

Complete three-loop QCD corrections to the decay $H \rightarrow \gamma\gamma$

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Abstract

We present the result for the three-loop singlet QCD corrections to the decay of a Higgs boson into two photons and improve the calculation for the non-singlet case. With the new result presented, the decay width $\Gamma(H \rightarrow \gamma\gamma)$ is completely known at $\mathcal{O}(G_F\alpha^2\alpha_s^2, G_F\alpha^3)$.

Key words: Perturbative calculations, Quantum Chromodynamics, Higgs

1 Introduction

After the discovery of a new boson at about 126 GeV at the LHC [1,2] it is imperative to study all decay modes of the Standard Model Higgs boson as precisely as possible. In this paper we are concerned with the decay of the Higgs boson into two photons which is the decay channel with the largest significance for the discovery of the Higgs boson at the LHC.

The decay of a Higgs boson into two photons is mediated through either charged gauge bosons or top quarks at one-loop order. The decay rate can be cast into the form

$$\Gamma(H \rightarrow \gamma\gamma) = \frac{M_H^3}{64\pi} |A_W + A_t|^2, \quad (1)$$

with the leading order values

$$\begin{aligned}
A_W^{(0)} &= -\frac{\alpha\sqrt{\sqrt{2}G_F}}{2\pi} \left(2 + \frac{3}{\tau_W} + \frac{3}{\tau_W} \left(2 - \frac{1}{\tau_W} \right) \arcsin^2 \sqrt{\tau_W} \right), \\
A_t^{(0)} &= \hat{A}_t \frac{3}{2\tau} \left(1 + \left(1 - \frac{1}{\tau} \right) \arcsin^2 \sqrt{\tau} \right), \\
\hat{A}_t &= N_c \frac{2\alpha\sqrt{\sqrt{2}G_F}}{3\pi} Q_t^2,
\end{aligned} \tag{2}$$

where $\tau_W = M_H^2/(4M_W^2)$ and $\tau = M_H^2/(4m_t^2)$. The former contribution is larger than the latter (by a factor 4.5) and opposite in sign. Both contributions have been investigated in great detail in the literature. The two-loop QCD corrections to the decay have first been evaluated in the heavy-top limit in Refs. [3–5] and later, keeping the full top-mass dependence, in Refs. [6–8]. The two-loop electroweak corrections have been investigated in [9–12]. Combining the two-loop QCD and electroweak corrections one observes a nearly complete cancellation between these two contributions for $M_H = 126$ GeV as discussed below.

At next-to-next-to-leading order (NNLO) the non-singlet QCD contributions (cf Fig. 1 (c)-(e)) have been calculated in the heavy top limit, including additional terms in an expansion in $\tau = M_H^2/(4m_t^2)$, in Ref. [13]. At this order a new class of diagrams, so-called singlet diagrams (cf Fig. 1 (f)-(g)), contributes for the first time. They can be characterized by the property, that the external lines are coupled to different fermion loops. The contribution from this kind of diagrams has not been taken into account up to now and there is no formal argument, that it should be suppressed compared to the non-singlet contribution, when considering the same order in α_s . In this letter we present the calculation of this last missing piece to obtain a complete NNLO QCD prediction for the decay of a Higgs boson into two photons. In addition, to improve the existing prediction and to check our setup, we recalculated the non-singlet contribution and added more terms in the expansion in τ .

This letter is organized as follows: In Section 2 we describe the necessary steps of the calculation and in Section 3 we present the full three-loop QCD corrections to the decay width. In Section 4 we give the numerical result and our conclusions.

