SFB/CPP-12-61, TTP12-30

Renormalization constants and beta functions for the gauge couplings of the Standard Model to three-loop order

Luminita N. Mihaila, Jens Salomon, Matthias Steinhauser[1]

[1] *Institut für Theoretische Teilchenphysik,*
Karlsruhe Institute of Technology (KIT), D-76128 Karlsruhe, Germany

We compute the beta functions for the three gauge couplings of the Standard Model in the minimal subtraction scheme to three loops. We take into account contributions from all sectors of the Standard Model. The calculation is performed using both Lorenz gauge in the unbroken phase of the Standard Model and background field gauge in the spontaneously broken phase. Furthermore, we describe in detail the treatment of γ_5 and present the automated setup which we use for the calculation of the Feynman diagrams. It starts with the generation of the Feynman rules and leads to the bare result for the Green's function of a given process.

PACS numbers: 11.10.Hi 11.15.Bt

I. INTRODUCTION

Renormalization group functions are fundamental quantities of each quantum field theory. They provide insights in the energy dependence of cross sections, hints to phase transitions and can provide evidence to the energy range in which a particular theory is valid. The renormalization group functions of the gauge couplings in the Standard Model (SM) are of particular importance in the context of Grand Unified Theories allowing the extrapolation of low-energy precision data to high energies, not accessible by collider experiments.

Important milestones for the calculation of the gauge coupling beta functions in the Standard Model are the following computations:

- The one-loop beta functions in gauge theories along with the discovery of asymptotic freedom have been presented in Refs. [1, 2].
- The corresponding two-loop corrections
  - in gauge theories without fermions [3, 4],
  - in gauge theories with fermions neglecting Yukawa couplings [5–7],
  - with corrections involving Yukawa couplings [8],

  are also available.
- The two-loop gauge coupling beta functions in an arbitrary quantum field theory have been considered in Ref. [9, 10].
- The contribution of the scalar self-interaction at three-loop order has been computed in [11, 12].
- The gauge coupling beta function in quantum chromodynamics (QCD) to three loops is known from Ref. [13, 14].
- The three-loop corrections to the gauge coupling beta function involving two strong and one top quark Yukawa coupling have been computed in Ref. [15].
- The three-loop corrections for a general quantum field theory based on a single gauge group have been computed in [16].
- The four-loop corrections in QCD are known from Refs. [17, 18].

Two-loop corrections to the renormalization group functions for the Yukawa and Higgs boson self-couplings in the Standard Model are also known [10, 19–23]. Recently the dominant three-loop corrections to the renormalization group functions of the top quark Yukawa and the Higgs boson self-coupling have been computed in Ref. [24]. In this calculation the gauge couplings and all Yukawa coupling except the one of the top quark are set to zero.

In this paper we present details to the three-loop calculation of the gauge coupling renormalization constants and the corresponding beta functions in the SM taking into account all sectors. The results have already been presented in Ref. [25].

The remainder of the paper is organized as follows: In the next Section we introduce our notation and describe in detail how we proceed to obtain the beta functions of the gauge couplings. In particular, we describe the calculation in Lorenz gauge and background field gauge (BFG), discuss our setup for an automated calculation, and explain our treatment of γ_5. The analytical results for the beta functions are presented in Section III. In contrast to Ref. [25] we show the results including all Yukawa couplings. Section IV is devoted to a description of the checks which have been performed to verify our result. A discussion of the numerical impact of the newly obtained corrections is given in Section V. We conclude in Section VI. Explicit results for the renormalization constants are relegated to Appendix A and Appendix B. In Appendix C we present three-loop results for the beta functions of the QED coupling constant and the weak mixing angle. Furthermore, we present in Appendix D translation rules which are useful in order to compare parts of our findings with the results of Ref. [16].

II. THE CALCULATION OF THE BETA FUNCTIONS

In this paper we present the beta functions for the three gauge couplings of the SM up to three loops in the modified minimal subtraction ($\overline{\text{MS}}$) renormalization scheme. In the corresponding calculation we took into account contributions involving the three gauge couplings of the SM, the top, the bottom, and the tau Yukawa couplings and the Higgs self-coupling. Let us mention that we were able to derive the beta functions for a general SM Yukawa sector from the calculation involving the aforementioned seven couplings. We postpone the discussion of this issue to the next Section. In the following, we give details on the computation at the three-loop order.

We define the beta functions as

$$\mu^2 \frac{d}{d\mu^2} \frac{\alpha_i}{\pi} = \beta_i(\{\alpha_j\}, \epsilon) \,, \tag{1}$$

where $\epsilon = (4-d)/2$ is the regulator of Dimensional Regularization with d being the space-time dimension used for the evaluation of the momentum integrals. The dependence of the couplings α_i on the renormalization scale is suppressed in the above equation.

The three gauge couplings α_1, α_2 and α_3 used in this paper are related to the quantities usually used in the SM by

$$\begin{aligned} \alpha_1 &= \frac{5}{3}\frac{\alpha_{\text{QED}}}{\cos^2\theta_W} \,, \\ \alpha_2 &= \frac{\alpha_{\text{QED}}}{\sin^2\theta_W} \,, \\ \alpha_3 &= \alpha_s \,, \end{aligned} \tag{2}$$

where α_{QED} is the fine structure constant, θ_W the weak mixing angle and α_s the strong coupling. We adopt the SU(5) normalization which leads to the factor 5/3 in the equation for α_1.

The Yukawa couplings are defined as

$$\alpha_x = \frac{\alpha_{\text{QED}} m_x^2}{2\sin^2\theta_W M_W^2} \quad \text{with} \quad x = t, b, \tau \,, \tag{3}$$

where m_x and M_W are the fermion and W boson mass, respectively.

We denote the Higgs boson self-coupling by $\hat{\lambda}$, where the Lagrange density contains the following term

$$\mathcal{L}_{\rm SM} = \ldots - (4\pi\hat{\lambda})(H^\dagger H)^2 + \ldots\,, \tag{4}$$

describing the quartic Higgs boson self-interaction.

The beta functions are obtained by calculating the renormalization constants relating bare and renormalized couplings via

$$\alpha_i^{\rm bare} = \mu^{2\epsilon} Z_{\alpha_i}(\{\alpha_j\},\epsilon)\alpha_i\,. \tag{5}$$

Taking into account that $\alpha_i^{\rm bare}$ does not depend on μ, Eqs. (1) and (5) lead to

$$\beta_i = -\left[\epsilon\frac{\alpha_i}{\pi} + \frac{\alpha_i}{Z_{\alpha_i}}\sum_{j=1,j\neq i}^{7}\frac{\partial Z_{\alpha_i}}{\partial \alpha_j}\beta_j\right]\left(1+\frac{\alpha_i}{Z_{\alpha_i}}\frac{\partial Z_{\alpha_i}}{\partial\alpha_i}\right)^{-1}\,, \tag{6}$$

where $i = 1, 2$ or 3. We furthermore set $\alpha_4 = \alpha_t$, $\alpha_5 = \alpha_b$, $\alpha_6 = \alpha_\tau$ and $\alpha_7 = \hat{\lambda}$.

The first term in the first factor of Eq. (6) originates from the term $\mu^{2\epsilon}$ in Eq. (5) and vanishes in four space-time dimensions. The second term in the first factor contains the beta functions of the remaining six couplings of the SM. Note that (for the gauge couplings) the one-loop term of Z_{α_i} only contains α_i, whereas at two loops all couplings are present except $\hat{\lambda}$. The latter appears for the first time at three-loop level. As a consequence, it is necessary to know β_j for $j = 4, 5, 6$ to one-loop order and only the ϵ-dependent term for β_7, namely $\beta_7 = -\epsilon\alpha_7/\pi$. From the second term in the first factor and the second factor of Eq. (6) one can read off that three-loop corrections to Z_{α_i} are required for the computation of β_i to the same loop order.

We have followed two distinct paths to obtain the results for the three-loop renormalization constants, which we discuss in the following Subsections, where we mention features and differences.

A. Lorenz gauge in the unbroken phase of the SM

The first method used for the calculation of the renormalization constants is based on Feynman rules derived for the SM in the unbroken phase in a general Lorenz gauge with three independent gauge parameters corresponding to the three simple gauge groups. All building blocks of our calculation are evaluated for general gauge parameters in order to have a strong check of the final results for the beta functions which have to be gauge parameter independent. It is possible to use the unbroken phase of the SM since the beta function in the $\overline{\rm MS}$ scheme is independent of all mass parameters and thus the spontaneous symmetry breaking does not affect the final result. Note that this choice is advantageous for the calculation because in the unbroken phase much less different types of vertices have to be considered as compared to the phase in which the spontaneous symmetry breaking is present.

In principle each vertex containing the coupling $g_i = \sqrt{4\pi\alpha_i}$ can be used in order to determine the corresponding renormalization constant via

$$Z_{\alpha_i} = \frac{(Z_{\rm vrtx})^2}{\prod_k Z_{k,{\rm wf}}}\,, \tag{7}$$

where $Z_{\rm vrtx}$ stands for the renormalization constant of the vertex and $Z_{k,{\rm wf}}$ for the wave function renormalization constant; k runs over all external particles.

We have computed Z_{α_3} using both the ghost-gluon and the three-gluon vertex. Z_{α_2} has been evaluated with the help of the ghost-W_3, the $W_1W_2W_3$ and the $\phi^+\phi^-W_3$ vertex where $\phi^\pm$ is the charged component of the Higgs doublet corresponding to the charged Goldstone boson in the broken phase and W_1, W_2 and W_3 are the components of the W boson. As to Z_{α_1}, a Ward identity guarantees that there is a cancellation between the vertex and some of the wave function renormalization constants yielding

$$Z_{\alpha_1} = \frac{1}{Z_B}, \tag{8}$$

where Z_B is the wave function renormalization constant for the gauge boson of the $U(1)$ subgroup of the SM in the unbroken phase.

In Fig. 1 we show several one-, two- and three-loop sample diagrams contributing to the considered two- and three-point functions.

We have not used vertices involving fermions as external particles as they may lead to problems in connection with the treatment of γ_5 in $d \neq 4$ dimensions. The vertices selected by us are safe in this respect. A detailed discussion of our prescription for γ_5 is given below.

In order to compute the individual renormalization constants entering Eq. (7) we proceed as outlined, e.g., in Ref. [15]. The underlying formula can be written in the form

$$Z_\Gamma = 1 - K_\epsilon \left(Z_\Gamma \Gamma\right) , \tag{9}$$

where Γ represents the two- or three-point function corresponding to the renormalization constant Z_Γ and the operator K_ϵ extracts the pole part of its argument. From the structure of Eq. (9) it is clear that Z_Γ is computed order-by-order in perturbation theory in a recursive way. It is understood that the bare parameters entering Γ on the right-hand side are replaced by the renormalized ones before applying K_ϵ. The corresponding counterterms are only needed to lower loop orders than the one which is requested for Γ. In our approach the three-loop calculation of Z_{α_i} requires — besides the result for Z_{α_i} to two loops — the one-loop renormalization constants for the other two gauge and the Yukawa couplings. Furthermore we have to renormalize the gauge parameters; the corresponding renormalization constants are given by the wave function renormalization constants of the corresponding gauge bosons which we anyway have to evaluate in the course of our calculation.

B. Background field gauge in the spontaneously broken phase

The second method that we used in order to get an independent result for the renormalization constants of the gauge couplings is a calculation in the BFG [26, 27]. The basic idea of the BFG is the splitting of all gauge fields in a "quantum" and a "classical" part where in practical calculations the latter only occurs as external particle.

The BFG has the advantage that Ward identities guarantee that renormalization constants for gauge couplings can be obtained from the exclusive knowledge of the corresponding wave function renormalization constant.[1] Thus we have the following formula

[1] In Lorenz gauge, this only works for $U(1)$ gauge groups, cf. Subsection II A.

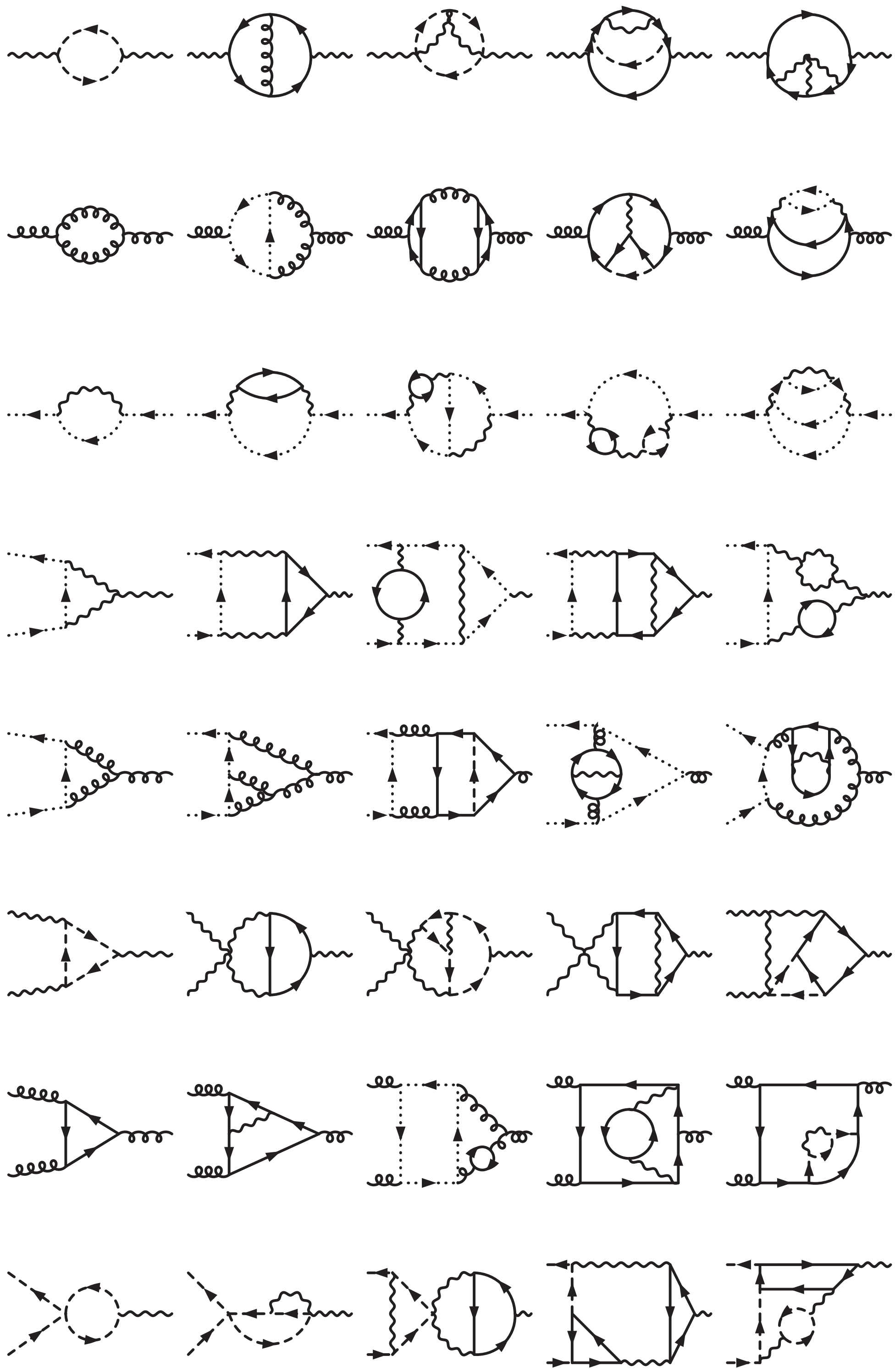


FIG. 1. Sample Feynman diagrams contributing to the Green's functions which have been used for the calculation of the renormalization constants of the gauge couplings. Solid, dashed, dotted, curly and wavy lines correspond to fermions, Higgs bosons, ghosts, gluons and electroweak gauge bosons, respectively.

$$Z_{\alpha_i} = \frac{1}{Z_{A_i,\mathrm{wf}}}\,, \tag{10}$$

where A denotes the gauge boson corresponding to the gauge coupling α_i.

In contrast to the calculation using Lorenz gauge, we performed the calculation in the BFG in the spontaneously broken phase of the SM. As discussed in the last Subsection, such a calculation is more involved than a calculation in the unbroken phase since more vertices are present. On the other hand it constitutes an additional check of our result, that allows us not only to compare the BFG and the Lorenz gauge but also to switch from the broken to the unbroken phase of the SM.

Since the calculation has been performed in the broken phase we have computed the transverse part of the two-point functions of the (background) photon, Z boson, photon-Z mixing, W boson and gluon which we denote by Π_γ, Π_Z, $\Pi_{\gamma Z}$, Π_W, Π_g, respectively. Sample Feynman diagrams up to three loops can be found in the first two lines of Fig. 1.

Π_W and Π_g can be used in analogy to Subsection II A in order to obtain the corresponding renormalization constants which leads in combination with Eq. (10) to the renormalization constants for α_2 and α_s. We found complete agreement with the calculation performed in Lorenz gauge.

As far as the self energies involving photon and Z boson are concerned, we consider at the bare level appropriate linear combinations in order to obtain the B and W boson self-energy contributions. To be precise, we have

$$\begin{aligned}
\Pi_B &= \cos^2\theta_W^{\mathrm{bare}}\Pi_\gamma + 2\cos\theta_W^{\mathrm{bare}}\sin\theta_W^{\mathrm{bare}}\Pi_{\gamma Z} + \sin^2\theta_W^{\mathrm{bare}}\Pi_Z\,,\\
\Pi_W &= \sin^2\theta_W^{\mathrm{bare}}\Pi_\gamma - 2\cos\theta_W^{\mathrm{bare}}\sin\theta_W^{\mathrm{bare}}\Pi_{\gamma Z} + \cos^2\theta_W^{\mathrm{bare}}\Pi_Z\,.
\end{aligned} \tag{11}$$

The second linear combination can immediately be compared with the bare result obtained from the charged W boson self energy and complete agreement up to the three-loop order has been found. This constitutes a strong consistency check on the implementation of the BFG Feynman rules. Π_B is used together with Eq. (10) in order to obtain the renormalization constant for α_1. Again, complete agreement with the calculation described in the previous Subsection has been found.

In our BFG calculation we want to adopt Landau gauge in order to avoid the renormalization of the gauge parameters ξ_i. However, it is not possible to choose Landau gauge from the very beginning since some Feynman rules for vertices involving a background gauge boson contain terms proportional to $1/\xi_i$ where $\xi_i = 0$ corresponds to Landau gauge. To circumvent this problem we evaluate the bare integrals for arbitrary gauge parameters. In the final result all inverse powers of ξ_i cancel and thus the limit $\xi_i = 0$ can be taken at the bare level.

C. Automated Calculation

Higher order calculations in the SM taking into account all contributions are quite involved. Apart from the complicated loop integrals, there are many different Feynman rules and plenty of Feynman diagrams which have to be considered. In our calculation we have used a setup which to a large extend avoids manual interventions in order to keep the error-proneness to a minimum.

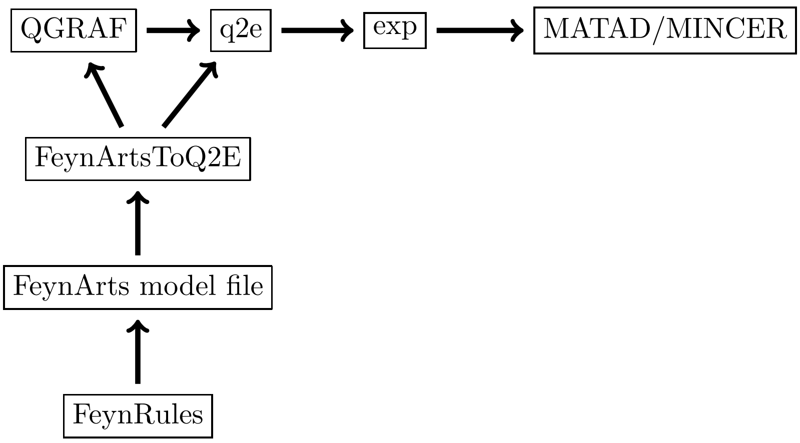


FIG. 2. Overview of our automated setup. Calling up the programs in the uppermost line determines and evaluates a given process in a given model. The vertical workflow leads to the implementation of a new model in the setup. The programs are discussed in more detail in the text.

As far as the loop integrals are concerned we exploit the fact that the beta function in the $\overline{\text{MS}}$ scheme is independent of the external momenta and the particle masses. Thus, we can choose a convenient kinematical configuration which leads to simple loop integrals as long as the infra-red structure is not modified. In our case we set all particle masses to zero and only keep one external momentum different from zero. We have checked that no infra-red divergences are introduced as we will discuss in detail in Section IV. In this way all loop-integrals are mapped to massless two-point functions that up to three loops can be computed with the help of the package `MINCER` [28].

As core of our setup we use a well-tested chain of programs that work hand-in-hand: `QGRAF` [29] generates all contributing Feynman diagrams. The output is passed via `q2e` [30, 31], which transforms Feynman diagrams into Feynman amplitudes, to `exp` [30, 31] that generates `FORM` [32] code. The latter is processed by `MINCER` [28] and/or `MATAD` [33] that compute the Feynman integrals and output the ϵ expansion of the result. The parallelization of the latter part is straightforward as the evaluation of each Feynman diagram corresponds to an independent calculation. We have also parallelized the part performed by `q2e` and `exp` which is essential for our calculation since it may happen that a few times 10^5 diagrams contribute at three-loop level to a single Green's function. The described workflow is illustrated on the top of Fig. 2.

In order to perform the calculation described in this paper we have extended the above setup by the vertical program chain in Fig. 2. The core of the new part is the program `FeynArtsToQ2E` which translates `FeynArts` [34] model files into model files processable by `QGRAF` and `q2e`. In this way we can exploit the well-tested input files of `FeynArts` in our effective and flexible setup based on `QGRAF`, `q2e`, `exp` and `MINCER`. This avoids the coding of the Feynman rules by hand which for the SM would require a dedicated debugging process.

For the part of our calculation based on the BFG we have used the `FeynArts` model files which come together with version 3.5. However, for Lorenz gauge in the unbroken phase there is no publicly available `FeynArts` model. For this reason we have used the package `FeynRules` [35] in order to generate such a file which is also indicated in Fig. 2.

Let us mention that `FeynArtsToQ2E` is not restricted to the SM but can process all model files available for `FeynArts`.

D. Treatment of γ_5

An important issue in multi-loop calculations is the definition of γ_5 away from $d = 4$ dimensions. A first possibility is the naive regularization that requires that γ_5 anti-commutes with all other γ-matrices. This approach has the advantage that its implementation is very simple. However, it can lead to wrong results, especially for Feynman diagrams involving several fermion loops. For example, the naive regularization of γ_5 leads to the problematic result (see, e.g., Ref. [36])

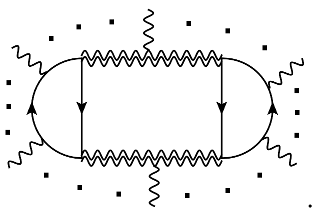
$$\mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_5) = 0 \qquad (d \neq 4)\,. \tag{12}$$

The limit of this expression for $d \to 4$ does not agree with its value in the physical case, when the regularization is turned off

$$\mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_5) = -4\mathrm{i}\varepsilon^{\mu\nu\rho\sigma} \ (d = 4)\,. \tag{13}$$

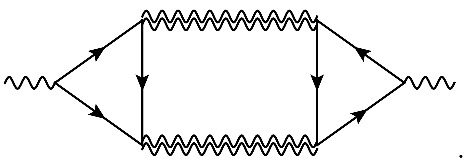
Here the totally anti-symmetric Levi-Civita tensor is defined by $\varepsilon^{0123} = 1$.

It is therefore reassuring that one can show explicitly that in the computation via the ghost-ghost-gauge boson vertex[2] all contributions stemming from this kind of traces vanish. To prove this, we notice that this kind of traces can only lead to non-vanishing contributions if there are at least two of them in a diagram. Only in this case the ε-tensors originating from Eq. (13) can be contracted, providing Lorentz structures that may contribute to the renormalization constants. We observe that the fermion loops can only yield problematic non-vanishing contributions if at least *three* lines are attached to them. Otherwise, there are too few external momenta and too few open Lorentz indices available. A general three-loop diagram with at least two closed fermion loops has the following form



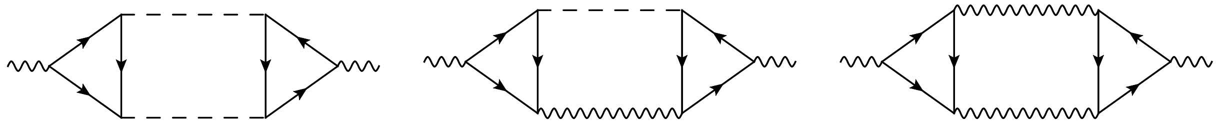
Here the solid lines represent fermions while the double lines can be either scalar bosons or gauge bosons.

One can easily show that one cannot attach ghosts as external particles in the above diagram, since there are no vertices involving ghosts and fermions. Thus, the ghost self-energy and the ghost-ghost-gauge boson vertex do not contain diagrams of this type. Therefore, we need to consider only diagrams with two external gauge bosons for the following discussion. So the diagrams which still have to be discussed have the structure



[2] We restrict the discussion in this Subsection to the ghost-ghost-gauge boson vertex. For all other vertices without external fermions the reasoning is in complete analogy.

There are three ways to replace the double lines:



.

The first diagram type cannot yield problematic contributions for the same reason as mentioned above. The second diagram type vanishes as the fermion traces involve exactly five γ-matrices, not counting γ_5 matrices. (Remember that we deal with diagrams in which all the propagators are massless.) Finally, the third diagram type involves fermion loops with three external gauge bosons. Such diagrams can indeed contain contributions originating from traces of γ_5 and an even number of other γ-matrices. However, the sum over all possible fermion species that can circulate in the loops, including also the diagrams in which the fermions circle in opposite directions, vanishes.[3] This is of course a consequence of the cancellation of the Adler-Bell-Jackiw anomaly [38, 39] within the SM, as required by gauge invariance. Therefore, we are allowed to calculate the Feynman diagrams contributing to Z_{α_i} using a naive regularization prescription for γ_5, in which the diagrams containing triangle anomalies are set to zero from the very beginning, according to Eq. (12) .

As an additional check of the calculation we implemented also a "semi-naive" regularization prescription for γ_5. Explicitly, we evaluate the expression $\mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_5)$ by applying the *formal* replacement

$$\mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_5) = -4\mathrm{i}\tilde{\varepsilon}^{\mu\nu\rho\sigma} + \mathcal{O}(\epsilon)\,. \tag{14}$$

The tensor $\tilde{\varepsilon}^{\mu\nu\rho\sigma}$ has some similarities with the four-dimensional Levi-Civita tensor: (i) it is completely antisymmetric in all indices; (ii) when contracted with a second one of its kind one obtains the following result

$$\tilde{\varepsilon}^{\mu\nu\rho\sigma}\tilde{\varepsilon}_{\mu'\nu'\rho'\sigma'} = g^{[\mu}_{[\mu'}\, g^{\ \nu}_{\ \nu'}\, g^{\ \rho}_{\ \rho'}\, g^{\ \sigma]}_{\ \sigma']}\,, \tag{15}$$

where the square brackets denote complete anti-symmetrization. When taking the limit $d \to 4$, $\tilde{\varepsilon}^{\mu\nu\rho\sigma}$ converts into the four-dimensional Levi-Civita tensor and Eqs. (14) and (15) ensure that it provides the correct four-dimensional result.

At this point a comment on Eqs. (14) is in order. It is straightforward to see that the combination of this equation and the cyclic property of traces leads to an ambiguity of order $\mathcal{O}(\epsilon)$. Therefore, we made sure that the terms that need to be treated in this way generate at most simple poles in ϵ and the above procedure can be applied directly without introducing additional finite counterterms.

Let us stress again that we find the same result for the renormalization constants Z_{α_i} both from the ghost–ghost–gauge boson vertex and by using other vertices and both by applying the "naive" as well as the "semi-naive" scheme. These findings strongly support the above reasoning.

E. Comparison of the methods

This Subsection is devoted to a brief comparison of the calculation via the Lorenz gauge and the one involving the BFG. As has been mentioned before, in the BFG it is sufficient

[3] The proof of this statement can be based only on considerations about group theoretic invariants. For details see Chapter 20 of Ref. [37].

