B Mixing in the Standard Model and Beyond[1]

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I present numerical updates of the Standard-Model predictions for the mass and width differences and the CP asymmetries in flavor-specific decays in $\mathrm{B_s}-\overline{\mathrm{B}}_\mathrm{s}$ and $\mathrm{B_d}-\overline{\mathrm{B}}_\mathrm{d}$ mixing. Then I discuss the current status of new physics in these mixing amplitudes.

1 $\mathrm{B}-\overline{\mathrm{B}}$ mixing: general formalism and Δm

$\mathrm{B_q}-\overline{\mathrm{B}}_\mathrm{q}$ mixing with $q=d$ or $q=s$ is governed by $M^q - i\Gamma^q/2$ with the hermitian 2×2 matrices M^q and Γ^q. The $(1,2)$ element of $M^q - i\Gamma^q/2$ induces $\overline{B}_q \to B_q$ transitions.

The mass matrix element M_{12}^q stems from the dispersive part of the box diagram in Fig. 1, which is obtained from the full diagram by replacing the loop integral with its real part. The decay matrix element Γ_{12}^q is calculated from the absorptive part of the box diagram, which instead involves the imaginary part of the loop integral. M_{12}^q is dominated by the top contribution, while Γ_{12}^q solely involves internal c,u quarks and $|\Gamma_{12}^q| \ll |M_{12}^q|$. $\mathrm{B_q}-\overline{\mathrm{B}}_\mathrm{q}$ mixing involves three physical quantities:

$$|M_{12}^q|\,, \quad |\Gamma_{12}^q|\,, \quad \phi_q \equiv \arg\left(-\frac{M_{12}^q}{\Gamma_{12}^q}\right) \tag{1}$$

The two eigenstates found by diagonalizing $M^q - i\,\Gamma^q/2$ differ in their masses and widths. A third observable is the CP asymmetry in flavor-specific decays (usually called semileptonic CP asymmetry), which quantifies CP violation in $\mathrm{B_q}-\overline{\mathrm{B}}_\mathrm{q}$ mixing. The three quantities in Eq. (1) can be determined from the following observables:

Mass difference:	$\Delta m_q \simeq 2|M_{12}^q|,$
Width difference:	$\Delta\Gamma_q \simeq 2|\Gamma_{12}^q|\cos\phi_q$
CP asymmetry in flavor-specific decays:	$a_{\mathrm{fs}}^q \simeq \dfrac{|\Gamma_{12}^q|}{|M_{12}^q|}\sin\phi_q$

[1]Proceedings of CKM 2012, the 7th International Workshop on the CKM Unitarity Triangle, University of Cincinnati, USA, 28 September - 2 October 2012

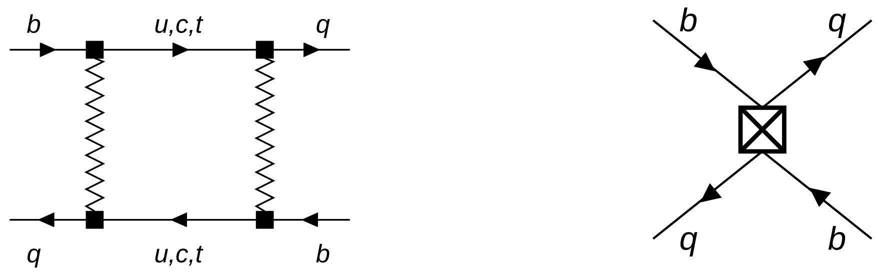

Figure 1: Left: Box diagram. Right: Local operator Q.

M_{12}^q is calculated with the help of an operator product expansion (OPE), expressing this transition amplitude in terms of Wilson coefficients and matrix elements of local four-quark operators. The Standard-Model prediction of M_{12}^q only involves a single operator Q:

$$M_{12} = (V_{tq}^* V_{tb})^2 \, C \, \langle \mathrm{B_q} | Q | \overline{\mathrm{B}}_q \rangle \tag{2}$$

Here V_{tq} and V_{tb} are the relevant elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The short-distance physics is contained in $C = C(m_t, \alpha_s)$, which depends on the top quark mass m_t and the QCD coupling constant α_s. C is known at the level of next-to-leading-order corrections in QCD [1] and suffers from very small theoretical uncertainties. The four-quark operator Q reads

$$Q \;\; = \;\; \overline{q}_L \gamma_\nu b_L \; \overline{q}_L \gamma^\nu b_L \tag{3}$$

and is depicted in Fig. 1.