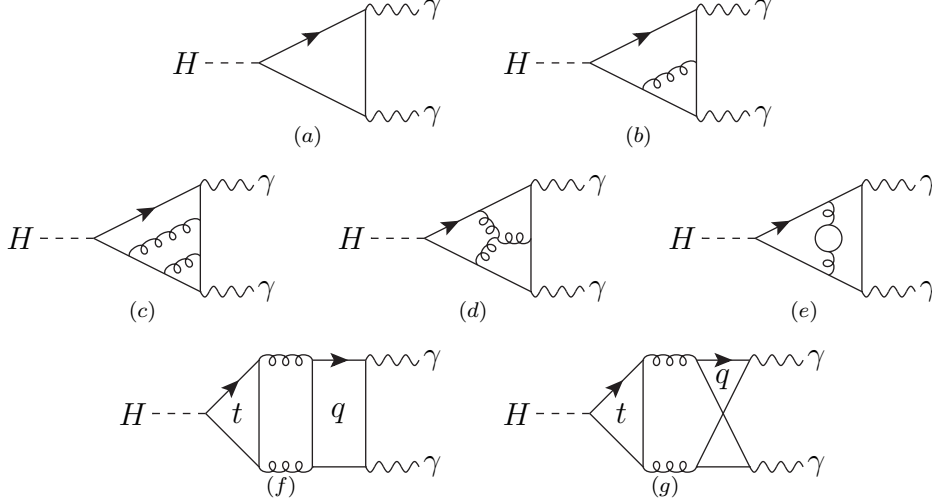


Fig. 1. Sample diagrams of the 1-loop (a), 2-loop (b), 3-loop non-singlet (c)-(e), and 3-loop singlet (f)-(g) top quark induced contribution to $H \rightarrow \gamma\gamma$.

2 Calculation

We start from the decay amplitude of a Higgs boson into two photons

$$\mathcal{A}^{\mu\nu} = H \text{---} \text{---} \begin{array}{c} \text{---} \mu \\ q_1 \\ \text{---} \nu \\ q_2 \end{array} . \quad (3)$$

Decomposing the Lorentz structure in the most general way the amplitude $\mathcal{A}^{\mu\nu}$ can be written as

$$\mathcal{A}^{\mu\nu} = q_1^\nu q_2^\mu A + g^{\mu\nu} B + q_1^\mu q_2^\nu C + q_1^\mu q_1^\nu D + q_2^\mu q_2^\nu E, \quad (4)$$

with scalar form factors A , B , C , D , and E , which can be calculated by applying the projectors

$$\begin{aligned} P_A^{\mu\nu} &= \frac{(d-1)q_1^\mu q_2^\nu + q_1^\nu q_2^\mu - q_1 q_2 g^{\mu\nu}}{(d-2)(q_1 q_2)^2}, \\ P_B^{\mu\nu} &= \frac{q_1 q_2 g^{\mu\nu} - q_1^\mu q_2^\nu - q_1^\nu q_2^\mu}{(d-2)q_1 q_2}, \\ P_C^{\mu\nu} &= \frac{(d-1)q_1^\nu q_2^\mu + q_1^\mu q_2^\nu - q_1 q_2 g^{\mu\nu}}{(d-2)(q_1 q_2)^2}, \\ P_D^{\mu\nu} &= \frac{q_2^\mu q_2^\nu}{(q_1 q_2)^2}, \\ P_E^{\mu\nu} &= \frac{q_1^\mu q_1^\nu}{(q_1 q_2)^2} \end{aligned} \quad (5)$$

on the amplitude (e. g. $A = P_A^{\mu\nu} \mathcal{A}_{\mu\nu}$). Due to gauge invariance the contractions $q_{1\mu} \mathcal{A}^{\mu\nu}$ and $q_{2\nu} \mathcal{A}^{\mu\nu}$ of the amplitude with the momenta of the external photons

vanish. This imposes the relations $B = -q_1 q_2 A$ and $D = E = 0$ on the form factors. In order to verify these relations explicitly we calculate A, \dots, E separately. The coefficient C , although non zero, does not contribute to the decay rate of the Higgs boson because of the transversality of the photon wave functions. Therefore the Lorentz structure of the physical amplitude simplifies to

$$\mathcal{A}^{\mu\nu} = (q_1^\nu q_2^\mu - q_1 q_2 g^{\mu\nu}) A. \quad (6)$$

To obtain the decay width, the combination $A = A_W + A_t$ is inserted into eq. (1) for the W and the top quark loop induced contributions.

