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Vacuum stability in the SM and the three-loop β-function for the Higgs self-interaction

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Abstract

In this article the stability of the Standard Model (SM) vacuum in the presence of radiative corrections and for a Higgs boson with a mass in the vicinity of 125 GeV is discussed. The central piece in this discussion will be the Higgs self-interaction λ and its evolution with the energy scale of a given physical process. This is described by the β-function to which we recently computed analytically the dominant three-loop contributions [1].[2] These are mainly the QCD and top-Yukawa corrections as well as the contributions from the Higgs self-interaction itself. We will see that for a Higgs boson with a mass of about 125 GeV the question whether the SM vacuum is stable and therefore whether the SM could be valid up to Planck scale cannot be answered with certainty due to large experimental uncertainties, mainly in the top quark mass.

1 The Higgs potential and the stability of the SM vacuum

In the SM the Higgs potential at tree-level appears as part of the Lagrangian for a scalar SU(2)-doublet field:

$$\mathcal{L}_\Phi = \partial_\mu \Phi^\dagger \partial^\mu \Phi - \underbrace{\left(m^2 \Phi^\dagger \Phi + \lambda \left(\Phi^\dagger \Phi \right)^2 \right)}_{V(\Phi)}, \qquad \Phi = \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}. \tag{1}$$

This doublet aquires a non-zero vacuum expectation value (VEV) under spontaneous symmetry breaking (SSB) and we get the Higgs field, three Goldstone bosons[3] and the

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[2]In [1] we also give the dominant contributions to the β-functions for the top-Yukawa coupling, the strong coupling and the anomalous dimensions of the scalar, gluon and quark fields in the unbroken phase of the Standard Model at three-loop level.

[3]These Goldstone bosons can be absorbed by the massive W and Z bosons.

masses of the SM particles:

$$\Phi = \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \xrightarrow{SSB} \begin{pmatrix} \Phi^+ \\ \frac{1}{\sqrt{2}}(v + H + i\chi) \end{pmatrix}, \qquad |\Phi_{SM}| = \sqrt{\frac{-m^2}{2\lambda}} = \frac{v}{\sqrt{2}} \tag{2}$$

At tree level the mass of the Higgs boson is then given by

$$M_H^2 = -2m^2 = 2\lambda v^2. \tag{3}$$

For $M_H = 125$ GeV the Higgs potential is shown in Fig. 1. If we now include radia-

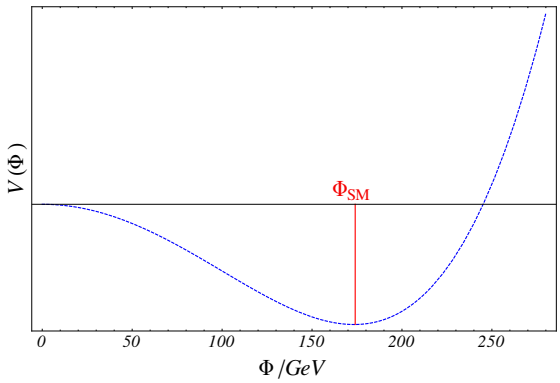

Figure 1: The SM Higgs potential at tree-level for $M_H = 125$ GeV

tive corrections we have to consider the effective potential $V_{eff}(\lambda(\Lambda), g_i(\Lambda), \Phi(\Lambda))$ as introduced in [2]. All couplings and fields undergo an evolution up to some scale Λ where the theory ceases to be valid.[4] Here we want to investigate whether a scenario in which the SM provides a good description of nature up to the Planck scale, i.e. $\Lambda \sim 10^{18}$ GeV, is possible.[5] The general shape of this effective potential for the SM is shown in Fig. 2 for the cases of a Higgs mass larger and smaller than a critical value m_{min}, the minimal stability bound[6] (see also [3]). If the Higgs mass is chosen below this critical value a second minimum develops which is lower than the SM one. This implies that the SM vacuum is no longer stable, i.e. it can tunnel into this energetically favoured state. Since this is in contradiction to our observation[7] we are led to the conclusion that our theory is incomplete and that new physics has to enter between Fermi and Planck scale. It has been demonstrated in [4] that for $\Phi \sim \Lambda \gg \mu_0$ a good approximation for the effective potential is

$$V_{eff}[\Phi] \approx \lambda(\Lambda)\Phi^4 + \mathcal{O}(\lambda^2(\Lambda), g_i^2(\Lambda)), \tag{4}$$

[4]With $t = \log\left(\frac{\Lambda}{\mu_0}\right)$ we have the field $\Phi(\Lambda) = \Phi_{cl} \cdot \exp\left(\int_0^t dt' \gamma_\Phi(\lambda(t'), g_i(t'))dt'\right)$ where Φ_{cl} is the classical field in the absence of radiative corrections. μ_0 is the scale at which we start the running of the paramters.

[5]An extended model is expected to be needed at this scale due to gravity.

[6]There is also an upper bound m_{max} on the Higgs mass from the requirement that no Landau pole appears at energies $\mu \leq \Lambda$.