The theoretical uncertainty of Δm_q is dominated by the hadronic matrix element, which is parametrized as

$$\langle \mathrm{B_q} | Q | \overline{\mathrm{B}}_q \rangle \;\; = \;\; \frac{2}{3} M_{B_q}^2 \, f_{B_q}^2 \, B_{B_q}. \tag{4}$$

Here M_{B_q} and f_{B_q} are mass and decay constant of B_q, respectively, and B_{B_q} is called "bag" parameter. The matrix element in Eq. (4) is calculated with lattice QCD. The prediction of Δm_s involves $|V_{ts}|$, which is essentially equal to the well-measured CKM element $|V_{cb}|$. With the result of Ref. [1] and present-day values of V_{cb}, m_t and α_s one finds

$$\Delta m_s = \left(\, 18.8 \pm 0.6_{V_{cb}} \pm 0.3_{m_t} \pm 0.1_{\alpha_s} \right) \, \mathrm{ps}^{-1} \, \frac{f_{B_s}^2 \, B_{B_s}}{(220 \, \mathrm{MeV})^2}$$

with individual errors from the indicated sources. Here the $\overline{\mathrm{MS}}$-NDR scheme for B_{B_s} is used and B_{B_s} is evaluated at the scale m_b. In the literature often the scheme-invariant $\widehat{B}_{B_s} = 1.51 B_{B_s}$ is used instead.

In phenomenological analyses usually also lattice results for the decay constant f_{B_s} are used. For instance, Ref. [2] uses the CKMfitter [3] averages of several lattice results,

$$f_{B_s} = (229 \pm 2 \pm 6)\,\text{MeV}, \qquad B_{B_s} = 0.85 \pm 0.02 \pm 0.02. \tag{5}$$

The quoted value for B_{B_s} is the average of the value in Ref. [4] and of the value obtained from the ratio of $f_{B_s}^2 B_{B_s}$ and $f_{B_s}^2$ calculated in Ref. [5]. With the numbers in Eq. (5) one finds $f_{B_s}^2 B_{B_s} = [(211 \pm 9)\,\text{MeV}]^2$ and

$$\Delta m_s = (17.3 \pm 1.5)\,\text{ps}^{-1} \tag{6}$$

complying excellently with the LHCb/CDF average [6]

$$\Delta m_s^{\text{exp}} = (17.719 \pm 0.043)\,\text{ps}^{-1}.$$

Bearing in mind that the prediction in Eq. (6) is essentially based on calculations of f_{B_s} rather than $f_{B_s}^2 B_{B_s}$ the quoted error cannot be considered conservative. Using the preliminary lattice result of the Fermilab/MILC collaboration [7], $f_{B_s}^2 B_{B_s} = 0.0559(68)\,\text{GeV}^2 \simeq [(237 \pm 14)\,\text{MeV}]^2$, one instead finds

$$\Delta m_s = (21.7 \pm 2.6)\,\text{ps}^{-1}.$$

Clearly, new lattice results for $f_{B_s}^2 B_{B_s}$ are highly desirable to decrease the uncertainty in Δm_s further.

