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Dimension 7 operators in the $b \to s$ transition.

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Abstract

We extend the low-energy effective field theory relevant for $b \to s$ transitions up to operators of mass-dimension 7 and compute the associated anomalous-dimension matrix. We then compare our findings to the known results for dimension 6 operators and derive a solution for the renormalization group equations involving operators of dimension 7. We finally apply our analysis to a particularly simple case where the Standard Model is extended by an electroweak-magnetic operator and consider limits on this scenario from the decays $B_s \to \mu^+\mu^-$ and $B \to K\nu\bar{\nu}$.

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1 Introduction

Flavour-violating processes are well-known as a central test of the Standard Model (SM). The pattern conceded to flavour transitions is indeed particularly constrained in this model, the Cabibbo-Kobayashi-Maskawa (CKM) matrix encoding the only source of flavour-breaking effects while only the charged currents of the weak interaction convey flavour-violation at tree-level. Consequently, flavour transitions also define closely watched observables in the quest for new physics and, due to the absence of positive deviations from the SM predictions, set serious constraints on the forms that physics beyond the SM (BSM) could take. One of the latest results is the observation by the LHCb collaboration of the decay $B_s^0 \to \mu^+\mu^-$ [?], with a branching ratio very compatible with the SM expectations (refer to [?] for a recent summary):

$$BR(B_s^0 \to \mu^+\mu^-)^{\text{exp.}} = (3.23 \pm 0.27) \cdot 10^{-9} \quad ; \quad BR(B_s^0 \to \mu^+\mu^-)^{\text{SM}} = (3.56 \pm 0.18) \cdot 10^{-9} \tag{1}$$

On the theoretical side, dedicated tools have been devised in order to study flavour-violating processes, in the form of low-energy effective field theories (EFT; refer to e.g. [?] for a review). These EFT's allow for a separation of the long distance, low-energy strong interaction effects and the short-distance, flavour-changing physics. In the context of B-physics, at the level of terms of dimension 4 and smaller, the relevant EFT consists in a QED$\times$QCD model with five quark-flavours (u, d, c, s, b) and 3 charged leptons (e, μ, τ), as well as neutrinos. Then, in the low-energy processes involving those fields, at a scale $\mu_b \sim M_B$, the impact of higher-energy (top/Electroweak/Higgs/BSM; we assume here that there are no further low-energy 'invisible' fields) physics can be essentially encoded within operators of dimension > 4, collectively defining the 'effective hamiltonian'. The strong-interaction problem then consists in evaluating the S-matrix elements driven by these operators among physical (mesonic/baryonic) states: this question is answered, either through lattice-QCD, QCD sum rules, heavy-quark expansions and other theoretical descriptions of the non-perturbative strong-interaction effects, or phenomenologically, through an identification of the decay-constants by comparison with a few standard channels. The short-distance problem is summarized within the couplings multiplying the operators. Those must be matched at high-energy with the predictions of the 'more fundamental' theory (Standard Model, supersymmetry-inspired models, etc.): scattering amplitudes in both the EFT and the 'full-theory' are equated at the matching scale $\mu_0 \gtrsim M_W$, hence defining a boundary condition for the parameters of the EFT in terms of those of the underlying model.
To relate these two scales, μ_b and μ_0, one relies on the Renormalization Group Equations (RGE) driven by the renormalization of the EFT: leading logarithmic contributions can thus be consistently resummed. It is mostly in this part of the procedure that we will be interested in the following.
To fix notations, let us write the lagrangian density of the EFT under consideration:

$$\mathcal{L}_{eff} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}G^a_{\mu\nu}G^{a\,\mu\nu} \;+\; \imath\bar{f}\left[\gamma^\mu D_\mu - m_f\right] f \;-\; \mathcal{H}_{eff}^{(\text{dim}\geq 5)} \tag{2}$$