Since the calculation of the decay amplitude including the full dependence on Higgs-boson and top-quark mass is not possible at the moment, only an expansion in $\tau = M_H^2/(4m_t^2)$ can be obtained. The expansion of the amplitude in τ can be performed following two conceptually different strategies. The first option, which is only applicable for the calculation of a few terms in the expansion, relies on the direct expansion of each Feynman diagram in τ and the subsequent reduction of the appearing integrals to master integrals. Since this method becomes very time-consuming when calculating higher terms in the expansion, a second approach has been devised. In this alternative approach, the full mass dependence is kept as long as possible, meaning that the reduction to master integrals is performed keeping the full mass dependence and only the master integrals are calculated in an expansion in τ . In the following we sketch the steps necessary to perform the calculation and explain how the results for the needed master integrals in an expansion in τ can be obtained.

The Feynman diagrams are generated with `qgraf` [14] and processed by a `Mathematica` program to map the integrals on 4 singlet and 15 non-singlet topologies. A sample of the contributing diagrams is depicted in Fig. 1. All appearing integrals are reduced to master integrals using `Crusher` [15]. To expand the master integrals $M_i(\tau)$ in $\tau = M_H^2/(4m_t^2)$ a series ansatz

$$M_i(\tau) = \sum_k \left(M_{i,0}^{(k)} + M_{i,1}^{(k)} (-\tau)^{-\epsilon} + M_{i,2}^{(k)} (-\tau)^{-2\epsilon} \right) \tau^k \quad (7)$$

is inserted into the differential equation [16]

$$\left(M_H^2 \frac{\partial}{\partial M_H^2} + m_t^2 \frac{\partial}{\partial m_t^2} - \frac{1}{2} \hat{D} \right) M_i(\tau) = 0, \quad (8)$$

where \hat{D} applied to $M_i(\tau)$ returns the mass dimension of M_i . The differential equation follows from the fact that $M_i(\tau)$ is a homogeneous function in its dimensionful parameters M_H^2 and m_t^2 . Quarks except for the top quark are treated as massless. Using the series ansatz results in a system of algebraic equations which are systematically solved for the coefficients $M_{i,j}^{(k)}$ by a `Mathematica` program which uses `Fermat` [17] to simplify rational functions.

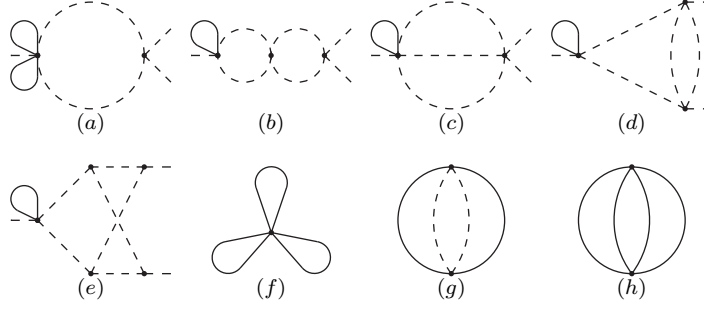


Fig. 2. Integrals which serve as boundary conditions for the solutions of the differential equations for the vertex master integrals. Dashed internal lines are massless, solid lines carry the mass m_t . External lines on the right side of the diagrams (a)-(e) are massless on-shell, the external line on the left side is on-shell with mass M_H . These diagrams only contribute to the singlet case.

By this procedure all coefficients $M_{i,j}^{(k)}$ are expressed as linear combinations of the integrals which are depicted in Fig. 2 and serve as boundary conditions. Coefficients $M_{i,1}^{(k)}$ and $M_{i,2}^{(k)}$ of non-integer powers of τ arise only in the singlet contribution due to the massless cuts of the singlet diagrams [18].

