strongly interacting SM and SUSY particles. The top mass m_t, the gluino mass $m_{\tilde g}$ and the strong coupling α_s are related to the top pole mass M_t, the gluino pole mass $M_{\tilde g}$ and the 5-flavor $\overline{\rm MS}$ coupling $\alpha^{(5)}_{s,\overline{\rm MS}}$ by

$$
\begin{aligned}
m_t(M_t) &= M_t\left\{1+\frac{\alpha^{(5)}_{s,\overline{\rm MS}}(M_t)}{3\pi}\right[5 \\
&\quad + \sum_{i=1}^{2}\Big(\bar B_1(M_t^2; M_{\tilde g}, M_{\tilde t_i}, M_t^2) \\
&\quad \left.\left. - (-1)^i \frac{M_{\tilde g} s_{2\theta}}{M_t}\bar B_0(M_t^2; M_{\tilde g}, M_{\tilde t_i}, M_t^2)\Big)\right]\right\}^{-1}, \\
m_{\tilde g}(M_{\tilde g}) &= M_{\tilde g}\left\{1+\frac{\alpha^{(5)}_{s,\overline{\rm MS}}(M_{\tilde g})}{4\pi}\right[15 \\
&\quad + \sum_{q,i}\Big(\bar B_1(M_{\tilde g}^2; M_q, M_{\tilde q_i}, M_{\tilde g}^2) \\
&\quad \left.\left. - (-1)^i \frac{M_q s_{2\theta_q}}{M_{\tilde g}}\bar B_0(M_{\tilde g}^2; M_q, M_{\tilde q_i}, M_{\tilde g}^2)\Big)\right]\right\}^{-1}, \\
\alpha_s(Q_0) &= \alpha^{(5)}_{s,\overline{\rm MS}}(Q_0)\left\{1+\frac{\alpha^{(5)}_{s,\overline{\rm MS}}(Q_0)}{\pi}\right[\frac{1}{6}\log\frac{Q_0^2}{M_t^2} \\
&\quad \left.\left. + \frac{1}{2}\log\frac{Q_0^2}{M_{\tilde g}^2} + \frac{1}{24}\sum_{\tilde q_i}\log\frac{Q_0^2}{M_{\tilde q_i}^2} + \frac{1}{4}\right]\right\}, \quad (15)
\end{aligned}
$$

where $M_{\tilde q_i}$ denotes the squark pole mass and θ_q the corresponding mixing angle generically (which have been defined via the tree level relations). The finite parts of the one-loop integrals can be cast into the form [18]

$$
\begin{aligned}
\bar B_{0[1]}(p^2; m_1, m_2, Q^2) &= \Re e\int_0^1 dx\,[-x]\times \\
&\log\frac{Q^2}{m_1^2 x + m_2^2(1-x) - p^2 x(1-x) - i\epsilon}\,, \quad (16)
\end{aligned}
$$

where the factor $-x$ has to be inserted for $\bar B_1$.

The numerical analysis of the Higgs self-couplings is performed in the "m_h^{mod+}" MSSM scenario [19] as a representative case:

$$
\begin{aligned}
{\rm tg}\beta &= 5,\ M_{\tilde Q_{L/R}} = 1\ {\rm TeV},\ M_{\tilde g} = 1.5\ {\rm TeV}, \\
A_b &= A_t = 1.64\ {\rm TeV},\ \mu = 200\ {\rm GeV}\,, \quad (17)
\end{aligned}
$$

where the parameters $M_{\tilde Q_{L/R}}, A_t, A_b$ are defined at the input scale $Q_0 = M_{\tilde Q_{L/R}}$. Within this scenario resonant Higgs production $gg \to H \to hh$ occurs with a sizable cross section. For the Higgs masses and couplings we used our calculation based on the $\mathcal{O}(\alpha_t\alpha_s)$-corrected effective potential. The top quark pole mass has been chosen as $M_t = 173.2$ GeV, while the strong coupling constant has been normalized to $\alpha^{(5)}_{s,\overline{\rm MS}}(M_Z) = 0.119$.