with $f = u, d, c, s, b, e, \mu, \tau, \nu_{e,\mu,\tau}$, $D_\mu = \partial_\mu - \imath e Q_f A_\mu - \imath g_S G^a_\mu$ the covariant derivative, m_f the mass of the fermion f, Q_f, its charge; $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $G_{\mu\nu} = G^a_{\mu\nu}T^a = (\partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_S f^{abc} G^b_\mu G^c_\nu)T^a$ represent the electromagnetic and gluonic field tensors, respectively; T^a and f^{abc} denote the $SU(3)_c$ generators and structure constants; e and g_S respectively stand for the elementary electric charge and the strong coupling constant; $\alpha \equiv \frac{e^2}{4\pi}$, $\alpha_S \equiv \frac{g_S^2}{4\pi}$; $\mathcal{H}_{eff}^{(\text{dim}\geq 5)}$ is the effective hamiltonian.
The question arising at this point is that of the operators that one needs to consider in order to describe $b \to s$ transitions. Sensibly, people have considered, up to now, only operators of the lowest possible mass-dimension, that is dimension-6 operators[3]. This approach is justified by the suppression factor of $\frac{m_b}{M_Z} \sim 5 \cdot 10^{-2}$ which is expected for higher-dimensional operators. Moreover, it was (with reason) regarded as sufficient to confine to the smallest subset of dimension-6 operators that would close under renormalization and for which the classical models (SM, supersymmetry-inspired, etc.) would generate a non-trivial contribution:

$$\mathcal{H}_{eff}^{(\text{dim}=6)} = \sum_i C_i(\mu) O_i^{(\text{dim}=6)}(\mu) \tag{3}$$

where the list of operators O_i can be read in e.g. [?], [?] or [?] (with small variations; note that a factor $G_F/\sqrt{2}$ is conventionally factored out in the usual notations). The determination at leading order of the anomalous-dimension matrix for the four-quark operators of this subset is quite old: [?, ?, ?, ?, ?]. The renormalization of the magnetic and chromomagnetic operators can be found in [?], while [?] included the mixing of those with the

[3]In fact, the magnetic and chromo-magnetic operators have mass-dimension 5. However, they always appear, e.g. in the SM, with an additional mass-suppression, lowering their scale to an apparent dimension 6.

four-quark operators (a two-loop effect). One may refer to [?, ?, ?] for the analysis of the semi-leptonic operators. Later works have focussed on next-to-leading order ($O(\alpha_S)$) [?, ?, ?, ?, ?, ?], electroweak ($O(\alpha)$; see e.g. [?]) and finally next-to-next-to-leading QCD ($O(\alpha_S^2)$; refer e.g. to the summary in [?]) effects: this formidable amout of work allows for a theoretical prediction, e.g. in the SM $\bar{B} \to X_s\gamma$ decay, competitive with experimental bounds.
In this paper we choose to adopt a different, more unprejudiced if somewhat more anecdotical, approach to the renormalization of the $b \to s$ EFT: we shall consider all possible (on-shell) operators up to mass-dimension 7 and compute the corresponding anomalous-dimension matrix at one-loop QCD order. As far as we know, no attempt has ever been made in that direction, due to the suppression of the order $\frac{m_b}{M_Z} \sim 5 \cdot 10^{-2}$ which one expects for higher-dimensional operators. In fact, if one considers the matching conditions in the particular case of the SM, with its restricted flavour-changing currents, the suppression would be even larger. The inclusion of higher-dimension operators hence admitedly appears in this concrete case more as a curiosity than a compelling necessity. There are however several reasons why analysing dimension 7 effects may not be completely irrelevant:

1. From the point of view of precision physics, one observes that $\alpha_S(M_Z) \sim \frac{m_b}{M_Z}$: with increasing precision in the SM evaluation as well as experimental measurements, observables shall eventually become sensitive to dimension 7 effects.