3 Results

The top quark loop induced amplitude A_t is expressed as a perturbative series in the strong coupling constant α_s with the one-, two-, and three-loop contributions $A_t^{(0)}$, $A_t^{(1)}$, and $A_t^{(2)}$,

$$A_t = \hat{A}_t \left[A_t^{(0)} + \frac{\alpha_s}{\pi} A_t^{(1)} + \left(\frac{\alpha_s}{\pi} \right)^2 A_t^{(2)} + \dots \right]. \quad (9)$$

The three-loop contribution is, furthermore, split into the non-singlet part $A_{t,0}^{(2)}$, the singlet part with two top-quark loops $A_{t,t}^{(2)}$, and the singlet part with a top-quark and light-quark loop $A_{t,q}^{(2)}$

$$A_t^{(2)} = A_{t,0}^{(2)} + A_{t,t}^{(2)} + (Q_t^{-2} \sum_{q \neq t} Q_q^2) A_{t,q}^{(2)} \quad (10)$$

Q_t and Q_q are the electromagnetic charges of the top quark and the light quarks ($q \in \{u, d, s, c, b\}$).

The full results with the expansion in τ up to τ^{20} including generic $SU(N)$ colour factors and renormalisation scale dependence are available online.¹ For the presentation of the results in this section we truncate the expansion after τ^5 and insert the $SU(3)$ colour factors explicitly. The top-quark mass is renormalized in the $\overline{\text{MS}}$ scheme. We use the abbreviations $L_\mu = \log \mu^2/m_t^2$ in the

¹ <http://www-ttp.particle.uni-karlsruhe.de/Progdata/ttp12/ttp12-046/>

non-singlet and $L_\tau = \log(-4\tau)$ in the singlet contribution. For completeness also the α_s^0 and α_s^1 contributions are given as an expansion in τ .

$$A_t^{(0)} = +1 + \tau\left(\frac{7}{30}\right) + \tau^2\left(\frac{2}{21}\right) + \tau^3\left(\frac{26}{525}\right) + \tau^4\left(\frac{512}{17325}\right) + \tau^5\left(\frac{1216}{63063}\right) \quad (11)$$

$$\begin{aligned} A_t^{(1)} = & -1 + \tau\left(\frac{38}{135} - \frac{7}{15}L_\mu\right) + \tau^2\left(\frac{1664}{14175} - \frac{8}{21}L_\mu\right) \\ & + \tau^3\left(\frac{12626}{496125} - \frac{52}{175}L_\mu\right) + \tau^4\left(-\frac{40664}{2338875} - \frac{4096}{17325}L_\mu\right) \\ & + \tau^5\left(-\frac{9128671204}{245827456875} - \frac{12160}{63063}L_\mu\right) \end{aligned} \quad (12)$$

$$\begin{aligned} A_{t,0}^{(2)} = & -\frac{31}{24} - \frac{7}{4}L_\mu \\ & + \tau\left(-\frac{22326329}{622080} + \frac{4116067}{138240}\zeta_3 - \frac{769}{1080}L_\mu + \frac{7}{120}L_\mu^2\right) \\ & + \tau^2\left(-\frac{68094821183}{2612736000} + \frac{508541309}{23224320}\zeta_3 - \frac{1241}{1575}L_\mu + \frac{3}{7}L_\mu^2\right) \\ & + \tau^3\left(-\frac{102458003430188113}{122903101440000} + \frac{190929277363}{275251200}\zeta_3 \right. \\ & \quad \left. - \frac{256363}{496125}L_\mu + \frac{221}{350}L_\mu^2\right) \\ & + \tau^4\left(-\frac{489470768471800920451}{202790117376000000} + \frac{547186023461087}{272498688000}\zeta_3 \right. \\ & \quad \left. - \frac{14462}{66825}L_\mu + \frac{512}{693}L_\mu^2\right) \\ & + \tau^5\left(-\frac{114742890543235030923359893}{6430815309390151680000} + \frac{1766782778485181879}{119027426918400}\zeta_3 \right. \\ & \quad \left. + \frac{307117801}{7449316875}L_\mu + \frac{1520}{1911}L_\mu^2\right) \end{aligned} \quad (13)$$

