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Two-loop static potential in $\mathcal{N} = 4$ supersymmetric Yang-Mills theory

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Abstract

We compute the soft contribution to the static energy of two heavy colour sources interacting via a $\mathcal{N} = 4$ supersymmetric Yang-Mills theory. Both singlet and octet colour configurations are considered. Our calculations complete recent considerations of the ultrasoft contributions.

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1 Introduction

Although $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) theory is not realized in nature it has received increasing attraction in the recent years. The main reason for this is the conjecture of a duality between $\mathcal{N} = 4$ SYM and a certain class of string theories which is usually called the AdS/CFT correspondence [1]. To test the correspondence it would be ideal to have all-order perturbative results which can be evaluated for large values of the coupling and then be compared to string theory calculations. However, in general only a few terms can be computed in the perturbative expansion and in the strong-coupling limit. One hopes to obtain information about the AdS/CFT correspondence from their comparison.

Among the interesting quantities which one can consider there is the static energy of two infinitely-heavy colour sources in the fundamental representation of $SU(N_c)$. It has been considered in the strong and weak coupling limit in Refs. [2, 3] and [4, 5], respectively. A systematic one-loop calculation in a framework analogue to the one applied in QCD has been performed in Ref. [6]. It has been noted that already at this loop-order ultrasoft contributions have to be taken into account which is due to the massless scalar particles present in $\mathcal{N} = 4$ SYM.

Recently, in Ref. [7] the two-loop ultrasoft contribution to the static energy has been computed. On its own it still contains poles in ϵ which have been subtracted using the $\overline{\text{MS}}$ scheme. In this paper we provide the soft contribution to the static energy both for singlet and octet colour configuration which combines with the ultrasoft result to a finite physical expression for the static energy.

In QCD, ultrasoft contributions arise for the first time at three-loop order [8–11] since the real radiation of gluons from ultrasoft quarks is suppressed by $v\sqrt{\alpha_s}$ where v is the velocity of the heavy quark. Thus, after taking into account the scaling rule $v \sim \alpha_s$ the combination of emission and absorption process scales like α_s^3.

In the next Section we provide some details to our calculation and the results are presented in Section 3.

2 Calculation

The theoretical framework convenient for the computation of the static energy within $\mathcal{N} = 4$ SYM has already been presented in Refs. [6, 7]. Let us for convenience repeat the main steps which are important for our calculation.

The Lagrange density for $\mathcal{N} = 4$ SYM theory reads

$$\mathcal{L}_{\mathcal{N}=4} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu\, a} + \frac{1}{2}\sum_{i=1}^{6}\left(D_\mu\Phi_i\right)^a\left(D^\mu\Phi_i\right)^a - \frac{i}{2}\sum_{j=1}^{4}\bar{\Psi}^a_j\gamma_\mu\left(D^\mu\Psi_j\right)^a + \ldots\,, \quad (1)$$

where Φ_i $(i = 1, \ldots, 6)$ represent six (pseudo) scalar particles and Ψ_j $(j = 1, \ldots, 4)$ four Majorana fermions in the adjoint representation of $SU(N_c)$, just like the gluon fields A_μ present in the field strength tensor $F^{\mu\nu}$.

Following [12] we introduce the Wilson loop

$$W_C = \frac{1}{N_c}\mathrm{Tr}\mathcal{P}\mathrm{exp}\left[-ig\oint_C \mathrm{d}\tau\left(A_\mu\dot{x}^\mu + \Phi_n|\dot{x}|\right)\right]\,. \quad (2)$$

The static energy is obtained by considering a rectangular path C and taking the limit of large temporal extension of the expression $(i/T)\ln\langle W_\Box\rangle$ [6].

The interaction with the static colour sources ψ and χ in the fundamental representation of $SU(N_c)$ is described via the Lagrange density

$$\mathcal{L}_{\rm stat} = \psi^\dagger\left(i\partial_0 - gA_0 - g\Phi_n\right)\psi + \chi_c^\dagger\left(i\partial_0 + gA_0^T - g\Phi_n^T\right)\chi_c\,. \quad (3)$$