2. Certain new-physics contributions are actually 'hidden' dimension 7 effects; a simple example lies in the famous Higgs-penguin contributions to dimension 6 $bsll$-operators [?], relevant e.g. in supersymmetric models at large $\tan\beta$: the corresponding coefficients actually contain a factor m_b, formally increasing their order to dimension 7. Note however that other dimension 7 effects are not expected to receive an equally large $\tan\beta$-enhancement, although they should be relevant already at subleading order [?].

3. New-physics in dimension 6 operators is already stringently constrained, essentially enforcing the usual 'Minimal Flavour Violation' condition. A possible strategy to account for this absence of new-physics in flavour observables would be to reject it on operators of higher dimension. Similar proposals have been made in the neutrino sector to concile neutrino masses, baryon/lepton-number and lepton-flavour violating processes [?]. Note that, here, we do not propose a mechanism that would ensure the suppression of new physics by rejecting it on operators of dimension ≥ 7 (although this might be achievable, e.g. by assigning adequate charges under a discrete symmetry), but we simply mention this possibility as a motivation to consider such operators.

4. If one parametrizes new physics blindly in an expansion of SM dimension 6 operators [?, ?], it turns out that certain operators would only have a dimension-7 signature at low energy. Including dimension 7 operators for the $b \to s$ transition thus naturally enters an unprejudiced analysis of physics BSM.

In the next section, we shall derive a list of all the (on-shell) operators of mass-dimension 5, 6 and 7 which may intervene in the $b \to s$ EFT. We shall then detail the calculation of their ultraviolet (UV)-divergences at one-loop QCD order, before we proceed to the renormalization and establish the RGE's. The following section will be dedicated to the solution of these RGE's in some specific cases. Finally, we shall illustrate our discussion by presenting a concrete, if naive, case where our analysis of dimension 7 operators apply. A short conclusion will eventually summarize our achievements.

2 Operators of dimension $5 - 7$ in the $b \to s$ transition

2.1 List of operators

The first step of our analysis consists in establishing a list of all the operators of dimension 5, 6 and 7 intervening in the $b \to s$ transition (but Lorentz + gauge invariant!). For simplicity, all the fields shall be taken on-shell, i.e. satisfy their equation of motion: this will be sufficient, at least for the leading-order calculation that we aim at. For a discussion concerning the relevance of on-shell EFT's and in particular the question of applying only 'naive' classical equations of motions (dismissing ghosts and gauge-fixing terms), we refer the reader to [?]. We can obviously distinguish among three categories of operators: four-quark, two-quark + two lepton (semi-leptonic) and $\bar{b}s$-Gauge operators.
Let us start with four-fermion operators: the fermion fields already account for a mass-dimension 6, leaving room for at most one covariant (for gauge-invariance) derivative, when one restricts to operators of mass-dimension ≤ 7. Considering chiral fermions (that is, we include projectors $P_{L,R} \equiv \frac{1 \mp \gamma_5}{2}$ in the fermion products), there are at most three ways to contract the spinor algebra: 1 provides scalar currents, γ^μ, vector currents and $\sigma^{\mu\nu} \equiv [\gamma^\mu, \gamma^\nu]$,

tensor currents[4]; any higher combination of γ-matrices can be reduced down to those three cases (through the use of the Levi-Civita tensor $\varepsilon_{\mu\nu\rho\sigma}$ and identities of the Dirac algebra): note that γ_5 only gives a sign, when applied on chiral fermions. Moreover, we may always keep the b and s of the flavour transition within the same current: other combinations are made redundant by the Fierz identities (see e.g. Appendix **??**).
Those considerations allow us to construct the relevant three classes of dimension 6 operators:

1. scalar $(\bar{b}P_{L,R}s)(\bar{f}P_{L,R}f)$;
2. vector $(\bar{b}\gamma^{\mu}P_{L,R}s)(\bar{f}\gamma_{\mu}P_{L,R}f)$;
3. tensor $(\bar{b}\sigma^{\mu\nu}P_{L,R}s)(\bar{f}\sigma_{\mu\nu}P_{L,R}f)$. Note that only two chiral combinations are possible for the tensor currents: $L\times L$ and $R\times R$; the other combinations, $L\times R$ and $R\times L$, are identically zero.