Lorenz gauge

# loops	1	2	3	4
BB	14	410	45 926	7 111 021
W_3W_3	17	502	55 063	8 438 172
gg	9	188	17 611	2 455 714
$c_g\bar{c}_g$	1	12	521	46 390
$c_{W_3}\bar{c}_{W_3}$	2	42	2 480	251 200
$\phi^+\phi^-$	10	429	46 418	6 918 256
BBB	34	2 172	358 716	73 709 886
$W_1W_2W_3$	34	2 216	382 767	79 674 008
ggg	21	946	118 086	20 216 024
$c_g\bar{c}_g g$	2	66	4 240	460 389
$c_{W_1}\bar{c}_{W_2}W_3$	2	107	10 577	1 517 631
$\phi^+\phi^-W_3$	24	2 353	387 338	77 292 771

BFG

# loops	1	2	3	4
$\gamma^B\gamma^B$	13	416	61 968	13 683 693
$\gamma^B Z^B$	13	604	100 952	23 640 897
$Z^B Z^B$	20	1064	183 465	44 049 196
$W^{+B}W^{-B}$	18	1438	252 162	42 423 978
$g^B g^B$	10	186	17 494	2 775 946

TABLE I. The number of Feynman diagrams contributing to the Green's functions evaluated in this work. Left table: two- and three-point functions computed in Lorenz gauge; right table: two-point functions computed in BFG. The superscript "B" denotes background fields. The first column indicates the external legs of the Green's function, the other columns show the number of diagrams at the individual loop orders. Note that the BBB vertex is computed in order to have a cross check as we will explain in Section IV.

to consider only the gauge boson propagators. This is advantageous as the use of Lorenz gauge also requires the evaluation of three- (or four-) point functions and in most cases it also demands the consideration of additional two-point functions apart from the gauge boson ones. The disadvantages of the BFG are the increased number of vertices and the more involved structure of the vertices containing a background field.

In Tab. I we list the number of diagrams for each Green's function contributing to the one-, two- and three-loop order. The number of diagrams computed in this work is obtained from the sum of the numbers in these columns. For comparison we also provide the corresponding number of diagrams which contribute to the four-loop order.

It is tempting to compare the number of contributing Feynman diagrams in Lorenz gauge and in BFG which is, however, not straightforward since we use the former in the broken and the latter in the unbroken phase. Nevertheless, one observes that in the case of β_3 the number of diagrams entering the BFG calculation is roughly the same as in case the gluon-ghost vertex is used in Lorenz gauge, even up to four-loop order. All other vertices lead to significantly more diagrams. In the case of β_2 there are about three to four times more diagrams to be considered in the BFG as compared to Lorenz gauge. Whereas at three-loop order the difference between approximately 70 000 and 250 000 diagrams is probably not substantial it is striking at four-loop order where the number of Feynman diagrams goes from about 10 000 000 (W_3W_3 , $c_{W_3}\bar{c}_{W_3}$ and $c_{W_1}\bar{c}_{W_2}W_3$ Green's function) to 42 000 000 ($W^{+B}W^{-B}$ Green's function) when switching from Lorenz gauge to BFG. Thus, starting from four loops it is probably less attractive to use the BFG.

Let us add that the precise number of Feynman diagrams depends on the detailed setup as, e.g., on the implementation of the four-particle vertices. Because of the colour structure we split in our calculation the four-gluon vertex into two cubic vertices by introducing non-propagating auxiliary particles, whereas all other four-particle vertices are left untouched.

Let us finally mention that the CPU time for the evaluation of an individual diagram ranges from less than a second to few minutes. For general gauge parameters it may take up to the order of an hour. Thus the use of about 100 cores leads to a wall-clock time which ranges from a few hours for a calculation in Feynman gauge up to about one day for general gauge parameters. For the preparation of the `FORM` files using `QGRAF`, `q2e` and `exp` also a few hours of CPU time are needed which is because of the large amount of Feynman diagrams. The use of about 100 cores leads to a wall-clock time of a few minutes.

III. ANALYTICAL RESULTS

In this Section we present the analytical results for the beta functions. As mentioned before, we are able to present the results involving all contributions of the SM Yukawa sector.

The SM Yukawa interactions are described by (see, e.g., Chapter 11 of Ref. [40])

$$\mathcal{L}_{\text{Yukawa}} = -\bar{Q}_i^L Y_{ij}^U \epsilon H^\star u_j^R - \bar{Q}_i^L Y_{ij}^D H d_j^R - \bar{L}_i^L Y_{ij}^L H l_j^R + \text{h.c.}\,, \tag{16}$$

where $Y^{U,D,L}$ are complex 3×3 matrices, i, j are generation labels, H denotes the Higgs field and ϵ is the 2×2 antisymmetric tensor. Q^L, L^L are the left-handed quark and lepton doublets, and u^R, d^R, l^R are the right-handed up- and down-type quark and lepton singlets, respectively. The physical mass-eigenstates are obtained by diagonalizing $Y^{U,D,L}$ by six unitary matrices $V_{L,R}^{U,D,L}$ as follows

$$\tilde{Y}_{\text{diag}}^f = V_L^f Y^f V_R^{f\dagger}\,, \quad f = U, D, L\,. \tag{17}$$

As a result the charged-current $W^\pm$ couples to the physical quark states with couplings parametrized by the Cabibbo-Kobayashi-Maskawa (CKM) matrix $V_{CKM} \equiv V_L^U V_L^{D\dagger}$. We furthermore introduce the notation

$$\hat{T} = \frac{1}{4\pi} Y^U Y^{U\dagger},\ \hat{B} = \frac{1}{4\pi} Y^D Y^{D\dagger},\ \hat{L} = \frac{1}{4\pi} Y^L Y^{L\dagger}. \tag{18}$$

In order to reconstruct the results for a general Yukawa sector, we multiplied each Feynman diagram by a factor $(n_h)^m$, where m denotes the number of fermion loops involving Yukawa couplings. After analyzing the structure of the diagrams that can arise, we could establish the following set of replacements that have to be performed in order to take into account a generalized Yukawa sector

$$\begin{aligned}
n_h\alpha_t &\to \text{tr}\hat{T}, & n_h\alpha_b &\to \text{tr}\hat{B},\\
n_h\alpha_\tau &\to \text{tr}\hat{L}, & n_h\alpha_t^2 &\to \text{tr}\hat{T}^2,\\
n_h\alpha_b^2 &\to \text{tr}\hat{B}^2, & n_h\alpha_t\alpha_b &\to \text{tr}\hat{T}\hat{B},\\
n_h\alpha_\tau^2 &\to \text{tr}\hat{L}^2, & n_h^2\alpha_t^2 &\to (\text{tr}\hat{T})^2,\\
n_h^2\alpha_b^2 &\to (\text{tr}\hat{B})^2, & n_h^2\alpha_\tau^2 &\to (\text{tr}\hat{L})^2,\\
n_h^2\alpha_t\alpha_b &\to \text{tr}\hat{T}\text{tr}\hat{B}, & n_h^2\alpha_t\alpha_\tau &\to \text{tr}\hat{T}\text{tr}\hat{L},\\
n_h^2\alpha_b\alpha_\tau &\to \text{tr}\hat{B}\text{tr}\hat{L}.
\end{aligned} \tag{19}$$

Of course, only traces over products of Yukawa matrices can occur because they arise from closed fermion loops. Using Eqs. (17) and (18) it is straightforward to see that in Eq. (19) only traces of diagonal matrices have to be taken except for $\mathrm{tr}\hat{T}\hat{B}$ which is given by

$$\mathrm{tr}\hat{T}\hat{B} = \mathrm{tr}\left[\begin{pmatrix}\alpha_u & 0 & 0\\ 0 & \alpha_c & 0\\ 0 & 0 & \alpha_t\end{pmatrix} V_{\mathrm{CKM}} \begin{pmatrix}\alpha_d & 0 & 0\\ 0 & \alpha_s & 0\\ 0 & 0 & \alpha_b\end{pmatrix} V_{\mathrm{CKM}}^{\dagger}\right]. \tag{20}$$

The addition of a fourth generation of fermions to the SM particle content can be also easily accounted for by this general notation. In this case, the Yukawa matrices become 4×4 dimensional. If we assume that the fourth generation is just a repetition of the existing generation pattern but much heavier and if we neglect all SM Yukawa interactions, then the explicit form of Yukawa matrices reads

$$\hat{F}_4 = \begin{pmatrix} 0_{3\times 3} & 0\\ 0 & \alpha_F\end{pmatrix}, \quad \text{with} \quad F = T, B, L\,. \tag{21}$$

Here T and B stand for the up- and down-type heavy quarks, and L for the heavy charged leptons, while α_F denotes the corresponding Yukawa couplings as defined in Eq. (3). Since in our calculation no Yukawa couplings for neutrinos have been introduced we cannot incorporate heavy neutrinos. This would require a dedicated calculation which, however, does not pose any principle problem.

We are now in the position to present the results for the beta functions of the gauge couplings which are given by

$$\begin{aligned}
\beta_1 &= \frac{\alpha_1^2}{(4\pi)^2}\left\{\frac{2}{5} + \frac{16n_G}{3}\right\}\\
&+ \frac{\alpha_1^2}{(4\pi)^3}\left\{\frac{18\alpha_1}{25} + \frac{18\alpha_2}{5} - \frac{34\mathrm{tr}\hat{T}}{5} - 2\mathrm{tr}\hat{B} - 6\mathrm{tr}\hat{L} + n_G\left[\frac{76\alpha_1}{15} + \frac{12\alpha_2}{5} + \frac{176\alpha_3}{15}\right]\right\}\\
&+ \frac{\alpha_1^2}{(4\pi)^4}\left\{\frac{489\alpha_1^2}{2000} + \frac{783\alpha_1\alpha_2}{200} + \frac{3401\alpha_2^2}{80} + \frac{54\alpha_1\hat{\lambda}}{25} + \frac{18\alpha_2\hat{\lambda}}{5} - \frac{36\hat{\lambda}^2}{5} - \frac{2827\alpha_1\mathrm{tr}\hat{T}}{200}\right.\\
&- \frac{471\alpha_2\mathrm{tr}\hat{T}}{8} - \frac{116\alpha_3\mathrm{tr}\hat{T}}{5} - \frac{1267\alpha_1\mathrm{tr}\hat{B}}{200} - \frac{1311\alpha_2\mathrm{tr}\hat{B}}{40} - \frac{68\alpha_3\mathrm{tr}\hat{B}}{5} - \frac{2529\alpha_1\mathrm{tr}\hat{L}}{200}\\
&- \frac{1629\alpha_2\mathrm{tr}\hat{L}}{40} + \frac{183\mathrm{tr}\hat{B}^2}{20} + \frac{51(\mathrm{tr}\hat{B})^2}{10} + \frac{157\mathrm{tr}\hat{B}\mathrm{tr}\hat{L}}{5} + \frac{261\mathrm{tr}\hat{L}^2}{20} + \frac{99(\mathrm{tr}\hat{L})^2}{10}\\
&+ \frac{3\mathrm{tr}\hat{T}\hat{B}}{2} + \frac{339\mathrm{tr}\hat{T}^2}{20} + \frac{177\mathrm{tr}\hat{T}\mathrm{tr}\hat{B}}{5} + \frac{199\mathrm{tr}\hat{T}\mathrm{tr}\hat{L}}{5} + \frac{303(\mathrm{tr}\hat{T})^2}{10}\\
&+ n_G\left[-\frac{232\alpha_1^2}{75} - \frac{7\alpha_1\alpha_2}{25} + \frac{166\alpha_2^2}{15} - \frac{548\alpha_1\alpha_3}{225} - \frac{4\alpha_2\alpha_3}{5} + \frac{1100\alpha_3^2}{9}\right]\\
&\left.+ n_G^2\left[-\frac{836\alpha_1^2}{135} - \frac{44\alpha_2^2}{15} - \frac{1936\alpha_3^2}{135}\right]\right\},
\end{aligned} \tag{22}$$

$$\beta_2 = \frac{\alpha_2^2}{(4\pi)^2}\left\{-\frac{86}{3} + \frac{16n_G}{3}\right\}$$

$$
\begin{aligned}
&+\frac{\alpha_2^2}{(4\pi)^3}\left\{\frac{6\alpha_1}{5}-\frac{518\alpha_2}{3}-6\mathrm{tr}\hat{T}-6\mathrm{tr}\hat{B}-2\mathrm{tr}\hat{L}+n_G\left[\frac{4\alpha_1}{5}+\frac{196\alpha_2}{3}+16\alpha_3\right]\right\}\\
&+\frac{\alpha_2^2}{(4\pi)^4}\left\{\frac{163\alpha_1^2}{400}+\frac{561\alpha_1\alpha_2}{40}-\frac{667111\alpha_2^2}{432}+\frac{6\alpha_1\hat{\lambda}}{5}+6\alpha_2\hat{\lambda}-12\hat{\lambda}^2-\frac{593\alpha_1\mathrm{tr}\hat{T}}{40}\right.\\
&-\frac{729\alpha_2\mathrm{tr}\hat{T}}{8}-28\alpha_3\mathrm{tr}\hat{T}-\frac{533\alpha_1\mathrm{tr}\hat{B}}{40}-\frac{729\alpha_2\mathrm{tr}\hat{B}}{8}-28\alpha_3\mathrm{tr}\hat{B}-\frac{51\alpha_1\mathrm{tr}\hat{L}}{8}\\
&-\frac{243\alpha_2\mathrm{tr}\hat{L}}{8}+\frac{57\mathrm{tr}\hat{B}^2}{4}+\frac{45(\mathrm{tr}\hat{B})^2}{2}+15\mathrm{tr}\hat{B}\mathrm{tr}\hat{L}+\frac{19\mathrm{tr}\hat{L}^2}{4}+\frac{5(\mathrm{tr}\hat{L})^2}{2}+\frac{27\mathrm{tr}\hat{T}\hat{B}}{2}\\
&+\frac{57\mathrm{tr}\hat{T}^2}{4}+45\mathrm{tr}\hat{T}\mathrm{tr}\hat{B}+15\mathrm{tr}\hat{T}\mathrm{tr}\hat{L}+\frac{45(\mathrm{tr}\hat{T})^2}{2}\\
&+n_G\left[-\frac{28\alpha_1^2}{15}+\frac{13\alpha_1\alpha_2}{5}+\frac{25648\alpha_2^2}{27}-\frac{4\alpha_1\alpha_3}{15}+52\alpha_2\alpha_3+\frac{500\alpha_3^2}{3}\right]\\
&\left.+n_G^2\left[-\frac{44\alpha_1^2}{45}-\frac{1660\alpha_2^2}{27}-\frac{176\alpha_3^2}{9}\right]\right\}
\end{aligned}
\tag{23}
$$

and

$$
\begin{aligned}
\beta_3=&\frac{\alpha_3^2}{(4\pi)^2}\left\{-44+\frac{16n_G}{3}\right\}\\
&+\frac{\alpha_3^2}{(4\pi)^3}\left\{-408\alpha_3-8\mathrm{tr}\hat{T}-8\mathrm{tr}\hat{B}+n_G\left[\frac{22\alpha_1}{15}+6\alpha_2+\frac{304\alpha_3}{3}\right]\right\}\\
&+\frac{\alpha_3^2}{(4\pi)^4}\left\{-5714\alpha_3^2-\frac{101\alpha_1\mathrm{tr}\hat{T}}{10}-\frac{93\alpha_2\mathrm{tr}\hat{T}}{2}-160\alpha_3\mathrm{tr}\hat{T}-\frac{89\alpha_1\mathrm{tr}\hat{B}}{10}-\frac{93\alpha_2\mathrm{tr}\hat{B}}{2}\right.\\
&-160\alpha_3\mathrm{tr}\hat{B}+18\mathrm{tr}\hat{B}^2+42(\mathrm{tr}\hat{B})^2+14\mathrm{tr}\hat{B}\mathrm{tr}\hat{L}-12\mathrm{tr}\hat{T}\hat{B}+18\mathrm{tr}\hat{T}^2+84\mathrm{tr}\hat{T}\mathrm{tr}\hat{B}\\
&+14\mathrm{tr}\hat{T}\mathrm{tr}\hat{L}+42(\mathrm{tr}\hat{T})^2\\
&+n_G\left[-\frac{13\alpha_1^2}{30}-\frac{\alpha_1\alpha_2}{10}+\frac{241\alpha_2^2}{6}+\frac{308\alpha_1\alpha_3}{45}+28\alpha_2\alpha_3+\frac{20132\alpha_3^2}{9}\right]\\
&\left.+n_G^2\left[-\frac{242\alpha_1^2}{135}-\frac{22\alpha_2^2}{3}-\frac{2600\alpha_3^2}{27}\right]\right\}.
\end{aligned}
\tag{24}
$$

In the above formulas n_G denotes the number of fermion generations. It is obtained by labeling the closed quark and lepton loops present in the diagrams.

To obtain the results for the three-loop gauge beta functions, one also needs the one-loop beta functions of the Yukawa couplings, cf. Eq. (6). They can be found in the literature, of course. Nevertheless we decided to re-calculate them as an additional check of our setup. For completeness, we present the analytical two-loop expressions which read

$$
\begin{aligned}
\beta_{\alpha_t}=&-\epsilon\frac{\alpha_t}{\pi}+\frac{\alpha_t}{(4\pi)^2}\left\{-6\alpha_b+6\alpha_t+4\mathrm{tr}\hat{L}+12\mathrm{tr}\hat{B}+12\mathrm{tr}\hat{T}-\frac{17}{5}\alpha_1-9\alpha_2-32\alpha_3\right\}\\
&+\frac{\alpha_t}{(4\pi)^3}\left\{\frac{9\alpha_1^2}{50}-\frac{9\alpha_1\alpha_2}{5}-35\alpha_2^2+\frac{76\alpha_1\alpha_3}{15}+36\alpha_2\alpha_3-\frac{1616\alpha_3^2}{3}+n_G\left[\frac{116\alpha_1^2}{45}\right.\right.\\
&\left.+4\alpha_2^2+\frac{320\alpha_3^2}{9}\right]+24\hat{\lambda}^2+\frac{393\alpha_1\alpha_t}{20}+\frac{225\alpha_2\alpha_t}{4}+144\alpha_3\alpha_t-48\hat{\lambda}\alpha_t-48\alpha_t^2
\end{aligned}
$$

$$+\frac{7\alpha_1\alpha_b}{20}+\frac{99\alpha_2\alpha_b}{4}+16\alpha_3\alpha_b-11\alpha_t\alpha_b-\alpha_b^2+\frac{15\alpha_1\alpha_\tau}{2}+\frac{15\alpha_2\alpha_\tau}{2}-9\alpha_t\alpha_\tau$$
$$+5\alpha_b\alpha_\tau-9\alpha_\tau^2\Bigg\}, \qquad (25)$$

$$\beta_{\alpha_b}= -\epsilon\frac{\alpha_b}{\pi}+\frac{\alpha_b}{(4\pi)^2}\Big\{6\alpha_b-6\alpha_t+4\mathrm{tr}\hat{L}+12\mathrm{tr}\hat{B}+12\mathrm{tr}\hat{T}-\alpha_1-9\alpha_2-32\alpha_3\Big\}$$
$$+\frac{\alpha_b}{(4\pi)^3}\Bigg\{\frac{-29\alpha_1^2}{50}-\frac{27\alpha_1\alpha_2}{5}-35\alpha_2^2+\frac{124\alpha_1\alpha_3}{15}+36\alpha_2\alpha_3-\frac{1616\alpha_3^2}{3}$$
$$+n_G\left[\frac{-4\alpha_1^2}{45}+4\alpha_2^2+\frac{320\alpha_3^2}{9}\right]+24\hat{\lambda}^2+\frac{91\alpha_1\alpha_t}{20}+\frac{99\alpha_2\alpha_t}{4}+16\alpha_3\alpha_t-\alpha_t^2+\frac{237\alpha_1\alpha_b}{20}$$
$$+\frac{225\alpha_2\alpha_b}{4}+144\alpha_3\alpha_b-48\hat{\lambda}\alpha_b-11\alpha_t\alpha_b-48\alpha_b^2+\frac{15\alpha_1\alpha_\tau}{2}+\frac{15\alpha_2\alpha_\tau}{2}+5\alpha_t\alpha_\tau$$
$$-9\alpha_b\alpha_\tau-9\alpha_\tau^2\Bigg\}, \qquad (26)$$

and

$$\beta_{\alpha_\tau}= -\epsilon\frac{\alpha_\tau}{\pi}+\frac{\alpha_\tau}{(4\pi)^2}\Big\{6\alpha_\tau+4\mathrm{tr}\hat{L}+12\mathrm{tr}\hat{B}+12\mathrm{tr}\hat{T}-9\alpha_1-9\alpha_2\Big\}$$
$$+\frac{\alpha_\tau}{(4\pi)^3}\Bigg\{\frac{51\alpha_1^2}{50}+\frac{27\alpha_1\alpha_2}{5}-35\alpha_2^2+n_G\left[\frac{44\alpha_1^2}{5}+4\alpha_2^2\right]+24\hat{\lambda}^2+\frac{17\alpha_1\alpha_t}{2}$$
$$+\frac{45\alpha_2\alpha_t}{2}+80\alpha_3\alpha_t-27\alpha_t^2+\frac{5\alpha_1\alpha_b}{2}+\frac{45\alpha_2\alpha_b}{2}+80\alpha_3\alpha_b+6\alpha_t\alpha_b-27\alpha_b^2$$
$$+\frac{537\alpha_1\alpha_\tau}{20}+\frac{165\alpha_2\alpha_\tau}{4}-48\hat{\lambda}\alpha_\tau-27\alpha_t\alpha_\tau-27\alpha_b\alpha_\tau-12\alpha_\tau^2\Bigg\}. \qquad (27)$$

The one-loop results have been expressed in terms of Yukawa matrices since these expressions enter the three-loop beta functions. At two-loop order, however, we refrain from reconstructing the general expression which would require an extension of the rules given in Eq. (19).

Our independent calculation of the two-loop Yukawa beta functions is also interesting as there is a discrepancy between [20] and [23] concerning the absence of terms proportional to $\alpha_b\alpha_t\hat{\lambda}$ in Eqs. (25) and (26). We were able to confirm the results in Ref. [23].

In Appendices A and B we provide the results for the renormalization constants which lead to the beta functions discussed in this Section.

IV. CHECKS

We successfully performed a number of consistency checks and compared our results with those already available in the literature. We describe these checks in detail in this Section.

The consistency checks show that all computed renormalization constants are local, that the renormalization constants of the gauge couplings are gauge parameter independent and that the beta functions are finite. We also find that the beta functions calculated by considering different vertices in Lorenz gauge agree among themselves and with the results of the computation in BFG.

In order to test that the program `FeynArtsToQ2E` correctly translates `FeynArts` model files into model files for `QGRAF/q2e`, we reproduced the beta function for the Higgs self-coupling to one-loop order and the beta functions for the top and bottom quark, and the tau lepton Yukawa couplings to two-loop order (cf. previous Section). We have even considered quantities within the Minimal Supersymmetric Standard Model, like the relation between the squark masses within one generation, which is quite involved in case electroweak interactions are kept non-zero. Furthermore, we find that the divergent loop corrections to the BBB vertex vanish in the Lorenz gauge, as expected since for this vertex no renormalization is required. We performed the latter check up to three-loop order.

Another check consists in verifying that in the vertex diagrams no infra-red divergences are introduced although one external momentum is set to zero. We do not have to consider two-point functions since they are infra-red safe. One can avoid infra-red divergences by introducing a common mass for the internal particles. Afterwards, the resulting integrals are evaluated in the limit $q^2 \gg m^2$ where q is the non-vanishing external momentum of the vertex diagrams. This is conveniently done by applying the rules of asymptotic expansion [41] which are encoded in the program `exp`. The setup described in Section II C is particularly useful for this test since `exp` takes over the task of generating `FORM` code for all relevant sub-diagrams which can be up to 35 for some of the diagrams; this makes the calculation significantly more complex. In our case the asymptotic expansion either leads to massless two-point functions or massive vacuum integrals or a combination of both. The former are computed with the help of the package `MINCER`, for the latter the package `MATAD` is used. As a result one obtains a series in m^2/q^2 where the coefficients contain numbers and $\ln(m^2/q^2)$ terms. For our purpose it is sufficient to restrict ourselves to the term $(m^2/q^2)^0$ and check that no logarithms appear in the final result. With this method we have explicitly checked that the $W_1W_2W_3$ and three-gluon vertices are free from infra-red divergences. Since the results for the gauge coupling renormalization constants agree with the ones obtained from the other vertices also the latter are infra-red safe.

Let us finally comment on the comparison of our findings with the literature. We have successfully compared our results for the two-loop gauge beta functions with [9] and the two-loop Yukawa beta functions with [23]. Even partial results for the three-loop gauge beta functions were available in the literature [16]. That paper comprises all hitherto known three-loop corrections in a general quantum field theory based on a single gauge group, however, the presentation of the results relies on a quite intricate notation. Its specification to the SM is straightforward, however, a bit tedious. For convenience, the translation rules needed to specify the notation of [16] to ours is given in Appendix D.

After specifying the notation of Ref. [16] to ours we find complete agreement, taking into account the following modifications in Eq. (33) of [16]:[4] The terms

$$\begin{aligned} &-\frac{7g^2\,\mathrm{tr}\left(Y^a\bar{Y}^b\right)\mathrm{tr}\left(\bar{Y}^bY^aC(R)\right)}{12r} \\ &+\frac{g^2\,\mathrm{tr}\left(Y^a\bar{Y}^b\right)\mathrm{tr}\left(Y^b\bar{Y}^c\right)C(S)_{ca}}{12r} \end{aligned} \tag{28}$$

have to be "symmetrized", so that they read

$$\frac{1}{2}\left(-\frac{7g^2\,\mathrm{tr}\left(Y^a\bar{Y}^b\right)\mathrm{tr}\left(\bar{Y}^bY^aC(R)\right)}{12r}-\frac{7g^2\,\mathrm{tr}\left(Y^a\bar{Y}^b\right)\mathrm{tr}\left(Y^b\bar{Y}^aC(R)\right)}{12r}\right)$$

[4] We want to thank the authors of [16] for pointing out the issue of symmetrization and for assistance in deriving the translation rules.

$$+\frac{1}{2}\left(+\frac{g^2\,\mathrm{tr}\left(Y^a\bar{Y}^b\right)\mathrm{tr}\left(Y^b\bar{Y}^c\right)C(S)_{ca}}{12r}-\frac{g^2\,\mathrm{tr}\left(Y^a\bar{Y}^b\right)\mathrm{tr}\left(\bar{Y}^bY^c\right)C(S)_{ca}}{12r}\right). \tag{29}$$

Furthermore, one has to correct the obvious misprint

$$+\frac{5g^4\,\mathrm{tr}\left(C(R)^2\bar{Y}^aY^b\right)}{12r}\;\rightarrow\;+\frac{5g^4\,\mathrm{tr}\left(C(R)^2\bar{Y}^aY^a\right)}{12r}. \tag{30}$$

V. NUMERICAL ANALYSIS

In this Section we discuss the numerical effect of the new contributions to the gauge beta functions. We solve the corresponding renormalization group equations of the gauge couplings numerically and take into account the contributions from the Yukawa couplings and the Higgs self-coupling to two-loops. As boundary conditions we choose

$$\begin{aligned}
\alpha_1^{\overline{\mathrm{MS}}}(M_Z) &= 0.0169225 \pm 0.0000039\,,\\
\alpha_2^{\overline{\mathrm{MS}}}(M_Z) &= 0.033735 \pm 0.000020\,,\\
\alpha_3^{\overline{\mathrm{MS}}}(M_Z) &= 0.1173 \pm 0.00069\,,\\
\alpha_t^{\overline{\mathrm{MS}}}(M_Z) &= 0.07514\,,\\
\alpha_b^{\overline{\mathrm{MS}}}(M_Z) &= 0.00002064\,,\\
\alpha_\tau^{\overline{\mathrm{MS}}}(M_Z) &= 8.077\cdot 10^{-6}\,,\\
4\pi\hat{\lambda} &= 0.13\,,
\end{aligned} \tag{31}$$

where the first six entries correspond to experimentally determined values while the value for the Higgs coupling is determined assuming a Higgs boson with mass 125 GeV:

$$4\pi\hat{\lambda}=\frac{m_H^2}{2v^2}\approx\frac{125^2\,\mathrm{GeV}^2}{2\cdot\left(\sqrt{2}\cdot 174\right)^2\,\mathrm{GeV}^2}\approx 0.13\,. \tag{32}$$

Note that the values in Eq. (31) are given in the full SM, the top quark being not integrated out. For a description how these values are determined from the knowledge of their directly measured counterparts [40], we refer to [40, 42].

It is noteworthy that for all three gauge couplings the sum of all three-loop terms involving at least one of the couplings α_b, α_τ or λ leads to corrections which are less than 0.1% of the difference between the two- and three-loop prediction.

In Fig. 3 the running of the couplings α_1, α_2 and α_3, is shown from $\mu = M_Z$ up to high energies. At this scale no difference between one, two and three loops is visible, all curves lie on top of each other.

The differences between the loop orders can be seen in Fig. 4 which magnifies the intersection point between α_1 and α_2. There is a clear jump between the one- (dotted) and two-loop (dashed) prediction. The difference between two and three loops (solid curves) is significantly smaller which suggests that perturbation theory converges very well.