$$\begin{aligned} A_{t,t}^{(2)} = & +\frac{1}{8} \\ & + \tau\left(-\frac{28777}{207360} + \frac{749}{3072}\zeta_3\right) \\ & + \tau^2\left(+\frac{34183679}{522547200} + \frac{18935}{221184}\zeta_3 - \frac{2}{45}L_\tau\right) \\ & + \tau^3\left(-\frac{2665898377}{390168576000} + \frac{1419929}{23592960}\zeta_3 - \frac{41}{2025}L_\tau\right) \\ & + \tau^4\left(-\frac{37746440955049}{1931334451200000} + \frac{11964631}{235929600}\zeta_3 - \frac{598}{42525}L_\tau\right) \\ & + \tau^5\left(-\frac{2381500300071333647}{231991894278144000000} + \frac{862749257}{28311552000}\zeta_3 - \frac{74924}{7016625}L_\tau\right) \end{aligned} \quad (14)$$

$$\begin{aligned}
A_{t,q}^{(2)} = & -\frac{13}{12} + \frac{2}{3}\zeta_3 + \frac{1}{6}L_\tau \\
& + \tau \left(-\frac{3493}{48600} + \frac{7}{45}\zeta_3 + \frac{19}{1620}L_\tau \right) \\
& + \tau^2 \left(-\frac{3953}{396900} + \frac{4}{63}\zeta_3 - \frac{1}{3780}L_\tau \right) \\
& + \tau^3 \left(-\frac{3668899}{2679075000} + \frac{52}{1575}\zeta_3 - \frac{1696}{1063125}L_\tau \right) \\
& + \tau^4 \left(+\frac{414962}{2210236875} + \frac{1024}{51975}\zeta_3 - \frac{136}{91125}L_\tau \right) \\
& + \tau^5 \left(+\frac{611578464557}{1409328810264375} + \frac{2432}{189189}\zeta_3 - \frac{7571576}{6257426175}L_\tau \right) \quad (15)
\end{aligned}$$

The results for $A_{t,0}^{(2)}$ agree with the results presented in Ref. [13]. In Tab. 1 we give the results for the first 20 coefficients of the series in τ in numerical form. For the singlet contribution we show the results for the constant and the logarithmic part proportional to L_τ separately. To illustrate the convergence of the series we show all 20 terms of the expansion of the three-loop contribution in graphical form in Fig. 3. Around $\tau = 0.14$, corresponding to the Higgs-boson mass favoured by the LHC experiments, the first four (five) terms in the expansion are sufficient to obtain an accurate result with a relative error of 10^{-5} wrt. the total 3-loop amplitude for the singlet (non-singlet) contribution. At larger values of τ more terms of the expansion have to be taken into account, e.g. at $\tau = 0.5$ nine (sixteen) terms are needed to obtain similar accuracy.

Collecting all available information the amplitude can be cast in the form

$$A_{H \rightarrow \gamma\gamma} = \underbrace{A_W^{(0)} + \hat{A}_t A_t^{(0)}}_{A_{\text{LO}}} + \frac{\alpha}{\pi} \underbrace{A_{\text{EW}}^{(1)}}_{A_{\text{NLO-EW}}} + \frac{\alpha_s}{\pi} \underbrace{\hat{A}_t A_t^{(1)}}_{A_{\text{NLO-QCD}}} + \left(\frac{\alpha_s}{\pi}\right)^2 \underbrace{\hat{A}_t A_t^{(2)}}_{A_{\text{NNLO}}}, \quad (16)$$

where $A_{\text{EW}}^{(1)}$ denotes the electroweak corrections to the W and top-quark induced processes combined. The partial decay width is then given by

$$\begin{aligned}
\Gamma_{H \rightarrow \gamma\gamma} = & \frac{M_H^3}{64\pi} \left(A_{\text{LO}}^2 + \frac{\alpha}{\pi} (2A_{\text{LO}}A_{\text{NLO-EW}}) + \frac{\alpha_s}{\pi} (2A_{\text{LO}}A_{\text{NLO-QCD}}) \right. \\
& \left. + \left(\frac{\alpha_s}{\pi}\right)^2 (2A_{\text{LO}}\text{Re}(A_{\text{NNLO}}) + A_{\text{NLO}}^2) \right). \quad (17)
\end{aligned}$$