One may then consider authentic dimension 7 operators by incorporating one covariant derivative in the fermion products. Two possibilities appear:

1. contracting the Lorentz index of D_μ with a vector current
2. contracting it with a tensor current, the second tensorial index being contracted with the second, vector current.

Note however that, using the equations of motion on fermions ($(\imath\not{D}-m_f)f=0$) and realizing partial integrations (i.e. adding a total derivative to the lagrangian density), the resulting set of operators is largely redundant (together with the dimension 6 operators). Indeed, the second possibility which we mentioned ('$D^{\mu}\gamma^{\nu}\otimes\sigma_{\mu\nu}$') can always be reduced down to operators of the first class ('$D_\mu\otimes\gamma^{\mu}$') plus dimension 6 operators (multiplying fermion masses). Additionally, certain combinations of the operators of the first class reduce to dimension 6 terms. We thus retain only two kinds of linearly independant operators:

1. $[\bar{b}\imath(\overrightarrow{D}-\overleftarrow{D})^{\mu}P_{L,R}s](\bar{f}\gamma_{\mu}P_{L,R}f)$;
2. $(\bar{b}\gamma^{\mu}P_{L,R}s)[\bar{f}\imath(\overrightarrow{D}-\overleftarrow{D})_{\mu}P_{L,R}f]$.

For semi-leptonic operators, i.e. when f is a lepton ($f=l$), the analysis is essentially over.
Let us therefore focus on four-quark operators ($f=q$). One should then also consider the contraction of the colour indices. Two possibilities arise:

1. product of two colour-singlet currents: colour-indices contracted between the $\bar{b}$ and s on one side, $\bar{q}$ and q on the other;
2. product of two octet currents: colour-indices contracted between the $\bar{b}$ and q on one side, $\bar{q}$ and s on the other.

In the special cases where $q=b,s$, however, Fierz identities make this distinction superfluous, so that we may consider only singlet products then (refer to Appendix **??**). This is our final word for four-fermion operators.
We now turn to $\bar{b}s$-Gauge operators. The methodology follows that of [?] for the dimension 6 operators: one may simply write all the possibilities to include covariant derivatives (at most four for operators of dimension ≤ 7) within the fermionic current. Using partial integration and equations of motion, it turns out that all these covariant derivatives can be combined in field-strength tensors. One thus simply needs to consider the possibilities to combine the indices of these field-strength tensors with those of the fermionic current:

- When only one field-strength is present, it can only contract with a (Lorentz) tensor current, and, in the case of the QCD field strength $G^a_{\mu\nu}$, with a $SU(3)_c$ octet current.
- When two field-strengths are present, we may either contract their Lorentz indices together – thus reducing the fermionic current to a scalar – via the metric or a $\varepsilon_{\mu\nu\rho\sigma}$ tensor –, or contract two of their Lorentz indices with a tensor current (the other two through the metric, or equivalently $\varepsilon_{\mu\nu\rho\sigma}$). In the case where only one QCD field-strength is involved (among the two), one needs again a $SU(3)_c$ octet on the fermionic side. When two QCD field-strengths are involved ($G^a_{\mu\nu}G^b_{\rho\sigma}$), the color indices may be contracted together (δ^{ab}: colour-singlet), with a symmetric tensor ($\{T^a,T^b\}$) or with an antisymmetric tensor ($[T^a,T^b]$) (superposition of colour-octets + singlet), depending of the compatibility of these structures with the Lorentz form of the fermionic current.

[4]Note that the definition of $\sigma^{\mu\nu}$ which we adopt here for simplicity differs somewhat, by a factor $\imath/2$, from the one the reader may have encountered in the literature.

We now have all the ingredients to present the list of relevant operators. Note that their normalization is a priori free: our choice can be justified a posteriori to ensure that all the RGE's intervene at the same, leading-order in α_S. Note however that this choice may be slightly misleading, for instance in the SM, for reasons that we will discuss in the next section, when we solve the RGE's.