The experimental uncertainties for $\alpha_1(M_Z)$ and $\alpha_2(M_Z)$ as given in Eq. (31) are reflected by the bands around the three-loop results. Defining the difference between the two- and

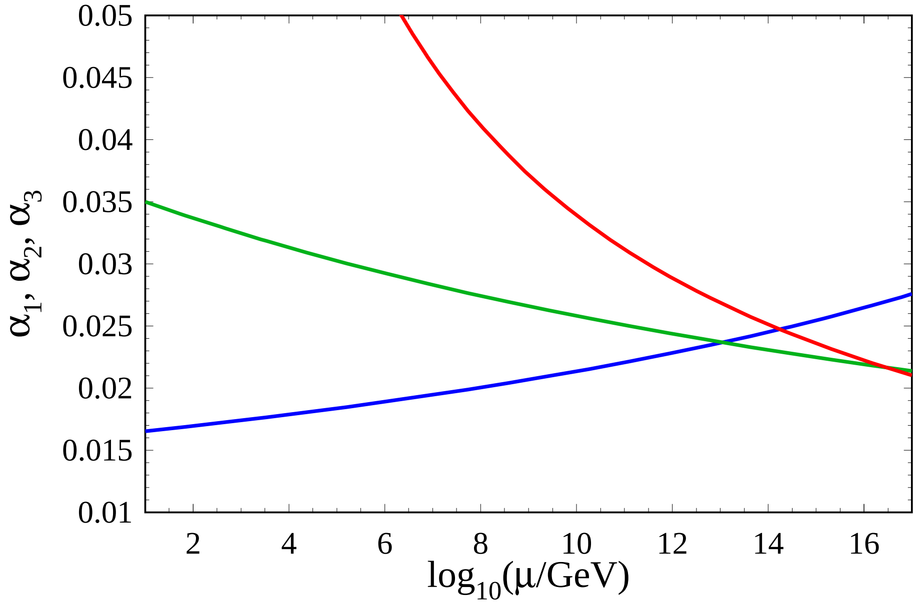


FIG. 3. The running of the gauge couplings at three loops. The curve with the smallest initial value corresponds to α_1, the middle curve to α_2, and the curve with the highest initial value to α_3.

three-loop result as theoretical uncertainty one observes that it is smaller than the experimental one, however, of the same order of magnitude. Without the new three-loop calculation performed in this paper the theory uncertainty is much larger than the experimental one.

Also in the case of α_3 perturbation theory seems to converge well. However, in contrast to α_1 and α_2 the experimental uncertainty turns out to be much larger than the theoretical uncertainty. This is not surprising as the relative experimental uncertainty of α_3 at the electroweak scale is quite large compared to its electroweak counterparts. The relative experimental and theoretical uncertainty of α_3 is plotted as a function of the renormalization scale in Fig. 5. Note that by construction we have that $\Delta\alpha_3/\alpha_3|_{\text{theory}}$ approaches zero for $\mu \to M_Z$.

Let us finally identify the numerically most important contributions. This is done by running from $\mu = M_Z$ to $\mu = 10^{16}$ GeV and by comparing the contribution of an individual term to the total difference between the two- and three-loop prediction. Similar results are also obtained for lower scales.

About 90% of the three-loop corrections to the running of the gauge couplings arises from only a few terms. In the case of α_1 there is only one term which dominates, namely the one of order $\alpha_1^2\alpha_3^2$. For α_2 one has a contribution of +56% from the $\mathcal{O}(\alpha_2^2\alpha_3^2)$ term, +13% from order $\alpha_2^3\alpha_3$ and +37% from $\mathcal{O}(\alpha_2^4)$. All other terms contribute at most 5% and partly also cancel each other. Except for the term of $\mathcal{O}(\alpha_2^4)$ all these terms are presented in this paper for the first time.

The beta function β_3 is dominated by the strong corrections, however, large cancellations between the α_3^4 (+137%), $\alpha_3^3\alpha_2$ (+45%), $\alpha_3^2\alpha_t^2$ (+28%) and $\alpha_3^2\alpha_2^2$ (+17%) terms on the one

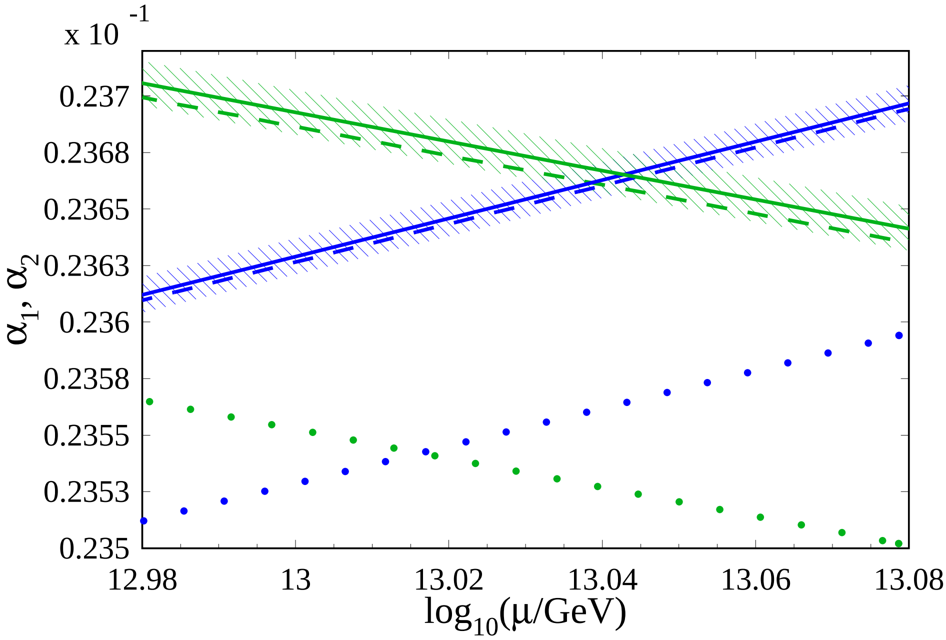


FIG. 4. The running of the electroweak gauge couplings in the SM. The lines with positive slope correspond to α_1, the lines with negative slope to α_2. The dotted, dashed and solid lines correspond to one-, two- and three-loop precision, respectively. The bands around the three-loop curves visualize the experimental uncertainty.

hand and the $\alpha_3^3\alpha_t$ (-112%) and $\alpha_3^2\alpha_2\alpha_t$ (-16%) on the hand are observed. It is worth noting that the four-loop term of order α_s^5 amounts to -58% of the difference between the two- and three-loop predictions. This number has been obtained by adding the four-loop QCD term [17, 18] to β_3.

VI. CONCLUSIONS

In this paper we present three-loop results for renormalization constants that are used to compute the three SM gauge coupling beta functions, taking into account all contributions. We have checked that our expressions agree with all partial results present in the literature. Furthermore the two-loop corrections to the Yukawa couplings have been computed. We have performed the calculation using both Lorenz gauge within the unbroken phase of the SM and BFG in the broken phase. Our final result is valid for a generic flavour structure with an arbitrary CKM matrix. It is furthermore sufficiently general to consider a fourth generation of quarks and leptons.

In order to perform the calculation in an automated way we have written an interface, `FeynArtsToQ2E`, between the package `FeynArts` and our chain of programs (`QGRAF`, `q2e`, `exp`, `MATAD`, `MINCER`) allowing to handle the $\mathcal{O}(10^6)$ Feynman diagrams, which have to be considered in the course of the calculation, in an effective way. Thus, we could perform several checks involving various different Green's functions. `FeynArtsToQ2E` is not limited

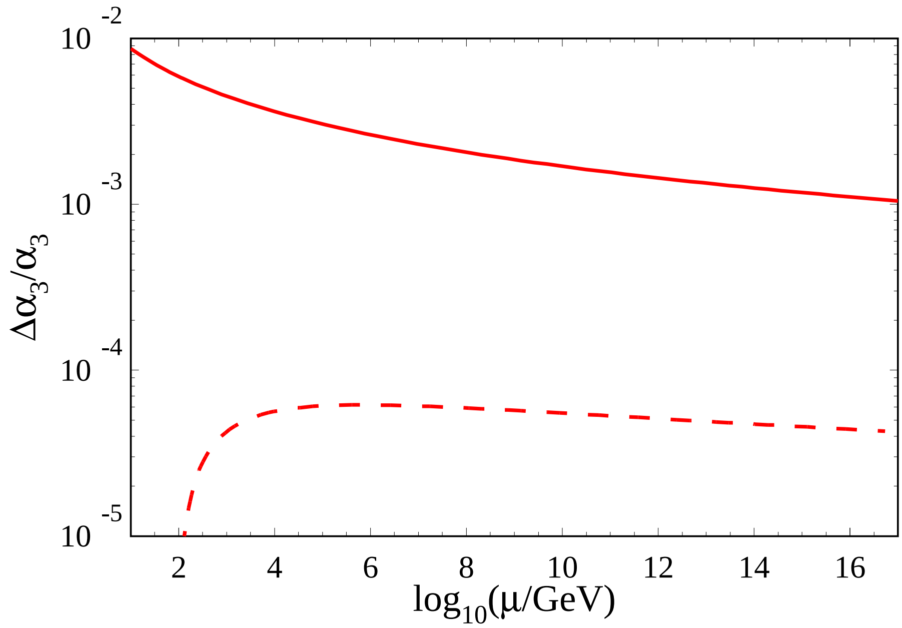


FIG. 5. Comparison of the relative experimental and theoretical uncertainty of α_3. The theoretical uncertainty is given by the dashed line, the solid curve corresponds to the experimental one.

to the SM but can easily be used for extensions like supersymmetric models.

ACKNOWLEDGMENTS

We thank David Kunz for tests concerning `FeynArtsToQ2E` and Mikołaj Misiak for suggesting to generalize the Yukawa structure of the final result. This work is supported by DFG through SFB/TR 9 "Computational Particle Physics".

Appendix A: Renormalization constants

In this Appendix we present analytical results for the renormalization constants Z_{α_1}, Z_{α_2}, Z_{α_3}, Z_B, Z_W, Z_G, Z_H, Z_{c_W}, Z_{c_G}, Z_{cCW}, Z_{cCG}, Z_{WWW}, Z_{GGG}, Z_{HHW}, where the definition of Z_{α_i} is given in Eq. (5) and the field renormalization is defined through

$$\begin{aligned} B^{\mathrm{bare}} &= Z_B B\,, & W^{\mathrm{bare}} &= Z_W W\,, & G^{\mathrm{bare}} &= Z_G G\,, \\ c_W^{\mathrm{bare}} &= Z_{c_W} c_W\,, & c_G^{\mathrm{bare}} &= Z_{c_G} c_G\,, & H^{\mathrm{bare}} &= Z_H H\,. \end{aligned} \tag{A1}$$

B, W, G, c_W and c_G denote the gauge boson and ghost fields. The scalar field H is defined in Eq. (D5). The renormalization constants for the three-particle vertices are also defined in a multiplicative way.

Some of the results listed below contain the gauge parameters ξ_B, ξ_W and ξ_G. They are conveniently defined via the corresponding gauge boson propagator which is given by

$$D_X^{\mu\nu}(q) = i\frac{-g^{\mu\nu} + (1-\xi_X)\,\frac{q^\mu q^\nu}{q^2}}{q^2}\,, \tag{A2}$$

with $X = B, W, G$. Note that $\xi_X = 1$ corresponds to Feynman and $\xi_X = 0$ to Landau gauge. Our analytical results read

$$\begin{aligned}
Z_{\alpha_1} = 1 &+ \frac{\alpha_1}{4\pi}\frac{1}{\epsilon}\left\{\frac{1}{10} + \frac{4n_G}{3}\right\} + \frac{\alpha_1}{(4\pi)^2}\left\{\frac{1}{\epsilon^2}\left[\frac{\alpha_1}{100} + \frac{4n_G\alpha_1}{15} + \frac{16n_G^2\alpha_1}{9}\right]\right.\\
&\left.+\frac{1}{\epsilon}\left[\frac{9\alpha_1}{100} + \frac{9\alpha_2}{20} - \frac{17\mathrm{tr}\hat{T}}{20} - \frac{\mathrm{tr}\hat{B}}{4} - \frac{3\mathrm{tr}\hat{L}}{4} + n_G\left(\frac{19\alpha_1}{30} + \frac{3\alpha_2}{10} + \frac{22\alpha_3}{15}\right)\right]\right\}\\
&+\frac{\alpha_1}{(4\pi)^3}\left\{\frac{1}{\epsilon^3}\left[\frac{\alpha_1^2}{1000} + \frac{n_G\alpha_1^2}{25} + \frac{8n_G^2\alpha_1^2}{15} + \frac{64n_G^3\alpha_1^2}{27}\right]\right.\\
&+\frac{1}{\epsilon^2}\left[\frac{21\alpha_1^2}{1000} + \frac{9\alpha_1\alpha_2}{100} - \frac{43\alpha_2^2}{40} + \frac{17\alpha_1\mathrm{tr}\hat{T}}{240} + \frac{51\alpha_2\mathrm{tr}\hat{T}}{80} + \frac{34\alpha_3\mathrm{tr}\hat{T}}{15} - \frac{7\alpha_1\mathrm{tr}\hat{B}}{240}\right.\\
&+\frac{3\alpha_2\mathrm{tr}\hat{B}}{16} + \frac{2\alpha_3\mathrm{tr}\hat{B}}{3} + \frac{33\alpha_1\mathrm{tr}\hat{L}}{80} + \frac{9\alpha_2\mathrm{tr}\hat{L}}{16} - \frac{\mathrm{tr}\hat{B}^2}{8} - \frac{(\mathrm{tr}\hat{B})^2}{4} - \frac{5\mathrm{tr}\hat{B}\mathrm{tr}\hat{L}}{6}\\
&-\frac{3\mathrm{tr}\hat{L}^2}{8} - \frac{(\mathrm{tr}\hat{L})^2}{4} + \frac{11\mathrm{tr}\hat{T}\hat{B}}{20} - \frac{17\mathrm{tr}\hat{T}^2}{40} - \frac{11\mathrm{tr}\hat{T}\mathrm{tr}\hat{B}}{10} - \frac{31\mathrm{tr}\hat{T}\mathrm{tr}\hat{L}}{30} - \frac{17(\mathrm{tr}\hat{T})^2}{20}\\
&+n_G\left(\frac{77\alpha_1^2}{180} + \frac{63\alpha_1\alpha_2}{50} - \frac{31\alpha_2^2}{60} + \frac{22\alpha_1\alpha_3}{75} - \frac{242\alpha_3^2}{45} - \frac{34\alpha_1\mathrm{tr}\hat{T}}{15} - \frac{2\alpha_1\mathrm{tr}\hat{B}}{3} - 2\alpha_1\mathrm{tr}\hat{L}\right)\\
&\left.+n_G^2\left(\frac{266\alpha_1^2}{135} + \frac{4\alpha_1\alpha_2}{5} + \frac{2\alpha_2^2}{15} + \frac{176\alpha_1\alpha_3}{45} + \frac{88\alpha_3^2}{135}\right)\right]\\
&+\frac{1}{\epsilon}\left[\frac{163\alpha_1^2}{8000} + \frac{261\alpha_1\alpha_2}{800} + \frac{3401\alpha_2^2}{960} + \frac{9\alpha_1\hat{\lambda}}{50} + \frac{3\alpha_2\hat{\lambda}}{10} - \frac{3\hat{\lambda}^2}{5} - \frac{2827\alpha_1\mathrm{tr}\hat{T}}{2400} - \frac{157\alpha_2\mathrm{tr}\hat{T}}{32}\right.\\
&-\frac{29\alpha_3\mathrm{tr}\hat{T}}{15} - \frac{1267\alpha_1\mathrm{tr}\hat{B}}{2400} - \frac{437\alpha_2\mathrm{tr}\hat{B}}{160} - \frac{17\alpha_3\mathrm{tr}\hat{B}}{15} - \frac{843\alpha_1\mathrm{tr}\hat{L}}{800} - \frac{543\alpha_2\mathrm{tr}\hat{L}}{160}\\
&+\frac{61\mathrm{tr}\hat{B}^2}{80} + \frac{17(\mathrm{tr}\hat{B})^2}{40} + \frac{157\mathrm{tr}\hat{B}\mathrm{tr}\hat{L}}{60} + \frac{87\mathrm{tr}\hat{L}^2}{80} + \frac{33(\mathrm{tr}\hat{L})^2}{40} + \frac{\mathrm{tr}\hat{T}\hat{B}}{8} + \frac{113\mathrm{tr}\hat{T}^2}{80}\\
&+\frac{59\mathrm{tr}\hat{T}\mathrm{tr}\hat{B}}{20} + \frac{199\mathrm{tr}\hat{T}\mathrm{tr}\hat{L}}{60} + \frac{101(\mathrm{tr}\hat{T})^2}{40}\\
&+n_G\left(-\frac{58\alpha_1^2}{225} - \frac{7\alpha_1\alpha_2}{300} + \frac{83\alpha_2^2}{90} - \frac{137\alpha_1\alpha_3}{675} - \frac{\alpha_2\alpha_3}{15} + \frac{275\alpha_3^2}{27}\right)\\
&\left.\left.+n_G^2\left(-\frac{209\alpha_1^2}{405} - \frac{11\alpha_2^2}{45} - \frac{484\alpha_3^2}{405}\right)\right]\right\},
\end{aligned} \tag{A3}$$

$$\begin{aligned}
Z_{\alpha_2} = 1 &+ \frac{\alpha_2}{4\pi}\frac{1}{\epsilon}\left\{-\frac{43}{6} + \frac{4n_G}{3}\right\} + \frac{\alpha_2}{(4\pi)^2}\left\{\frac{1}{\epsilon^2}\left[\frac{1849\alpha_2}{36} - \frac{172n_G\alpha_2}{9} + \frac{16n_G^2\alpha_2}{9}\right]\right.\\
&\left.+\frac{1}{\epsilon}\left[\frac{3\alpha_1}{20} - \frac{259\alpha_2}{12} - \frac{3\mathrm{tr}\hat{T}}{4} - \frac{3\mathrm{tr}\hat{B}}{4} - \frac{\mathrm{tr}\hat{L}}{4} + n_G\left(\frac{\alpha_1}{10} + \frac{49\alpha_2}{6} + 2\alpha_3\right)\right]\right\}\\
&+\frac{\alpha_2}{(4\pi)^3}\left\{\frac{1}{\epsilon^3}\left[-\frac{79507\alpha_2^2}{216} + \frac{1849n_G\alpha_2^2}{9} - \frac{344n_G^2\alpha_2^2}{9} + \frac{64n_G^3\alpha_2^2}{27}\right]\right.
\end{aligned}$$

$$
\begin{aligned}
&+\frac{1}{\epsilon^2}\bigg[\frac{\alpha_1^2}{200}-\frac{43\alpha_1\alpha_2}{20}+\frac{77959\alpha_2^2}{216}+\frac{17\alpha_1\mathrm{tr}\hat{T}}{80}+\frac{181\alpha_2\mathrm{tr}\hat{T}}{16}+2\alpha_3\mathrm{tr}\hat{T}+\frac{\alpha_1\mathrm{tr}\hat{B}}{16}\\
&+\frac{181\alpha_2\mathrm{tr}\hat{B}}{16}+2\alpha_3\mathrm{tr}\hat{B}+\frac{3\alpha_1\mathrm{tr}\hat{L}}{16}+\frac{181\alpha_2\mathrm{tr}\hat{L}}{48}-\frac{3\mathrm{tr}\hat{B}^2}{8}-\frac{3(\mathrm{tr}\hat{B})^2}{4}-\frac{\mathrm{tr}\hat{B}\mathrm{tr}\hat{L}}{2}\\
&-\frac{(\mathrm{tr}\hat{L})^2}{12}+\frac{3\mathrm{tr}\hat{T}\hat{B}}{4}-\frac{3\mathrm{tr}\hat{T}^2}{8}-\frac{3\mathrm{tr}\hat{T}\mathrm{tr}\hat{B}}{2}-\frac{\mathrm{tr}\hat{T}\mathrm{tr}\hat{L}}{2}-\frac{3(\mathrm{tr}\hat{T})^2}{4}-\frac{\mathrm{tr}\hat{L}^2}{8}\\
&+n_G\bigg(\frac{7\alpha_1^2}{100}-\frac{31\alpha_1\alpha_2}{30}-\frac{22001\alpha_2^2}{108}-\frac{86\alpha_2\alpha_3}{3}-\frac{22\alpha_3^2}{3}-2\alpha_2\mathrm{tr}\hat{T}-2\alpha_2\mathrm{tr}\hat{B}-\frac{2\alpha_2\mathrm{tr}\hat{L}}{3}\bigg)\\
&+n_G^2\bigg(\frac{2\alpha_1^2}{45}+\frac{4\alpha_1\alpha_2}{15}+\frac{686\alpha_2^2}{27}+\frac{16\alpha_2\alpha_3}{3}+\frac{8\alpha_3^2}{9}\bigg)\bigg]\\
&+\frac{1}{\epsilon}\bigg[\frac{163\alpha_1^2}{4800}+\frac{187\alpha_1\alpha_2}{160}-\frac{667111\alpha_2^2}{5184}+\frac{\alpha_1\hat{\lambda}}{10}+\frac{\alpha_2\hat{\lambda}}{2}-\hat{\lambda}^2-\frac{593\alpha_1\mathrm{tr}\hat{T}}{480}-\frac{243\alpha_2\mathrm{tr}\hat{T}}{32}\\
&-\frac{7\alpha_3\mathrm{tr}\hat{T}}{3}-\frac{533\alpha_1\mathrm{tr}\hat{B}}{480}-\frac{243\alpha_2\mathrm{tr}\hat{B}}{32}-\frac{7\alpha_3\mathrm{tr}\hat{B}}{3}-\frac{17\alpha_1\mathrm{tr}\hat{L}}{32}-\frac{81\alpha_2\mathrm{tr}\hat{L}}{32}+\frac{19\mathrm{tr}\hat{B}^2}{16}\\
&+\frac{15(\mathrm{tr}\hat{B})^2}{8}+\frac{5\mathrm{tr}\hat{B}\mathrm{tr}\hat{L}}{4}+\frac{19\mathrm{tr}\hat{L}^2}{48}+\frac{5(\mathrm{tr}\hat{L})^2}{24}+\frac{9\mathrm{tr}\hat{T}\hat{B}}{8}+\frac{19\mathrm{tr}\hat{T}^2}{16}+\frac{15\mathrm{tr}\hat{T}\mathrm{tr}\hat{B}}{4}\\
&+\frac{5\mathrm{tr}\hat{T}\mathrm{tr}\hat{L}}{4}+\frac{15(\mathrm{tr}\hat{T})^2}{8}+n_G\bigg(-\frac{7\alpha_1^2}{45}+\frac{13\alpha_1\alpha_2}{60}+\frac{6412\alpha_2^2}{81}-\frac{\alpha_1\alpha_3}{45}+\frac{13\alpha_2\alpha_3}{3}+\frac{125\alpha_3^2}{9}\bigg)\\
&+n_G^2\bigg(-\frac{11\alpha_1^2}{135}-\frac{415\alpha_2^2}{81}-\frac{44\alpha_3^2}{27}\bigg)\bigg]\bigg\}, \qquad\qquad \text{(A4)}
\end{aligned}
$$

$$
\begin{aligned}
Z_{\alpha_3}=\;&1+\frac{\alpha_3}{4\pi}\frac{1}{\epsilon}\bigg\{-11+\frac{4n_G}{3}\bigg\}+\frac{\alpha_3}{(4\pi)^2}\bigg\{\frac{1}{\epsilon^2}\bigg[121\alpha_3-\frac{88n_G\alpha_3}{3}+\frac{16n_G^2\alpha_3}{9}\bigg]\\
&+\frac{1}{\epsilon}\bigg[-51\alpha_3-\mathrm{tr}\hat{T}-\mathrm{tr}\hat{B}+n_G\bigg(\frac{11\alpha_1}{60}+\frac{3\alpha_2}{4}+\frac{38\alpha_3}{3}\bigg)\bigg]\bigg\}\\
&+\frac{\alpha_3}{(4\pi)^3}\bigg\{\frac{1}{\epsilon^3}\bigg[-1331\alpha_3^2+484n_G\alpha_3^2-\frac{176n_G^2\alpha_3^2}{3}+\frac{64n_G^3\alpha_3^2}{27}\bigg]\\
&+\frac{1}{\epsilon^2}\bigg[1309\alpha_3^2+\frac{17\alpha_1\mathrm{tr}\hat{T}}{60}+\frac{3\alpha_2\mathrm{tr}\hat{T}}{4}+\frac{74\alpha_3\mathrm{tr}\hat{T}}{3}+\frac{\alpha_1\mathrm{tr}\hat{B}}{12}+\frac{3\alpha_2\mathrm{tr}\hat{B}}{4}+\frac{74\alpha_3\mathrm{tr}\hat{B}}{3}\\
&-\frac{\mathrm{tr}\hat{B}^2}{2}-(\mathrm{tr}\hat{B})^2-\frac{\mathrm{tr}\hat{B}\mathrm{tr}\hat{L}}{3}+\mathrm{tr}\hat{T}\hat{B}-\frac{\mathrm{tr}\hat{T}^2}{2}-2\mathrm{tr}\hat{T}\mathrm{tr}\hat{B}-\frac{\mathrm{tr}\hat{T}\mathrm{tr}\hat{L}}{3}-(\mathrm{tr}\hat{T})^2\\
&+n_G\bigg(\frac{11\alpha_1^2}{1800}-\frac{43\alpha_2^2}{24}-\frac{121\alpha_1\alpha_3}{30}-\frac{33\alpha_2\alpha_3}{2}-\frac{4354\alpha_3^2}{9}-\frac{8\alpha_3\mathrm{tr}\hat{T}}{3}-\frac{8\alpha_3\mathrm{tr}\hat{B}}{3}\bigg)\\
&+n_G^2\bigg(\frac{11\alpha_1^2}{135}+\frac{\alpha_2^2}{3}+\frac{22\alpha_1\alpha_3}{45}+2\alpha_2\alpha_3+\frac{1064\alpha_3^2}{27}\bigg)\bigg]\\
&+\frac{1}{\epsilon}\bigg[-\frac{2857\alpha_3^2}{6}-\frac{101\alpha_1\mathrm{tr}\hat{T}}{120}-\frac{31\alpha_2\mathrm{tr}\hat{T}}{8}-\frac{40\alpha_3\mathrm{tr}\hat{T}}{3}-\frac{89\alpha_1\mathrm{tr}\hat{B}}{120}-\frac{31\alpha_2\mathrm{tr}\hat{B}}{8}-\frac{40\alpha_3\mathrm{tr}\hat{B}}{3}\\
&+\frac{3\mathrm{tr}\hat{B}^2}{2}+\frac{7(\mathrm{tr}\hat{B})^2}{2}+\frac{7\mathrm{tr}\hat{B}\mathrm{tr}\hat{L}}{6}-\mathrm{tr}\hat{T}\hat{B}+\frac{3\mathrm{tr}\hat{T}^2}{2}+7\mathrm{tr}\hat{T}\mathrm{tr}\hat{B}+\frac{7\mathrm{tr}\hat{T}\mathrm{tr}\hat{L}}{6}+\frac{7(\mathrm{tr}\hat{T})^2}{2}
\end{aligned}
$$

$$+ n_G\left(- \frac{13\alpha_1^2}{360} - \frac{\alpha_1\alpha_2}{120} + \frac{241\alpha_2^2}{72} + \frac{77\alpha_1\alpha_3}{135} + \frac{7\alpha_2\alpha_3}{3} + \frac{5033\alpha_3^2}{27}\right)$$
$$+ n_G^2\left(- \frac{121\alpha_1^2}{810} - \frac{11\alpha_2^2}{18} - \frac{650\alpha_3^2}{81}\right)\Bigg]\Bigg\}, \tag{A5}$$