[7]It is however not possible to exclude that we live in metastable universe, i.e. the lifetime of the (local) SM minimum could be longer than the age of the universe.

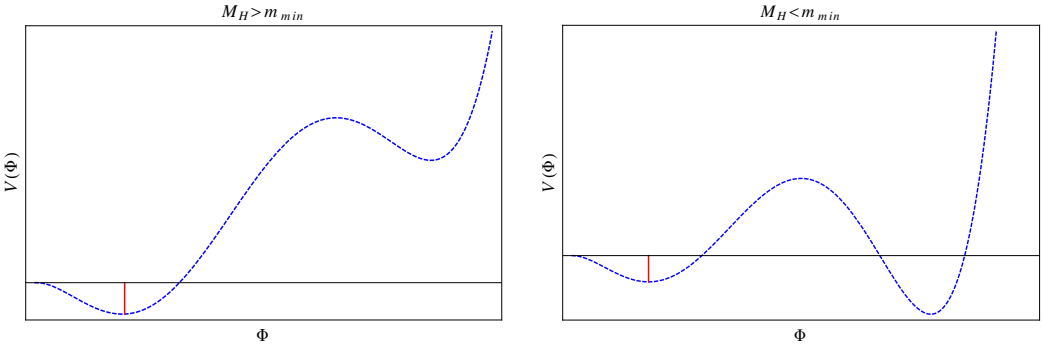

Figure 2: The effective Higgs potential

which means that the stability of the SM vacuum is approximately equivalent to the question whether λ stays positive up to the scale Λ (see also [5, 6]). Fig. 3 shows the

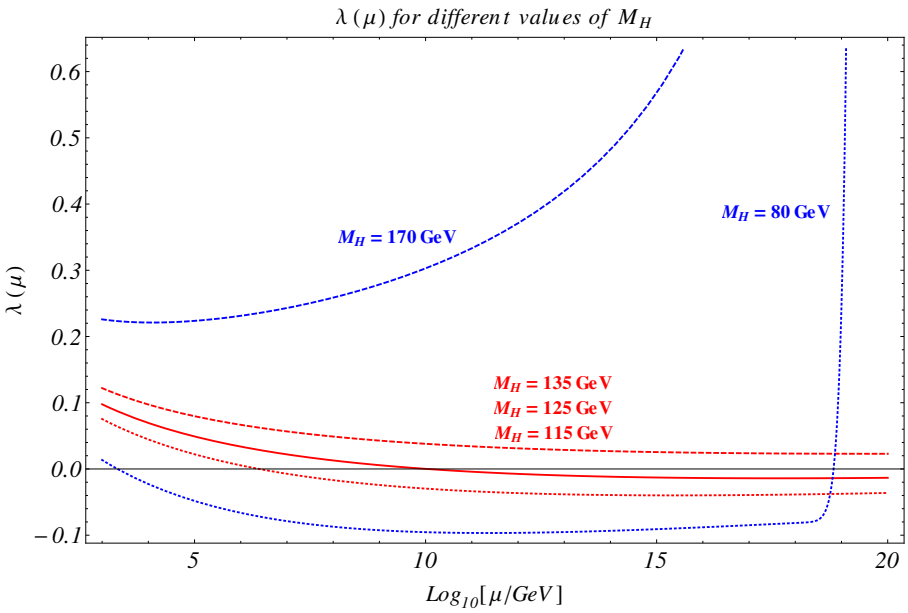

Figure 3: The behaviour of $\lambda(\mu)$ for different Higgs mass values

evolution of λ for different Higgs mass values. For a large enough Higgs mass λ diverges quickly[8] and for a small Higgs mass λ would become negative at a relatively low scale before eventually reaching a Landau pole.[9] The interesting region is around 125 GeV where λ is very close to zero at the Planck scale! As the minimal stability bound has been estimated to be about 129 ± 3 GeV [3, 10], which is very close to the mass of the boson recently descovered at the LHC, the question of vacuum stability in the SM

[8]We find a Landau pole below the Planck scale for $M_H > m_{max} \approx 175$ GeV (see also [7, 5, 8, 9]).

[9]The reason for the fact that λ becomes positive again and diverges is the evolution of the gauge couplings and y_t. At large scales (above 10^{16} GeV) the electroweak couplings, especially the $U(1)$ coupling g_1, start to be the dominant contributions. In contrast y_t, which is responsible for the decrease of λ at lower scales, becomes small. This is also the reason why the effective potential goes up again at large field strengths Φ beyond the second minimum.

becomes one for precision physics. This serves as a strong motivation for calculating the three-loop β-function of the Higgs self-interaction which describes the evolution of this crucial coupling as well as the β-functions of the relevant SM parameters on which β_λ depends.