Turning to Δm_d, I discuss the SM prediction for the ratio $\Delta m_d/\Delta m_s$, from which $|V_{cb}|$, the short-distance coefficient C and some hadronic uncertainties drop out: The hadronic quantity needed is

$$\xi^2 = \frac{f_{B_s}^2 B_{B_s}}{f_{B_d}^2 B_{B_d}}$$

and the dependence on the CKM parameters reads:

$$\frac{\Delta m_d}{\Delta m_s} \propto \frac{|V_{td}|^2}{|V_{ts}|^2} \propto R_t^2 \tag{7}$$

Here R_t is one side of the unitarity triangle shown in Fig. 2. The usual way to probe the Standard Model with Δm_d is to perform a global fit to the unitarity triangle [3,8]. A shortcut which reproduces the result of the global fit in an accurate way exploits the calculation of R_t from two angles of the UT triangle:

$$R_t = \frac{\sin\gamma}{\sin\alpha} = \frac{\sin(\alpha+\beta)}{\sin\alpha}$$

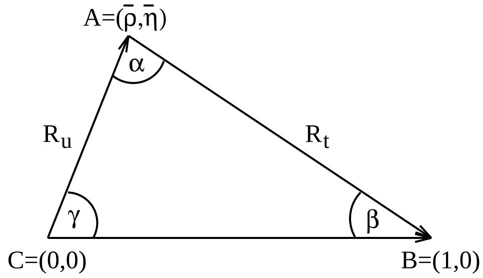

Figure 2: Unitarity triangle.

and the experimental data [6]

$$\beta = 21.4^\circ \pm 0.8^\circ, \qquad \alpha = 88.7^\circ{}^{+4.6^\circ}_{-4.2^\circ}$$

give

$$R_t = 0.939 \pm 0.027. \tag{8}$$

This number can be directly compared with the value of R_t found from $\Delta m_d/\Delta m_s$. Inverting the relation sketched in Eq. (7) yields [9, p.354]:

$$R_t \;=\; 0.880 \frac{\xi}{1.16} \sqrt{\frac{\Delta m_d}{0.49\,\mathrm{ps}^{-1}}} \sqrt{\frac{17\,\mathrm{ps}^{-1}}{\Delta m_s}} \frac{0.22}{|V_{us}|} (1 + 0.050\overline{\rho}) \tag{9}$$

With the Fermilab/MILC result [10] $\xi = 1.268 \pm 0.063$ we find

$$R_t = 0.942 \pm 0.047_\xi \pm 0.006_{\mathrm{rest}}$$

which agrees very well with Eq. (8). Both numbers also agree with the CKMfitter global fit result $R_t = 0.926^{+0.027}_{-0.028}$ (using different lattice input) obtained a few days before this conference. Ignoring the small deviation of B_{B_s}/B_{B_d} from 1 the quantity ξ equals the ratio f_{B_s}/f_{B_d}, for which also a result obtained with QCD sum rules is available, $\xi \simeq f_{B_s}/f_{B_d} = 1.16 \pm 0.04$ [11]. Data are now starting to challenge such low values of ξ.

2 Decay matrix Γ_{12}: prediction of $\Delta\Gamma$ and a_{fs}

Γ_{12}^q, $q = d, s$, is needed for the prediction of the width difference $\Delta\Gamma_q \simeq 2|\Gamma_{12}^q| \cos\phi_q$ and the semileptonic CP asymmetry $a_{\mathrm{fs}}^q = \dfrac{|\Gamma_{12}^q|}{|M_{12}^q|} \sin\phi_q$. For the calculation of Γ_{12}^q another OPE, the so-called Heavy Quark Expansion (HQE) is employed. The HQE expresses the u, c contributions to the $\overline{B}_q \to B_q$ transition amplitude in inverse powers

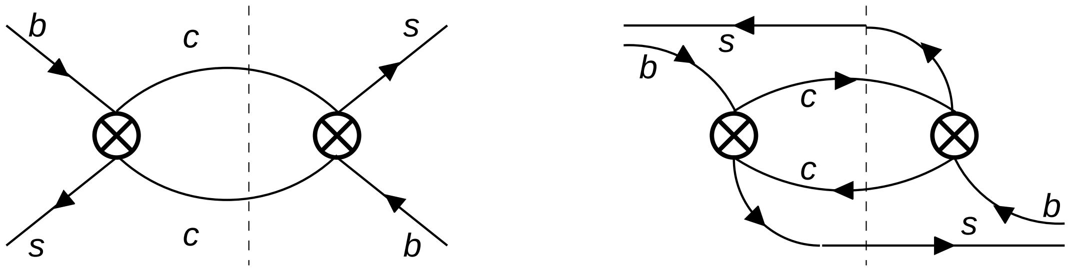

Figure 3: Γ_{12} in the leading order of QCD.