Our aim is the computation of the static energy between two static sources, one in the fundamental and one in the anti-fundamental representation, to two-loop accuracy. It can be written as

$$E_{s,o} = V_{s,o} + \delta E^{\rm US}_{s,o}\,, \quad (4)$$

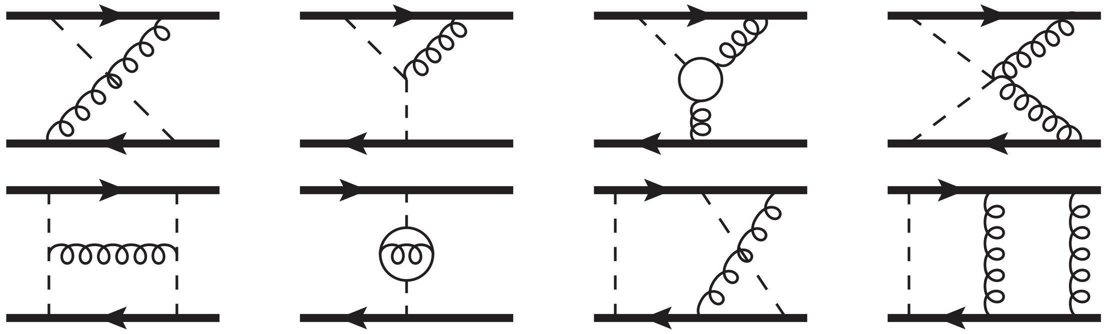


Figure 1: One- and two-loop Feynman diagrams contributing to V_s and V_o. Thick solid lines represent heavy colour sources, thin solid lines massless fermions, curled lines gluons, and dashed lines massless scalar particles.

where the subscripts "s" and "o" represent the singlet and octet representation of the source-anti-source system, respectively. The one-loop results for V_s, V_o and $\delta E_s^{\rm US}$ have been obtained in Ref. [6] and the one- and two-loop results for $\delta E_s^{\rm US}$ and $\delta E_o^{\rm US}$ have been computed in Ref. [7]. In this paper we complete the next-to-next-to-leading order calculation and compute V_s and V_o to two loops. Furthermore, we add a two-loop diagram to the expression for $\delta E_o^{\rm US}$ in [7] which was omitted in that reference.

We perform the calculation of V_s and V_o in momentum space, in close analogy to the calculations performed in the context of QCD which are discussed in detail in the literature [13–21]. The potential is obtained from the one-particle-irreducible contributions to four-point functions with momentum exchange $\vec{q}$ between the static sources. Sample Feynman diagrams are shown in Fig. 1. For the singlet contribution only non-abelian contributions have to be considered. As a consequence there are no contributions which contain so-called pinch-singularities of the form

$$\frac{1}{(k_0 + i0)(k_0 - i0)}\,, \tag{5}$$

where k_0 is the 0-component of the loop momentum and thus all two-loop integrals can be reduced to one of the families shown in Fig. 2. We perform the reduction of the scalar integrals to master integrals with the help of `FIRE` [22]. The analytic results for the master integrals are taken from Ref. [23].

The colour-octet potential needs special attention since Feynman diagrams with pinch contributions (cf. Eq. (5)) contribute to V_o. The corresponding integrals cannot be computed directly but can be reduced to integrals without pinches using the methods described in Refs. [17, 18]. In our calculation we have exploited the exponentiation of the colour-singlet potential in order to establish relations between Feynman integrals with the same colour factor. As an example let us consider the ladder-type diagrams which have colour factors C_F^3, $C_F^2 C_A$ and $C_F C_A^2$. Feynman diagrams with pinches are only present in the first two cases. Exponentiation requires that the sum of all contributions proportional

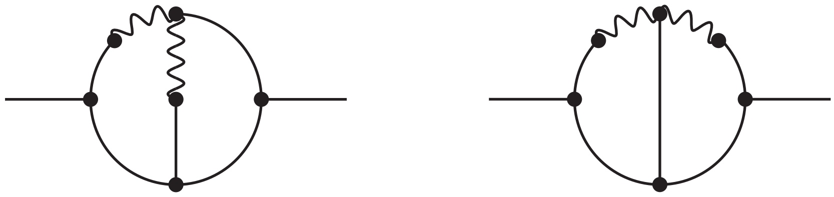


Figure 2: Families of scalar two-loop Feynman integrals. Solid and wavy lines represent relativistic massless and static propagators, respectively.