2 The three-loop β-function for the Higgs self-interaction

The β-function is defined as the derivative of the coupling with respect to the renormalization scale:

$$\beta_\lambda(\lambda, y_t, g_i, \ldots) = \mu^2 \frac{d}{d\mu^2}\lambda(\mu). \tag{5}$$

This object has been known at the two-loop level for a while including the dependence on all the gauge couplings g_1,g_2 and g_s, the quartic Higgs self-interaction λ and the Yukawa couplings which give mass to the fermions [11,12,13,14]. The one-loop and two-loop results for all SM couplings have been known for a long time [15, 16, 17, 18, 19, 20, 21, 22, 11, 23, 24, 12, 25, 14, 13] as have been partial three-loop results [26, 27, 28, 29, 30, 31]. Four-loop β-functions are available for QCD [32, 33] and the purely scalar part of the SM [34, 35, 36].

As there are many Feynman diagrams at the three-loop level and the treatment of γ_5 matrices in dimensional regularization poses a serious problem at this order for the Higgs and Yukawa sector we try to find the dominant contributions to the evolution of the Higgs self-interaction at the scales that we are intersted in. At the scale of the top mass $\mu = M_t = 172.9$ GeV we find the strong coupling $g_s \approx 1.17$ and the top-Yukawa coupling $y_t \approx 0.93$ to be much larger than the electroweak couplings $g_2 \approx 0.65$ and $g_1 \approx 0.36$. The Higgs self-interaction for a Higgs boson around 125 GeV is $\lambda(M_H) \approx 0.13$. [10] From this we conclude that a simplified model containing only g_s, y_t and λ will give the numerically largest terms for β_λ at three-loop level. When combinen with the full SM result at one and two-loop level this leads to the evolution of λ to the highest precision so far. The Lagrangian of our model consists of three pieces:

$$\mathcal{L} = \mathcal{L}_{QCD} + \mathcal{L}_\Phi + \mathcal{L}_{y_t}. \tag{6}$$

with the standard QCD Lagrangian, the Higgs part as defined in (1) and the top-Yukawa sector

$$\mathcal{L}_{y_t} = -y_t \left\{ \bar{t}_R \left(\Phi_2, -\Phi_1\right) \cdot \begin{pmatrix} t \\ b \end{pmatrix}_L + \left(\bar{t}, \bar{b}\right)_L \cdot \begin{pmatrix} \Phi_2^* \\ -\Phi_1^* \end{pmatrix} t_R \right\} \tag{7}$$

[10] These couplings enter at every loop order with characteristic factors $\frac{g_s^2}{4\pi} \approx 0.11$, $\frac{g_2^2}{4\pi} \approx 0.03$, $\frac{g_1^2}{4\pi} \approx 0.01$ and $\frac{y_t^2}{4\pi} \approx 0.07$. The Higgs self-coupling is already present at tree level and enters at each order linearly as $\frac{\lambda}{4\pi} \approx 0.01$. The second-largest Yukawa coupling to be considered would be $y_b = \sqrt{2}\frac{M_b}{v} \approx 0.02$ which is negligible in comparison.

The indices L and R indicate the left- and right-handed part of the Dirac fermion fields as obtained by the projectors

$$P_L = \frac{1}{2}(1-\gamma_5) \qquad P_R = \frac{1}{2}(1+\gamma_5)\,. \tag{8}$$

Hence γ_5 enters into our calculation and has to be treated carefully as described below.

2.1 Calculation

In order to obtain the β-functions for a coupling, here λ or y_t, we have to compute the renormalization constant for a vertex involving this coupling and the field renormalization constants for the external legs of this vertex. The latter is done by computing loop corrections to the respective propagators. The renormalization constant for y_t for