$$Z_B = 1 + \frac{\alpha_1}{4\pi}\frac{1}{\epsilon}\left\{ - \frac{1}{10} - \frac{4n_G}{3}\right\} + \frac{\alpha_1}{(4\pi)^2}\frac{1}{\epsilon}\Bigg\{ - \frac{9\alpha_1}{100} - \frac{9\alpha_2}{20} + \frac{17\mathrm{tr}\hat{T}}{20} + \frac{\mathrm{tr}\hat{B}}{4} + \frac{3\mathrm{tr}\hat{L}}{4}$$
$$+ n_G\left[- \frac{19\alpha_1}{30} - \frac{3\alpha_2}{10} - \frac{22\alpha_3}{15}\right]\Bigg\}$$
$$+ \frac{\alpha_1}{(4\pi)^3}\Bigg\{\frac{1}{\epsilon^2}\Bigg[- \frac{3\alpha_1^2}{1000} + \frac{43\alpha_2^2}{40} - \frac{289\alpha_1\mathrm{tr}\hat{T}}{1200} - \frac{51\alpha_2\mathrm{tr}\hat{T}}{80} - \frac{34\alpha_3\mathrm{tr}\hat{T}}{15} - \frac{\alpha_1\mathrm{tr}\hat{B}}{48}$$
$$- \frac{3\alpha_2\mathrm{tr}\hat{B}}{16} - \frac{2\alpha_3\mathrm{tr}\hat{B}}{3} - \frac{9\alpha_1\mathrm{tr}\hat{L}}{16} - \frac{9\alpha_2\mathrm{tr}\hat{L}}{16} + \frac{\mathrm{tr}\hat{B}^2}{8} + \frac{(\mathrm{tr}\hat{B})^2}{4} + \frac{5\mathrm{tr}\hat{B}\mathrm{tr}\hat{L}}{6} + \frac{3\mathrm{tr}\hat{L}^2}{8}$$
$$+ \frac{(\mathrm{tr}\hat{L})^2}{4} - \frac{11\mathrm{tr}\hat{T}\hat{B}}{20} + \frac{17\mathrm{tr}\hat{T}^2}{40} + \frac{11\mathrm{tr}\hat{T}\mathrm{tr}\hat{B}}{10} + \frac{31\mathrm{tr}\hat{T}\mathrm{tr}\hat{L}}{30} + \frac{17(\mathrm{tr}\hat{T})^2}{20}$$
$$+ n_G\left(- \frac{11\alpha_1^2}{180} + \frac{31\alpha_2^2}{60} + \frac{242\alpha_3^2}{45}\right) + n_G^2\left(- \frac{38\alpha_1^2}{135} - \frac{2\alpha_2^2}{15} - \frac{88\alpha_3^2}{135}\right)\Bigg]$$
$$+ \frac{1}{\epsilon}\Bigg[- \frac{163\alpha_1^2}{8000} - \frac{261\alpha_1\alpha_2}{800} - \frac{3401\alpha_2^2}{960} - \frac{9\alpha_1\hat{\lambda}}{50} - \frac{3\alpha_2\hat{\lambda}}{10} + \frac{3\hat{\lambda}^2}{5}$$
$$+ \frac{2827\alpha_1\mathrm{tr}\hat{T}}{2400} + \frac{157\alpha_2\mathrm{tr}\hat{T}}{32} + \frac{29\alpha_3\mathrm{tr}\hat{T}}{15} + \frac{1267\alpha_1\mathrm{tr}\hat{B}}{2400} + \frac{437\alpha_2\mathrm{tr}\hat{B}}{160} + \frac{17\alpha_3\mathrm{tr}\hat{B}}{15}$$
$$+ \frac{843\alpha_1\mathrm{tr}\hat{L}}{800} + \frac{543\alpha_2\mathrm{tr}\hat{L}}{160} - \frac{61\mathrm{tr}\hat{B}^2}{80} - \frac{17(\mathrm{tr}\hat{B})^2}{40} - \frac{157\mathrm{tr}\hat{B}\mathrm{tr}\hat{L}}{60} - \frac{87\mathrm{tr}\hat{L}^2}{80}$$
$$- \frac{33(\mathrm{tr}\hat{L})^2}{40} - \frac{\mathrm{tr}\hat{T}\hat{B}}{8} - \frac{113\mathrm{tr}\hat{T}^2}{80} - \frac{59\mathrm{tr}\hat{T}\mathrm{tr}\hat{B}}{20} - \frac{199\mathrm{tr}\hat{T}\mathrm{tr}\hat{L}}{60} - \frac{101(\mathrm{tr}\hat{T})^2}{40}$$
$$+ n_G\left(\frac{58\alpha_1^2}{225} + \frac{7\alpha_1\alpha_2}{300} - \frac{83\alpha_2^2}{90} + \frac{137\alpha_1\alpha_3}{675} + \frac{\alpha_2\alpha_3}{15} - \frac{275\alpha_3^2}{27}\right)$$
$$+ n_G^2\left(\frac{209\alpha_1^2}{405} + \frac{11\alpha_2^2}{45} + \frac{484\alpha_3^2}{405}\right)\Bigg]\Bigg\}, \tag{A6}$$

$$Z_W = 1 + \frac{\alpha_2}{4\pi}\frac{1}{\epsilon}\left\{\frac{25}{6} - \frac{4n_G}{3} - \xi_W\right\}$$
$$+ \frac{\alpha_2}{(4\pi)^2}\Bigg\{\frac{1}{\epsilon^2}\Bigg[- \frac{25\alpha_2}{4} - \frac{8\xi_W\alpha_2}{3} + \xi_W^2\alpha_2 + n_G\left(2\alpha_2 + \frac{4\xi_W\alpha_2}{3}\right)\Bigg]$$
$$+ \frac{1}{\epsilon}\Bigg[- \frac{3\alpha_1}{20} + \frac{113\alpha_2}{8} + \frac{3\mathrm{tr}\hat{T}}{4} + \frac{3\mathrm{tr}\hat{B}}{4} + \frac{\mathrm{tr}\hat{L}}{4} - \frac{11\xi_W\alpha_2}{4} - \frac{\xi_W^2\alpha_2}{2}$$
$$+ n_G\left(- \frac{\alpha_1}{10} - \frac{13\alpha_2}{2} - 2\alpha_3\right)\Bigg]\Bigg\}$$
$$+ \frac{\alpha_2}{(4\pi)^3}\Bigg\{\frac{1}{\epsilon^3}\Bigg[\frac{1525\alpha_2^2}{72} + \frac{8n_G^2\alpha_2^2}{9} + \frac{89\xi_W\alpha_2^2}{12} + \frac{7\xi_W^2\alpha_2^2}{6} - \xi_W^3\alpha_2^2$$

$$
\begin{aligned}
&+ n_G\biggl(- \frac{86\alpha_2^2}{9} - \frac{10\xi_W\alpha_2^2}{3} - \frac{4\xi_W^2\alpha_2^2}{3}\biggr)\biggr] \\
&+ \frac{1}{\epsilon^2}\biggl[- \frac{\alpha_1^2}{200} + \frac{3\alpha_1\alpha_2}{20} - \frac{29629\alpha_2^2}{432} - \frac{17\alpha_1\mathrm{tr}\hat{T}}{80} - \frac{21\alpha_2\mathrm{tr}\hat{T}}{16} - 2\alpha_3\mathrm{tr}\hat{T} \\
&- \frac{\alpha_1\mathrm{tr}\hat{B}}{16} - \frac{21\alpha_2\mathrm{tr}\hat{B}}{16} - 2\alpha_3\mathrm{tr}\hat{B} - \frac{3\alpha_1\mathrm{tr}\hat{L}}{16} - \frac{7\alpha_2\mathrm{tr}\hat{L}}{16} + \frac{3\mathrm{tr}\hat{B}^2}{8} + \frac{3(\mathrm{tr}\hat{B})^2}{4} \\
&+ \frac{\mathrm{tr}\hat{B}\mathrm{tr}\hat{L}}{2} + \frac{\mathrm{tr}\hat{L}^2}{8} + \frac{(\mathrm{tr}\hat{L})^2}{12} - \frac{3\mathrm{tr}\hat{T}\hat{B}}{4} + \frac{3\mathrm{tr}\hat{T}^2}{8} + \frac{3\mathrm{tr}\hat{T}\mathrm{tr}\hat{B}}{2} + \frac{\mathrm{tr}\hat{T}\mathrm{tr}\hat{L}}{2} + \frac{3(\mathrm{tr}\hat{T})^2}{4} \\
&+ \xi_W\biggl(\frac{3\alpha_1\alpha_2}{20} - \frac{271\alpha_2^2}{24} - \frac{3\alpha_2\mathrm{tr}\hat{T}}{4} - \frac{3\alpha_2\mathrm{tr}\hat{B}}{4} - \frac{\alpha_2\mathrm{tr}\hat{L}}{4}\biggr) + \frac{53\xi_W^2\alpha_2^2}{12} + \frac{7\xi_W^3\alpha_2^2}{6} \\
&+ n_G\biggl(- \frac{7\alpha_1^2}{100} + \frac{\alpha_1\alpha_2}{10} + \frac{4273\alpha_2^2}{108} + 2\alpha_2\alpha_3 + \frac{22\alpha_3^2}{3} \\
&+ \xi_W\biggl(\frac{\alpha_1\alpha_2}{10} + \frac{47\alpha_2^2}{6} + 2\alpha_2\alpha_3\biggr) + \frac{2\xi_W^2\alpha_2^2}{3}\biggr) + n_G^2\biggl(- \frac{2\alpha_1^2}{45} - \frac{118\alpha_2^2}{27} - \frac{8\alpha_3^2}{9}\biggr)\biggr] \\
&+ \frac{1}{\epsilon}\biggl[- \frac{163\alpha_1^2}{4800} - \frac{11\alpha_1\alpha_2}{32} - \frac{3\zeta_3\alpha_1\alpha_2}{10} + \frac{143537\alpha_2^2}{1728} + \frac{\zeta_3\alpha_2^2}{2} - \frac{\alpha_1\hat{\lambda}}{10} - \frac{\alpha_2\hat{\lambda}}{2} + \hat{\lambda}^2 \\
&+ \frac{593\alpha_1\mathrm{tr}\hat{T}}{480} + \frac{79\alpha_2\mathrm{tr}\hat{T}}{32} + \frac{7\alpha_3\mathrm{tr}\hat{T}}{3} + \frac{533\alpha_1\mathrm{tr}\hat{B}}{480} + \frac{79\alpha_2\mathrm{tr}\hat{B}}{32} + \frac{7\alpha_3\mathrm{tr}\hat{B}}{3} + \frac{17\alpha_1\mathrm{tr}\hat{L}}{32} \\
&+ \frac{79\alpha_2\mathrm{tr}\hat{L}}{96} - \frac{19\mathrm{tr}\hat{B}^2}{16} - \frac{15(\mathrm{tr}\hat{B})^2}{8} - \frac{5\mathrm{tr}\hat{B}\mathrm{tr}\hat{L}}{4} - \frac{19\mathrm{tr}\hat{L}^2}{48} - \frac{5(\mathrm{tr}\hat{L})^2}{24} - \frac{9\mathrm{tr}\hat{T}\hat{B}}{8} \\
&- \frac{19\mathrm{tr}\hat{T}^2}{16} - \frac{15\mathrm{tr}\hat{T}\mathrm{tr}\hat{B}}{4} - \frac{5\mathrm{tr}\hat{T}\mathrm{tr}\hat{L}}{4} - \frac{15(\mathrm{tr}\hat{T})^2}{8} \\
&+ \xi_W\biggl(- \frac{105\alpha_2^2}{8} - 2\zeta_3\alpha_2^2\biggr) + \xi_W^2\biggl(- \frac{11\alpha_2^2}{4} - \frac{\zeta_3\alpha_2^2}{2}\biggr) - \frac{7\xi_W^3\alpha_2^2}{12} \\
&+ n_G\biggl(\frac{7\alpha_1^2}{45} + \frac{8\alpha_1\alpha_2}{15} - \frac{4\zeta_3\alpha_1\alpha_2}{5} - \frac{7025\alpha_2^2}{108} + 12\zeta_3\alpha_2^2 + \frac{\alpha_1\alpha_3}{45} + \frac{32\alpha_2\alpha_3}{3} \\
&- 16\zeta_3\alpha_2\alpha_3 - \frac{125\alpha_3^2}{9} + \frac{8\xi_W\alpha_2^2}{3}\biggr) + n_G^2\biggl(\frac{11\alpha_1^2}{135} + \frac{185\alpha_2^2}{27} + \frac{44\alpha_3^2}{27}\biggr)\biggr]\biggr\}\,, \qquad \text{(A7)}
\end{aligned}
$$

$$
\begin{aligned}
Z_G &= 1 + \frac{\alpha_3}{4\pi}\frac{1}{\epsilon}\biggl\{\frac{13}{2} - \frac{4n_G}{3} - \frac{3\xi_G}{2}\biggr\} \\
&+ \frac{\alpha_3}{(4\pi)^2}\biggl\{\frac{1}{\epsilon^2}\biggl[- \frac{117\alpha_3}{8} - \frac{51\xi_G\alpha_3}{8} + \frac{9\xi_G^2\alpha_3}{4} + n_G\biggl(3\alpha_3 + 2\xi_G\alpha_3\biggr)\biggr] \\
&+ \frac{1}{\epsilon}\biggl[+ \frac{531\alpha_3}{16} + \mathrm{tr}\hat{T} + \mathrm{tr}\hat{B} - \frac{99\xi_G\alpha_3}{16} - \frac{9\xi_G^2\alpha_3}{8} + n_G\biggl(- \frac{11\alpha_1}{60} - \frac{3\alpha_2}{4} - \frac{61\alpha_3}{6}\biggr)\biggr]\biggr\} \\
&+ \frac{\alpha_3}{(4\pi)^3}\biggl\{\frac{1}{\epsilon^3}\biggl[\frac{1209\alpha_3^2}{16} + \frac{4n_G^2\alpha_3^2}{3} + \frac{423\xi_G\alpha_3^2}{16} + \frac{9\xi_G^2\alpha_3^2}{2} - \frac{27\xi_G^3\alpha_3^2}{8} \\
&+ n_G\biggl(- 22\alpha_3^2 - \frac{15\xi_G\alpha_3^2}{2} - 3\xi_G^2\alpha_3^2\biggr)\biggr] \\
&+ \frac{1}{\epsilon^2}\biggl[- \frac{7957\alpha_3^2}{32} - \frac{17\alpha_1\mathrm{tr}\hat{T}}{60} - \frac{3\alpha_2\mathrm{tr}\hat{T}}{4} - \frac{25\alpha_3\mathrm{tr}\hat{T}}{6} - \frac{\alpha_1\mathrm{tr}\hat{B}}{12} - \frac{3\alpha_2\mathrm{tr}\hat{B}}{4} - \frac{25\alpha_3\mathrm{tr}\hat{B}}{6}
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{\mathrm{tr}\hat{B}^2}{2}+(\mathrm{tr}\hat{B})^2+\frac{\mathrm{tr}\hat{B}\mathrm{tr}\hat{L}}{3}-\mathrm{tr}\hat{T}\hat{B}+\frac{\mathrm{tr}\hat{T}^2}{2}+2\mathrm{tr}\hat{T}\mathrm{tr}\hat{B}+\frac{\mathrm{tr}\hat{T}\mathrm{tr}\hat{L}}{3}+(\mathrm{tr}\hat{T})^2\\
&+\xi_G\bigg(-\frac{1287\alpha_3^2}{32}-\frac{3\alpha_3\mathrm{tr}\hat{T}}{2}-\frac{3\alpha_3\mathrm{tr}\hat{B}}{2}\bigg)+\frac{117\xi_G^2\alpha_3^2}{8}+\frac{63\xi_G^3\alpha_3^2}{16}\\
&+n_G\bigg(-\frac{11\alpha_1^2}{1800}+\frac{43\alpha_2^2}{24}+\frac{11\alpha_1\alpha_3}{40}+\frac{9\alpha_2\alpha_3}{8}+\frac{1691\alpha_3^2}{18}\\
&+\xi_G\bigg(\frac{11\alpha_1\alpha_3}{40}+\frac{9\alpha_2\alpha_3}{8}+\frac{73\alpha_3^2}{4}\bigg)+\frac{3\xi_G^2\alpha_3^2}{2}\bigg)+n_G^2\bigg(-\frac{11\alpha_1^2}{135}-\frac{\alpha_2^2}{3}-\frac{182\alpha_3^2}{27}\bigg)\bigg]\\
&+\frac{1}{\epsilon}\bigg[\frac{9965\alpha_3^2}{32}-\frac{81\zeta_3\alpha_3^2}{16}+\frac{101\alpha_1\mathrm{tr}\hat{T}}{120}+\frac{31\alpha_2\mathrm{tr}\hat{T}}{8}+\frac{91\alpha_3\mathrm{tr}\hat{T}}{12}+\frac{89\alpha_1\mathrm{tr}\hat{B}}{120}+\frac{31\alpha_2\mathrm{tr}\hat{B}}{8}\\
&+\frac{91\alpha_3\mathrm{tr}\hat{B}}{12}-\frac{3\mathrm{tr}\hat{B}^2}{2}-\frac{7(\mathrm{tr}\hat{B})^2}{2}-\frac{7\mathrm{tr}\hat{B}\mathrm{tr}\hat{L}}{6}+\mathrm{tr}\hat{T}\hat{B}-\frac{3\mathrm{tr}\hat{T}^2}{2}-7\mathrm{tr}\hat{T}\mathrm{tr}\hat{B}-\frac{7\mathrm{tr}\hat{T}\mathrm{tr}\hat{L}}{6}\\
&-\frac{7(\mathrm{tr}\hat{T})^2}{2}+\xi_G\bigg(-\frac{1503\alpha_3^2}{32}-\frac{27\zeta_3\alpha_3^2}{4}\bigg)+\xi_G^2\bigg(-\frac{297\alpha_3^2}{32}-\frac{27\zeta_3\alpha_3^2}{16}\bigg)-\frac{63\xi_G^3\alpha_3^2}{32}\\
&+n_G\bigg(\frac{13\alpha_1^2}{360}+\frac{\alpha_1\alpha_2}{120}-\frac{241\alpha_2^2}{72}+\frac{3223\alpha_1\alpha_3}{2160}-\frac{11\zeta_3\alpha_1\alpha_3}{5}+\frac{293\alpha_2\alpha_3}{48}-9\zeta_3\alpha_2\alpha_3\\
&-\frac{8155\alpha_3^2}{54}+22\zeta_3\alpha_3^2+6\xi_G\alpha_3^2\bigg)+n_G^2\bigg(\frac{121\alpha_1^2}{810}+\frac{11\alpha_2^2}{18}+\frac{860\alpha_3^2}{81}\bigg)\bigg]\bigg\}\,, \qquad \text{(A8)}
\end{aligned}
$$

$$
\begin{aligned}
Z_{c_W}&=1+\frac{\alpha_2}{4\pi}\frac{1}{\epsilon}\bigg\{\frac{3}{2}-\frac{\xi_W}{2}\bigg\}\\
&+\frac{\alpha_2}{(4\pi)^2}\bigg\{\frac{1}{\epsilon^2}\bigg[-\frac{17\alpha_2}{4}+n_G\alpha_2+\frac{3\xi_W^2\alpha_2}{8}\bigg]+\frac{1}{\epsilon}\bigg[\frac{179\alpha_2}{48}-\frac{5n_G\alpha_2}{6}+\frac{\xi_W\alpha_2}{8}\bigg]\bigg\}\\
&+\frac{\alpha_2}{(4\pi)^3}\bigg\{\frac{1}{\epsilon^3}\bigg[\frac{1309\alpha_2^2}{72}+\frac{8n_G^2\alpha_2^2}{9}+\frac{17\xi_W\alpha_2^2}{24}-\frac{9\xi_W^2\alpha_2^2}{16}-\frac{5\xi_W^3\alpha_2^2}{16}\\
&+n_G\bigg(-\frac{145\alpha_2^2}{18}-\frac{\xi_W\alpha_2^2}{6}\bigg)\bigg]+\frac{1}{\epsilon^2}\bigg[\frac{3\alpha_1\alpha_2}{20}-\frac{29209\alpha_2^2}{864}-\frac{20n_G^2\alpha_2^2}{27}-\frac{3\alpha_2\mathrm{tr}\hat{T}}{4}-\frac{3\alpha_2\mathrm{tr}\hat{B}}{4}\\
&-\frac{\alpha_2\mathrm{tr}\hat{L}}{4}+\frac{37\xi_W\alpha_2^2}{96}+\frac{13\xi_W^2\alpha_2^2}{16}+\frac{\xi_W^3\alpha_2^2}{6}+n_G\bigg(\frac{\alpha_1\alpha_2}{10}+\frac{1535\alpha_2^2}{108}+2\alpha_2\alpha_3-\frac{\xi_W\alpha_2^2}{12}\bigg)\bigg]\\
&+\frac{1}{\epsilon}\bigg[-\frac{33\alpha_1\alpha_2}{80}+\frac{3\zeta_3\alpha_1\alpha_2}{20}+\frac{59125\alpha_2^2}{2592}-\frac{70n_G^2\alpha_2^2}{81}-\frac{\zeta_3\alpha_2^2}{4}+\frac{41\alpha_2\mathrm{tr}\hat{T}}{16}+\frac{41\alpha_2\mathrm{tr}\hat{B}}{16}\\
&+\frac{41\alpha_2\mathrm{tr}\hat{L}}{48}+\xi_W\bigg(-\frac{29\alpha_2^2}{24}+\zeta_3\alpha_2^2\bigg)+\xi_W^2\bigg(-\frac{\alpha_2^2}{4}+\frac{\zeta_3\alpha_2^2}{4}\bigg)-\frac{\xi_W^3\alpha_2^2}{8}\\
&+n_G\bigg(-\frac{3\alpha_1\alpha_2}{8}+\frac{2\zeta_3\alpha_1\alpha_2}{5}-\frac{4573\alpha_2^2}{648}-6\zeta_3\alpha_2^2-\frac{15\alpha_2\alpha_3}{2}+8\zeta_3\alpha_2\alpha_3+\frac{7\xi_W\alpha_2^2}{6}\bigg)\bigg]\bigg\}\,, \qquad \text{(A9)}
\end{aligned}
$$

$$
Z_{c_G}=1+\frac{\alpha_3}{4\pi}\frac{1}{\epsilon}\bigg\{\frac{9}{4}-\frac{3\xi_G}{4}\bigg\}
$$

$$+\frac{\alpha_3}{(4\pi)^2}\left\{\frac{1}{\epsilon^2}\left[-\frac{315\alpha_3}{32}+\frac{3n_G\alpha_3}{2}+\frac{27\xi_G^2\alpha_3}{32}\right]+\frac{1}{\epsilon}\left[\frac{285\alpha_3}{32}-\frac{5n_G\alpha_3}{4}+\frac{9\xi_G\alpha_3}{32}\right]\right\}$$
$$+\frac{\alpha_3}{(4\pi)^3}\left\{\frac{1}{\epsilon^3}\left[\frac{8295\alpha_3^2}{128}+\frac{4n_G^2\alpha_3^2}{3}+\frac{315\xi_G\alpha_3^2}{128}-\frac{243\xi_G^2\alpha_3^2}{128}-\frac{135\xi_G^3\alpha_3^2}{128}\right.\right.$$
$$\left.+n_G\left(-\frac{149\alpha_3^2}{8}-\frac{3\xi_G\alpha_3^2}{8}\right)\right]+\frac{1}{\epsilon^2}\left[-\frac{15587\alpha_3^2}{128}-\frac{10n_G^2\alpha_3^2}{9}-\frac{3\alpha_3\mathrm{tr}\hat{T}}{2}-\frac{3\alpha_3\mathrm{tr}\hat{B}}{2}\right.$$
$$\left.+\frac{45\xi_G\alpha_3^2}{32}+\frac{351\xi_G^2\alpha_3^2}{128}+\frac{9\xi_G^3\alpha_3^2}{16}+n_G\left(\frac{11\alpha_1\alpha_3}{40}+\frac{9\alpha_2\alpha_3}{8}+\frac{1597\alpha_3^2}{48}-\frac{3\xi_G\alpha_3^2}{16}\right)\right]$$
$$+\frac{1}{\epsilon}\left[\frac{15817\alpha_3^2}{192}-\frac{35n_G^2\alpha_3^2}{27}+\frac{81\zeta_3\alpha_3^2}{32}+\frac{23\alpha_3\mathrm{tr}\hat{T}}{8}+\frac{23\alpha_3\mathrm{tr}\hat{B}}{8}\right.$$
$$+\xi_G\left(-\frac{153\alpha_3^2}{32}+\frac{27\zeta_3\alpha_3^2}{8}\right)+\xi_G^2\left(-\frac{27\alpha_3^2}{32}+\frac{27\zeta_3\alpha_3^2}{32}\right)-\frac{27\xi_G^3\alpha_3^2}{64}$$
$$\left.\left.+n_G\left(-\frac{33\alpha_1\alpha_3}{32}+\frac{11\zeta_3\alpha_1\alpha_3}{10}-\frac{135\alpha_2\alpha_3}{32}+\frac{9\zeta_3\alpha_2\alpha_3}{2}-\frac{637\alpha_3^2}{36}+\frac{21\xi_G\alpha_3^2}{8}-11\zeta_3\alpha_3^2\right)\right]\right\}, \tag{A10}$$

$$Z_H=1+\frac{1}{4\pi}\frac{1}{\epsilon}\left\{\frac{9\alpha_1}{20}+\frac{9\alpha_2}{4}-3\mathrm{tr}\hat{T}-3\mathrm{tr}\hat{B}-\mathrm{tr}\hat{L}-\frac{3\xi_B\alpha_1}{20}-\frac{3\xi_W\alpha_2}{4}\right\}$$
$$+\frac{1}{(4\pi)^2}\left\{\frac{1}{\epsilon^2}\left[\frac{99\alpha_1^2}{800}+\frac{81\alpha_1\alpha_2}{80}-\frac{177\alpha_2^2}{32}-\frac{3\alpha_1\mathrm{tr}\hat{T}}{40}-\frac{27\alpha_2\mathrm{tr}\hat{T}}{8}+12\alpha_3\mathrm{tr}\hat{T}-\frac{39\alpha_1\mathrm{tr}\hat{B}}{40}\right.\right.$$
$$-\frac{27\alpha_2\mathrm{tr}\hat{B}}{8}+12\alpha_3\mathrm{tr}\hat{B}+\frac{27\alpha_1\mathrm{tr}\hat{L}}{40}-\frac{9\alpha_2\mathrm{tr}\hat{L}}{8}-\frac{9\mathrm{tr}\hat{B}^2}{4}-\frac{3\mathrm{tr}\hat{L}^2}{4}+\frac{9\mathrm{tr}\hat{T}\hat{B}}{2}-\frac{9\mathrm{tr}\hat{T}^2}{4}$$
$$+\xi_B\left(-\frac{27\alpha_1^2}{400}-\frac{27\alpha_1\alpha_2}{80}+\frac{9\xi_W\alpha_1\alpha_2}{80}+\frac{9\alpha_1\mathrm{tr}\hat{T}}{20}+\frac{9\alpha_1\mathrm{tr}\hat{B}}{20}+\frac{3\alpha_1\mathrm{tr}\hat{L}}{20}\right)+\frac{9\xi_B^2\alpha_1^2}{800}$$
$$+\xi_W\left(-\frac{27\alpha_1\alpha_2}{80}-\frac{9\alpha_2^2}{16}+\frac{9\alpha_2\mathrm{tr}\hat{T}}{4}+\frac{9\alpha_2\mathrm{tr}\hat{B}}{4}+\frac{3\alpha_2\mathrm{tr}\hat{L}}{4}\right)+\frac{21\xi_W^2\alpha_2^2}{32}$$
$$\left.+n_G\left(\frac{3\alpha_1^2}{10}+\frac{3\alpha_2^2}{2}\right)\right]$$
$$+\frac{1}{\epsilon}\left[-\frac{93\alpha_1^2}{1600}-\frac{27\alpha_1\alpha_2}{160}+\frac{511\alpha_2^2}{64}-3\hat{\lambda}^2-\frac{17\alpha_1\mathrm{tr}\hat{T}}{16}-\frac{45\alpha_2\mathrm{tr}\hat{T}}{16}-10\alpha_3\mathrm{tr}\hat{T}-\frac{5\alpha_1\mathrm{tr}\hat{B}}{16}\right.$$
$$-\frac{45\alpha_2\mathrm{tr}\hat{B}}{16}-10\alpha_3\mathrm{tr}\hat{B}-\frac{15\alpha_1\mathrm{tr}\hat{L}}{16}-\frac{15\alpha_2\mathrm{tr}\hat{L}}{16}+\frac{27\mathrm{tr}\hat{B}^2}{8}+\frac{9\mathrm{tr}\hat{L}^2}{8}-\frac{3\mathrm{tr}\hat{T}\hat{B}}{4}+\frac{27\mathrm{tr}\hat{T}^2}{8}$$
$$\left.\left.-\frac{3\xi_W\alpha_2^2}{2}-\frac{3\xi_W^2\alpha_2^2}{16}+n_G\left(-\frac{\alpha_1^2}{4}-\frac{5\alpha_2^2}{4}\right)\right]\right\}$$
$$+\frac{1}{(4\pi)^3}\left\{\frac{1}{\epsilon^3}\left[\frac{429\alpha_1^3}{16000}+\frac{891\alpha_1^2\alpha_2}{3200}-\frac{1593\alpha_1\alpha_2^2}{640}+\frac{8555\alpha_2^3}{384}-\frac{93\alpha_1^2\mathrm{tr}\hat{T}}{800}-\frac{9\alpha_1\alpha_2\mathrm{tr}\hat{T}}{16}\right.\right.$$
$$+\frac{435\alpha_2^2\mathrm{tr}\hat{T}}{32}-\frac{7\alpha_1\alpha_3\mathrm{tr}\hat{T}}{5}+9\alpha_2\alpha_3\mathrm{tr}\hat{T}-76\alpha_3^2\mathrm{tr}\hat{T}-\frac{177\alpha_1^2\mathrm{tr}\hat{B}}{800}-\frac{99\alpha_1\alpha_2\mathrm{tr}\hat{B}}{80}$$
$$+\frac{435\alpha_2^2\mathrm{tr}\hat{B}}{32}+\frac{17\alpha_1\alpha_3\mathrm{tr}\hat{B}}{5}+9\alpha_2\alpha_3\mathrm{tr}\hat{B}-76\alpha_3^2\mathrm{tr}\hat{B}-\frac{339\alpha_1^2\mathrm{tr}\hat{L}}{800}+\frac{27\alpha_1\alpha_2\mathrm{tr}\hat{L}}{80}$$