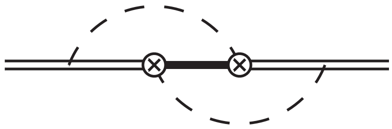
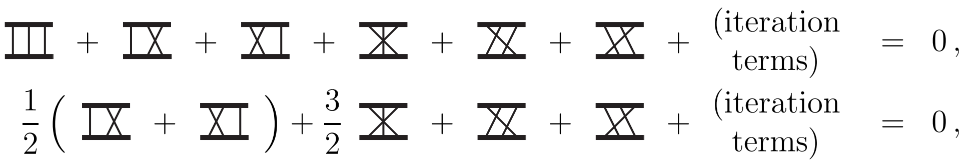


Figure 3: Feynman diagram which contributes to $\delta E_o^{\rm US}$. Single and double lines correspond to singlet and octet Greens functions, respectively. Dashed lines represent ultrasoft scalars.

to C_F^3 or $C_F^2 C_A$ vanish which can be expressed through the following graphical equations[1]



where the Feynman diagrams represent momentum-space expressions with stripped-off colour factors. Thick and thin lines represent static sources and massless particles (scalars and gluons), respectively. In practice we can ignore the contributions denoted by "iteration terms" since they are generated by the logarithm of W_C (see text below Eq. (2)). The equations can be solved for the Feynman integrals involving pinches which one in turn inserts into the expression for V_o where they get multiplied by the corresponding colour octet colour factor.

We have performed our calculation in general R_ξ gauge and have checked that the final results for V_s and V_o are independent of ξ which constitutes a welcome check.

As stated in Eq. (4) it is necessary to add the ultrasoft contribution to the potential in order to arrive at a finite quantity. Actually the individual contributions are divergent and contain poles in ϵ. They cancel in the sum which is a strong check both for $V_{s,o}$ and $\delta E_{s,o}^{\rm US}$.

Using the results for $\delta E_s^{\rm US}$ from Ref. [7] we indeed arrive at a finite result for E_s. However, the poles do not cancel in the octet case. After examining the calculation of $\delta E_o^{\rm US}$ we have realized that the Feynman diagram in Fig. 3 has not been considered in [7]. Only

[1] For simplicity we only consider ladder-type diagrams.

after taking it into account the result for E_o becomes finite. For completeness we provide the result of the missing contribution which completes the list given in Appendix B of [7]. Our result, which has been obtained in $D = 4 - 2\epsilon$ dimensions, reads

$$(\text{Fig. } 3) = \frac{i}{4\pi^D} g^4 \left(\frac{C_A}{2} - C_F\right) (8C_F - 3C_A) (-\Delta V)^{2D-7} \frac{\Gamma^2\left(\frac{D}{2}-1\right)\Gamma(7-2D)}{(D-3)^2}, \qquad (6)$$

where $\Delta V = V_o - V_s$, $C_A = N_c, C_F = (N_c^2 - 1)/2/N_c$, and the coupling g is defined in Eq. (3).

3 Results and conclusions

In a first step we present results for the dimensionally regularized potentials V_s and V_o which we parametrize in momentum space as

$$\begin{aligned}
\tilde{V}_s &= -\frac{8\pi C_F \alpha}{\vec{q}^2}\left[1 + \frac{\alpha}{\pi}\tilde{a}_s^{(1)} + \left(\frac{\alpha}{\pi}\right)^2 \tilde{a}_s^{(2)} + \mathcal{O}\left(\alpha^3\right)\right], \\
\tilde{V}_o &= -\frac{8\pi \left(C_F - \frac{C_A}{2}\right)\alpha}{\vec{q}^2}\left[1 + \frac{\alpha}{\pi}\tilde{a}_o^{(1)} + \left(\frac{\alpha}{\pi}\right)^2 \tilde{a}_o^{(2)} + \mathcal{O}\left(\alpha^3\right)\right], \qquad (7)
\end{aligned}$$

where $\alpha = g^2/4\pi$. After adding all contributing diagrams we obtain for the colour singlet case

$$\begin{aligned}
\tilde{a}_s^{(1)} &= C_A\left[\frac{1}{\epsilon} + \ln\left(\frac{4\pi\mu^2}{e^\gamma \vec{q}^2}\right) + \epsilon\left(\frac{1}{2}\ln^2\left(\frac{4\pi\mu^2}{e^\gamma \vec{q}^2}\right) - \frac{\pi^2}{12}\right)\right] + \mathcal{O}(\epsilon^2), \\
\tilde{a}_s^{(2)} &= C_A^2\Bigg\{\frac{1}{2\epsilon^2} + \left[\frac{1}{2} + \frac{\pi^2}{6} + \ln\left(\frac{4\pi\mu^2}{e^\gamma \vec{q}^2}\right)\right]\frac{1}{\epsilon} \\
&\quad -1 - \frac{\pi^2}{12} + \frac{1}{2}\zeta(3) + \left(1 + \frac{\pi^2}{3}\right)\ln\left(\frac{4\pi\mu^2}{e^\gamma \vec{q}^2}\right) + \ln^2\left(\frac{4\pi\mu^2}{e^\gamma \vec{q}^2}\right)\Bigg\} + \mathcal{O}(\epsilon), \qquad (8)
\end{aligned}$$