The scale dependences of the trilinear Higgs couplings λ_{hhh} and λ_{Hhh} are displayed in Fig. 1 at one- and two-loop order. The central scale is chosen as half the SUSY scale, i.e. $\mu_R = M_{\tilde Q_{L/R}}/2 = 500$ GeV. We obtain a significant reduction of the scale dependence from $\mathcal{O}(10\%)$ at one-loop order to the per-cent level at two-loop order and thus a large reduction of the theoretical uncertainties. Moreover a broad maximum/minimum develops at about the chosen central scale in contrast to the monotonous scale dependences at one-loop order. In the "m_h^{mod+}" scenario the one-loop corrections are large and positive, increasing the trilinear self-couplings by about a factor of 2. The two-loop corrections amount to a few per cent for the central scale choices. The strong reduction of the residual scale dependences is also visible in Fig. 2, which displays the trilinear Higgs couplings λ_{hhh} and λ_{Hhh} as a function of the pseudoscalar Higgs mass

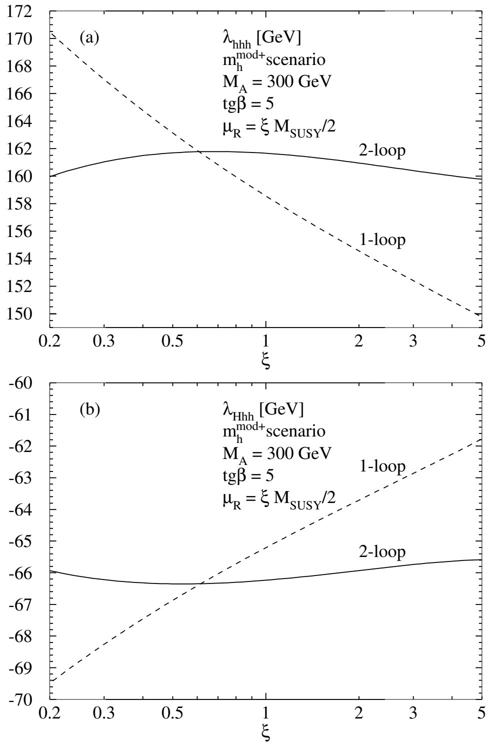

FIG. 1: *Scale dependence of the trilinear Higgs couplings λ_{hhh} (a) and λ_{Hhh} (b) at one- and two-loop order in the m_h^{mod+} scenario for $M_A = 300$ GeV and* $\mathrm{tg}\beta = 5$.

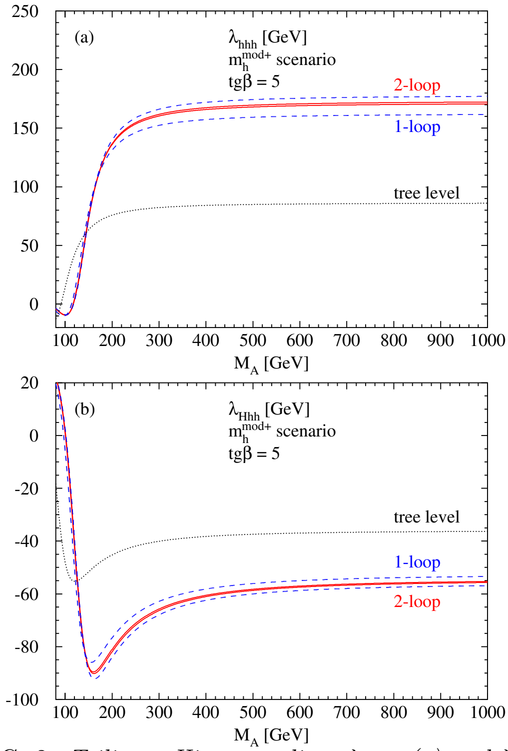

FIG. 2: *Trilinear Higgs couplings λ_{hhh} (a) and λ_{Hhh} (b) at tree level, one- and two-loop order in the m_h^{mod+} scenario for* $\mathrm{tg}\beta = 5$. *The bands indicate the scale uncertainties within 1/3 and 3 times the central scale $M_{SUSY}/2$.*

M_A. The one- and two-loop bands show the minimal and maximal values of the Higgs couplings if the scale is varied between 1/3 and 3 times the central scale.

In summary, the significant scale dependence of $\mathcal{O}(10\%)$ of the one-loop predictions for the trilinear MSSM Higgs self-couplings requires the inclusion of two-loop corrections. For the corrected trilinear and quartic Higgs couplings, we find a reduction of the scale dependence to the per-cent level at $\mathcal{O}(\alpha_t\alpha_s)$. The improved predictions for these couplings can thus be taken as a base for experimental analyses at the LHC and the ILC.

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[20] Note that due to a sign typo the formula for the running coupling α_s of Ref. [16] differs from Eq. (13).