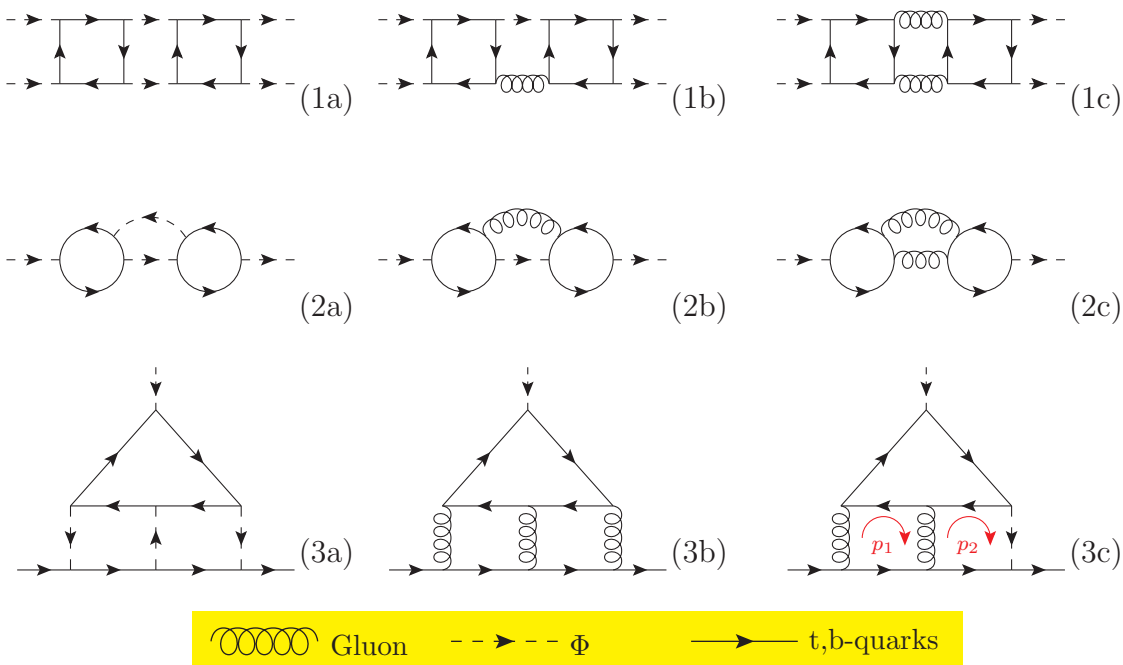

Figure 4: Sample Feynman diagrams for the calculation of renormalization constants for the quartic Higgs vertex, the Higgs self-energy and a Yukawa vertex at three-loop order.

example can be computed as[11]

$$Z_{y_t} = \frac{Z^{(\text{Yukawa vertex})}}{\sqrt{Z_L^{(\text{top field})} Z_R^{(\text{top field})} Z^{(\text{Higgs field})}}}. \tag{9}$$

The β-function β_{y_t} can then be computed from the requirement that the bare Yukawa coupling

$$y_t^{bare} = Z_{y_t}[g_i(\mu), y_t(\mu), \lambda(\mu)]\, y_t(\mu) \tag{10}$$

[11]Note that in general left and right-handed fields have to be renormalized separately.

is independent of the renormalization scale μ and hence its μ^2-derivative must vanish.

For the purpose of calculating renormalization constants and β-functions we only need the UV-divergent part of all these diagrams. Nevertheless, there are two issues which have to be considered carefully in a computation of this type. The first problem is the complicated nature of our integrals when we have arbitrary momenta flowing into the external legs of our Feynman diagrams. A nice feature of the $\overline{\text{MS}}$-scheme however is that renormalization constants do not depend on external momenta[12], so we can set those to zero. Unfortunately, this introduces artificial IR-divergences into our diagrams which cannot be distinguished from the UV ones in dimensional regularization. In many cases this problem can be avoided by setting all external momenta to zero except for one which enters at one leg and exits at another. If we have no masses, as we do here, the propagator-like integrals resulting from this method can be computed with the FORM package MINCER [37] up to three-loop order. For the λ-vertex and its radiative corrections this fails, however, as there are still IR-divergent diagrams. A method to compute only the UV-divergences without having to worry about the IR ones has been described in [38, 39]. The trick is to introduce the same auxiliary mass into every propagator denominator, Taylor expand in all the external momenta and introduce all possible counterterms for the auxiliary mass in order to cancel subdivergences arising from this new mass. The resulting massive tadpole integrals can be computed with the FORM package MATAD [40]. This method will yield the correct UV-pole part of the calculated Feynman diagrams (but not the correct finite part) which is enough for the computation of β-functions. Where possible both methods have been used for the calculation in order to have an independent check.

The second problem is the treatment of γ_5 matrices, which arise here from the projectors in the Yukawa sector, in d space-time dimensions. This matrix is only well-defined in four dimensions:

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \frac{i}{4!}\varepsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma \quad \text{where} \quad \varepsilon_{0123} = 1 = -\varepsilon^{0123}. \tag{11}$$

A naive treatment of γ_5, i.e. using the relations $\{\gamma_5, \gamma_\mu\} = 0$ and $\gamma_5^2 = \mathbb{1}$ to eliminate as many γ_5 as possible and then discarding all terms which still have one γ_5 in them, can only be applied to external fermion lines and closed fermion loops with less than four Lorentz indices or momenta flowing in or out of the fermion loop in question.[13] An example for a problematic diagram is shown in Fig. 4 (3c). The Lorentz indices of the two gluons connected to the closed fermion loop and the two loop momenta p_1 and p_2 offer the possibility of a non-trivial γ_5 contribution from such a diagram. And indeed, if we apply the treatment suggested by 't Hooft and Veltman in [41], i.e. using the definition with $\varepsilon_{\mu\nu\rho\sigma}$ from (11) and contracting the ε-tensor from the closed fermion loop with the one from the external fermion line or a projector $\propto \gamma_5$ acting on this

[12]To be precise, UV-divergent terms can only depend polynomially on external momenta (and masses).

[13]Four different Lorentz indices are needed to support a non-vanishing $\varepsilon_{\mu\nu\rho\sigma}$ after the trace over the fermion line has been performed. These can be the Lorentz indices of gauge boson vertices or they can be contracted with internal momenta from other loops. External momenta can be set to zero without changing the UV-divergent part and can hence be ignored in this consideration.

external line to make the integral scalar, we find a sizable contribution. But even this treatment is not exact, in fact it is only correct up to an error of $\mathcal{O}(\varepsilon)$. Fortunately, there are only first order poles in ε from these integrals which makes the UV-divergent part and therefore our renormalization constants correct. Once again, the finite part calculated for these diagrams is unreliable but luckily not needed here.