where $\gamma \approx 0.57721\ldots$ is the EulerMascheroni constant. The results for the colour-octet case read

$$\begin{aligned}
\tilde{a}_o^{(1)} &= \tilde{a}_s^{(1)}, \\
\tilde{a}_o^{(2)} &= \tilde{a}_s^{(2)} + \delta\tilde{a}_o^{(2)}, \\
\delta\tilde{a}_o^{(2)} &= -C_A^2\pi^2\left[\frac{1}{2\epsilon} + \ln\left(\frac{4\pi\mu^2}{e^\gamma \vec{q}^2}\right)\right] + \mathcal{O}(\epsilon). \qquad (9)
\end{aligned}$$

One observes that, as for QCD, the one-loop results agree up to the change of the global colour factor from C_F to $C_F - C_A/2$ and that at two loops there is an additional term proportional to $C_A^2\pi^2$. However, in contrast to QCD, this term only contains a pole in ϵ and the corresponding logarithm.

In coordinate space we introduce V_s and V_o as

$$
\begin{aligned}
V_s &= -\frac{2C_F\alpha}{r}\left[a_s^{(0)}+\frac{\alpha}{\pi}a_s^{(1)}+\left(\frac{\alpha}{\pi}\right)^2 a_s^{(2)}+\mathcal{O}\left(\alpha^3\right)\right], \\
V_o &= -\frac{2\left(C_F-\frac{C_A}{2}\right)\alpha}{r}\left[a_o^{(0)}+\frac{\alpha}{\pi}a_o^{(1)}+\left(\frac{\alpha}{\pi}\right)^2 a_o^{(2)}+\mathcal{O}\left(\alpha^3\right)\right], \qquad (10)
\end{aligned}
$$

and obtain for the coefficients

$$
\begin{aligned}
a_s^{(0)} &= 1+\epsilon\ln\left(4\pi\mu^2e^\gamma r^2\right)+\epsilon^2\left(\frac{\pi^2}{4}+\frac{1}{2}\ln^2\left(4\pi\mu^2e^\gamma r^2\right)\right)+\mathcal{O}\left(\epsilon^3\right), \\
a_s^{(1)} &= C_A\left[\frac{1}{\epsilon}+2\ln\left(4\pi\mu^2e^\gamma r^2\right)+\epsilon\left(\frac{5\pi^2}{6}+2\ln^2\left(4\pi\mu^2e^\gamma r^2\right)\right)\right]+\mathcal{O}(\epsilon^2), \\
a_s^{(2)} &= C_A^2\Bigg[\frac{1}{2\epsilon^2}+\left(\frac{1}{2}+\frac{\pi^2}{6}+\frac{3}{2}\ln\left(4\pi\mu^2e^\gamma r^2\right)\right)\frac{1}{\epsilon} \\
&\quad -1+\frac{7\pi^2}{8}+\frac{1}{2}\zeta(3)+\left(\frac{3}{2}+\frac{\pi^2}{2}\right)\ln\left(4\pi\mu^2e^\gamma r^2\right)+\frac{9}{4}\ln^2\left(4\pi\mu^2e^\gamma r^2\right)\Bigg]+\mathcal{O}(\epsilon), \\
a_o^{(0)} &= a_s^{(0)}, \\
a_o^{(1)} &= a_s^{(1)}, \\
a_o^{(2)} &= a_s^{(2)}+\delta a_o^{(2)}, \\
\delta a_o^{(2)} &= -C_A^2\pi^2\left[\frac{1}{2\epsilon}+\frac{3}{2}\ln\left(4\pi\mu^2e^\gamma r^2\right)\right]+\mathcal{O}(\epsilon). \qquad (11)
\end{aligned}
$$

The $\mathcal{O}(\epsilon)$ terms in $a_s^{(0)}$ and $a_o^{(0)}$ arise due to the D-dimensional Fourier transformation and are needed when inserting ΔV into the ultrasoft expression.