$$
\begin{aligned}
&+\frac{145\alpha_2^2\mathrm{tr}\hat{L}}{32}-\frac{9\mathrm{tr}\hat{B}^3}{4}-\frac{9\alpha_1\mathrm{tr}\hat{B}^2}{20}+18\alpha_3\mathrm{tr}\hat{B}^2-\frac{9\mathrm{tr}\hat{T}\mathrm{tr}\hat{B}^2}{4}-\frac{9\mathrm{tr}\hat{B}\mathrm{tr}\hat{B}^2}{4}-\frac{3\mathrm{tr}\hat{L}\mathrm{tr}\hat{B}^2}{4}\\
&-\frac{3\mathrm{tr}\hat{L}^3}{4}+\frac{27\alpha_1\mathrm{tr}\hat{L}^2}{20}-\frac{3\mathrm{tr}\hat{T}\mathrm{tr}\hat{L}^2}{4}-\frac{3\mathrm{tr}\hat{B}\mathrm{tr}\hat{L}^2}{4}-\frac{\mathrm{tr}\hat{L}\mathrm{tr}\hat{L}^2}{4}+\frac{9\mathrm{tr}\hat{T}^2\hat{B}}{4}-\frac{9\mathrm{tr}\hat{T}^3}{4}\\
&-\frac{9\alpha_1\mathrm{tr}\hat{T}\hat{B}}{20}-36\alpha_3\mathrm{tr}\hat{T}\hat{B}+\frac{9\mathrm{tr}\hat{T}\mathrm{tr}\hat{T}\hat{B}}{2}+\frac{9\mathrm{tr}\hat{B}\mathrm{tr}\hat{T}\hat{B}}{2}+\frac{3\mathrm{tr}\hat{L}\mathrm{tr}\hat{T}\hat{B}}{2}+\frac{9\mathrm{tr}\hat{T}\hat{B}^2}{4}\\
&+\frac{9\alpha_1\mathrm{tr}\hat{T}^2}{10}+18\alpha_3\mathrm{tr}\hat{T}^2-\frac{9\mathrm{tr}\hat{T}\mathrm{tr}\hat{T}^2}{4}-\frac{9\mathrm{tr}\hat{B}\mathrm{tr}\hat{T}^2}{4}-\frac{3\mathrm{tr}\hat{L}\mathrm{tr}\hat{T}^2}{4}\\
&+\xi_B\Bigg(-\frac{297\alpha_1^3}{16000}-\frac{243\alpha_1^2\alpha_2}{1600}+\frac{531\alpha_1\alpha_2^2}{640}-\frac{63\xi_W^2\alpha_1\alpha_2^2}{640}+\frac{9\alpha_1^2\mathrm{tr}\hat{T}}{800}+\frac{81\alpha_1\alpha_2\mathrm{tr}\hat{T}}{160}\\
&-\frac{9\alpha_1\alpha_3\mathrm{tr}\hat{T}}{5}+\frac{117\alpha_1^2\mathrm{tr}\hat{B}}{800}+\frac{81\alpha_1\alpha_2\mathrm{tr}\hat{B}}{160}-\frac{9\alpha_1\alpha_3\mathrm{tr}\hat{B}}{5}-\frac{81\alpha_1^2\mathrm{tr}\hat{L}}{800}+\frac{27\alpha_1\alpha_2\mathrm{tr}\hat{L}}{160}\\
&+\frac{27\alpha_1\mathrm{tr}\hat{B}^2}{80}+\frac{9\alpha_1\mathrm{tr}\hat{L}^2}{80}-\frac{27\alpha_1\mathrm{tr}\hat{T}\hat{B}}{40}+\frac{27\alpha_1\mathrm{tr}\hat{T}^2}{80}\\
&+\xi_W\Bigg(\frac{81\alpha_1^2\alpha_2}{1600}+\frac{27\alpha_1\alpha_2^2}{320}-\frac{27\alpha_1\alpha_2\mathrm{tr}\hat{T}}{80}-\frac{27\alpha_1\alpha_2\mathrm{tr}\hat{B}}{80}-\frac{9\alpha_1\alpha_2\mathrm{tr}\hat{L}}{80}\Bigg)\Bigg)\\
&+\xi_W\Bigg(-\frac{297\alpha_1^2\alpha_2}{3200}-\frac{81\alpha_1\alpha_2^2}{320}+\frac{367\alpha_2^3}{128}+\frac{9\alpha_1\alpha_2\mathrm{tr}\hat{T}}{160}-\frac{27\alpha_2^2\mathrm{tr}\hat{T}}{32}-9\alpha_2\alpha_3\mathrm{tr}\hat{T}\\
&+\frac{117\alpha_1\alpha_2\mathrm{tr}\hat{B}}{160}-\frac{27\alpha_2^2\mathrm{tr}\hat{B}}{32}-9\alpha_2\alpha_3\mathrm{tr}\hat{B}-\frac{81\alpha_1\alpha_2\mathrm{tr}\hat{L}}{160}-\frac{9\alpha_2^2\mathrm{tr}\hat{L}}{32}+\frac{27\alpha_2\mathrm{tr}\hat{B}^2}{16}\\
&+\frac{9\alpha_2\mathrm{tr}\hat{L}^2}{16}-\frac{27\alpha_2\mathrm{tr}\hat{T}\hat{B}}{8}+\frac{27\alpha_2\mathrm{tr}\hat{T}^2}{16}\Bigg)\\
&+\xi_B^2\Bigg(\frac{81\alpha_1^3}{16000}+\frac{81\alpha_1^2\alpha_2}{3200}-\frac{27\xi_W\alpha_1^2\alpha_2}{3200}-\frac{27\alpha_1^2\mathrm{tr}\hat{T}}{800}-\frac{27\alpha_1^2\mathrm{tr}\hat{B}}{800}-\frac{9\alpha_1^2\mathrm{tr}\hat{L}}{800}\Bigg)\\
&+\xi_W^2\Bigg(\frac{189\alpha_1\alpha_2^2}{640}-\frac{63\alpha_2^3}{128}-\frac{63\alpha_2^2\mathrm{tr}\hat{T}}{32}-\frac{63\alpha_2^2\mathrm{tr}\hat{B}}{32}-\frac{21\alpha_2^2\mathrm{tr}\hat{L}}{32}\Bigg)-\frac{9\xi_B^3\alpha_1^3}{16000}-\frac{77\xi_W^3\alpha_2^3}{128}\\
&+n_G\Bigg(\frac{7\alpha_1^3}{40}+\frac{27\alpha_1^2\alpha_2}{40}+\frac{27\alpha_1\alpha_2^2}{40}-\frac{263\alpha_2^3}{24}-\frac{\alpha_1^2\mathrm{tr}\hat{T}}{3}-3\alpha_2^2\mathrm{tr}\hat{T}+\frac{16\alpha_3^2\mathrm{tr}\hat{T}}{3}-\frac{11\alpha_1^2\mathrm{tr}\hat{B}}{15}\\
&-3\alpha_2^2\mathrm{tr}\hat{B}+\frac{16\alpha_3^2\mathrm{tr}\hat{B}}{3}+\frac{\alpha_1^2\mathrm{tr}\hat{L}}{5}-\alpha_2^2\mathrm{tr}\hat{L}+\xi_B\Bigg(-\frac{9\alpha_1^3}{200}-\frac{9\alpha_1\alpha_2^2}{40}\Bigg)\\
&+\xi_W\Bigg(-\frac{9\alpha_1^2\alpha_2}{40}-\frac{5\alpha_2^3}{8}\Bigg)\Bigg)+n_G^2\Bigg(\frac{4\alpha_1^3}{15}+\frac{4\alpha_2^3}{3}\Bigg)\Bigg]\\
&+\frac{1}{\epsilon^2}\Bigg[-\frac{97\alpha_1^3}{32000}-\frac{99\alpha_1^2\alpha_2}{1280}+\frac{4917\alpha_1\alpha_2^2}{1280}-\frac{121093\alpha_2^3}{2304}-\frac{27\alpha_1^2\hat{\lambda}}{200}-\frac{9\alpha_1\alpha_2\hat{\lambda}}{20}-\frac{9\alpha_2^2\hat{\lambda}}{8}\\
&+\frac{9\alpha_1\hat{\lambda}^2}{20}+\frac{9\alpha_2\hat{\lambda}^2}{4}-24\hat{\lambda}^3-\frac{541\alpha_1^2\mathrm{tr}\hat{T}}{1600}-\frac{177\alpha_1\alpha_2\mathrm{tr}\hat{T}}{160}-\frac{885\alpha_2^2\mathrm{tr}\hat{T}}{64}-\frac{\alpha_1\alpha_3\mathrm{tr}\hat{T}}{10}\\
&-\frac{33\alpha_2\alpha_3\mathrm{tr}\hat{T}}{2}+198\alpha_3^2\mathrm{tr}\hat{T}-9\hat{\lambda}^2\mathrm{tr}\hat{T}+\frac{191\alpha_1^2\mathrm{tr}\hat{B}}{1600}+\frac{57\alpha_1\alpha_2\mathrm{tr}\hat{B}}{160}-\frac{885\alpha_2^2\mathrm{tr}\hat{B}}{64}\\
&-\frac{49\alpha_1\alpha_3\mathrm{tr}\hat{B}}{10}-\frac{33\alpha_2\alpha_3\mathrm{tr}\hat{B}}{2}+198\alpha_3^2\mathrm{tr}\hat{B}-9\hat{\lambda}^2\mathrm{tr}\hat{B}-\frac{3\alpha_1^2\mathrm{tr}\hat{L}}{1600}-\frac{45\alpha_1\alpha_2\mathrm{tr}\hat{L}}{32}
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{295\alpha_2^2\mathrm{tr}\hat{L}}{64}-3\hat{\lambda}^2\mathrm{tr}\hat{L}+\frac{15\mathrm{tr}\hat{B}^3}{8}-\frac{123\alpha_1\mathrm{tr}\hat{B}^2}{80}-\frac{117\alpha_2\mathrm{tr}\hat{B}^2}{16}-39\alpha_3\mathrm{tr}\hat{B}^2+18\hat{\lambda}\mathrm{tr}\hat{B}^2\\
&+\frac{81\mathrm{tr}\hat{B}\mathrm{tr}\hat{B}^2}{8}+\frac{27\mathrm{tr}\hat{L}\mathrm{tr}\hat{B}^2}{8}+\frac{5\mathrm{tr}\hat{L}^3}{8}-\frac{261\alpha_1\mathrm{tr}\hat{L}^2}{80}-\frac{39\alpha_2\mathrm{tr}\hat{L}^2}{16}+6\hat{\lambda}\mathrm{tr}\hat{L}^2\\
&+\frac{27\mathrm{tr}\hat{T}\mathrm{tr}\hat{L}^2}{8}+\frac{27\mathrm{tr}\hat{B}\mathrm{tr}\hat{L}^2}{8}+\frac{9\mathrm{tr}\hat{L}\mathrm{tr}\hat{L}^2}{8}+\frac{15\mathrm{tr}\hat{T}^3}{8}+\frac{43\alpha_1\mathrm{tr}\hat{T}\hat{B}}{20}\\
&+\frac{9\alpha_2\mathrm{tr}\hat{T}\hat{B}}{8}+46\alpha_3\mathrm{tr}\hat{T}\hat{B}-\frac{11\mathrm{tr}\hat{L}\mathrm{tr}\hat{T}\hat{B}}{4}-\frac{297\alpha_1\mathrm{tr}\hat{T}^2}{80}\\
&-\frac{117\alpha_2\mathrm{tr}\hat{T}^2}{16}-39\alpha_3\mathrm{tr}\hat{T}^2+18\hat{\lambda}\mathrm{tr}\hat{T}^2+\frac{81\mathrm{tr}\hat{T}\mathrm{tr}\hat{T}^2}{8}+\frac{27\mathrm{tr}\hat{L}\mathrm{tr}\hat{T}^2}{8}\\
&+\xi_B\Bigg(\frac{279\alpha_1^3}{32000}+\frac{81\alpha_1^2\alpha_2}{3200}-\frac{1533\alpha_1\alpha_2^2}{1280}+\frac{9\xi_W\alpha_1\alpha_2^2}{40}+\frac{9\xi_W^2\alpha_1\alpha_2^2}{320}+\frac{9\alpha_1\hat{\lambda}^2}{20}+\frac{51\alpha_1^2\mathrm{tr}\hat{T}}{320}\\
&+\frac{27\alpha_1\alpha_2\mathrm{tr}\hat{T}}{64}+\frac{3\alpha_1\alpha_3\mathrm{tr}\hat{T}}{2}+\frac{3\alpha_1^2\mathrm{tr}\hat{B}}{64}+\frac{27\alpha_1\alpha_2\mathrm{tr}\hat{B}}{64}+\frac{3\alpha_1\alpha_3\mathrm{tr}\hat{B}}{2}+\frac{9\alpha_1^2\mathrm{tr}\hat{L}}{64}\\
&+\frac{9\alpha_1\alpha_2\mathrm{tr}\hat{L}}{64}-\frac{81\alpha_1\mathrm{tr}\hat{B}^2}{160}-\frac{27\alpha_1\mathrm{tr}\hat{L}^2}{160}+\frac{9\alpha_1\mathrm{tr}\hat{T}\hat{B}}{80}-\frac{81\alpha_1\mathrm{tr}\hat{T}^2}{160}\Bigg)\\
&+\xi_W\Bigg(\frac{279\alpha_1^2\alpha_2}{6400}-\frac{351\alpha_1\alpha_2^2}{640}-\frac{141\alpha_2^3}{256}+\frac{9\alpha_2\hat{\lambda}^2}{4}+\frac{51\alpha_1\alpha_2\mathrm{tr}\hat{T}}{64}+\frac{423\alpha_2^2\mathrm{tr}\hat{T}}{64}\\
&+\frac{15\alpha_2\alpha_3\mathrm{tr}\hat{T}}{2}+\frac{15\alpha_1\alpha_2\mathrm{tr}\hat{B}}{64}+\frac{423\alpha_2^2\mathrm{tr}\hat{B}}{64}+\frac{15\alpha_2\alpha_3\mathrm{tr}\hat{B}}{2}+\frac{45\alpha_1\alpha_2\mathrm{tr}\hat{L}}{64}+\frac{141\alpha_2^2\mathrm{tr}\hat{L}}{64}\\
&-\frac{81\alpha_2\mathrm{tr}\hat{B}^2}{32}-\frac{27\alpha_2\mathrm{tr}\hat{L}^2}{32}+\frac{9\alpha_2\mathrm{tr}\hat{T}\hat{B}}{16}-\frac{81\alpha_2\mathrm{tr}\hat{T}^2}{32}\Bigg)\\
&+\xi_W^2\Bigg(-\frac{27\alpha_1\alpha_2^2}{320}+\frac{189\alpha_2^3}{64}+\frac{9\alpha_2^2\mathrm{tr}\hat{T}}{16}+\frac{9\alpha_2^2\mathrm{tr}\hat{B}}{16}+\frac{3\alpha_2^2\mathrm{tr}\hat{L}}{16}\Bigg)+\frac{33\xi_W^3\alpha_2^3}{64}\\
&+n_G\Bigg(\frac{11\alpha_1^3}{1200}-\frac{219\alpha_1^2\alpha_2}{400}-\frac{39\alpha_1\alpha_2^2}{80}+\frac{3241\alpha_2^3}{144}+\frac{11\alpha_1^2\alpha_3}{25}+3\alpha_2^2\alpha_3-\frac{11\alpha_1^2\mathrm{tr}\hat{T}}{30}+\frac{3\alpha_2^2\mathrm{tr}\hat{T}}{2}\\
&-\frac{40\alpha_3^2\mathrm{tr}\hat{T}}{3}+\frac{19\alpha_1^2\mathrm{tr}\hat{B}}{30}+\frac{3\alpha_2^2\mathrm{tr}\hat{B}}{2}-\frac{40\alpha_3^2\mathrm{tr}\hat{B}}{3}-\frac{9\alpha_1^2\mathrm{tr}\hat{L}}{10}+\frac{\alpha_2^2\mathrm{tr}\hat{L}}{2}+\xi_B\Bigg(\frac{3\alpha_1^3}{80}+\frac{3\alpha_1\alpha_2^2}{16}\Bigg)\\
&+\xi_W\Bigg(\frac{3\alpha_1^2\alpha_2}{16}-\frac{9\alpha_2^3}{16}\Bigg)\Bigg)+n_G^2\Bigg(-\frac{2\alpha_1^3}{9}-\frac{10\alpha_2^3}{9}\Bigg)-3\alpha_b\alpha_t^2-3\alpha_b^2\alpha_t\Bigg]\\
&+\frac{1}{\epsilon}\Bigg[-\frac{413\alpha_1^3}{6000}+\frac{27\zeta_3\alpha_1^3}{2000}-\frac{279\alpha_1^2\alpha_2}{800}-\frac{27\zeta_3\alpha_1^2\alpha_2}{400}-\frac{51\alpha_1\alpha_2^2}{64}+\frac{9\zeta_3\alpha_1\alpha_2^2}{80}+\frac{70519\alpha_2^3}{1728}\\
&+\frac{69\zeta_3\alpha_2^3}{16}-\frac{117\alpha_1^2\hat{\lambda}}{400}+\frac{27\zeta_3\alpha_1^2\hat{\lambda}}{50}-\frac{39\alpha_1\alpha_2\hat{\lambda}}{40}+\frac{9\zeta_3\alpha_1\alpha_2\hat{\lambda}}{5}-\frac{39\alpha_2^2\hat{\lambda}}{16}+\frac{9\zeta_3\alpha_2^2\hat{\lambda}}{2}\\
&-3\alpha_1\hat{\lambda}^2-15\alpha_2\hat{\lambda}^2+12\hat{\lambda}^3+\frac{52831\alpha_1^2\mathrm{tr}\hat{T}}{28800}-\frac{\zeta_3\alpha_1^2\mathrm{tr}\hat{T}}{100}-\frac{371\alpha_1\alpha_2\mathrm{tr}\hat{T}}{320}\\
&-\frac{27\zeta_3\alpha_1\alpha_2\mathrm{tr}\hat{T}}{10}-\frac{2433\alpha_2^2\mathrm{tr}\hat{T}}{128}+\frac{63\zeta_3\alpha_2^2\mathrm{tr}\hat{T}}{4}+\frac{2419\alpha_1\alpha_3\mathrm{tr}\hat{T}}{180}-\frac{68\zeta_3\alpha_1\alpha_3\mathrm{tr}\hat{T}}{5}\\
&+\frac{163\alpha_2\alpha_3\mathrm{tr}\hat{T}}{4}-36\zeta_3\alpha_2\alpha_3\mathrm{tr}\hat{T}-\frac{910\alpha_3^2\mathrm{tr}\hat{T}}{9}+8\zeta_3\alpha_3^2\mathrm{tr}\hat{T}+\frac{45\hat{\lambda}^2\mathrm{tr}\hat{T}}{2}+\frac{27\alpha_1(\mathrm{tr}\hat{T})^2}{20}
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{27\alpha_2(\mathrm{tr}\hat{T})^2}{4}+\frac{5479\alpha_1^2\mathrm{tr}\hat{B}}{28800}+\frac{29\zeta_3\alpha_1^2\mathrm{tr}\hat{B}}{100}-\frac{671\alpha_1\alpha_2\mathrm{tr}\hat{B}}{320}+\frac{9\zeta_3\alpha_1\alpha_2\mathrm{tr}\hat{B}}{5}\\
&-\frac{2433\alpha_2^2\mathrm{tr}\hat{B}}{128}+\frac{63\zeta_3\alpha_2^2\mathrm{tr}\hat{B}}{4}+\frac{991\alpha_1\alpha_3\mathrm{tr}\hat{B}}{180}-4\zeta_3\alpha_1\alpha_3\mathrm{tr}\hat{B}+\frac{163\alpha_2\alpha_3\mathrm{tr}\hat{B}}{4}\\
&-36\zeta_3\alpha_2\alpha_3\mathrm{tr}\hat{B}-\frac{910\alpha_3^2\mathrm{tr}\hat{B}}{9}+8\zeta_3\alpha_3^2\mathrm{tr}\hat{B}+\frac{45\hat{\lambda}^2\mathrm{tr}\hat{B}}{2}+\frac{27\alpha_1\mathrm{tr}\hat{T}\mathrm{tr}\hat{B}}{10}+\frac{27\alpha_2\mathrm{tr}\hat{T}\mathrm{tr}\hat{B}}{2}\\
&+\frac{27\alpha_1(\mathrm{tr}\hat{B})^2}{20}+\frac{27\alpha_2(\mathrm{tr}\hat{B})^2}{4}+\frac{8517\alpha_1^2\mathrm{tr}\hat{L}}{3200}-\frac{117\zeta_3\alpha_1^2\mathrm{tr}\hat{L}}{100}+\frac{411\alpha_1\alpha_2\mathrm{tr}\hat{L}}{320}-\frac{18\zeta_3\alpha_1\alpha_2\mathrm{tr}\hat{L}}{5}\\
&-\frac{811\alpha_2^2\mathrm{tr}\hat{L}}{128}+\frac{21\zeta_3\alpha_2^2\mathrm{tr}\hat{L}}{4}+\frac{15\hat{\lambda}^2\mathrm{tr}\hat{L}}{2}+\frac{9\alpha_1\mathrm{tr}\hat{T}\mathrm{tr}\hat{L}}{10}+\frac{9\alpha_2\mathrm{tr}\hat{T}\mathrm{tr}\hat{L}}{2}+\frac{9\alpha_1\mathrm{tr}\hat{B}\mathrm{tr}\hat{L}}{10}\\
&+\frac{9\alpha_2\mathrm{tr}\hat{B}\mathrm{tr}\hat{L}}{2}+\frac{3\alpha_1(\mathrm{tr}\hat{L})^2}{20}+\frac{3\alpha_2(\mathrm{tr}\hat{L})^2}{4}+\frac{25\mathrm{tr}\hat{B}^3}{16}-3\zeta_3\mathrm{tr}\hat{B}^3+\frac{303\alpha_1\mathrm{tr}\hat{B}^2}{80}-\frac{9\zeta_3\alpha_1\mathrm{tr}\hat{B}^2}{5}\\
&+\frac{279\alpha_2\mathrm{tr}\hat{B}^2}{16}-9\zeta_3\alpha_2\mathrm{tr}\hat{B}^2-\frac{5\alpha_3\mathrm{tr}\hat{B}^2}{2}+24\zeta_3\alpha_3\mathrm{tr}\hat{B}^2-15\hat{\lambda}\mathrm{tr}\hat{B}^2\\
&-18\mathrm{tr}\hat{B}\mathrm{tr}\hat{B}^2-6\mathrm{tr}\hat{L}\mathrm{tr}\hat{B}^2+\frac{25\mathrm{tr}\hat{L}^3}{48}-\zeta_3\mathrm{tr}\hat{L}^3+\frac{33\alpha_1\mathrm{tr}\hat{L}^2}{80}+\frac{9\zeta_3\alpha_1\mathrm{tr}\hat{L}^2}{5}+\frac{93\alpha_2\mathrm{tr}\hat{L}^2}{16}\\
&-3\zeta_3\alpha_2\mathrm{tr}\hat{L}^2-5\hat{\lambda}\mathrm{tr}\hat{L}^2-6\mathrm{tr}\hat{T}\mathrm{tr}\hat{L}^2-6\mathrm{tr}\hat{B}\mathrm{tr}\hat{L}^2-2\mathrm{tr}\hat{L}\mathrm{tr}\hat{L}^2+\frac{25\mathrm{tr}\hat{T}^3}{16}\\
&-3\zeta_3\mathrm{tr}\hat{T}^3+\frac{31\alpha_1\mathrm{tr}\hat{T}\hat{B}}{40}-\frac{8\zeta_3\alpha_1\mathrm{tr}\hat{T}\hat{B}}{5}+\frac{21\alpha_2\mathrm{tr}\hat{T}\hat{B}}{8}-19\alpha_3\mathrm{tr}\hat{T}\hat{B}+16\zeta_3\alpha_3\mathrm{tr}\hat{T}\hat{B}\\
&+\frac{\mathrm{tr}\hat{L}\mathrm{tr}\hat{T}\hat{B}}{2}+\frac{211\alpha_1\mathrm{tr}\hat{T}^2}{80}+\frac{3\zeta_3\alpha_1\mathrm{tr}\hat{T}^2}{5}\\
&+\frac{279\alpha_2\mathrm{tr}\hat{T}^2}{16}-9\zeta_3\alpha_2\mathrm{tr}\hat{T}^2-\frac{5\alpha_3\mathrm{tr}\hat{T}^2}{2}+24\zeta_3\alpha_3\mathrm{tr}\hat{T}^2-15\hat{\lambda}\mathrm{tr}\hat{T}^2-18\mathrm{tr}\hat{T}\mathrm{tr}\hat{T}^2\\
&-6\mathrm{tr}\hat{L}\mathrm{tr}\hat{T}^2-\frac{507\xi_W\alpha_2^3}{64}+\xi_W^2\left(-\frac{39\alpha_2^3}{32}-\frac{3\zeta_3\alpha_2^3}{8}\right)-\frac{5\xi_W^3\alpha_2^3}{16}\\
&+n_G\left(-\frac{158\alpha_1^3}{225}+\frac{19\zeta_3\alpha_1^3}{25}-\frac{3\alpha_1^2\alpha_2}{40}+\frac{9\zeta_3\alpha_1^2\alpha_2}{25}-\frac{3\alpha_1\alpha_2^2}{10}+\frac{3\zeta_3\alpha_1\alpha_2^2}{5}-\frac{2381\alpha_2^3}{216}\right.\\
&-15\zeta_3\alpha_2^3-\frac{33\alpha_1^2\alpha_3}{20}+\frac{44\zeta_3\alpha_1^2\alpha_3}{25}-\frac{45\alpha_2^2\alpha_3}{4}+12\zeta_3\alpha_2^2\alpha_3+\frac{127\alpha_1^2\mathrm{tr}\hat{T}}{120}+\frac{21\alpha_2^2\mathrm{tr}\hat{T}}{8}\\
&\left.+\frac{32\alpha_3^2\mathrm{tr}\hat{T}}{3}+\frac{31\alpha_1^2\mathrm{tr}\hat{B}}{120}+\frac{21\alpha_2^2\mathrm{tr}\hat{B}}{8}+\frac{32\alpha_3^2\mathrm{tr}\hat{B}}{3}+\frac{39\alpha_1^2\mathrm{tr}\hat{L}}{40}+\frac{7\alpha_2^2\mathrm{tr}\hat{L}}{8}+\frac{17\xi_W\alpha_2^3}{8}\right)\\
&\left.\left.+n_G^2\left(-\frac{7\alpha_1^3}{27}-\frac{35\alpha_2^3}{27}\right)-\frac{277\alpha_t\alpha_b^2}{16}-\frac{277\alpha_b\alpha_t^2}{16}\right]\right\}, \qquad \text{(A11)}
\end{aligned}
$$

$$
\begin{aligned}
Z_{cCW} &= 1-\frac{\xi_W\alpha_2}{4\pi}\frac{1}{\epsilon}+\frac{\xi_W\alpha_2}{(4\pi)^2}\left\{\frac{1}{\epsilon}\left[-\frac{5\alpha_2}{4}-\frac{\xi_W\alpha_2}{4}\right]+\frac{1}{\epsilon^2}\left[\frac{3\alpha_2}{2}+\xi_W\alpha_2\right]\right\}\\
&+\frac{\xi_W\alpha_2}{(4\pi)^3}\left\{\frac{1}{\epsilon^3}\left[-\frac{61\alpha_2^2}{12}+\frac{2n_G\alpha_2^2}{3}-3\xi_W\alpha_2^2-\xi_W^2\alpha_2^2\right]\right.\\
&+\frac{1}{\epsilon^2}\left[\frac{221\alpha_2^2}{24}-\frac{5n_G\alpha_2^2}{3}+4\xi_W\alpha_2^2+\frac{3\xi_W^2\alpha_2^2}{4}\right]
\end{aligned}
$$