2.2 Result

The result for the β-function of the Higgs self-interaction

$$\mu^2 \frac{d}{d\mu^2}\lambda(\mu) = \beta_\lambda(g_s, y_t, \lambda) = \sum_{n=1}^{\infty} \frac{1}{(16\pi^2)^n}\, \beta_\lambda^{(n)}(g_s, y_t, \lambda) \tag{12}$$

in our simplified version of the Standard Model is given by

$$\begin{aligned}
\beta_\lambda^{(1)} =& 12\,\lambda^2 + 6\,y_t^2\lambda - 3\,y_t^4,\\
\beta_\lambda^{(2)} =& -156\,\lambda^3 - 72\,y_t^2\lambda^2 - \frac{3}{2}\,y_t^4\lambda + 15\,y_t^6 + 40\,g_s^2 y_t^2\lambda - 16\,g_s^2 y_t^4,\\
\beta_\lambda^{(3)} =& \lambda^4\,(3588 + 2016\zeta_3) + 873\,y_t^2\lambda^3 + y_t^4\lambda^2\left(\frac{1719}{2} + 756\zeta_3\right)\\
&+ y_t^6\lambda\left(\frac{117}{8} - 198\zeta_3\right) - y_t^8\left(\frac{1599}{8} + 36\zeta_3\right)\\
&+ g_s^2 y_t^2\lambda^2\,(-1224 + 1152\zeta_3) + g_s^2 y_t^4\lambda\,(895 - 1296\zeta_3)\\
&+ g_s^2 y_t^6\,(-38 + 240\zeta_3) + g_s^4 y_t^2\lambda\left(\frac{1820}{3} - 32n_f - 48\zeta_3\right)\\
&+ g_s^4 y_t^4\left(-\frac{626}{3} + 20n_f + 32\zeta_3\right)
\end{aligned}$$

in the $\overline{\text{MS}}$-scheme. $\beta_\lambda^{(1)}$, $\beta_\lambda^{(2)}$ and $\beta_\lambda^{(3)}$ are the one, two and three-loop results respectively. To get an idea of the size of the individual terms and the overall result we evaluate β_λ at the scale $\mu = M_Z$ (with an assumed Higgs mass of 125 GeV and the number of fermion flavours $n_f = 6$) which yields a value of $\beta_\lambda \sim (-0.01)$ at the one-loop level. The two and three-loop contributions change this result by $\sim 1\%$ and $\sim (-0.04)\%$ respectively which is quite small e.g. in comparison with the β-function for the top-Yukawa coupling where we get a value of $\beta_{y_t} \sim (-0.023)$ and corrections of $\sim 16.6\%$ and $\sim 0.7\%$ at two and three-loop level. The full result for β_{y_t} can be found in [1]. If we have a look at the the numerically largest individual terms in (2.2) we find a curious behaviour:

$$\frac{\beta_\lambda^{(3)}}{(16\pi^2)^3}(\mu = M_Z) = (\underbrace{+7.9}_{g_s^2 y_t^6}\quad \underbrace{-4.8}_{y_t^8}\quad \underbrace{-3.1}_{g_s^2 y_t^4\lambda}\quad \underbrace{-2.5}_{g_s^4 y_t^4}\quad \underbrace{+2.6}_{g_s^4 y_t^2\lambda}) \times 10^{-5}. \tag{13}$$

There is a strong cancellation between these terms making the overall effect almost two orders of magnitude smaller than the largest individual contributions.[14] This significantly improves the convergence of the perturbation series for β_λ and makes the remaining theoretical uncertainty small.

[14]Note that for a significantly different Higgs mass this would not happen to such extent.

3 The evolution of the Higgs self-coupling

Now we want to investigate the effect of the new three-loop result $\beta_\lambda^{(3)}$ on the running of λ and therefore its effect on the stability of the electroweak vacuum in the SM. For