Note that there is no counterterm contribution since the beta function vanishes in $\mathcal{N}=4$ SYM. The poles in Eqs. (8), (9) and (11) are of infra-red type and cancel against the ultraviolet poles of the ultrasoft contribution. Within dimensional regularization such an identification is necessary since scaleless integrals are set to zero (see, e.g., Ref. [24]).

For completeness we also display the ultrasoft result in coordinate space which is given by [6, 7]

$$
\begin{aligned}
\delta E_s^{\mathrm{US}} &= -\frac{2C_F\alpha}{r}\left[\frac{\alpha}{\pi}b_s^{(1)}+\left(\frac{\alpha}{\pi}\right)^2 b_s^{(2)}+\mathcal{O}\left(\alpha^3\right)\right], \\
\delta E_o^{\mathrm{US}} &= -\frac{2\left(C_F-\frac{C_A}{2}\right)\alpha}{r}\left[\frac{\alpha}{\pi}b_o^{(1)}+\left(\frac{\alpha}{\pi}\right)^2 b_o^{(2)}+\mathcal{O}\left(\alpha^3\right)\right], \qquad (12)
\end{aligned}
$$

with

$$
\begin{aligned}
b_s^{(1)} &= C_A\left[-\frac{1}{\epsilon}-2+2\ln\left(2C_A\alpha e^{\gamma}\right)-2\ln\left(4\pi\mu^2e^{\gamma}r^2\right)\right]\,,\\
b_s^{(2)} &= C_A^2\Bigg[-\frac{1}{2\epsilon^2}-\left(\frac{1}{2}+\frac{\pi^2}{6}+\frac{3}{2}\ln\left(4\pi\mu^2e^{\gamma}r^2\right)\right)\frac{1}{\epsilon}\\
&\quad-\left(\frac{3}{2}+\frac{\pi^2}{2}\right)\ln\left(4\pi\mu^2e^{\gamma}r^2\right)-\frac{9}{4}\ln^2\left(4\pi\mu^2e^{\gamma}r^2\right)\\
&\quad+\left(2+\frac{2\pi^2}{3}\right)\ln\left(2C_A\alpha e^{\gamma}\right)+2\ln^2\left(2C_A\alpha e^{\gamma}\right)\\
&\quad-6-\frac{17\pi^2}{24}+4\zeta(3)\Bigg]-2C_FC_A\frac{\pi^2}{3}\,,\\
b_o^{(1)} &= b_s^{(1)}\,,\\
b_o^{(2)} &= b_s^{(2)}+\delta b_o^{(2)}\,,\\
\delta b_o^{(2)} &= C_A^2\pi^2\left[\frac{1}{2\epsilon}-2\ln\left(2C_A\alpha e^{\gamma}\right)+\frac{3}{2}\ln\left(4\pi\mu^2e^{\gamma}r^2\right)\right]\,. \qquad (13)
\end{aligned}
$$

As compared to the result given in [7] there is a change in $b_o^{(2)}$ which is due to the missing Feynman diagram discussed in the previous Section. Note that initially the ultrasoft contribution depends on ΔV (cf. Eq. (6)) since V_s and V_o are present in the ultrasoft singlet and octet propagators, respectively, see Appendix A of Ref. [7]. In order to arrive at Eqs. (13) the perturbative expansion of ΔV has been inserted and an expansion in α has been performed.

The comparison of the results in Eqs. (10) and (12) shows that in the sum the pole parts and the dependence on μr cancels and we arrive at the following results for the static energies

$$
\begin{aligned}
E_s &= -\frac{2C_F\alpha}{r}\left[1+\frac{\alpha}{\pi}e_s^{(1)}+\left(\frac{\alpha}{\pi}\right)^2e_s^{(2)}+\mathcal{O}\left(\alpha^3\right)\right]\,,\\
E_o &= -\frac{2\left(C_F-\frac{C_A}{2}\right)\alpha}{r}\left[1+\frac{\alpha}{\pi}e_o^{(1)}+\left(\frac{\alpha}{\pi}\right)^2e_o^{(2)}+\mathcal{O}\left(\alpha^3\right)\right]\,, \qquad (14)
\end{aligned}
$$

with

$$
\begin{aligned}
e_s^{(1)} &= 2C_A\left[\ln\left(2C_A\alpha e^{\gamma}\right)-1\right]\,,\\
e_s^{(2)} &= 2C_A^2\left[\ln^2\left(2C_A\alpha e^{\gamma}\right)+\left(1+\frac{\pi^2}{3}\right)\ln\left(2C_A\alpha e^{\gamma}\right)+\frac{\pi^2}{12}-\frac{7}{2}+\frac{9}{4}\zeta(3)\right]-2C_AC_F\frac{\pi^2}{3}\,,\\
e_o^{(1)} &= e_s^{(1)}\,,\\
e_o^{(2)} &= e_s^{(2)}+\delta e_o^{(2)}\,,\\
\delta e_o^{(2)} &= -2C_A^2\pi^2\ln\left(2C_A\alpha e^{\gamma}\right)\,. \qquad (15)
\end{aligned}
$$