For a Higgs boson with a mass of $M_H = 126 \text{ GeV}$ ($\tau = 0.14$) this evaluates numerically to

$$\begin{aligned}
\Gamma_{H \rightarrow \gamma\gamma} = & (9.398 \cdot 10^{-6} - 1.48 \cdot 10^{-7} + 1.68 \cdot 10^{-7} + 7.93 \cdot 10^{-9}) \text{ GeV} \quad (18) \\
= & 9.425 \cdot 10^{-6} \text{ GeV},
\end{aligned}$$

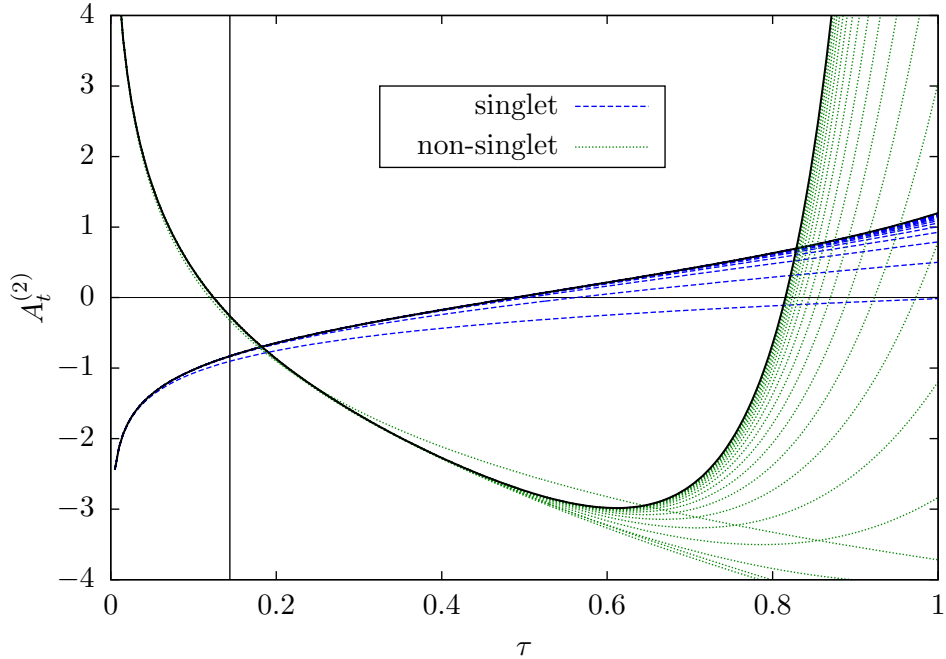


Fig. 3. Numerical results for the non-singlet and singlet contribution at three-loop order. In the plot the first 20 terms in the expansion in τ are shown. The singlet plot shows the sum $A_{t,t}^{(2)} + \sum_{q \neq t} Q_q^2/Q_t^2 A_{t,q}^{(2)}$. For the plot of the non-singlet contribution $A_{t,0}^{(2)}$ we set the renormalization scale $\mu = M_H$. The vertical line at $\tau = 0.14$ marks the value of τ for $M_H = 126$ GeV. The convergence of the expansion for $\tau \rightarrow 1$ can be improved by using the on-shell scheme for the top-quark mass.

where we used $m_t(M_H) = 166$ GeV², $\alpha_s(M_H)/\pi = 0.0358$, $G_F = 1.16637 \cdot 10^{-5}$ GeV⁻², and $\alpha = \alpha(0) = 1/137$ as input parameters. The value for the two-loop electroweak correction was taken from Ref. [10]. It has to be noted, that there is a partial cancellation between the two-loop QCD and electroweak corrections. To assess the influence of the singlet diagrams the next-to-next-to-leading order term can be further decomposed as

$$\Gamma_{H \rightarrow \gamma\gamma}|_{\text{NNLO}} = (7.5 \cdot 10^{-10}|_{\text{NLO}^2} + 1.73 \cdot 10^{-9}|_{\text{non-singlet}} + 5.45 \cdot 10^{-9}|_{\text{singlet}}) \text{ GeV},$$

which shows, that the singlet diagrams are a factor of three larger than the non-singlet ones and thus the most important three-loop contribution.