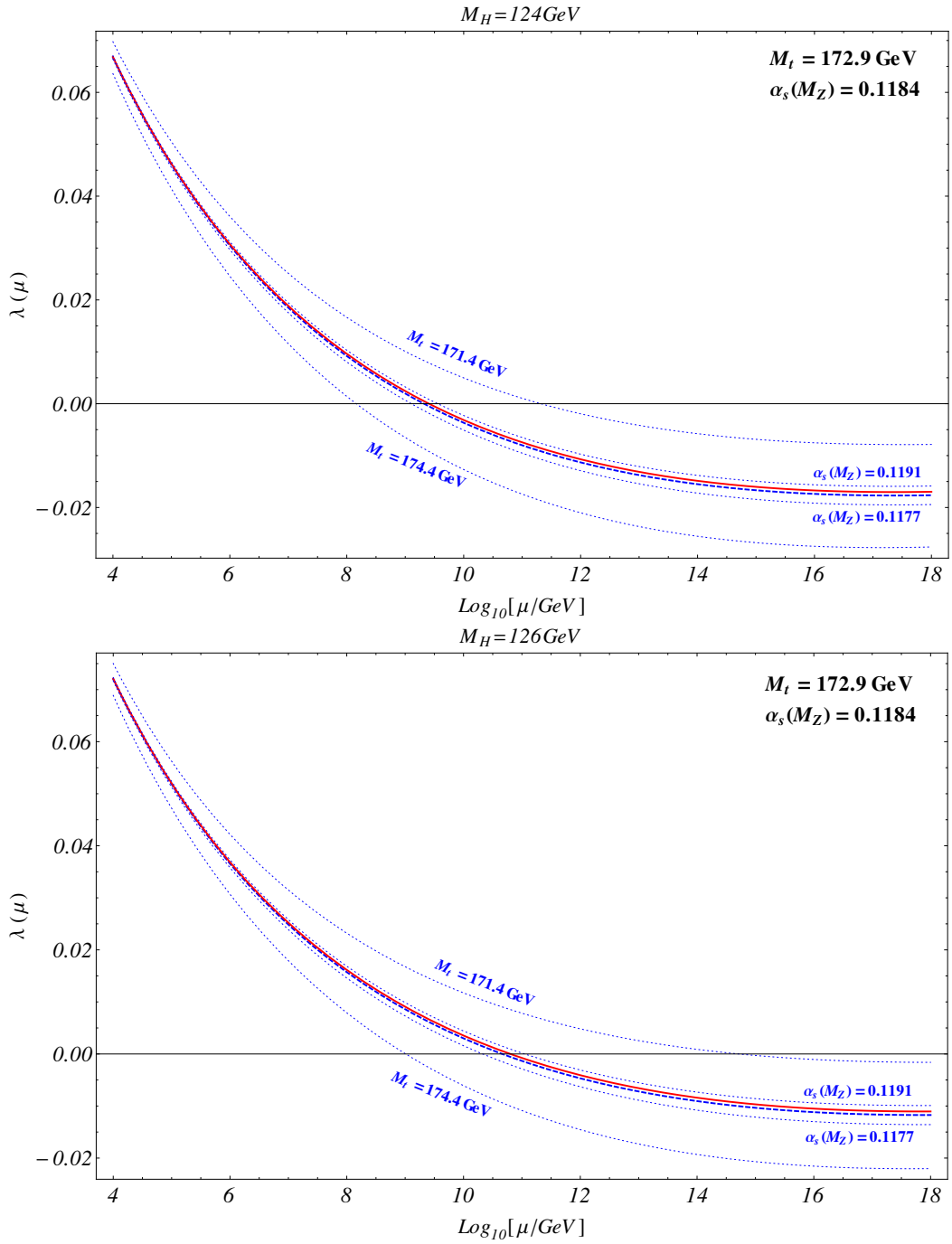

Figure 5: Evolution of λ with the scale μ: 2 loop (dashed, blue) and 3 loop (continuous, red) results; Uncertainties with respect to the two-loop result: $\pm 1\sigma_{\alpha_s}$, $\pm 1\sigma_{M_t}$ (dotted)

this we also include the electroweak contributions up to the two-loop level. For recent

detailed discussions of the SM vacuum stability and its dependence on key parameters like the Higgs and the top mass see for example [42, 43, 3, 44, 1]. We start the evolution of λ at the scale of the top mass and go up to the Planck scale at 10^{18} GeV. To find the starting values in the $\overline{\text{MS}}$-scheme we have to match the physical parameters, like e.g. pole masses, to their $\overline{\text{MS}}$ counterparts.[15] These matching relations depend on the exact values of the $\overline{\text{MS}}$ parameter $\alpha_s(M_Z)$, the pole mass M_t of the top quark and of course the mass of the Higgs boson M_H. We use [48]

$$\alpha_s(M_Z) = 0.1184 \pm 0.0007, \qquad M_t = 172.9 \pm 0.6 \pm 0.9 \text{ GeV}. \tag{14}$$

Fig. 5 shows the evolution of λ for the two cases $M_H = 124$ GeV and $M_H = 126$ GeV.. The dependence of the λ-running on the parameters $\alpha_s(M_Z)$ and M_t can be estimated from the shifted curves where these parameters are changed by $\pm\sigma$ as given in eq. (14). Note that there is a considerable difference between $M_H = 124$ GeV and $M_H = 126$ GeV which means that the evolution of λ is very sensitive to the value of the Higgs mass. Given a fixed value for M_H the largest uncertainty lies in the exact value of the top mass. The second largest uncertainty comes from α_s. The total effect due to the three-loop part of the β-functions, an extension of the vacuum stability to larger scales, is a little smaller than the α_s uncertainty. Note that an analysis including higher order corrections for the matching between the pole masses and $\overline{\text{MS}}$-masses has been performed in [3, 10].

In conclusion it can be said that for a Higgs mass in the vicinity of 125 GeV the question of vacuum stability cannot be resolved with certainty. Although it looks as if λ becomes indeed negative at high scales the experimental uncertainties, mainly on the Higgs mass and the top mass, are too large at the moment. However, this is a very good motivation to attempt higher precision experiments and calculations in the near future.