of Λ_{QCD}/m_b, the ratio of some hadronic scale $\Lambda_{QCD} \sim 500$ MeV and the bottom quark mass. The leading contribution to Γ_{12}^q is depicted in Fig. 3. The HQE results in a simultaneous expansion of Γ_{12}^q in Λ_{QCD}/m_b and $\alpha_s(m_b)$. Corrections of order Λ_{QCD}/m_b were calculated in Ref. [12,13], those of order α_s were obtained in Ref. [14–16]. In Ref. [18] these NLO results have been expressed in terms of a new operator basis, which leads to a better numerical stability by rendering an important Λ_{QCD}/m_b correction color-suppressed. Furthermore, in Ref. [18] the all-order re-summation of $\alpha_s^n z \ln^n z$ terms (with $z = m_c^2/m_b^2$ and $n = 1, 2, \ldots$) (developed in Ref. [17]) has been applied to Γ_{12}^q. To leading order in Λ_{QCD}/m_b one encounters two operators, Q defined in Eq. (3) and

$$\widetilde{Q}_S = \overline{s}_L^\alpha b_R^\beta \, \overline{s}_L^\beta b_R^\alpha. \tag{10}$$

with color indices α and β. The matrix element is parametrized as

$$\langle B_s | \widetilde{Q}_S | \overline{B}_s \rangle = \frac{1}{12} M_{B_s}^2 \, f_{B_s}^2 \widetilde{B}'_{S,B_s}. \tag{11}$$

In the ratio $\Delta\Gamma_s/\Delta m_s$ the dependence on $f_{B_s}^2 B_{B_s}$ drops out. We predict $\Delta\Gamma_s$ by combining the theory prediction of this quantity with $\Delta m_s^{\rm exp} = 17.719\,\text{ps}^{-1}$. For $m_t(m_t) = 165.8$ GeV, $m_b(m_b) = 4.248$ GeV, $m_c(m_c) = 1.286$ GeV, $m_s(m_b) = 85$ MeV and $\alpha_s(M_Z) = 0.1184$ (all in $\overline{\rm MS}$ scheme) one finds

$$\frac{\Delta\Gamma_s}{\Delta m_s} \Delta m_s^{\rm exp} = \left[0.082 \pm 0.007 + (0.019 \pm 0.001) \frac{\widetilde{B}'_{S,B_s}}{B_{B_s}} - (0.027 \pm 0.003) \frac{B_R}{B_{B_s}} \right] \text{ps}^{-1}$$

Here B_R is a generic bag parameter for the operators appearing at order Λ_{QCD}/m_b and the quoted errors are the perturbative uncertainties estimated by varying the renormalization scale. All SM predictions presented in this talk are an average of two renormalization schemes, using either the $\overline{\rm MS}$ or pole definition for the overall factor of m_b^2 appearing in $\Delta\Gamma$ and $a_{\rm fs}$ (while re-summing $\alpha_s^n z \ln^n z$ terms in both cases). Parametric errors (like those of the quark masses) are of minor relevance.

With the preliminary Fermilab/MILC result [7],

$$\frac{\widetilde{B}'_{S,B_s}}{B_{B_s}} = 1.50 \pm 0.30,$$

and the estimate $B_R = 1 \pm 0.5$ of the unknown higher-order bag parameters one finds:

$$\frac{\Delta\Gamma_s}{\Delta m_s}\Delta m^{\rm exp} = \left[0.078 \pm 0.016_{B_R/B} \pm 0.012_{\rm scale} \pm 0.008_{\widetilde{B}/B}\right] \,\mathrm{ps}^{-1}, \tag{12}$$

which complies well with the LHCb measurement [19]

$$\Delta\Gamma_s^{\rm LHCb} = [0.116 \pm 0.018_{\rm stat} \pm 0.006_{\rm syst}]\,\mathrm{ps}^{-1}$$

and the LHCb/CDF/DØ average found by the HFAG [6]:

$$\Delta\Gamma_s^{\rm exp} = [0.089 \pm 0.012]\,\mathrm{ps}^{-1}.$$

For the same input as used in Eq. (12) the CP asymmetry in flavor-specific decays equals

$$a_{\rm fs}^s \;=\; (1.8 \pm 0.3) \cdot 10^{-5} \tag{13}$$

Here also the values of the CKM parameter are relevant, the quoted number corresponds to $|V_{ub}| = 3.49 \cdot 10^{-3}$, $|V_{cb}| = 40.89 \cdot 10^{-3}$, and $\gamma = 67.7^\circ$ [3].