$$
+\frac{1}{\epsilon}\bigg[-\frac{373\alpha_2^2}{48}+\frac{5n_G\alpha_2^2}{2}-\frac{13\xi_W\alpha_2^2}{8}-\frac{5\xi_W^2\alpha_2^2}{12}\bigg]\bigg\}, \tag{A12}
$$

$$
\begin{aligned}
Z_{cCG} = 1 &- \frac{3}{2}\frac{\xi_G\alpha_3}{4\pi}\frac{1}{\epsilon}+\frac{\xi_G\alpha_3}{(4\pi)^2}\bigg\{\frac{1}{\epsilon^2}\bigg[\frac{27\alpha_3}{8}+\frac{9\xi_G\alpha_3}{4}\bigg]+\frac{1}{\epsilon}\bigg[-\frac{45\alpha_3}{16}-\frac{9\xi_G\alpha_3}{16}\bigg]\bigg\}\\
&+\frac{\xi_G\alpha_3}{(4\pi)^3}\bigg\{\frac{1}{\epsilon^3}\bigg[-\frac{279\alpha_3^2}{16}+\frac{3n_G\alpha_3^2}{2}-\frac{81\xi_G\alpha_3^2}{8}-\frac{27\xi_G^2\alpha_3^2}{8}\bigg]\\
&+\frac{1}{\epsilon^2}\bigg[\frac{1035\alpha_3^2}{32}-\frac{15n_G\alpha_3^2}{4}+\frac{27\xi_G\alpha_3^2}{2}+\frac{81\xi_G^2\alpha_3^2}{32}\bigg]\\
&+\frac{1}{\epsilon}\bigg[-\frac{1809\alpha_3^2}{64}+\frac{45n_G\alpha_3^2}{8}-\frac{351\xi_G\alpha_3^2}{64}-\frac{45\xi_G^2\alpha_3^2}{32}\bigg]\bigg\},
\end{aligned} \tag{A13}
$$

$$
\begin{aligned}
Z_{WWW} = 1 &+ \frac{1}{\epsilon}\frac{\alpha_2}{4\pi}\bigg\{\frac{8}{3}-\frac{4n_G}{3}-\frac{3\xi_W}{2}\bigg\}\\
&+\frac{\alpha_2}{(4\pi)^2}\bigg\{\frac{1}{\epsilon^2}\bigg[-6\alpha_2-\frac{7\xi_W\alpha_2}{4}+\frac{15\xi_W^2\alpha_2}{8}+n_G\Big(3\alpha_2+2\xi_W\alpha_2\Big)\bigg]\\
&+\frac{1}{\epsilon}\bigg[-\frac{3\alpha_1}{20}+\frac{499\alpha_2}{48}+\frac{3\mathrm{tr}\hat{T}}{4}+\frac{3\mathrm{tr}\hat{B}}{4}+\frac{\mathrm{tr}\hat{L}}{4}-\frac{33\xi_W\alpha_2}{8}-\frac{3\xi_W^2\alpha_2}{4}\\
&+n_G\Big(-\frac{\alpha_1}{10}-\frac{17\alpha_2}{3}-2\alpha_3\Big)\bigg]\bigg\}\\
&+\frac{\alpha_2}{(4\pi)^3}\bigg\{\frac{1}{\epsilon^3}\bigg[\frac{70\alpha_2^2}{3}+\frac{4n_G^2\alpha_2^2}{3}+\frac{59\xi_W\alpha_2^2}{8}-\frac{5\xi_W^2\alpha_2^2}{8}-\frac{35\xi_W^3\alpha_2^2}{16}\\
&+n_G\Big(-\frac{43\alpha_2^2}{3}-\frac{13\xi_W\alpha_2^2}{2}-\frac{5\xi_W^2\alpha_2^2}{2}\Big)\bigg]\\
&+\frac{1}{\epsilon^2}\bigg[-\frac{\alpha_1^2}{200}+\frac{9\alpha_1\alpha_2}{40}-\frac{26057\alpha_2^2}{432}-\frac{17\alpha_1\mathrm{tr}\hat{T}}{80}-\frac{27\alpha_2\mathrm{tr}\hat{T}}{16}-2\alpha_3\mathrm{tr}\hat{T}-\frac{\alpha_1\mathrm{tr}\hat{B}}{16}\\
&-\frac{27\alpha_2\mathrm{tr}\hat{B}}{16}-2\alpha_3\mathrm{tr}\hat{B}-\frac{3\alpha_1\mathrm{tr}\hat{L}}{16}-\frac{9\alpha_2\mathrm{tr}\hat{L}}{16}+\frac{3\mathrm{tr}\hat{B}^2}{8}+\frac{3(\mathrm{tr}\hat{B})^2}{4}+\frac{\mathrm{tr}\hat{B}\mathrm{tr}\hat{L}}{2}+\frac{\mathrm{tr}\hat{L}^2}{8}\\
&+\frac{(\mathrm{tr}\hat{L})^2}{12}-\frac{3\mathrm{tr}\hat{T}\hat{B}}{4}+\frac{3\mathrm{tr}\hat{T}^2}{8}+\frac{3\mathrm{tr}\hat{T}\mathrm{tr}\hat{B}}{2}+\frac{\mathrm{tr}\hat{T}\mathrm{tr}\hat{L}}{2}+\frac{3(\mathrm{tr}\hat{T})^2}{4}\\
&+\xi_W\Big(\frac{9\alpha_1\alpha_2}{40}-\frac{165\alpha_2^2}{32}-\frac{9\alpha_2\mathrm{tr}\hat{T}}{8}-\frac{9\alpha_2\mathrm{tr}\hat{B}}{8}-\frac{3\alpha_2\mathrm{tr}\hat{L}}{8}\Big)+\frac{157\xi_W^2\alpha_2^2}{16}+\frac{17\xi_W^3\alpha_2^2}{8}\\
&+n_G\Big(-\frac{7\alpha_1^2}{100}+\frac{3\alpha_1\alpha_2}{20}+\frac{4433\alpha_2^2}{108}+3\alpha_2\alpha_3+\frac{22\alpha_3^2}{3}\\
&+\xi_W\Big(\frac{3\alpha_1\alpha_2}{20}+\frac{21\alpha_2^2}{2}+3\alpha_2\alpha_3\Big)+\xi_W^2\alpha_2^2\Big)+n_G^2\Big(-\frac{2\alpha_1^2}{45}-\frac{128\alpha_2^2}{27}-\frac{8\alpha_3^2}{9}\Big)\bigg]\\
&+\frac{1}{\epsilon}\bigg[-\frac{163\alpha_1^2}{4800}+\frac{11\alpha_1\alpha_2}{160}-\frac{9\zeta_3\alpha_1\alpha_2}{20}+\frac{312361\alpha_2^2}{5184}+\frac{3\zeta_3\alpha_2^2}{4}-\frac{\alpha_1\hat{\lambda}}{10}-\frac{\alpha_2\hat{\lambda}}{2}+\hat{\lambda}^2\\
&+\frac{593\alpha_1\mathrm{tr}\hat{T}}{480}-\frac{3\alpha_2\mathrm{tr}\hat{T}}{32}+\frac{7\alpha_3\mathrm{tr}\hat{T}}{3}+\frac{533\alpha_1\mathrm{tr}\hat{B}}{480}-\frac{3\alpha_2\mathrm{tr}\hat{B}}{32}+\frac{7\alpha_3\mathrm{tr}\hat{B}}{3}+\frac{17\alpha_1\mathrm{tr}\hat{L}}{32}
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{\alpha_2\mathrm{tr}\hat{L}}{32}-\frac{19\mathrm{tr}\hat{B}^2}{16}-\frac{15(\mathrm{tr}\hat{B})^2}{8}-\frac{5\mathrm{tr}\hat{B}\mathrm{tr}\hat{L}}{4}-\frac{19\mathrm{tr}\hat{L}^2}{48}-\frac{5(\mathrm{tr}\hat{L})^2}{24}-\frac{9\mathrm{tr}\hat{T}\hat{B}}{8}\\
&-\frac{19\mathrm{tr}\hat{T}^2}{16}-\frac{15\mathrm{tr}\hat{T}\mathrm{tr}\hat{B}}{4}-\frac{5\mathrm{tr}\hat{T}\mathrm{tr}\hat{L}}{4}-\frac{15(\mathrm{tr}\hat{T})^2}{8}+\xi_W\bigg(-\frac{315\alpha_2^2}{16}-3\zeta_3\alpha_2^2\bigg)\\
&+\xi_W^2\bigg(-\frac{33\alpha_2^2}{8}-\frac{3\zeta_3\alpha_2^2}{4}\bigg)-\frac{7\xi_W^3\alpha_2^2}{8}+n_G\bigg(\frac{7\alpha_1^2}{45}+\frac{109\alpha_1\alpha_2}{120}-\frac{6\zeta_3\alpha_1\alpha_2}{5}\\
&-\frac{37577\alpha_2^2}{648}+18\zeta_3\alpha_2^2+\frac{\alpha_1\alpha_3}{45}+\frac{109\alpha_2\alpha_3}{6}-24\zeta_3\alpha_2\alpha_3-\frac{125\alpha_3^2}{9}+4\xi_W\alpha_2^2\bigg)\\
&+n_G^2\bigg(\frac{11\alpha_1^2}{135}+\frac{625\alpha_2^2}{81}+\frac{44\alpha_3^2}{27}\bigg)\bigg]\bigg\}, \qquad \text{(A14)}
\end{aligned}
$$

$$
\begin{aligned}
Z_{GGG}=1&+\frac{1}{\epsilon}\frac{\alpha_3}{4\pi}\bigg\{\frac{17}{4}-\frac{4n_G}{3}-\frac{9\xi_G}{4}\bigg\}\\
&+\frac{\alpha_3}{(4\pi)^2}\bigg\{\frac{1}{\epsilon^2}\bigg[-\frac{459\alpha_3}{32}-\frac{9\xi_G\alpha_3}{2}+\frac{135\xi_G^2\alpha_3}{32}+n_G\bigg(\frac{9\alpha_3}{2}+3\xi_G\alpha_3\bigg)\bigg]\\
&+\frac{1}{\epsilon}\bigg[\frac{777\alpha_3}{32}+\mathrm{tr}\hat{T}+\mathrm{tr}\hat{B}-\frac{297\xi_G\alpha_3}{32}-\frac{27\xi_G^2\alpha_3}{16}+n_G\bigg(-\frac{11\alpha_1}{60}-\frac{3\alpha_2}{4}-\frac{107\alpha_3}{12}\bigg)\bigg]\bigg\}\\
&+\frac{\alpha_3}{(4\pi)^3}\bigg\{\frac{1}{\epsilon^3}\bigg[\frac{10863\alpha_3^2}{128}+2n_G^2\alpha_3^2+\frac{3537\xi_G\alpha_3^2}{128}-\frac{135\xi_G^2\alpha_3^2}{128}-\frac{945\xi_G^3\alpha_3^2}{128}\\
&+n_G\bigg(-33\alpha_3^2-\frac{117\xi_G\alpha_3^2}{8}-\frac{45\xi_G^2\alpha_3^2}{8}\bigg)\bigg]\\
&+\frac{1}{\epsilon^2}\bigg[-\frac{28079\alpha_3^2}{128}-\frac{17\alpha_1\mathrm{tr}\hat{T}}{60}-\frac{3\alpha_2\mathrm{tr}\hat{T}}{4}-\frac{59\alpha_3\mathrm{tr}\hat{T}}{12}-\frac{\alpha_1\mathrm{tr}\hat{B}}{12}-\frac{3\alpha_2\mathrm{tr}\hat{B}}{4}-\frac{59\alpha_3\mathrm{tr}\hat{B}}{12}\\
&+\frac{\mathrm{tr}\hat{B}^2}{2}+(\mathrm{tr}\hat{B})^2+\frac{\mathrm{tr}\hat{B}\mathrm{tr}\hat{L}}{3}-\mathrm{tr}\hat{T}\hat{B}+\frac{\mathrm{tr}\hat{T}^2}{2}+2\mathrm{tr}\hat{T}\mathrm{tr}\hat{B}+\frac{\mathrm{tr}\hat{T}\mathrm{tr}\hat{L}}{3}+(\mathrm{tr}\hat{T})^2\\
&+\xi_G\bigg(-\frac{621\alpha_3^2}{32}-\frac{9\alpha_3\mathrm{tr}\hat{T}}{4}-\frac{9\alpha_3\mathrm{tr}\hat{B}}{4}\bigg)+\frac{4185\xi_G^2\alpha_3^2}{128}+\frac{459\xi_G^3\alpha_3^2}{64}\\
&+n_G\bigg(-\frac{11\alpha_1^2}{1800}+\frac{43\alpha_2^2}{24}+\frac{33\alpha_1\alpha_3}{80}+\frac{27\alpha_2\alpha_3}{16}+\frac{14101\alpha_3^2}{144}+\frac{9\xi_G^2\alpha_3^2}{4}\\
&+\xi_G\bigg(\frac{33\alpha_1\alpha_3}{80}+\frac{27\alpha_2\alpha_3}{16}+\frac{393\alpha_3^2}{16}\bigg)\bigg)+n_G^2\bigg(-\frac{11\alpha_1^2}{135}-\frac{\alpha_2^2}{3}-\frac{197\alpha_3^2}{27}\bigg)\bigg]\\
&+\frac{1}{\epsilon}\bigg[\frac{43973\alpha_3^2}{192}-\frac{243\zeta_3\alpha_3^2}{32}+\frac{101\alpha_1\mathrm{tr}\hat{T}}{120}+\frac{31\alpha_2\mathrm{tr}\hat{T}}{8}+\frac{113\alpha_3\mathrm{tr}\hat{T}}{24}+\frac{89\alpha_1\mathrm{tr}\hat{B}}{120}+\frac{31\alpha_2\mathrm{tr}\hat{B}}{8}\\
&+\frac{113\alpha_3\mathrm{tr}\hat{B}}{24}-\frac{3\mathrm{tr}\hat{B}^2}{2}-\frac{7(\mathrm{tr}\hat{B})^2}{2}-\frac{7\mathrm{tr}\hat{B}\mathrm{tr}\hat{L}}{6}+\mathrm{tr}\hat{T}\hat{B}-\frac{3\mathrm{tr}\hat{T}^2}{2}-7\mathrm{tr}\hat{T}\mathrm{tr}\hat{B}-\frac{7\mathrm{tr}\hat{T}\mathrm{tr}\hat{L}}{6}\\
&-\frac{7(\mathrm{tr}\hat{T})^2}{2}+\xi_G\bigg(-\frac{4509\alpha_3^2}{64}-\frac{81\zeta_3\alpha_3^2}{8}\bigg)+\xi_G^2\bigg(-\frac{891\alpha_3^2}{64}-\frac{81\zeta_3\alpha_3^2}{32}\bigg)-\frac{189\xi_G^3\alpha_3^2}{64}\\
&+n_G\bigg(\frac{13\alpha_1^2}{360}+\frac{\alpha_1\alpha_2}{120}-\frac{241\alpha_2^2}{72}+\frac{10901\alpha_1\alpha_3}{4320}-\frac{33\zeta_3\alpha_1\alpha_3}{10}+\frac{991\alpha_2\alpha_3}{96}-\frac{27\zeta_3\alpha_2\alpha_3}{2}
\end{aligned}
$$

$$- \frac{14399\alpha_3^2}{108} + 9\xi_G\alpha_3^2 + 33\zeta_3\alpha_3^2\Bigg) + n_G^2\Bigg(\frac{121\alpha_1^2}{810} + \frac{11\alpha_2^2}{18} + \frac{965\alpha_3^2}{81}\Bigg)\Bigg]\Bigg\}, \tag{A15}$$

$$\begin{aligned}
Z_{HHW} =& 1 + \frac{1}{4\pi}\frac{1}{\epsilon}\Bigg\{\frac{9\alpha_1}{20} + \frac{3\alpha_2}{4} - 3\mathrm{tr}\hat{T} - 3\mathrm{tr}\hat{B} - \mathrm{tr}\hat{L} - \frac{3\xi_B\alpha_1}{20} - \frac{5\xi_W\alpha_2}{4}\Bigg\} \\
&+ \frac{1}{(4\pi)^2}\Bigg\{\frac{1}{\epsilon^2}\Bigg[\frac{99\alpha_1^2}{800} + \frac{27\alpha_1\alpha_2}{80} - \frac{77\alpha_2^2}{32} - \frac{3\alpha_1\mathrm{tr}\hat{T}}{40} + \frac{9\alpha_2\mathrm{tr}\hat{T}}{8} + 12\alpha_3\mathrm{tr}\hat{T} - \frac{39\alpha_1\mathrm{tr}\hat{B}}{40} \\
&+ \frac{9\alpha_2\mathrm{tr}\hat{B}}{8} + 12\alpha_3\mathrm{tr}\hat{B} + \frac{27\alpha_1\mathrm{tr}\hat{L}}{40} + \frac{3\alpha_2\mathrm{tr}\hat{L}}{8} - \frac{9\mathrm{tr}\hat{B}^2}{4} - \frac{3\mathrm{tr}\hat{L}^2}{4} + \frac{9\mathrm{tr}\hat{T}\hat{B}}{2} - \frac{9\mathrm{tr}\hat{T}^2}{4} \\
&+ \xi_B\Bigg(-\frac{27\alpha_1^2}{400} - \frac{9\alpha_1\alpha_2}{80} + \frac{9\alpha_1\mathrm{tr}\hat{T}}{20} + \frac{9\alpha_1\mathrm{tr}\hat{B}}{20} + \frac{3\alpha_1\mathrm{tr}\hat{L}}{20} + \frac{3\xi_W\alpha_1\alpha_2}{16}\Bigg) \\
&+ \xi_W\Bigg(-\frac{9\alpha_1\alpha_2}{16} + \frac{15\alpha_2^2}{16} + \frac{15\alpha_2\mathrm{tr}\hat{T}}{4} + \frac{15\alpha_2\mathrm{tr}\hat{B}}{4} + \frac{5\alpha_2\mathrm{tr}\hat{L}}{4}\Bigg) + \frac{9\xi_B^2\alpha_1^2}{800} + \frac{45\xi_W^2\alpha_2^2}{32} \\
&+ n_G\Bigg(\frac{3\alpha_1^2}{10} + \frac{\alpha_2^2}{2}\Bigg)\Bigg] \\
&+ \frac{1}{\epsilon}\Bigg[-\frac{93\alpha_1^2}{1600} - \frac{27\alpha_1\alpha_2}{160} + \frac{817\alpha_2^2}{192} - 3\hat{\lambda}^2 - \frac{17\alpha_1\mathrm{tr}\hat{T}}{16} - \frac{45\alpha_2\mathrm{tr}\hat{T}}{16} - 10\alpha_3\mathrm{tr}\hat{T} \\
&- \frac{5\alpha_1\mathrm{tr}\hat{B}}{16} - \frac{45\alpha_2\mathrm{tr}\hat{B}}{16} - 10\alpha_3\mathrm{tr}\hat{B} - \frac{15\alpha_1\mathrm{tr}\hat{L}}{16} - \frac{15\alpha_2\mathrm{tr}\hat{L}}{16} + \frac{27\mathrm{tr}\hat{B}^2}{8} + \frac{9\mathrm{tr}\hat{L}^2}{8} \\
&- \frac{3\mathrm{tr}\hat{T}\hat{B}}{4} + \frac{27\mathrm{tr}\hat{T}^2}{8} - \frac{23\xi_W\alpha_2^2}{8} - \frac{7\xi_W^2\alpha_2^2}{16} + n_G\Bigg(-\frac{\alpha_1^2}{4} - \frac{5\alpha_2^2}{12}\Bigg)\Bigg]\Bigg\} \\
&+ \frac{1}{(4\pi)^3}\Bigg\{\frac{1}{\epsilon^3}\Bigg[\frac{429\alpha_1^3}{16000} + \frac{297\alpha_1^2\alpha_2}{3200} - \frac{693\alpha_1\alpha_2^2}{640} + \frac{12551\alpha_2^3}{1152} - \frac{93\alpha_1^2\mathrm{tr}\hat{T}}{800} - \frac{9\alpha_1\alpha_2\mathrm{tr}\hat{T}}{20} \\
&- \frac{27\alpha_2^2\mathrm{tr}\hat{T}}{32} - \frac{7\alpha_1\alpha_3\mathrm{tr}\hat{T}}{5} - 9\alpha_2\alpha_3\mathrm{tr}\hat{T} - 76\alpha_3^2\mathrm{tr}\hat{T} - \frac{177\alpha_1^2\mathrm{tr}\hat{B}}{800} + \frac{9\alpha_1\alpha_2\mathrm{tr}\hat{B}}{40} \\
&- \frac{27\alpha_2^2\mathrm{tr}\hat{B}}{32} + \frac{17\alpha_1\alpha_3\mathrm{tr}\hat{B}}{5} - 9\alpha_2\alpha_3\mathrm{tr}\hat{B} - 76\alpha_3^2\mathrm{tr}\hat{B} - \frac{339\alpha_1^2\mathrm{tr}\hat{L}}{800} - \frac{27\alpha_1\alpha_2\mathrm{tr}\hat{L}}{40} \\
&- \frac{9\alpha_2^2\mathrm{tr}\hat{L}}{32} - \frac{9\mathrm{tr}\hat{B}^3}{4} - \frac{9\alpha_1\mathrm{tr}\hat{B}^2}{20} + \frac{27\alpha_2\mathrm{tr}\hat{B}^2}{8} + 18\alpha_3\mathrm{tr}\hat{B}^2 - \frac{9\mathrm{tr}\hat{T}\mathrm{tr}\hat{B}^2}{4} \\
&- \frac{9\mathrm{tr}\hat{B}\mathrm{tr}\hat{B}^2}{4} - \frac{3\mathrm{tr}\hat{L}\mathrm{tr}\hat{B}^2}{4} - \frac{3\mathrm{tr}\hat{L}^3}{4} + \frac{27\alpha_1\mathrm{tr}\hat{L}^2}{20} + \frac{9\alpha_2\mathrm{tr}\hat{L}^2}{8} - \frac{3\mathrm{tr}\hat{T}\mathrm{tr}\hat{L}^2}{4} \\
&- \frac{3\mathrm{tr}\hat{B}\mathrm{tr}\hat{L}^2}{4} - \frac{\mathrm{tr}\hat{L}\mathrm{tr}\hat{L}^2}{4} + \frac{9\mathrm{tr}\hat{T}^2\hat{B}}{4} - \frac{9\mathrm{tr}\hat{T}^3}{4} - \frac{9\alpha_1\mathrm{tr}\hat{T}\hat{B}}{20} - \frac{27\alpha_2\mathrm{tr}\hat{T}\hat{B}}{4} \\
&- 36\alpha_3\mathrm{tr}\hat{T}\hat{B} + \frac{9\mathrm{tr}\hat{T}\mathrm{tr}\hat{T}\hat{B}}{2} + \frac{9\mathrm{tr}\hat{B}\mathrm{tr}\hat{T}\hat{B}}{2} + \frac{3\mathrm{tr}\hat{L}\mathrm{tr}\hat{T}\hat{B}}{2} + \frac{9\mathrm{tr}\hat{T}\hat{B}^2}{4} + \frac{9\alpha_1\mathrm{tr}\hat{T}^2}{10} \\
&+ \frac{27\alpha_2\mathrm{tr}\hat{T}^2}{8} + 18\alpha_3\mathrm{tr}\hat{T}^2 - \frac{9\mathrm{tr}\hat{T}\mathrm{tr}\hat{T}^2}{4} - \frac{9\mathrm{tr}\hat{B}\mathrm{tr}\hat{T}^2}{4} - \frac{3\mathrm{tr}\hat{L}\mathrm{tr}\hat{T}^2}{4} \\
&+ \xi_B\Bigg(-\frac{297\alpha_1^3}{16000} - \frac{81\alpha_1^2\alpha_2}{1600} + \frac{231\alpha_1\alpha_2^2}{640} + \frac{9\alpha_1^2\mathrm{tr}\hat{T}}{800} - \frac{27\alpha_1\alpha_2\mathrm{tr}\hat{T}}{160} \\
&- \frac{9\alpha_1\alpha_3\mathrm{tr}\hat{T}}{5} + \frac{117\alpha_1^2\mathrm{tr}\hat{B}}{800} - \frac{27\alpha_1\alpha_2\mathrm{tr}\hat{B}}{160} - \frac{9\alpha_1\alpha_3\mathrm{tr}\hat{B}}{5} - \frac{81\alpha_1^2\mathrm{tr}\hat{L}}{800}
\end{aligned}$$

$$
\begin{aligned}
&-\frac{9\alpha_1\alpha_2\mathrm{tr}\hat{L}}{160}+\frac{27\alpha_1\mathrm{tr}\hat{B}^2}{80}+\frac{9\alpha_1\mathrm{tr}\hat{L}^2}{80}-\frac{27\alpha_1\mathrm{tr}\hat{T}\hat{B}}{40}+\frac{27\alpha_1\mathrm{tr}\hat{T}^2}{80}\\
&+\xi_W\left(\frac{27\alpha_1^2\alpha_2}{320}-\frac{9\alpha_1\alpha_2^2}{64}-\frac{9\alpha_1\alpha_2\mathrm{tr}\hat{T}}{16}-\frac{9\alpha_1\alpha_2\mathrm{tr}\hat{B}}{16}-\frac{3\alpha_1\alpha_2\mathrm{tr}\hat{L}}{16}\right)-\frac{27\xi_W^2\alpha_1\alpha_2^2}{128}\Bigg)\\
&+\xi_W\Bigg(-\frac{99\alpha_1^2\alpha_2}{640}+\frac{27\alpha_1\alpha_2^2}{64}-\frac{745\alpha_2^3}{384}+\frac{3\alpha_1\alpha_2\mathrm{tr}\hat{T}}{32}-\frac{225\alpha_2^2\mathrm{tr}\hat{T}}{32}-15\alpha_2\alpha_3\mathrm{tr}\hat{T}\\
&+\frac{39\alpha_1\alpha_2\mathrm{tr}\hat{B}}{32}-\frac{225\alpha_2^2\mathrm{tr}\hat{B}}{32}-15\alpha_2\alpha_3\mathrm{tr}\hat{B}-\frac{27\alpha_1\alpha_2\mathrm{tr}\hat{L}}{32}-\frac{75\alpha_2^2\mathrm{tr}\hat{L}}{32}+\frac{45\alpha_2\mathrm{tr}\hat{B}^2}{16}\\
&+\frac{15\alpha_2\mathrm{tr}\hat{L}^2}{16}-\frac{45\alpha_2\mathrm{tr}\hat{T}\hat{B}}{8}+\frac{45\alpha_2\mathrm{tr}\hat{T}^2}{16}\Bigg)+\xi_B^2\Bigg(\frac{81\alpha_1^3}{16000}+\frac{27\alpha_1^2\alpha_2}{3200}\\
&-\frac{27\alpha_1^2\mathrm{tr}\hat{T}}{800}-\frac{27\alpha_1^2\mathrm{tr}\hat{B}}{800}-\frac{9\alpha_1^2\mathrm{tr}\hat{L}}{800}-\frac{9\xi_W\alpha_1^2\alpha_2}{640}\Bigg)\\
&+\xi_W^2\left(\frac{81\alpha_1\alpha_2^2}{128}-\frac{405\alpha_2^3}{128}-\frac{135\alpha_2^2\mathrm{tr}\hat{T}}{32}-\frac{135\alpha_2^2\mathrm{tr}\hat{B}}{32}-\frac{45\alpha_2^2\mathrm{tr}\hat{L}}{32}\right)\\
&-\frac{9\xi_B^3\alpha_1^3}{16000}-\frac{195\xi_W^3\alpha_2^3}{128}\\
&+n_G\Bigg(\frac{7\alpha_1^3}{40}+\frac{9\alpha_1^2\alpha_2}{40}+\frac{9\alpha_1\alpha_2^2}{40}-\frac{317\alpha_2^3}{72}-\frac{\alpha_1^2\mathrm{tr}\hat{T}}{3}+\frac{16\alpha_3^2\mathrm{tr}\hat{T}}{3}-\frac{11\alpha_1^2\mathrm{tr}\hat{B}}{15}\\
&+\frac{16\alpha_3^2\mathrm{tr}\hat{B}}{3}+\frac{\alpha_1^2\mathrm{tr}\hat{L}}{5}+\xi_B\left(-\frac{9\alpha_1^3}{200}-\frac{3\alpha_1\alpha_2^2}{40}\right)+\xi_W\left(-\frac{3\alpha_1^2\alpha_2}{8}+\frac{5\alpha_2^3}{24}\right)\Bigg)\\
&+n_G^2\left(\frac{4\alpha_1^3}{15}+\frac{4\alpha_2^3}{9}\right)\Bigg]\\
&+\frac{1}{\epsilon^2}\Bigg[-\frac{97\alpha_1^3}{32000}+\frac{63\alpha_1^2\alpha_2}{6400}+\frac{2901\alpha_1\alpha_2^2}{1280}-\frac{193057\alpha_2^3}{6912}-\frac{27\alpha_1^2\hat{\lambda}}{200}-\frac{9\alpha_1\alpha_2\hat{\lambda}}{20}\\
&-\frac{9\alpha_2^2\hat{\lambda}}{8}+\frac{9\alpha_1\hat{\lambda}^2}{20}+\frac{27\alpha_2\hat{\lambda}^2}{4}-24\hat{\lambda}^3-\frac{541\alpha_1^2\mathrm{tr}\hat{T}}{1600}+\frac{39\alpha_1\alpha_2\mathrm{tr}\hat{T}}{80}+\frac{149\alpha_2^2\mathrm{tr}\hat{T}}{64}\\
&-\frac{\alpha_1\alpha_3\mathrm{tr}\hat{T}}{10}-\frac{3\alpha_2\alpha_3\mathrm{tr}\hat{T}}{2}+198\alpha_3^2\mathrm{tr}\hat{T}-9\hat{\lambda}^2\mathrm{tr}\hat{T}+\frac{191\alpha_1^2\mathrm{tr}\hat{B}}{1600}+\frac{33\alpha_1\alpha_2\mathrm{tr}\hat{B}}{40}\\
&+\frac{149\alpha_2^2\mathrm{tr}\hat{B}}{64}-\frac{49\alpha_1\alpha_3\mathrm{tr}\hat{B}}{10}-\frac{3\alpha_2\alpha_3\mathrm{tr}\hat{B}}{2}+198\alpha_3^2\mathrm{tr}\hat{B}-9\hat{\lambda}^2\mathrm{tr}\hat{B}-\frac{3\alpha_1^2\mathrm{tr}\hat{L}}{1600}\\
&+\frac{149\alpha_2^2\mathrm{tr}\hat{L}}{192}-3\hat{\lambda}^2\mathrm{tr}\hat{L}+\frac{15\mathrm{tr}\hat{B}^3}{8}-\frac{123\alpha_1\mathrm{tr}\hat{B}^2}{80}-\frac{99\alpha_2\mathrm{tr}\hat{B}^2}{8}-39\alpha_3\mathrm{tr}\hat{B}^2\\
&+18\hat{\lambda}\mathrm{tr}\hat{B}^2+\frac{81\mathrm{tr}\hat{B}\mathrm{tr}\hat{B}^2}{8}+\frac{27\mathrm{tr}\hat{L}\mathrm{tr}\hat{B}^2}{8}+\frac{5\mathrm{tr}\hat{L}^3}{8}-\frac{261\alpha_1\mathrm{tr}\hat{L}^2}{80}\\
&-\frac{33\alpha_2\mathrm{tr}\hat{L}^2}{8}+6\hat{\lambda}\mathrm{tr}\hat{L}^2+\frac{27\mathrm{tr}\hat{T}\mathrm{tr}\hat{L}^2}{8}+\frac{27\mathrm{tr}\hat{B}\mathrm{tr}\hat{L}^2}{8}+\frac{9\mathrm{tr}\hat{L}\mathrm{tr}\hat{L}^2}{8}\\
&+\frac{15\mathrm{tr}\hat{T}^3}{8}+\frac{43\alpha_1\mathrm{tr}\hat{T}\hat{B}}{20}+\frac{9\alpha_2\mathrm{tr}\hat{T}\hat{B}}{4}+46\alpha_3\mathrm{tr}\hat{T}\hat{B}+\frac{81\mathrm{tr}\hat{T}\mathrm{tr}\hat{T}^2}{8}+\frac{27\mathrm{tr}\hat{L}\mathrm{tr}\hat{T}^2}{8}\\
&-\frac{11\mathrm{tr}\hat{L}\mathrm{tr}\hat{T}\hat{B}}{4}-\frac{297\alpha_1\mathrm{tr}\hat{T}^2}{80}-\frac{99\alpha_2\mathrm{tr}\hat{T}^2}{8}-39\alpha_3\mathrm{tr}\hat{T}^2+18\hat{\lambda}\mathrm{tr}\hat{T}^2
\end{aligned}
$$