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References

[1] K. Chetyrkin and M. Zoller, *Three-loop β-functions for top-Yukawa and the Higgs self-interaction in the Standard Model.* JHEP **1206** (2012) 033, `arXiv:1205.2892 [hep-ph]`.

[2] S. Coleman and E. Weinberg, *Radiative Corrections as the Origin of Spontaneous Symmetry Breaking.* Phys. Rev. D **7** (1973) 1888–1910.

[15] We take the electroweak corrections at one-loop and the QCD ones at two-loop level from [45, 46, 47].

[3] F. Bezrukov, M. Y. Kalmykov, B. A. Kniehl, and M. Shaposhnikov, *Higgs boson mass and new physics*. arXiv:1205.2893 [hep-ph].

[4] G. Altarelli and G. Isidori, *Lower limit on the Higgs mass in the standard model: An update*. Physics Letters B **337** (1994) no. 1-2, 141–144.

[5] N. Cabibbo, L. Maiani, G. Parisi, and R. Petronzio, *Bounds on the Fermions and Higgs Boson Masses in Grand Unified Theories*. Nucl. Phys. **B158** (1979) 295–305.

[6] C. Ford, D. Jones, P. Stephenson, and M. Einhorn, *The Effective potential and the renormalization group*. Nucl. Phys. **B395** (1993) 17–34, arXiv:hep-lat/9210033 [hep-lat].

[7] L. Maiani, G. Parisi, and R. Petronzio, *Bounds on the Number and Masses of Quarks and Leptons*. Nucl. Phys. **B136** (1978) 115.

[8] M. Lindner, *Implications of Triviality for the Standard Model*. Z. Phys. **C31** (1986) 295.

[9] T. Hambye and K. Riesselmann, *Matching conditions and Higgs mass upper bounds revisited*. Phys.Rev. **D55** (1997) 7255–7262, arXiv:hep-ph/9610272 [hep-ph].

[10] J. Elias-Miro, J. R. Espinosa, G. F. Giudice, G. Isidori, A. Riotto, *et al.*, *Higgs mass implications on the stability of the electroweak vacuum*. Phys. Lett. **B709** (2012) 222–228, arXiv:1112.3022 [hep-ph].

[11] M. Fischler and J. Oliensis, *Two-loop corrections to the beta function for the Higgs-Yukawa coupling constant*. Phys. Lett. B **119** (1982) no. 4, 385–386.

[12] M. E. Machacek and M. T. Vaughn, *Two-loop renormalization group equations in a general quantum field theory (II). Yukawa couplings*. Nucl. Phys. B **236** (1984) no. 1, 221–232.

[13] C. Ford, I. Jack, and D. Jones, *The Standard model effective potential at two loops*. Nucl.Phys. **B387** (1992) 373–390, arXiv:hep-ph/0111190 [hep-ph].

[14] M.-x. Luo and Y. Xiao, *Two loop renormalization group equations in the standard model*. Phys. Rev. Lett. **90** (2003) 011601, arXiv:hep-ph/0207271 [hep-ph].

[15] D. J. Gross and F. Wilczek, *Ultraviolet Behavior of Non-Abelian Gauge Theories*. Phys. Rev. Lett. **30** (1973) 1343–1346.

[16] H. D. Politzer, *Reliable Perturbative Results for Strong Interactions?* Phys. Rev. Lett. **30** (1973) 1346–1349.

[17] D. Jones, *Two-loop diagrams in Yang-Mills theory*. Nuclear Physics B **75** (1974) no. 3, 531–538.

[18] O. Tarasov and A. Vladimirov, *Two Loop Renormalization of the Yang-Mills Theory in an Arbitrary Gauge*. Sov.J.Nucl.Phys. **25** (1977) 585.

[19] W. E. Caswell, *Asymptotic Behavior of Non-Abelian Gauge Theories to*

Two-Loop Order. Phys.Rev.Lett. **33** (1974) 244–246.

[20] E. Egorian and O. Tarasov, *Two loop renormalization of the QCD in an arbitrary gauge*. Teor.Mat.Fiz. **41** (1979) 26–32.

[21] D. R. T. Jones, *Two-loop β function for a $G_1 \times G_2$ gauge theory.* Phys. Rev. D **25** (1982) 581–582.

[22] M. S. Fischler and C. T. Hill, *Effects of Large Mass Fermions on M_X and* $\sin^2\theta_W$. Nucl.Phys. **B193** (1981) 53.

[23] I. Jack and H. Osborn, *General background field calculations with fermion fields.* Nucl. Phys. B **249** (1985) no. 3, 472–506.

[24] M. E. Machacek and M. T. Vaughn, *Two-loop renormalization group equations in a general quantum field theory: (I). Wave function renormalization.* Nucl. Phys. B **222** (1983) no. 1, 83–103.