The result for $e_s^{(1)}$ and the quadratic logarithm in $e_s^{(2)}$ agree with Ref. [6] and the linear logarithm in $e_s^{(2)}$ coincides with [7]. It is interesting to note that the two-loop singlet and octet coefficients only differ by a term proportional to π^2 multiplied by a logarithm which originates from the ultrasoft contribution in Eq. (13).

In the expressions for $e_o^{(1)}$ and $e_o^{(2)}$ as presented above only the real part has been considered. As discussed in Ref. [7] there is a nonzero imaginary part in the ultrasoft contribution which can be interpreted as the decay rate of the octet state into the singlet state and massless particles. The result given in [7] changes due to the additional Feynman diagram of Fig. 3 (cf. Eq. (6)). The corrected expression for $\Gamma_o = -2\mathrm{Im}[E_o(r)]$ reads

$$\Gamma_o = -\frac{8\alpha^2}{r}C_A\left(C_F-\frac{C_A}{2}\right)\left\{1+\frac{\alpha}{\pi}C_A\left[2\ln\left(2\alpha C_A e^{\gamma}\right)+1-\frac{2\pi^2}{3}\right]\right\}.$$

The results discussed so far are obtained from a Wilson loop containing both the coupling to the vector bosons and scalar fields of $\mathcal{N}=4$ SYM. Alternatively, it is also possible to consider the "ordinary" Wilson loop which is obtained by nullifying the term $\Phi_n|\dot{x}|$ in Eq. (2), i.e. there is no interaction of the static sources and the scalars. In analogy to Eq. (10) we write

$$\begin{aligned}
\bar{V}_s &= -\frac{C_F\alpha}{r}\left[1+\frac{\alpha}{\pi}\bar{a}_s^{(1)}+\left(\frac{\alpha}{\pi}\right)^2\bar{a}_s^{(2)}+\mathcal{O}\left(\alpha^3\right)\right],\\
\bar{V}_o &= -\frac{\left(C_F-\frac{C_A}{2}\right)\alpha}{r}\left[1+\frac{\alpha}{\pi}\bar{a}_o^{(1)}+\left(\frac{\alpha}{\pi}\right)^2\bar{a}_o^{(2)}+\mathcal{O}\left(\alpha^3\right)\right].
\end{aligned}\tag{16}$$

The results for the coefficients read

$$\begin{aligned}
\bar{a}_s^{(1)} &= -C_A\,,\\
\bar{a}_s^{(2)} &= C_A^2\left[\frac{5}{4}+\frac{\pi^2}{4}-\frac{\pi^4}{64}\right],\\
\bar{a}_o^{(1)} &= \bar{a}_s^{(1)}\,,\\
\bar{a}_o^{(2)} &= \bar{a}_s^{(2)}+\delta\bar{a}_o^{(2)}\,,\\
\delta\bar{a}_o^{(2)} &= -C_A^2\left[\frac{3\pi^2}{4}-\frac{\pi^4}{16}\right].
\end{aligned}\tag{17}$$

As expected, these results are free from ultrasoft effects which contribute starting from three loops. The expression for $\bar{a}_s^{(1)}$ agrees with the literature [6, 25],[2] the other three results are new.

To conclude, we have computed two-loop corrections to the static potential in $\mathcal{N}=4$ SYM. Together with previously computed ultrasoft contributions two-loop expressions for the energy of two static sources are obtained where we consider the latter both in a colour-singlet and colour-octet configuration. Results are presented both for the ordinary Wilson loop and the one involving only the interaction of the static sources and the vector field.

[2]There seems to be a misprint in the explicit result for $\bar{a}_s^{(1)}$ as given below Eq. (31) of Ref. [6] since agreement with Ref. [25] is claimed. However, in [25] a different sign between the tree-level and one-loop result is obtained.

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