$$
\begin{aligned}
&+\xi_B\bigg(\frac{279\alpha_1^3}{32000}+\frac{81\alpha_1^2\alpha_2}{3200}-\frac{817\alpha_1\alpha_2^2}{1280}+\frac{9\alpha_1\hat{\lambda}^2}{20}+\frac{51\alpha_1^2\mathrm{tr}\hat{T}}{320}+\frac{27\alpha_1\alpha_2\mathrm{tr}\hat{T}}{64}\\
&+\frac{3\alpha_1\alpha_3\mathrm{tr}\hat{T}}{2}+\frac{3\alpha_1^2\mathrm{tr}\hat{B}}{64}+\frac{27\alpha_1\alpha_2\mathrm{tr}\hat{B}}{64}+\frac{3\alpha_1\alpha_3\mathrm{tr}\hat{B}}{2}+\frac{9\alpha_1^2\mathrm{tr}\hat{L}}{64}+\frac{9\alpha_1\alpha_2\mathrm{tr}\hat{L}}{64}\\
&-\frac{81\alpha_1\mathrm{tr}\hat{B}^2}{160}-\frac{27\alpha_1\mathrm{tr}\hat{L}^2}{160}+\frac{9\alpha_1\mathrm{tr}\hat{T}\hat{B}}{80}-\frac{81\alpha_1\mathrm{tr}\hat{T}^2}{160}+\frac{69\xi_W\alpha_1\alpha_2^2}{160}+\frac{21\xi_W^2\alpha_1\alpha_2^2}{320}\bigg)\\
&+\xi_W\bigg(\frac{93\alpha_1^2\alpha_2}{1280}-\frac{693\alpha_1\alpha_2^2}{640}+\frac{6515\alpha_2^3}{768}+\frac{15\alpha_2\hat{\lambda}^2}{4}+\frac{85\alpha_1\alpha_2\mathrm{tr}\hat{T}}{64}+\frac{777\alpha_2^2\mathrm{tr}\hat{T}}{64}\\
&+\frac{25\alpha_2\alpha_3\mathrm{tr}\hat{T}}{2}+\frac{25\alpha_1\alpha_2\mathrm{tr}\hat{B}}{64}+\frac{777\alpha_2^2\mathrm{tr}\hat{B}}{64}+\frac{25\alpha_2\alpha_3\mathrm{tr}\hat{B}}{2}+\frac{75\alpha_1\alpha_2\mathrm{tr}\hat{L}}{64}\\
&+\frac{259\alpha_2^2\mathrm{tr}\hat{L}}{64}-\frac{135\alpha_2\mathrm{tr}\hat{B}^2}{32}-\frac{45\alpha_2\mathrm{tr}\hat{L}^2}{32}+\frac{15\alpha_2\mathrm{tr}\hat{T}\hat{B}}{16}-\frac{135\alpha_2\mathrm{tr}\hat{T}^2}{32}\bigg)\\
&+\xi_W^2\bigg(-\frac{63\alpha_1\alpha_2^2}{320}+\frac{473\alpha_2^3}{64}+\frac{21\alpha_2^2\mathrm{tr}\hat{T}}{16}+\frac{21\alpha_2^2\mathrm{tr}\hat{B}}{16}+\frac{7\alpha_2^2\mathrm{tr}\hat{L}}{16}\bigg)+\frac{241\xi_W^3\alpha_2^3}{192}\\
&+n_G\bigg(\frac{11\alpha_1^3}{1200}-\frac{69\alpha_1^2\alpha_2}{400}-\frac{17\alpha_1\alpha_2^2}{80}+\frac{4123\alpha_2^3}{432}+\frac{11\alpha_1^2\alpha_3}{25}+\alpha_2^2\alpha_3-\frac{11\alpha_1^2\mathrm{tr}\hat{T}}{30}\\
&-\alpha_2^2\mathrm{tr}\hat{T}-\frac{40\alpha_3^2\mathrm{tr}\hat{T}}{3}+\frac{19\alpha_1^2\mathrm{tr}\hat{B}}{30}-\alpha_2^2\mathrm{tr}\hat{B}-\frac{40\alpha_3^2\mathrm{tr}\hat{B}}{3}-\frac{9\alpha_1^2\mathrm{tr}\hat{L}}{10}-\frac{\alpha_2^2\mathrm{tr}\hat{L}}{3}\\
&+\xi_B\bigg(\frac{3\alpha_1^3}{80}+\frac{\alpha_1\alpha_2^2}{16}\bigg)+\xi_W\bigg(\frac{5\alpha_1^2\alpha_2}{16}-\frac{103\alpha_2^3}{48}\bigg)\bigg)+n_G^2\bigg(-\frac{2\alpha_1^3}{9}-\frac{10\alpha_2^3}{27}\bigg)\\
&-3\alpha_t\alpha_b^2-3\alpha_t^2\alpha_b\bigg]\\
&+\frac{1}{\epsilon}\bigg[-\frac{413\alpha_1^3}{6000}+\frac{27\zeta_3\alpha_1^3}{2000}-\frac{279\alpha_1^2\alpha_2}{800}-\frac{27\zeta_3\alpha_1^2\alpha_2}{400}-\frac{123\alpha_1\alpha_2^2}{320}-\frac{3\zeta_3\alpha_1\alpha_2^2}{80}\\
&+\frac{93307\alpha_2^3}{5184}+\frac{73\zeta_3\alpha_2^3}{16}-\frac{117\alpha_1^2\hat{\lambda}}{400}+\frac{27\zeta_3\alpha_1^2\hat{\lambda}}{50}-\frac{39\alpha_1\alpha_2\hat{\lambda}}{40}+\frac{9\zeta_3\alpha_1\alpha_2\hat{\lambda}}{5}-\frac{39\alpha_2^2\hat{\lambda}}{16}\\
&+\frac{9\zeta_3\alpha_2^2\hat{\lambda}}{2}-3\alpha_1\hat{\lambda}^2-15\alpha_2\hat{\lambda}^2+12\hat{\lambda}^3+\frac{52831\alpha_1^2\mathrm{tr}\hat{T}}{28800}-\frac{\zeta_3\alpha_1^2\mathrm{tr}\hat{T}}{100}-\frac{371\alpha_1\alpha_2\mathrm{tr}\hat{T}}{320}\\
&-\frac{27\zeta_3\alpha_1\alpha_2\mathrm{tr}\hat{T}}{10}-\frac{2761\alpha_2^2\mathrm{tr}\hat{T}}{128}+\frac{63\zeta_3\alpha_2^2\mathrm{tr}\hat{T}}{4}+\frac{2419\alpha_1\alpha_3\mathrm{tr}\hat{T}}{180}-\frac{68\zeta_3\alpha_1\alpha_3\mathrm{tr}\hat{T}}{5}\\
&+\frac{163\alpha_2\alpha_3\mathrm{tr}\hat{T}}{4}-36\zeta_3\alpha_2\alpha_3\mathrm{tr}\hat{T}-\frac{910\alpha_3^2\mathrm{tr}\hat{T}}{9}+8\zeta_3\alpha_3^2\mathrm{tr}\hat{T}+\frac{45\hat{\lambda}^2\mathrm{tr}\hat{T}}{2}\\
&+\frac{27\alpha_1(\mathrm{tr}\hat{T})^2}{20}+\frac{27\alpha_2(\mathrm{tr}\hat{T})^2}{4}+\frac{5479\alpha_1^2\mathrm{tr}\hat{B}}{28800}+\frac{29\zeta_3\alpha_1^2\mathrm{tr}\hat{B}}{100}-\frac{671\alpha_1\alpha_2\mathrm{tr}\hat{B}}{320}\\
&+\frac{9\zeta_3\alpha_1\alpha_2\mathrm{tr}\hat{B}}{5}-\frac{2761\alpha_2^2\mathrm{tr}\hat{B}}{128}+\frac{63\zeta_3\alpha_2^2\mathrm{tr}\hat{B}}{4}+\frac{991\alpha_1\alpha_3\mathrm{tr}\hat{B}}{180}-4\zeta_3\alpha_1\alpha_3\mathrm{tr}\hat{B}\\
&+\frac{163\alpha_2\alpha_3\mathrm{tr}\hat{B}}{4}-36\zeta_3\alpha_2\alpha_3\mathrm{tr}\hat{B}-\frac{910\alpha_3^2\mathrm{tr}\hat{B}}{9}+8\zeta_3\alpha_3^2\mathrm{tr}\hat{B}+\frac{45\hat{\lambda}^2\mathrm{tr}\hat{B}}{2}\\
&+\frac{27\alpha_1\mathrm{tr}\hat{T}\mathrm{tr}\hat{B}}{10}+\frac{27\alpha_2\mathrm{tr}\hat{T}\mathrm{tr}\hat{B}}{2}+\frac{27\alpha_1(\mathrm{tr}\hat{B})^2}{20}+\frac{27\alpha_2(\mathrm{tr}\hat{B})^2}{4}+\frac{8517\alpha_1^2\mathrm{tr}\hat{L}}{3200}
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{117\zeta_3\alpha_1^2\mathrm{tr}\hat{L}}{100}+\frac{411\alpha_1\alpha_2\mathrm{tr}\hat{L}}{320}-\frac{18\zeta_3\alpha_1\alpha_2\mathrm{tr}\hat{L}}{5}-\frac{2761\alpha_2^2\mathrm{tr}\hat{L}}{384}+\frac{21\zeta_3\alpha_2^2\mathrm{tr}\hat{L}}{4}\\
&+\frac{15\hat{\lambda}^2\mathrm{tr}\hat{L}}{2}+\frac{9\alpha_1\mathrm{tr}\hat{T}\mathrm{tr}\hat{L}}{10}+\frac{9\alpha_2\mathrm{tr}\hat{T}\mathrm{tr}\hat{L}}{2}+\frac{9\alpha_1\mathrm{tr}\hat{B}\mathrm{tr}\hat{L}}{10}+\frac{9\alpha_2\mathrm{tr}\hat{B}\mathrm{tr}\hat{L}}{2}\\
&+\frac{3\alpha_1(\mathrm{tr}\hat{L})^2}{20}+\frac{3\alpha_2(\mathrm{tr}\hat{L})^2}{4}+\frac{25\mathrm{tr}\hat{B}^3}{16}-3\zeta_3\mathrm{tr}\hat{B}^3+\frac{303\alpha_1\mathrm{tr}\hat{B}^2}{80}-\frac{9\zeta_3\alpha_1\mathrm{tr}\hat{B}^2}{5}\\
&+\frac{279\alpha_2\mathrm{tr}\hat{B}^2}{16}-9\zeta_3\alpha_2\mathrm{tr}\hat{B}^2-\frac{5\alpha_3\mathrm{tr}\hat{B}^2}{2}+24\zeta_3\alpha_3\mathrm{tr}\hat{B}^2-15\hat{\lambda}\mathrm{tr}\hat{B}^2\\
&-18\mathrm{tr}\hat{B}\mathrm{tr}\hat{B}^2-6\mathrm{tr}\hat{L}\mathrm{tr}\hat{B}^2+\frac{25\mathrm{tr}\hat{L}^3}{48}-\zeta_3\mathrm{tr}\hat{L}^3+\frac{33\alpha_1\mathrm{tr}\hat{L}^2}{80}\\
&+\frac{9\zeta_3\alpha_1\mathrm{tr}\hat{L}^2}{5}+\frac{93\alpha_2\mathrm{tr}\hat{L}^2}{16}-3\zeta_3\alpha_2\mathrm{tr}\hat{L}^2-5\hat{\lambda}\mathrm{tr}\hat{L}^2-6\mathrm{tr}\hat{T}\mathrm{tr}\hat{L}^2-6\mathrm{tr}\hat{B}\mathrm{tr}\hat{L}^2\\
&-2\mathrm{tr}\hat{L}\mathrm{tr}\hat{L}^2+\frac{25\mathrm{tr}\hat{T}^3}{16}-3\zeta_3\mathrm{tr}\hat{T}^3+\frac{31\alpha_1\mathrm{tr}\hat{T}\hat{B}}{40}-\frac{8\zeta_3\alpha_1\mathrm{tr}\hat{T}\hat{B}}{5}\\
&+\frac{21\alpha_2\mathrm{tr}\hat{T}\hat{B}}{8}-19\alpha_3\mathrm{tr}\hat{T}\hat{B}+16\zeta_3\alpha_3\mathrm{tr}\hat{T}\hat{B}\\
&+\frac{\mathrm{tr}\hat{L}\mathrm{tr}\hat{T}\hat{B}}{2}+\frac{211\alpha_1\mathrm{tr}\hat{T}^2}{80}+\frac{3\zeta_3\alpha_1\mathrm{tr}\hat{T}^2}{5}+\frac{279\alpha_2\mathrm{tr}\hat{T}^2}{16}-9\zeta_3\alpha_2\mathrm{tr}\hat{T}^2\\
&-\frac{5\alpha_3\mathrm{tr}\hat{T}^2}{2}+24\zeta_3\alpha_3\mathrm{tr}\hat{T}^2-15\hat{\lambda}\mathrm{tr}\hat{T}^2-18\mathrm{tr}\hat{T}\mathrm{tr}\hat{T}^2-6\mathrm{tr}\hat{L}\mathrm{tr}\hat{T}^2\\
&+\xi_W\Bigg(-\frac{927\alpha_2^3}{64}-\zeta_3\alpha_2^3\Bigg)+\xi_W^2\Bigg(-\frac{83\alpha_2^3}{32}-\frac{5\zeta_3\alpha_2^3}{8}\Bigg)-\frac{29\xi_W^3\alpha_2^3}{48}\\
&+n_G\Bigg(-\frac{158\alpha_1^3}{225}+\frac{19\zeta_3\alpha_1^3}{25}-\frac{3\alpha_1^2\alpha_2}{40}+\frac{9\zeta_3\alpha_1^2\alpha_2}{25}+\frac{3\alpha_1\alpha_2^2}{40}+\frac{\zeta_3\alpha_1\alpha_2^2}{5}-\frac{1285\alpha_2^3}{324}\\
&-9\zeta_3\alpha_2^3-\frac{33\alpha_1^2\alpha_3}{20}+\frac{44\zeta_3\alpha_1^2\alpha_3}{25}-\frac{15\alpha_2^2\alpha_3}{4}+4\zeta_3\alpha_2^2\alpha_3+\frac{127\alpha_1^2\mathrm{tr}\hat{T}}{120}+\frac{21\alpha_2^2\mathrm{tr}\hat{T}}{8}\\
&+\frac{32\alpha_3^2\mathrm{tr}\hat{T}}{3}+\frac{31\alpha_1^2\mathrm{tr}\hat{B}}{120}+\frac{21\alpha_2^2\mathrm{tr}\hat{B}}{8}+\frac{32\alpha_3^2\mathrm{tr}\hat{B}}{3}+\frac{39\alpha_1^2\mathrm{tr}\hat{L}}{40}+\frac{7\alpha_2^2\mathrm{tr}\hat{L}}{8}+\frac{83\xi_W\alpha_2^3}{24}\Bigg)\\
&+n_G^2\Bigg(-\frac{7\alpha_1^3}{27}-\frac{35\alpha_2^3}{81}\Bigg)-\frac{277\alpha_t\alpha_b^2}{16}-\frac{277\alpha_b\alpha_t^2}{16}\Bigg]\Bigg\}\,.
\end{aligned}
\tag{A16}
$$

Note that in the results for Z_H and Z_{HHW} there are terms which contain explicitly α_b and α_t since we have not been able to reconstruct the corresponding expressions in terms of $\hat{B}$ and $\hat{T}$. They drop out in the final result for Z_{α_2}.

Appendix B: Two-loop Yukawa coupling renormalization constants

This Subsection contains the two-loop results for the Yukawa coupling renormalization constants defined through

$$
\alpha_i^{\mathrm{bare}}=Z_{\alpha_i}\alpha_i\,,
\tag{B1}
$$

with $i = t, b, \tau$. They read

$$
\begin{aligned}
Z_{\alpha_t} = {} & 1 + \frac{1}{4\pi}\frac{1}{\epsilon}\bigg\{ -\frac{17\alpha_1}{20} - \frac{9\alpha_2}{4} - 8\alpha_3 + \frac{3\alpha_t}{2} - \frac{3\alpha_b}{2} + 3\mathrm{tr}\hat{T} + 3\mathrm{tr}\hat{B} + \mathrm{tr}\hat{L}\bigg\} \\
& + \frac{1}{(4\pi)^2}\bigg\{ \frac{1}{\epsilon^2}\bigg[\frac{51\alpha_1^2}{160} + \frac{153\alpha_1\alpha_2}{80} + \frac{339\alpha_2^2}{32} + \frac{34\alpha_1\alpha_3}{5} + 18\alpha_2\alpha_3 + 76\alpha_3^2 - \frac{459\alpha_1\alpha_t}{80} \\
& - \frac{243\alpha_2\alpha_t}{16} - 54\alpha_3\alpha_t + \frac{81\alpha_t^2}{4} - \frac{117\alpha_1\alpha_b}{80} - \frac{81\alpha_2\alpha_b}{16} - 18\alpha_3\alpha_b + \frac{45\alpha_t\alpha_b}{4} + \frac{9\alpha_b^2}{2} \\
& - \frac{79\alpha_1\alpha_\tau}{40} - \frac{27\alpha_2\alpha_\tau}{8} - 8\alpha_3\alpha_\tau + \frac{33\alpha_t\alpha_\tau}{4} + \frac{15\alpha_b\alpha_\tau}{4} + \frac{7\alpha_\tau^2}{4} \\
& + n_G\bigg(-\frac{17\alpha_1^2}{30} - \frac{3\alpha_2^2}{2} - \frac{16\alpha_3^2}{3}\bigg)\bigg] \\
& + \frac{1}{\epsilon}\bigg[\frac{9\alpha_1^2}{400} - \frac{9\alpha_1\alpha_2}{40} - \frac{35\alpha_2^2}{8} + \frac{19\alpha_1\alpha_3}{30} + \frac{9\alpha_2\alpha_3}{2} - \frac{202\alpha_3^2}{3} + 3\hat{\lambda}^2 + \frac{393\alpha_1\alpha_t}{160} \\
& + \frac{225\alpha_2\alpha_t}{32} + 18\alpha_3\alpha_t - 6\hat{\lambda}\alpha_t - 6\alpha_t^2 + \frac{7\alpha_1\alpha_b}{160} + \frac{99\alpha_2\alpha_b}{32} + 2\alpha_3\alpha_b - \frac{11\alpha_t\alpha_b}{8} - \frac{\alpha_b^2}{8} \\
& + \frac{15\alpha_1\alpha_\tau}{16} + \frac{15\alpha_2\alpha_\tau}{16} - \frac{9\alpha_t\alpha_\tau}{8} + \frac{5\alpha_b\alpha_\tau}{8} - \frac{9\alpha_\tau^2}{8} + n_G\bigg(\frac{29\alpha_1^2}{90} + \frac{\alpha_2^2}{2} + \frac{40\alpha_3^2}{9}\bigg)\bigg]\bigg\},
\end{aligned}
\tag{B2}
$$

$$
\begin{aligned}
Z_{\alpha_b} = {} & 1 + \frac{1}{4\pi}\frac{1}{\epsilon}\bigg\{ -\frac{\alpha_1}{4} - \frac{9\alpha_2}{4} - 8\alpha_3 - \frac{3\alpha_t}{2} + \frac{3\alpha_b}{2} + 3\mathrm{tr}\hat{T} + 3\mathrm{tr}\hat{B} + \mathrm{tr}\hat{L}\bigg\} \\
& + \frac{1}{(4\pi)^2}\bigg\{ \frac{1}{\epsilon^2}\bigg[\frac{3\alpha_1^2}{160} + \frac{9\alpha_1\alpha_2}{16} + \frac{339\alpha_2^2}{32} + 2\alpha_1\alpha_3 + 18\alpha_2\alpha_3 + 76\alpha_3^2 - \frac{81\alpha_1\alpha_t}{80} - \frac{81\alpha_2\alpha_t}{16} \\
& - 18\alpha_3\alpha_t + \frac{9\alpha_t^2}{2} - \frac{27\alpha_1\alpha_b}{16} - \frac{243\alpha_2\alpha_b}{16} - 54\alpha_3\alpha_b + \frac{45\alpha_t\alpha_b}{4} + \frac{81\alpha_b^2}{4} - \frac{11\alpha_1\alpha_\tau}{8} \\
& - \frac{27\alpha_2\alpha_\tau}{8} - 8\alpha_3\alpha_\tau + \frac{15\alpha_t\alpha_\tau}{4} + \frac{33\alpha_b\alpha_\tau}{4} + \frac{7\alpha_\tau^2}{4} + n_G\bigg(-\frac{\alpha_1^2}{6} - \frac{3\alpha_2^2}{2} - \frac{16\alpha_3^2}{3}\bigg)\bigg] \\
& + \frac{1}{\epsilon}\bigg[-\frac{29\alpha_1^2}{400} - \frac{27\alpha_1\alpha_2}{40} - \frac{35\alpha_2^2}{8} + \frac{31\alpha_1\alpha_3}{30} + \frac{9\alpha_2\alpha_3}{2} - \frac{202\alpha_3^2}{3} + 3\hat{\lambda}^2 + \frac{91\alpha_1\alpha_t}{160} \\
& + \frac{99\alpha_2\alpha_t}{32} + 2\alpha_3\alpha_t - \frac{\alpha_t^2}{8} + \frac{237\alpha_1\alpha_b}{160} + \frac{225\alpha_2\alpha_b}{32} + 18\alpha_3\alpha_b - 6\hat{\lambda}\alpha_b - \frac{11\alpha_t\alpha_b}{8} - 6\alpha_b^2 \\
& + \frac{15\alpha_1\alpha_\tau}{16} + \frac{15\alpha_2\alpha_\tau}{16} + \frac{5\alpha_t\alpha_\tau}{8} - \frac{9\alpha_b\alpha_\tau}{8} - \frac{9\alpha_\tau^2}{8} + n_G\bigg(-\frac{\alpha_1^2}{90} + \frac{\alpha_2^2}{2} + \frac{40\alpha_3^2}{9}\bigg)\bigg]\bigg\},
\end{aligned}
\tag{B3}
$$

$$
\begin{aligned}
Z_{\alpha_\tau} = {} & 1 + \frac{1}{4\pi}\frac{1}{\epsilon}\bigg\{ -\frac{9\alpha_1}{4} - \frac{9\alpha_2}{4} + \frac{3\alpha_\tau}{2} + 3\mathrm{tr}\hat{T} + 3\mathrm{tr}\hat{B} + \mathrm{tr}\hat{L}\bigg\} \\
& + \frac{1}{(4\pi)^2}\bigg\{ \frac{1}{\epsilon^2}\bigg[\frac{387\alpha_1^2}{160} + \frac{81\alpha_1\alpha_2}{16} + \frac{339\alpha_2^2}{32} - \frac{321\alpha_1\alpha_t}{40} - \frac{81\alpha_2\alpha_t}{8} - 12\alpha_3\alpha_t + \frac{45\alpha_t^2}{4} \\
& - \frac{57\alpha_1\alpha_b}{8} - \frac{81\alpha_2\alpha_b}{8} - 12\alpha_3\alpha_b + \frac{27\alpha_t\alpha_b}{2} + \frac{45\alpha_b^2}{4} - \frac{135\alpha_1\alpha_\tau}{16} - \frac{135\alpha_2\alpha_\tau}{16} \\
& + \frac{51\alpha_t\alpha_\tau}{4} + \frac{51\alpha_b\alpha_\tau}{4} + \frac{25\alpha_\tau^2}{4} + n_G\bigg(-\frac{3\alpha_1^2}{2} - \frac{3\alpha_2^2}{2}\bigg)\bigg]
\end{aligned}
$$

$$
+\frac{1}{\epsilon}\Bigg[\frac{51\alpha_1^2}{400}+\frac{27\alpha_1\alpha_2}{40}-\frac{35\alpha_2^2}{8}+3\hat{\lambda}^2+\frac{17\alpha_1\alpha_t}{16}+\frac{45\alpha_2\alpha_t}{16}+10\alpha_3\alpha_t-\frac{27\alpha_t^2}{8}
+\frac{5\alpha_1\alpha_b}{16}+\frac{45\alpha_2\alpha_b}{16}+10\alpha_3\alpha_b+\frac{3\alpha_t\alpha_b}{4}-\frac{27\alpha_b^2}{8}+\frac{537\alpha_1\alpha_\tau}{160}+\frac{165\alpha_2\alpha_\tau}{32}-6\hat{\lambda}\alpha_\tau
-\frac{27\alpha_t\alpha_\tau}{8}-\frac{27\alpha_b\alpha_\tau}{8}-\frac{3\alpha_\tau^2}{2}+n_G\left(\frac{11\alpha_1^2}{10}+\frac{\alpha_2^2}{2}\right)\Bigg]\Bigg\}\,. \tag{B4}
$$