- $(\bar{b}s)(\bar{l}l)$ operators[5]:

$$\begin{cases} S^l_{L,R\;L,R} = \frac{\alpha}{\alpha_S} m_b (\bar{b} P_{L,R} s)(\bar{l} P_{L,R} l) \\ V^l_{L,R\;L,R} = \frac{\alpha}{\alpha_S} m_b (\bar{b}\gamma^\mu P_{L,R} s)(\bar{l}\gamma_\mu P_{L,R} l) \\ T^l_{L,R\;L,R} = \frac{\alpha}{\alpha_S} m_b (\bar{b}\sigma^{\mu\nu} P_{L,R} s)(\bar{l}\sigma_{\mu\nu} P_{L,R} l) \quad ; \; \sigma^{\mu\nu} \equiv [\gamma^\mu, \gamma^\nu] \\ H^l_{L,R\;L,R} = \frac{\alpha}{\alpha_S} \left[\bar{b} \left(\imath \overrightarrow{\partial} - \imath \overleftarrow{\partial} + 2Q_d eA + 2g_S T^a G^a \right)^\mu P_{L,R} s \right] (\bar{l}\gamma_\mu P_{L,R} l) \\ \tilde{H}^l_{L,R\;L,R} = \frac{\alpha}{\alpha_S} (\bar{b}\gamma^\mu P_{L,R} s) \left[\bar{l} \left(\imath \overrightarrow{\partial} - \imath \overleftarrow{\partial} + 2Q_l eA \right)_\mu P_{L,R} l \right] \end{cases} \tag{4}$$

- $(\bar{b}s)(\bar{q}q)$ operators[6]:

$$\begin{cases} S^q_{L,R\;L,R} = m_b (\bar{b} P_{L,R} s)(\bar{q} P_{L,R} q) \\ V^q_{L,R\;L,R} = m_b (\bar{b}\gamma^\mu P_{L,R} s)(\bar{q}\gamma_\mu P_{L,R} q) \\ T^q_{L,R\;L,R} = m_b (\bar{b}\sigma^{\mu\nu} P_{L,R} s)(\bar{q}\sigma_{\mu\nu} P_{L,R} q) \\ H^q_{L,R\;L,R} = \left[\bar{b}\imath \left(\overrightarrow{D} - \overleftarrow{D} \right)^\mu P_{L,R} s \right] (\bar{q}\gamma_\mu P_{L,R} q) \\ \tilde{H}^q_{L,R\;L,R} = (\bar{b}\gamma^\mu P_{L,R} s) \left[\bar{q}\imath \left(\overrightarrow{D} - \overleftarrow{D} \right)_\mu P_{L,R} q \right] \\ \mathcal{S}^q_{L,R\;L,R} = m_b (\bar{b}_\alpha P_{L,R} s_\beta)(\bar{q}_\beta P_{L,R} q_\alpha) \\ \mathcal{V}^q_{L,R\;L,R} = m_b (\bar{b}_\alpha \gamma^\mu P_{L,R} s_\beta)(\bar{q}_\beta \gamma_\mu P_{L,R} q_\alpha) \\ \mathcal{T}^q_{L,R\;L,R} = m_b (\bar{b}_\alpha \sigma^{\mu\nu} P_{L,R} s_\beta)(\bar{q}_\beta \sigma_{\mu\nu} P_{L,R} q_\alpha) \\ \mathcal{H}^q_{L,R\;L,R} = \left[\bar{b}_\alpha \imath \left(\overrightarrow{D} - \overleftarrow{D} \right)^\mu P_{L,R} s_\beta \right] (\bar{q}_\beta \gamma_\mu P_{L,R} q_\alpha) \\ \tilde{\mathcal{H}}^q_{L,R\;L,R} = (\bar{b}_\alpha \gamma^\mu P_{L,R} s_\beta) \left[\bar{q}_\beta \imath \left(\overrightarrow{D} - \overleftarrow{D} \right)_\mu P_{L,R} q_\alpha \right] \end{cases} \tag{5}$$