4 Conclusion

We presented new results for the singlet diagrams which contribute to the decay of a Higgs boson into two photons at next-to-next-to-leading order,

² The $\overline{\text{MS}}$ mass $m_t(M_H)$ was obtained from the on-shell mass $M_t = 172.64$ GeV using the $\overline{\text{MS}}$ -on-shell relation at 3-loop accuracy [19–21].

n	$A_{t,0}^{(2)}$	$A_{t,q}^{(2)}[0]$	$A_{t,q}^{(2)}[L_\tau]$	$A_{t,t}^{(2)}[0]$	$A_{t,t}^{(2)}[L_\tau]$
0	-1.29166667	-0.28196206	0.16666667	0.12500000	0.00000000
1	-0.09881144	0.11511420	0.01172840	0.15430166	0.00000000
2	0.25870689	0.06636139	-0.00026455	0.16832244	-0.04444444
3	0.16373113	0.03831749	-0.00159530	0.06551243	-0.02024691
4	0.08688592	0.02387041	-0.00149246	0.04141534	-0.01406232
5	0.05709451	0.01588624	-0.00121001	0.02636532	-0.01067807
6	0.05798951	0.01114079	-0.00095670	0.01555892	-0.00622762
7	0.07545680	0.00814321	-0.00075859	0.01272091	-0.00587319
8	0.10086885	0.00615332	-0.00060811	0.00751470	-0.00333698
9	0.12935841	0.00477756	-0.00049383	0.00708618	-0.00353585
10	0.15827468	0.00379378	-0.00040622	0.00423275	-0.00200757
11	0.18622498	0.00307013	-0.00033822	0.00437893	-0.00229504
12	0.21252358	0.00252489	-0.00028474	0.00264217	-0.00130721
13	0.23687981	0.00210551	-0.00024213	0.00291761	-0.00157907
14	0.25922151	0.00177712	-0.00020779	0.00177454	-0.00090205
15	0.27959438	0.00151591	-0.00017979	0.00205621	-0.00113694
16	0.29810454	0.00130527	-0.00015673	0.00125855	-0.00065080
17	0.31488570	0.00113331	-0.00013755	0.00151292	-0.00084866
18	0.33008045	0.00099137	-0.00012148	0.00093081	-0.00048637
19	0.34383005	0.00087306	-0.00010789	0.00115159	-0.00065225
20	0.35626884	0.00077356	-0.00009631	0.00071159	-0.00037400

Table 1

Numerical values of the first 20 terms in the expansion in τ for the three-loop non-singlet $A_{t,0}^{(2)}$ and singlet contribution $A_{t,t}^{(2)}$ and $A_{t,q}^{(2)}$. For the non-singlet part we set $\mu = m_t$. In case of the singlet contribution the results for the constant part ($A_t^{(2)}[0]$) and the logarithmic part ($A_t^{(2)}[L_\tau]$) are shown separately.

which up to now have not been considered. The corrections to the decay rate due to singlet diagrams are about a factor of three larger than the non-singlet ones. An improved prediction for the non-singlet contribution reduces the error of the three-loop contribution, which can be neglected for a Higgs-boson mass of 126 GeV. The total partial decay width is $\Gamma_{H \rightarrow \gamma\gamma} = \Gamma_{\text{LO}} + \Gamma_{\text{NLO-EW}} + \Gamma_{\text{NLO-QCD}} + \Gamma_{\text{NNLO}} = (9.398 \cdot 10^{-6} - 1.48 \cdot 10^{-7} + 1.68 \cdot 10^{-7} + 7.93 \cdot 10^{-9}) \text{ GeV} = 9.425 \cdot 10^{-6} \text{ GeV}$.

Acknowledgements

We are grateful to S. Uccirati and C. Sturm for providing the exact numerical value for the two-loop electroweak contribution. We like to thank N. Zerf for crosschecks on the non-singlet part and J.H. Kühn, M. Steinhauser and T. Kasprzik for carefully reading the manuscript. This work was supported by the DFG through the SFB/TR 9 “Computational Particle Physics”. The work of Ph.M. was supported by the Swiss National Science Foundation.

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