In the B_d system the central value of $\Delta\Gamma/\Delta m$ has slightly shifted downwards from the 2006 value of $5.3 \cdot 10^{-3}$ [18] to $\Delta\Gamma_d/\Delta m_d = 4.7 \cdot 10^{-3}$ with an error of roughly 20%. With $\Delta m_d^{\rm exp} = 0.507\,\mathrm{ps}^{-1}$ this means $\Delta\Gamma_d = 2.4\,\mathrm{ns}^{-1}$ which is challenging to measure. The numerical prediction for the CP asymmetry is

$$a_{\rm fs}^d \;=\; (-4.0 \pm 0.6) \cdot 10^{-4}, \tag{14}$$

essentially unchanged from the update in [20]. The quoted numbers use $\widetilde{B}'_{S,B_d}/B_{B_d} = 1.4 \pm 0.4$ inferred from [7]. Finally the CP phases in Eq. (1) read

$$\phi_s \;=\; 0.24^\circ \pm 0.06^\circ, \qquad\qquad \phi_d \;=\; -4.9^\circ \pm 1.4^\circ. \tag{15}$$

3 New physics

The DØ experiment has measured [21, 22]

$$\begin{aligned} A_{\rm SL}^{\rm D0} &= (0.532 \pm 0.039) a_{\rm fs}^d + (0.468 \pm 0.039) a_{\rm fs}^s \\ &= (-7.87 \pm 1.72 \pm 0.93) \cdot 10^{-3}, \end{aligned} \tag{16}$$

which is 3.9σ off the SM prediction inferred from Eqs. (13) and (14),

$$A_{\rm SL} \;=\; (-0.20 \pm 0.03) \cdot 10^{-3}. \tag{17}$$

The prefactors of $a_{\rm fs}^d$ and $a_{\rm fs}^s$ in Eq. (16) are taken from Ref. [23], in which they were calculated from the $B_{d,s}$ production fractions obtained by HFAG [6].

From a theoretical point of view, it is natural for new physics to affect M_{12}^s and M_{12}^d, which are sensitive to scales of 100 TeV and above. Parametrising the new-physics contribution as [18]

$$M_{12}^q \;\equiv\; M_{12}^{\rm SM,q} \cdot \Delta_q\,, \qquad \Delta_q \;\equiv\; |\Delta_q| e^{i\phi_q^\Delta}, \qquad\qquad q = d, s,$$

one can perform a global fit of $\Delta_{d,s}$ and the CKM elements to all relevant data. This has been done in summer 2010 [24] and spring 2012 [23], before and after the precise LHCb measurement of the CP violating phase in $B_s \to J/\psi\phi$, respectively. In 2010 the scenario with new physics in $\mathrm{B}-\overline{\mathrm{B}}$ mixing and $\mathrm{K}-\overline{\mathrm{K}}$ mixing (and taking SM formulas for tree-dominated observables) gave an excellent fit with a large, $\mathcal{O}(1)$ deviation of Δ_s from its SM value $\Delta_s = 1$ [24], with the SM point $\Delta_d = \Delta_s = 1$ disfavored by 3.6σ. The new CP phase ϕ_q^Δ enters $a_{\rm fs}^q$ as [18]

$$\begin{aligned} a_{\rm fs}^s &= (4.4 \pm 1.2) \cdot 10^{-3} \cdot \frac{\sin\left(\phi_s^{\rm SM} + \phi_s^\Delta\right)}{|\Delta_s|}, \\ a_{\rm fs}^d &= (4.7 \pm 1.4) \cdot 10^{-3} \cdot \frac{\sin\left(\phi_d^{\rm SM} + \phi_d^\Delta\right)}{|\Delta_d|}. \end{aligned} \tag{18}$$