Appendix C: Beta functions for α_{QED} and $\sin^2\theta_W$

This Appendix contains explicit results up to three-loop order for the QED coupling α_{QED} and the weak mixing angle. We refrain from providing expressions for the renormalization constants but directly list the beta functions. They are obtained in a straightforward way from Eq. (2) and are given by

$$
\begin{aligned}
\beta_{\alpha_{\mathrm{QED}}} = {}& \frac{\alpha_{\mathrm{QED}}^2}{(4\pi)^2}\left\{-28+\frac{128n_G}{9}\right\}+\frac{\alpha_{\mathrm{QED}}^2}{(4\pi)^3}\left\{-\frac{500\alpha_{\mathrm{QED}}}{3\sin^2\theta_W}+\frac{4\alpha_{\mathrm{QED}}}{\cos^2\theta_W}\right.\\
&\left.-\frac{52\mathrm{tr}\hat{T}}{3}-\frac{28\mathrm{tr}\hat{B}}{3}-12\mathrm{tr}\hat{L}+n_G\left[\frac{208\alpha_{\mathrm{QED}}}{3\sin^2\theta_W}+\frac{416\alpha_{\mathrm{QED}}}{27\cos^2\theta_W}+\frac{320\alpha_3}{9}\right]\right\}\\
&+\frac{\alpha_{\mathrm{QED}}^2}{(4\pi)^4}\left\{\frac{137\alpha_{\mathrm{QED}}^2}{4\sin^2\theta_W\cos^2\theta_W}-\frac{318251\alpha_{\mathrm{QED}}^2}{216\sin^4\theta_W}+\frac{163\alpha_{\mathrm{QED}}^2}{72\cos^4\theta_W}+\frac{12\alpha_{\mathrm{QED}}\hat{\lambda}}{\sin^2\theta_W}+\frac{8\alpha_{\mathrm{QED}}\hat{\lambda}}{\cos^2\theta_W}\right.\\
&-24\hat{\lambda}^2-\frac{757\alpha_{\mathrm{QED}}\mathrm{tr}\hat{T}}{4\sin^2\theta_W}-\frac{2303\alpha_{\mathrm{QED}}\mathrm{tr}\hat{T}}{36\cos^2\theta_W}-\frac{200\alpha_3\mathrm{tr}\hat{T}}{3}-\frac{583\alpha_{\mathrm{QED}}\mathrm{tr}\hat{B}}{4\sin^2\theta_W}-\frac{1433\alpha_{\mathrm{QED}}\mathrm{tr}\hat{B}}{36\cos^2\theta_W}\\
&-\frac{152\alpha_3\mathrm{tr}\hat{B}}{3}-\frac{393\alpha_{\mathrm{QED}}\mathrm{tr}\hat{L}}{4\sin^2\theta_W}-\frac{183\alpha_{\mathrm{QED}}\mathrm{tr}\hat{L}}{4\cos^2\theta_W}+\frac{59\mathrm{tr}\hat{B}^2}{2}+31(\mathrm{tr}\hat{B})^2+\frac{202\mathrm{tr}\hat{B}\mathrm{tr}\hat{L}}{3}\\
&+\frac{53\mathrm{tr}\hat{L}^2}{2}+19(\mathrm{tr}\hat{L})^2+16\mathrm{tr}\hat{T}\hat{B}+\frac{85\mathrm{tr}\hat{T}^2}{2}+104\mathrm{tr}\hat{T}\mathrm{tr}\hat{B}+\frac{244\mathrm{tr}\hat{T}\mathrm{tr}\hat{L}}{3}+73(\mathrm{tr}\hat{T})^2\\
&+n_G\left[\frac{32\alpha_{\mathrm{QED}}^2}{9\sin^2\theta_W\cos^2\theta_W}+\frac{26146\alpha_{\mathrm{QED}}^2}{27\sin^4\theta_W}-\frac{1580\alpha_{\mathrm{QED}}^2}{81\cos^4\theta_W}+\frac{152\alpha_{\mathrm{QED}}\alpha_3}{3\sin^2\theta_W}\right.\\
&\left.\left.-\frac{584\alpha_{\mathrm{QED}}\alpha_3}{81\cos^2\theta_W}+\frac{10000\alpha_3^2}{27}\right]+n_G^2\left[-\frac{1792\alpha_{\mathrm{QED}}^2}{27\sin^4\theta_W}-\frac{22880\alpha_{\mathrm{QED}}^2}{729\cos^4\theta_W}-\frac{3520\alpha_3^2}{81}\right]\right\}\,,
\end{aligned}\tag{C1}
$$

$$
\begin{aligned}
\beta_{\sin^2\theta_W} = {}& \mu^2\frac{\mathrm{d}\sin^2\theta_W}{\mathrm{d}\mu^2}\\
={}& \frac{\alpha_{\mathrm{QED}}}{4\pi}\left\{\frac{\sin^2\theta_W}{6}+\frac{43\cos^2\theta_W}{6}+n_G\left[\frac{20\sin^2\theta_W}{9}-\frac{4\cos^2\theta_W}{3}\right]\right\}\\
&+\frac{\alpha_{\mathrm{QED}}}{(4\pi)^2}\left\{\alpha_{\mathrm{QED}}+\frac{\alpha_{\mathrm{QED}}\tan^2\theta_W}{2}+\frac{259\alpha_{\mathrm{QED}}\cot^2\theta_W}{6}-\frac{17\sin^2\theta_W\mathrm{tr}\hat{T}}{6}\right.\\
&+\frac{3\cos^2\theta_W\mathrm{tr}\hat{T}}{2}-\frac{5\sin^2\theta_W\mathrm{tr}\hat{B}}{6}+\frac{3\cos^2\theta_W\mathrm{tr}\hat{B}}{2}-\frac{5\sin^2\theta_W\mathrm{tr}\hat{L}}{2}+\frac{\cos^2\theta_W\mathrm{tr}\hat{L}}{2}\\
&\left.+n_G\left[\frac{2\alpha_{\mathrm{QED}}}{3}+\frac{95\alpha_{\mathrm{QED}}\tan^2\theta_W}{27}-\frac{49\alpha_{\mathrm{QED}}\cot^2\theta_W}{3}+\frac{44\sin^2\theta_W\alpha_3}{9}-4\cos^2\theta_W\alpha_3\right]\right\}
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{\alpha_{\rm QED}}{(4\pi)^3}\Bigg\{\frac{2279\alpha_{\rm QED}^2}{192\sin^2\theta_W}+\frac{1403\alpha_{\rm QED}^2}{576\cos^2\theta_W}+\frac{163\alpha_{\rm QED}^2\tan^2\theta_W}{576\cos^2\theta_W}+\frac{667111\alpha_{\rm QED}^2\cot^2\theta_W}{1728\sin^2\theta_W}\\
&+\alpha_{\rm QED}\hat\lambda+\frac{3\alpha_{\rm QED}\tan^2\theta_W\hat\lambda}{2}-\frac{3\alpha_{\rm QED}\cot^2\theta_W\hat\lambda}{2}-3\sin^2\theta_W\hat\lambda^2+3\cos^2\theta_W\hat\lambda^2\\
&-\frac{881\alpha_{\rm QED}{\rm tr}\hat T}{48}-\frac{2827\alpha_{\rm QED}\tan^2\theta_W{\rm tr}\hat T}{288}+\frac{729\alpha_{\rm QED}\cot^2\theta_W{\rm tr}\hat T}{32}-\frac{29\sin^2\theta_W\alpha_3{\rm tr}\hat T}{3}\\
&+7\cos^2\theta_W\alpha_3{\rm tr}\hat T-\frac{389\alpha_{\rm QED}{\rm tr}\hat B}{48}-\frac{1267\alpha_{\rm QED}\tan^2\theta_W{\rm tr}\hat B}{288}+\frac{729\alpha_{\rm QED}\cot^2\theta_W{\rm tr}\hat B}{32}\\
&-\frac{17\sin^2\theta_W\alpha_3{\rm tr}\hat B}{3}+7\cos^2\theta_W\alpha_3{\rm tr}\hat B-\frac{229\alpha_{\rm QED}{\rm tr}\hat L}{16}-\frac{281\alpha_{\rm QED}\tan^2\theta_W{\rm tr}\hat L}{32}\\
&+\frac{243\alpha_{\rm QED}\cot^2\theta_W{\rm tr}\hat L}{32}+\frac{61\sin^2\theta_W{\rm tr}\hat B^2}{16}-\frac{57\cos^2\theta_W{\rm tr}\hat B^2}{16}+\frac{17\sin^2\theta_W({\rm tr}\hat B)^2}{8}\\
&-\frac{45\cos^2\theta_W({\rm tr}\hat B)^2}{8}+\frac{157\sin^2\theta_W{\rm tr}\hat B{\rm tr}\hat L}{12}-\frac{15\cos^2\theta_W{\rm tr}\hat B{\rm tr}\hat L}{4}+\frac{87\sin^2\theta_W{\rm tr}\hat L^2}{16}\\
&-\frac{19\cos^2\theta_W{\rm tr}\hat L^2}{16}+\frac{33\sin^2\theta_W({\rm tr}\hat L)^2}{8}-\frac{5\cos^2\theta_W({\rm tr}\hat L)^2}{8}+\frac{5\sin^2\theta_W{\rm tr}\hat T\hat B}{8}\\
&-\frac{27\cos^2\theta_W{\rm tr}\hat T\hat B}{8}+\frac{113\sin^2\theta_W{\rm tr}\hat T^2}{16}-\frac{57\cos^2\theta_W{\rm tr}\hat T^2}{16}+\frac{59\sin^2\theta_W{\rm tr}\hat T{\rm tr}\hat B}{4}\\
&-\frac{45\cos^2\theta_W{\rm tr}\hat T{\rm tr}\hat B}{4}+\frac{199\sin^2\theta_W{\rm tr}\hat T{\rm tr}\hat L}{12}-\frac{15\cos^2\theta_W{\rm tr}\hat T{\rm tr}\hat L}{4}\\
&+\frac{101\sin^2\theta_W({\rm tr}\hat T)^2}{8}-\frac{45\cos^2\theta_W({\rm tr}\hat T)^2}{8}+n_G\Bigg[\frac{127\alpha_{\rm QED}^2}{36\sin^2\theta_W}+\frac{119\alpha_{\rm QED}^2}{108\cos^2\theta_W}\\
&-\frac{290\alpha_{\rm QED}^2\tan^2\theta_W}{81\cos^2\theta_W}-\frac{6412\alpha_{\rm QED}^2\cot^2\theta_W}{27\sin^2\theta_W}-\frac{2\alpha_{\rm QED}\alpha_3}{9}-\frac{137\alpha_{\rm QED}\tan^2\theta_W\alpha_3}{81}\\
&-13\alpha_{\rm QED}\cot^2\theta_W\alpha_3+\frac{1375\sin^2\theta_W\alpha_3^2}{27}-\frac{125\cos^2\theta_W\alpha_3^2}{3}\Bigg]\\
&+n_G^2\Bigg[-\frac{11\alpha_{\rm QED}^2}{9\sin^2\theta_W}+\frac{55\alpha_{\rm QED}^2}{81\cos^2\theta_W}-\frac{5225\alpha_{\rm QED}^2\tan^2\theta_W}{729\cos^2\theta_W}+\frac{415\alpha_{\rm QED}^2\cot^2\theta_W}{27\sin^2\theta_W}\\
&-\frac{484\sin^2\theta_W\alpha_3^2}{81}+\frac{44\cos^2\theta_W\alpha_3^2}{9}\Bigg]\Bigg\}\,. \qquad \text{(C2)}
\end{aligned}
$$

Appendix D: Comparison with Ref. [16]

In this Appendix we provide the explicit form of the expressions needed for the comparison with Ref. [16]. In this paper two-component Weyl spinors were used. To make contact with our convention based on four-component Dirac spinors we define

$$
\Psi_D=\begin{pmatrix}\chi\\ \xi^\dagger\end{pmatrix}\,, \qquad \text{(D1)}
$$

where ξ and χ are left-handed Weyl spinors and Ψ_D denotes a Dirac spinor. Thus, the Lagrange density of the SM can be expressed in terms of 45 Weyl spinors:

$$
\chi_t\,,\xi_t\,,\chi_b\,,\xi_b\,,\chi_\tau\,,\xi_\tau\,,\chi_{\nu_\tau}\,,\chi_c\,,\xi_c\,,\chi_s,\xi_s,\chi_\mu,\xi_\mu,\chi_{\nu_\mu},\chi_u,\xi_u,\chi_d,\xi_d,\chi_e,\xi_e,\chi_{\nu_e}\,. \qquad \text{(D2)}
$$

For simplicity we have suppressed the $SU(3)$ color indices for all quark spinors. Of course, each quark spinor has to be understood as a triplet in color space. In this basis the Yukawa matrices become 45×45 dimensional.

In the notation of [16] the part of the Lagrange density describing the Yukawa couplings is given by

$$-\frac{1}{2}\left(Y^{aij}\phi^a\psi_i\psi_j + \bar{Y}^a_{ij}\phi^a\bar{\psi}^i\bar{\psi}^j\right) , \tag{D3}$$

where Y^a and $\bar{Y}^a$ are the (complex conjugated) Yukawa matrices, ϕ^a are real scalar fields, ψ^i and $\bar{\psi}^i$ are (Hermitian conjugated) spinor fields. There are four real scalar fields in the SM, which means that we have four Yukawa matrices. They are given by

$$Y^1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0_{3\times3} & y_t\mathbb{1}_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & \cdots \\ y_t\mathbb{1}_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & \cdots \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & y_b\mathbb{1}_{3\times3} & 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & \cdots \\ 0_{3\times3} & 0_{3\times3} & y_b\mathbb{1}_{3\times3} & 0_{3\times3} & 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & \cdots \\ 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0 & y_\tau & 0 & \cdots \\ 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & y_\tau & 0 & 0 & \cdots \\ 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix},$$

$$Y^2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & y_b\mathbb{1}_{3\times3} & 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & \cdots \\ 0_{3\times3} & 0_{3\times3} & -y_t\mathbb{1}_{3\times3} & 0_{3\times3} & 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & \cdots \\ 0_{3\times3} & -y_t\mathbb{1}_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & \cdots \\ y_b\mathbb{1}_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & \cdots \\ 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0 & 0 & 0 & \cdots \\ 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0 & 0 & y_\tau & \cdots \\ 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0 & y_\tau & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix},$$

$$Y^3 = -\frac{1}{\sqrt{2}}\begin{pmatrix} 0_{3\times3} & -\mathrm{i}y_t\mathbb{1}_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & \cdots \\ -\mathrm{i}y_t\mathbb{1}_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & \cdots \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & \mathrm{i}y_b\mathbb{1}_{3\times3} & 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & \cdots \\ 0_{3\times3} & 0_{3\times3} & \mathrm{i}y_b\mathbb{1}_{3\times3} & 0_{3\times3} & 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & \cdots \\ 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0 & \mathrm{i}y_\tau & 0 & \cdots \\ 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & \mathrm{i}y_\tau & 0 & 0 & \cdots \\ 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix},$$

$$Y^4 = -\frac{1}{\sqrt{2}} \begin{pmatrix} 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & \mathrm{i}y_b \mathbb{1}_{3\times3} & 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & \cdots \\ 0_{3\times3} & 0_{3\times3} & \mathrm{i}y_t \mathbb{1}_{3\times3} & 0_{3\times3} & 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & \cdots \\ 0_{3\times3} & \mathrm{i}y_t \mathbb{1}_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & \cdots \\ \mathrm{i}y_b \mathbb{1}_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & \cdots \\ 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0 & 0 & 0 & \cdots \\ 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0 & 0 & \mathrm{i}y_\tau & \cdots \\ 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0 & \mathrm{i}y_\tau & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} . \tag{D4}$$

In the above formulas, all matrix elements not explicitly given are zero. Y^1/Y^3 is the Yukawa matrix of the real/the imaginary part of the isospin down component of the SM Higgs doublet. Y^2/Y^4 is the Yukawa matrix of the real/the imaginary part of the isospin up component of the SM Higgs doublet. So we have

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi^2 + \mathrm{i}\phi^4 \\ \phi^1 + \mathrm{i}\phi^3 \end{pmatrix} . \tag{D5}$$

The part of the Lagrange density describing the Higgs self-interaction reads in the notation of Ref. [16]

$$-\frac{1}{4!}\lambda_{abcd}\phi^a\phi^b\phi^c\phi^d . \tag{D6}$$

In the SM we have

$$\begin{aligned} \lambda_{aaaa} &= 6 \times (4\pi\hat{\lambda}), \\ \lambda_{aabb} = \lambda_{abab} = \lambda_{abba} &= 2 \times (4\pi\hat{\lambda}) \ (a \neq b), \\ \lambda_{abcd} &= 0 \quad (\text{otherwise}) . \end{aligned} \tag{D7}$$

The expressions given above are generic for all three gauge groups. However, there are some expressions which depend on the specific gauge group one wants to consider. The remainder of this Section lists these expressions. For each of the three gauge groups, we give the expressions for the generators in the representations of the scalar fields, S^A, and of the Weyl spinors, R^A. We also give expressions for the invariants $T(S), C(S), T(R), C(R), C(G), r$, all of which are symbols used in Ref. [16]. They are defined as

$$\begin{aligned} \mathrm{Tr}\left(S^A S^B\right) &= \delta^{AB} T(S), & S^A_{ac} S^A_{cb} &= C(S)_{ab}, \\ \mathrm{Tr}\left(R^A R^B\right) &= \delta^{AB} T(R), & R_i^{Ak} R_k^{Aj} &= C(R)_i^j, \\ f^{ACD} f^{BCD} &= \delta^{AB} C(G), & \delta^{AA} &= r . \end{aligned} \tag{D8}$$

f^{ABC} are the structure constants of the respective gauge group. The indices A, B, C take values in the range $1, \ldots, r$, where r gives the dimension of the group.

1. $U(1)$

For the case of $U(1)$ gauge group, we have

$$S^1 = \frac{\mathrm{i}}{2} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} , \tag{D9}$$

$$R^1 = \mathrm{Diag}\left(\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, -\frac{2}{3}, -\frac{2}{3}, -\frac{2}{3}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -\frac{1}{2}, 1, -\frac{1}{2}, \dots \right) , \tag{D10}$$

where the ellipsis is to be replaced twice by the first 15 entries. The derivation of S^1 is given below. The entries in R^1 are the hypercharges Y of the respective spinors. They can be derived by using the relation $Y = Q - I_3$, where Q corresponds to the electric charge and I_3 is the weak isospin. Furthermore, we have

$$\begin{aligned} T(S) &= 1\,, & C(S) &= \frac{1}{4}\mathbb{1}_{4\times 4}\,, \\ T(R) &= 10\,, & C(R) &= \left(R^1\right)^2\,, \\ C(G) &= 0\,, & r &= 1\,. \end{aligned} \tag{D11}$$

Let us now we show how to derive Eq. (D9). We want to find the $U(1)$ representation transforming the four real scalar fields of the Higgs doublet of the SM (see Eq. (D5). The matrix S^1 is the generator of this transformation,

$$\begin{pmatrix} \phi_1' \\ \phi_2' \\ \phi_3' \\ \phi_4' \end{pmatrix} = \mathrm{e}^{\mathrm{i}\omega S^1} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix} = \left(\mathbb{1} + \mathrm{i}\omega \mathrm{S}^1 + \mathcal{O}(\omega^2)\right) \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix} . \tag{D12}$$

with ω being the transformation parameter. In a next step we take advantage of the fact that it is known how the SM Higgs doublet transforms in order to determine S^1. As the SM Higgs doublet has hypercharge 1/2, we have

$$\begin{aligned} \frac{1}{\sqrt{2}} \begin{pmatrix} \phi'^2 + \mathrm{i}\phi'^4 \\ \phi'^1 + \mathrm{i}\phi'^3 \end{pmatrix} &= H' = \mathrm{e}^{\mathrm{i}\omega\left(\frac{1}{2}\mathbb{1}\right)} H = \mathrm{e}^{\mathrm{i}\omega\left(\frac{1}{2}\mathbb{1}\right)} \frac{1}{\sqrt{2}} \begin{pmatrix} \phi^2 + \mathrm{i}\phi^4 \\ \phi^1 + \mathrm{i}\phi^3 \end{pmatrix} \\ &= \left[\mathbb{1} + \mathrm{i}\omega \left(\frac{1}{2}\mathbb{1}\right) + \mathcal{O}(\omega^2)\right] \frac{1}{\sqrt{2}} \begin{pmatrix} \phi^2 + \mathrm{i}\phi^4 \\ \phi^1 + \mathrm{i}\phi^3 \end{pmatrix} \\ &= H + \omega \frac{1}{2} \frac{1}{\sqrt{2}} \begin{pmatrix} -\phi^4 + \mathrm{i}\phi^2 \\ -\phi^3 + \mathrm{i}\phi^1 \end{pmatrix} + \mathcal{O}(\omega^2)\,. \end{aligned} \tag{D13}$$

With the help of the last equation one can determine the transformation of the SM Higgs doublet, like, e.g., $\phi_1' = \phi_1 - \frac{\omega}{2}\phi_3 + \mathcal{O}(\omega^2)$. It is then straightforward to determine S^1 by inserting the equations found in this way in Eq. (D12).

2. $SU(2)$

For the $SU(2)$ group, we have

$$S^1 = -\frac{\mathrm{i}}{2}\begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \; S^2 = -\frac{\mathrm{i}}{2}\begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \; S^3 = -\frac{\mathrm{i}}{2}\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \tag{D14}$$

and

$$R^A = 1/2\begin{pmatrix} \sigma^A_{1,1}\mathbb{1}_{3\times3} & 0_{3\times3} & \sigma^A_{1,2}\mathbb{1}_{3\times3} & 0_{3\times3} & 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & \cdots \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & \cdots \\ \sigma^A_{2,1}\mathbb{1}_{3\times3} & 0_{3\times3} & \sigma^A_{2,2}\mathbb{1}_{3\times3} & 0_{3\times3} & 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & \cdots \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & \cdots \\ 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & \sigma^A_{2,2} & 0 & \sigma^A_{2,1} & \cdots \\ 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0 & 0 & 0 & \cdots \\ 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & \sigma^A_{1,2} & 0 & \sigma^A_{1,1} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \tag{D15}$$

In the last equation σ^A are the Pauli matrices. Not all of the matrix elements left out in R^A vanish, but these elements play no role for the comparison of our results to [16]. The derivation of the generators S^A proceeds in analogy to the derivation of S^1 explained in the former Subsection. One merely has to substitute the generator of $U(1)$, $\frac{1}{2}\mathbb{1}$, by the generators of SU(2), $\frac{1}{2}\sigma^A$, in Eq. (D13).

We furthermore have

$$\begin{aligned} T(S) &= 1, & C(S) &= \frac{3}{4}\mathbb{1}_{4\times4}, \\ T(R) &= 6, & C(R) &= \frac{3}{4}\,\mathrm{Diag}\left(1,1,1,0,0,0,1,1,1,0,0,0,1,0,1,\dots\right), \\ C(G) &= 2, & r &= 3\,. \end{aligned} \tag{D16}$$

The ellipsis has to be replaced by the first entries twice.

3. $SU(3)$

In the SM the matrices S^A vanish for the group $SU(3)$. The matrices R^A are block-diagonal and read

$$R^A = \frac{1}{2}\,\mathrm{BlockDiag}\left(\Lambda^A, -(\Lambda^A)^T, \Lambda^A, -(\Lambda^A)^T, 0, 0, 0, \dots\right). \tag{D17}$$

The ellipsis has to be replaced twice by the former entries, which contain the Gell-Mann matrices Λ^A.

Finally, we have

$$T(S) = 0, \qquad C(S) = 0_{4\times4},$$

$$
\begin{aligned}
T(R) &= 6, \qquad & C(R) &= \frac{4}{3}\,\mathrm{BlockDiag}\left(\mathbb{1}_{12\times12}, 0_{3\times3}, \mathbb{1}_{12\times12}, 0_{3\times3}, \mathbb{1}_{12\times12}, 0_{3\times3}\right), \\
C(G) &= 3, \qquad & r &= 8\,.
\end{aligned} \tag{D18}
$$

---

[1] D. J. Gross and F. Wilczek, Phys. Rev. Lett. **30** (1973) 1343.

[2] H. D. Politzer, Phys. Rev. Lett. **30** (1973) 1346.

[3] D. R. T. Jones, Nucl. Phys. B **75**, 531 (1974).

[4] O. V. Tarasov and A. A. Vladimirov, Sov. J. Nucl. Phys. **25** (1977) 585 [Yad. Fiz. **25** (1977) 1104].

[5] W. E. Caswell, Phys. Rev. Lett. **33** (1974) 244.

[6] E. Egorian and O. V. Tarasov, Teor. Mat. Fiz. **41** (1979) 26 [Theor. Math. Phys. **41** (1979) 863].

[7] D. R. T. Jones, Phys. Rev. D **25** (1982) 581.

[8] M. S. Fischler and C. T. Hill, Nucl. Phys. B **193** (1981) 53.

[9] M. E. Machacek and M. T. Vaughn, Nucl. Phys. B **222** (1983) 83.

[10] I. Jack and H. Osborn, Nucl. Phys. B **249** (1985) 472.

[11] T. Curtright, Phys. Rev. D **21** (1980) 1543.

[12] D. R. T. Jones, Phys. Rev. D **22** (1980) 3140.

[13] O. V. Tarasov, A. A. Vladimirov and A. Y. Zharkov, Phys. Lett. B **93** (1980) 429.

[14] S. A. Larin and J. A. M. Vermaseren, Phys. Lett. B **303** (1993) 334 [hep-ph/9302208].

[15] M. Steinhauser, Phys. Rev. D **59** (1999) 054005 [arXiv:hep-ph/9809507].

[16] A. G. M. Pickering, J. A. Gracey and D. R. T. Jones, Phys. Lett. B **510** (2001) 347 [Phys. Lett. B **512** (2001 ERRAT,B535,377.2002) 230] [arXiv:hep-ph/0104247].

[17] T. van Ritbergen, J. A. M. Vermaseren and S. A. Larin, Phys. Lett. B **400** (1997) 379 [hep-ph/9701390].

[18] M. Czakon, Nucl. Phys. B **710** (2005) 485 [arXiv:hep-ph/0411261].

[19] M. Fischler and J. Oliensis, Phys. Lett. B **119** (1982) 385.

[20] M. E. Machacek and M. T. Vaughn, Nucl. Phys. B **236** (1984) 221.

[21] M. E. Machacek and M. T. Vaughn, Nucl. Phys. B **249** (1985) 70.

[22] C. Ford, I. Jack and D. R. T. Jones, Nucl. Phys. B **387** (1992) 373 [Erratum-ibid. B **504** (1997) 551] [arXiv:hep-ph/0111190].

[23] M. x. Luo and Y. Xiao, Phys. Rev. Lett. **90** (2003) 011601 [arXiv:hep-ph/0207271].

[24] K. G. Chetyrkin and M. F. Zoller, JHEP **1206** (2012) 033 [arXiv:1205.2892 [hep-ph]].

[25] L. N. Mihaila, J. Salomon and M. Steinhauser, Phys. Rev. Lett. **108** (2012) 151602 [arXiv:1201.5868 [hep-ph]].

[26] L. F. Abbott, Nucl. Phys. B **185** (1981) 189.

[27] A. Denner, G. Weiglein and S. Dittmaier, Nucl. Phys. B **440** (1995) 95 [hep-ph/9410338].

[28] S. A. Larin, F. V. Tkachov and J. A. M. Vermaseren, preprint NIKHEF-H-91-18 (1991).

[29] P. Nogueira, J. Comput. Phys. **105** (1993) 279.

[30] R. Harlander, T. Seidensticker and M. Steinhauser, Phys. Lett. B **426** (1998) 125 [hep-ph/9712228].

[31] T. Seidensticker, hep-ph/9905298.

[32] J. A. M. Vermaseren, arXiv:math-ph/0010025.

[33] M. Steinhauser, Comput. Phys. Commun. **134** (2001) 335 [arXiv:hep-ph/0009029].

[34] T. Hahn, Comput. Phys. Commun. **140** (2001) 418 [hep-ph/0012260].

[35] N. D. Christensen and C. Duhr, Comput. Phys. Commun. **180** (2009) 1614 [arXiv:0806.4194 [hep-ph]].

[36] J. C. Collins, “Renormalization,” Cambridge, Uk: Univ. Pr. (1984) 380p

[37] M. E. Peskin and D. V. Schroeder, *Reading, USA: Addison-Wesley (1995) 842 p*

[38] S. L. Adler, Phys. Rev. **177** (1969) 2426.

[39] J. S. Bell und R. Jackiw, Nuovo Cim. A **60** (1969) 47.

[40] K. Nakamura *et al.* [Particle Data Group Collaboration], J. Phys. G G **37** (2010) 075021.

[41] V. A. Smirnov, “Applied asymptotic expansions in momenta and masses,” Springer Tracts Mod. Phys. **177** (2002) 1.

[42] W. Martens, L. Mihaila, J. Salomon und M. Steinhauser, Phys. Rev. D **82** (2010) 095013 [arXiv:1008.3070 [hep-ph]].