- $(\bar{b}s)$-Gauge operators:

$$\begin{cases} E_{L,R} = \frac{\imath e m_b^2}{4\pi\alpha_S} \bar{b}\sigma^{\mu\nu} P_{L,R} s\, F_{\mu\nu} \\ Q_{L,R} = \frac{\imath m_b^2}{g_S} \bar{b}\sigma^{\mu\nu} T^a P_{L,R} s\, G^a_{\mu\nu} \\ \mathcal{E}^S_{L,R} = \frac{\alpha}{\alpha_S} \bar{b} P_{L,R} s\, F_{\mu\nu} F^{\mu\nu} \\ \tilde{\mathcal{E}}^S_{L,R} = \imath \frac{\alpha}{\alpha_S} \bar{b} P_{L,R} s\, F_{\mu\nu} \tilde{F}^{\mu\nu} \\ \mathcal{E}^T_{L,R} = \frac{\alpha}{\alpha_S} \bar{b}\sigma^{\nu\rho} P_{L,R} s\, F_{\mu\nu} F^\mu_{\ \rho} \\ \mathcal{H}^S_{L,R} = \frac{e}{g_S} \bar{b} T^a P_{L,R} s\, F_{\mu\nu} G^{a\,\mu\nu} \\ \tilde{\mathcal{H}}^S_{L,R} = \frac{\imath e}{g_S} \bar{b} T^a P_{L,R} s\, F_{\mu\nu} \tilde{G}^{a\,\mu\nu} \\ \mathcal{H}^T_{L,R} = \frac{e}{g_S} \bar{b}\sigma^{\nu\rho} T^a P_{L,R} s\, F^\mu_{\ \nu} G^a_{\mu\rho} \\ \mathcal{Q}^D_{L,R} = \bar{b} P_{L,R} s\, G^a_{\mu\nu} G^{a\,\mu\nu} \\ \tilde{\mathcal{Q}}^D_{L,R} = \imath \bar{b} P_{L,R} s\, G^a_{\mu\nu} \tilde{G}^{a\,\mu\nu} \\ \mathcal{Q}^S_{L,R} = \bar{b} \frac{\{T^a, T^b\}}{2} P_{L,R} s\, G^a_{\mu\nu} G^{b\,\mu\nu} \\ \tilde{\mathcal{Q}}^S_{L,R} = \imath \bar{b} \frac{\{T^a, T^b\}}{2} P_{L,R} s\, G^a_{\mu\nu} \tilde{G}^{b\,\mu\nu} \\ \mathcal{Q}^T_{L,R} = \bar{b}\sigma^{\nu\rho} \frac{[T^a, T^b]}{2} P_{L,R} s\, G^a_{\mu\nu} G^{b\,\mu}_{\ \ \rho} \end{cases} \tag{6}$$

This list determines the effective hamiltonian of our EFT (the generic notation O^i spans the whole list):

$$\mathcal{H}^{(\mathrm{dim} \leq 7)}_{eff} = \sum_i C_i(\mu) O^i(\mu) + h.c. \tag{7}$$

[5]We repeat that, for the tensor operators, only the $L \times L$ and $R \times R$ combinations are relevant, which may not be obvious from our notation.

[6]We discard $\mathcal{S}, \mathcal{V}, \mathcal{T}, \mathcal{H}, \tilde{\mathcal{H}}^q$ for $q = b, s$ since they reduce to $S, V, T, H, \tilde{H}^q$ due to Fierz transformations which are provided in Appendix **??**. The colour indices α, β of the fermions are displayed when not trivially contracted. Note finally that the ambiguous notation '$\overrightarrow{D}_\mu q_\alpha$' stands actually for $(D_\mu q)_\alpha \ldots$