The tagged analysis of $B_s \to J/\psi\phi$ determines $\phi_s^\Delta - 2\beta_s$ with $2\beta_s = 2.1°$. With the LHCb data placing tight bounds on $|\phi_d^\Delta|$, the new-physics scenario described above cannot accommodate the DØ result anymore, the SM point $\Delta_d = \Delta_s = 1$ is merely disfavored by 1σ. The September-2012 update (see Ref. [3]) of the plots presented in Ref. [24] are shown in Fig. 4. The pull value for $A_{\rm SL}$ is found as 3.3σ, showing that the improvement compared to the SM is small. The fit prefers $\phi_d^\Delta < 0$ to loosen the tension with $A_{\rm SL}$ and to accommodate the world average for $B(B \to \tau\nu)$ [25–28]. This branching ratio prefers a larger value of $|V_{ub}|$ (despite of the recent Belle result complying with the SM [27]), implying larger values of R_u (see Fig. 2) and β. Since the CP asymmetry in $B_d \to J/\psi K_S$ precisely fixes $2\beta + \phi_d^\Delta = 42.8° \pm 1.6°$, a larger $|V_{ub}|$ entails $\beta > 21.4°$ and therefore $\phi_d^\Delta < 0$ [23, 24].

Note, however, that the 95% CL region in the upper plot of Fig. 4 is compatible with new physics in M_{12}^s of the order of 30% of the SM amplitude. While the upcoming better LHCb data on $B_s \to J/\psi\phi$ will reduce this allowed region, these data will only constrain ϕ_s^Δ and not $|\Delta_s|$. To this end better lattice results for $f_{B_s}^2 B_{B_s}$ are urgently needed.