[25] M. E. Machacek and M. T. Vaughn, *Two-loop renormalization group equations in a general quantum field theory: (III). Scalar quartic couplings.* Nucl. Phys. B **249** (1985) no. 1, 70–92.

[26] T. Curtright, *Three loop charge renormalization effects due to quartic scalar selfinteractions*. Phys.Rev. **D21** (1980) 1543.

[27] D. Jones, *Comment on the charge renormalization effects of quartic scalar selfinteractions*. Phys.Rev. **D22** (1980) 3140–3141.

[28] O. Tarasov, A. Vladimirov, and A. Y. Zharkov, *The Gell-Mann-Low Function of QCD in the Three Loop Approximation*. Phys.Lett. **B93** (1980) 429–432.

[29] S. Larin and J. Vermaseren, *The Three loop QCD Beta function and anomalous dimensions*. Phys. Lett. **B303** (1993) 334–336, `arXiv:hep-ph/9302208 [hep-ph]`.

[30] M. Steinhauser, *Higgs decay into gluons up to O(alpha**3(s) G(F)m**2(t)).* Phys.Rev. **D59** (1999) 054005, `arXiv:hep-ph/9809507 [hep-ph]`.

[31] A. Pickering, J. Gracey, and D. Jones, *Three loop gauge beta function for the most general single gauge coupling theory*. Phys.Lett. **B510** (2001) 347–354, `arXiv:hep-ph/0104247 [hep-ph]`.

[32] T. van Ritbergen, J. Vermaseren, and S. Larin, *The Four loop beta function in quantum chromodynamics*. Phys. Lett. **B400** (1997) 379–384, `arXiv:hep-ph/9701390 [hep-ph]`.

[33] M. Czakon, *The Four-loop QCD beta-function and anomalous dimensions.* Nucl.Phys. **B710** (2005) 485–498, `arXiv:hep-ph/0411261 [hep-ph]`.

[34] E. Brezin, J. C. Le Guillou, J. Zinn-Justin, and B. G. Nickel, *Higher oder contributions to critical exponents*. Phys. Lett. **44A** (1973) 227–228.

[35] E. Brezin, J. C. Le Guillou, and J. Zinn-Justin, *Addendum to wilson's theory of critical phenomena and callan-symanzik equations in 4-epsilon dimensions.*

Phys. Rev. **D9** (1974) 1121–1124.

[36] D. I. Kazakov, O. V. Tarasov, and A. A. Vladimirov, *Calculation Of Critical Exponents By Quantum Field Theory Methods*. Sov.Phys. **JETP 50** (1979) 521.

[37] S. G. Gorishnii, S. A. Larin, L. R. Surguladze, and F. V. Tkachov, *MINCER: Program for multiloop calculations in quantum field theory for the SCHOONSCHIP system*. Comput. Phys. Commun. **55** (1989) 381–408.

[38] M. Misiak and M. Munz, *Two loop mixing of dimension five flavor changing operators*. Phys. Lett. **B344** (1995) 308–318, `arXiv:hep-ph/9409454`.

[39] K. G. Chetyrkin, M. Misiak, and M. Munz, *Beta functions and anomalous dimensions up to three loops*. Nucl. Phys. **B518** (1998) 473–494, `arXiv:hep-ph/9711266 [hep-ph]`.

[40] M. Steinhauser, *MATAD: A program package for the computation of massive tadpoles*. Comput. Phys. Commun. **134** (2001) 335–364, `arXiv:hep-ph/0009029`.

[41] G. 't Hooft and M. J. G. Veltman, *Regularization and Renormalization of Gauge Fields*. Nucl. Phys. **B44** (1972) 189–213.

[42] I. Masina, *The Higgs boson and Top quark masses as tests of Electroweak Vacuum Stability*. `arXiv:1209.0393 [hep-ph]`.

[43] M. Holthausen, K. S. Lim, and M. Lindner, *Planck scale Boundary Conditions and the Higgs Mass*. JHEP **1202** (2012) 037, `arXiv:1112.2415 [hep-ph]`.

[44] G. Degrassi, S. Di Vita, J. Elias-Miro, J. R. Espinosa, G. F. Giudice, *et al.*, *Higgs mass and vacuum stability in the Standard Model at NNLO*. JHEP **1208** (2012) 098, `arXiv:1205.6497 [hep-ph]`.

[45] J. Espinosa, G. Giudice, and A. Riotto, *Cosmological implications of the Higgs mass measurement*. JCAP **0805** (2008) 002, `arXiv:0710.2484 [hep-ph]`.

[46] R. Hempfling and B. A. Kniehl, *On the relation between the fermion pole mass and MS Yukawa coupling in the standard model*. Phys.Rev. **D51** (1995) 1386–1394, `arXiv:hep-ph/9408313 [hep-ph]`.

[47] A. Sirlin and R. Zucchini, *Dependence of the Higgs coupling hMS(M) on mH and the possible onset of new physics*. Nucl. Phys. B **266** (1986) no. 2, 389–409.

[48] K. Nakamura *et al.*, *The review of particle physics*. J. Phys. **G** (2010) no. 37, 075021.