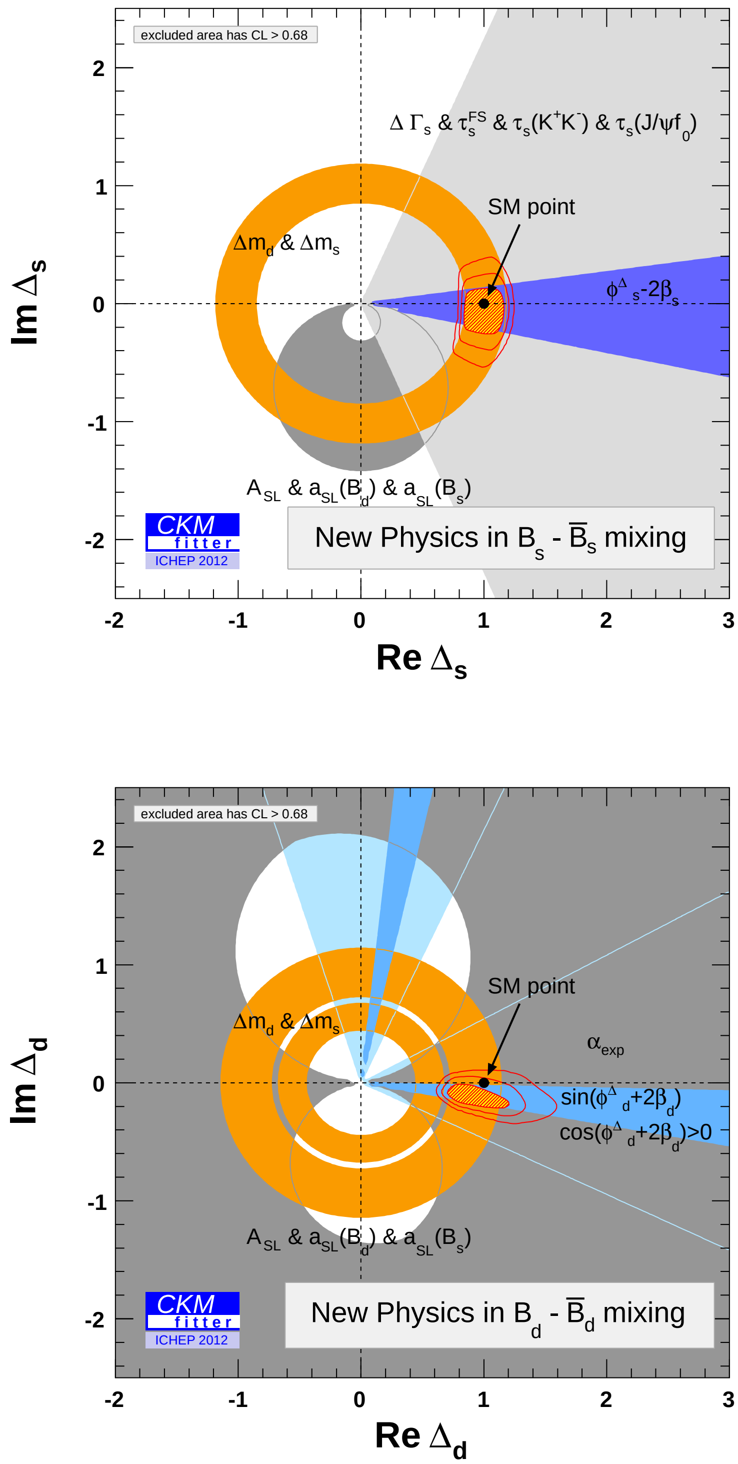

Figure 4: Allowed regions for the new-physics parameters Δ_s and Δ_d [3, 23].

Several authors have considered the possibility of new physics in Γ_{12}^s from the yet unobserved decay mode $B_s \to \tau^+\tau^-$ [29–31]. The idea of new physics in B_s decays can reconcile the DØ result in Eq. (16) with the LHCb measurement of $\phi_s^\Delta - 2\beta_s$ because $a_{\rm fs}^s = \mathrm{Im}\,(\Gamma_{12}^s/M_{12}^s)$ grows with new contributions to Γ_{12}^s. However, the LHCb experiment has measured the average width Γ_s of the two B_s eigenstates with high accuracy [19]. With the PDT value [32] for the B_d width $\Gamma_d = 1/\tau_{B_d}$ one finds

$$\frac{\Gamma_d}{\Gamma_s} = 0.997 \pm 0.013$$

in excellent agreement with the SM prediction $\Gamma_d/\Gamma_s = 0.998 \pm 0.003$ [12, 20, 33]. This result precludes a sizable new B_s decay rate into $\tau^+\tau^-$ or other undetected final states [23, 24]. However, phenomenologically, sizable new physics in the doubly Cabibbo-suppressed quantity Γ_{12}^d is still allowed [23], but requires somewhat contrived models of new physics.

4 Conclusions

In this proceedings article I have updated several quantities related to $\mathrm{B_s}-\overline{\mathrm{B}}_\mathrm{s}$ and $\mathrm{B_d}-\overline{\mathrm{B}}_\mathrm{d}$ mixing. It is stressed that the commonly used prediction of Δm_s relies on just two lattice calculations [4, 5], which date back to 2003 and 2009. In this article I have used the newer, but still preliminary results of the Fermilab/MILC collaboration presented in Refs. [7, 10]. Significant numerical differences with respect to the last update in Ref. [20] only occur for Δm_s.

The $\mathrm{B_s}-\overline{\mathrm{B}}_\mathrm{s}$ mixing and $\mathrm{B_d}-\overline{\mathrm{B}}_\mathrm{d}$ mixing amplitudes are highly sensitive to new physics. The LHCb measurements in $B_s \to J/\psi\phi$ have placed tight bounds on the CP phase in M_{12}^s, but $\mathcal{O}(30\%)$ new physics in M_{12}^s is still possible. However, unlike in 2010 [24] the hypothesis of new physics only in M_{12}^d and M_{12}^s can no more explain the DØ result for $A_{\rm SL}$ [23].

I am grateful to Alex Lenz for the opportunity to give this talk. The presented work is supported by BMBF under grant no. 05H12VKF.

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