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Status of the fourth fermion generation before ICHEP2012: Higgs data and electroweak precision observables

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We perform a global fit of the parameters of the Standard Model with a sequential fourth generation (SM4) to LHC and Tevatron Higgs data and electroweak precision data. Using several likelihood ratio tests we compare the performance of the SM4 and SM3 at describing the measured data. Since the SM3 and SM4 are not *nested* (i.e. the SM3 can not be considered as a special case of the SM4 with some parameters fixed) the usual analytical formulae for p-values in likelihood ratio tests do not hold. We thus apply a new method to compute these p-values. For a Higgs mass of 126.5 GeV and fourth-generation quark masses above 600 GeV we find that the SM4 is excluded at 3.1σ.

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METHOD AND INPUTS

In this paper we study the SM4, which differs from the established Standard Model (denoted by SM3) by an additional fermion generation. We treat the masses of the extra fermions as free parameters and allow for arbitrary flavor mixings among the quarks of the four generations in our fits. Large mixings of the fourth-generation lepton doublet with those of the first three generations are ruled out [1] from data on lepton-flavor violating decays and lepton-flavor universality [2]. Recent NA62 data constrain these mixing angles even further [3]. Including lepton mixing within the allowed range has a negligible impact on the electroweak precision observables (EWPOs). In the absence of lepton mixing the decay of the Higgs boson into neutrinos is invisible as long as the fourth-generation charged lepton is heavier than the corresponding neutrino. This invisible Higgs decay mode increases the total Higgs width and potentially counterbalances the effect of the enhanced $gg \to H$ production mechanism [4, 5], because the branching fractions into the observed final states are reduced [6, 7]. Allowing for (even small) mixing of the fourth with the other lepton doublets can render the neutrino decay mode visible. Since we want to quantify the level at which the SM4 is ruled out, we may confine ourselves to the most conservative scenario with an unmixed fourth-generation lepton doublet. Like the SM3, the SM4 can be studied with Dirac or Majorana neutrinos. In the fits presented in this paper we use Dirac neutrinos. In our conclusions we briefly discuss the (marginal) changes in the results expected for Majorana neutrinos. From a model-building point of view, the hierarchy between three almost massless neutrinos and a fourth neutrino with mass of order of the electroweak scale can be motivated by a symmetry enforcing massless neutrinos in the exact symmetry limit: e.g. three right-handed neutrino fields might carry some U(1) charge while the fourth neutrino field and the left-handed lepton doublets are uncharged under this new symmetry. The Yukawa couplings are small spurions breaking this symmetry, leading to three tiny neutrino masses and tiny mixings between the fourth and the other generations.

A sequential fourth generation of fermions decouples neither from the production cross section $\sigma(gg \to H)$ nor from the Higgs decay rate into photons. Consequently, current LHC Higgs data put the SM4 under serious pressure [7–10]. In a recent publication [7] we presented a global fit of the SM4 parameters to EWPOs and Higgs signal strengths measured at Tevatron and the LHC. The signal strength is defined as

$$\hat{\mu}(X \to H \to Y) = \frac{\sigma(X \to H)\mathcal{B}(H \to Y)|_{\mathrm{SM4}}}{\sigma(X \to H)\mathcal{B}(H \to Y)|_{\mathrm{SM3}}} \quad . \tag{1}$$

Here we update our results with all available data and analyse the status of the SM4 prior to the ICHEP2012 conference. We also compute the statistical significance (p-value) at which the SM4 is excluded. As explained in [7] the computation of the p-value is non-trivial: due to the non-decoupling nature of the fourth-generation fermions the

SM3 can not be regarded as a special case of the SM4, i.e. the two models are not *nested*. Analytical formulae for p-values only hold for nested models and thus the p-value of the SM4 has to be computed numerically. To this end, a new C++ framework for maximum likelihood fits and likelihood ratio tests called *my*Fitter [11] was written. The implementation is discussed in [12].

In total, the following aspects of our previous analysis have been improved:

1. The masses of all four fourth-generation fermions are now consistently treated as free parameters. To avoid non-perturbative Yukawa couplings and constraints from direct searches of fourth-generation quarks we require $600\,\text{GeV} \leq m_{t'}, m_{b'} \leq 800\,\text{GeV}$. We are aware that for fermion masses of 800 GeV the validity of perturbation theory is questionable at best. However, reducing the upper limit for the fermion masses can only lead to larger χ^2 values in the SM4 and thus to smaller p-values. In this sense, the upper limit of 800 GeV is a *conservative* estimate.

2. The signal strength for $pp \to H \to \tau\tau$ measured at the LHC [13] is included in the analysis.

3. In the global fit, the Higgs mass is no longer fixed at 125 GeV, but is allowed to float in the range where experimental data on the Higgs signal strengths is available, i.e. $115\,\text{GeV} \leq m_H \leq 150\,\text{GeV}$.[**?**]

4. Since, for a variable Higgs mass, no separate $H \to \gamma\gamma$ signal strengths for the gluon fusion and vector boson fusion production modes are available we only use the combined signal strength for $pp \to H \to \gamma\gamma$ as input.

5. For the two cases $m_H = 126.5\,\text{GeV}$ (the preferred Higgs mass of the SM3) and $m_H = 147\,\text{GeV}$ (the preferred Higgs mass of the SM4) we perform likelihood ratio tests to compare the performance of the SM3 and SM4 at describing the measured data.

Regarding the last point, a few more comments are in order. In likelihood ratio tests the difference $\Delta\chi^2$ of minimal χ^2 values obtained in the SM3 and the SM4 is used as a test statistic. One then assumes that the measured observables are random variables distributed around the prediction of one model (e.g. the SM4) with a spread determined by their errors and computes the probability (p-value) that a random set of "toy-observables" leads to a $\Delta\chi^2$ which is more extreme (e.g. more SM3-like) than the $\Delta\chi^2$-value obtained from the real data. Note that this is different from the goodness-of-fit analysis presented in [9], which used the χ^2 value of the SM4 as a test statistic and therefore did not compare the performance of the SM3 and the SM4. Also, the $H \to \tau\tau$ signal strengths were not included in their analysis.

Unfortunately, the likelihood ratio tests can not be done (by us) if the Higgs mass is treated as a free parameter. In that case, the signal strengths measured in each invariant mass bin of each Higgs decay mode would have to be treated as separate observables, and we do not have any information on statistical correlations between adjacent bins. Thus we only perform likelihood ratio tests for *specialisations* of the SM3 and SM4, where the Higgs mass is fixed to $m_H = 126.5\,\text{GeV}$ (the value preferred by the global SM3 fit) or $m_H = 147\,\text{GeV}$ (the value preferred by the global SM4 fit). Then only the signal strengths at $m_H = 126.5\,\text{GeV}$ and $m_H = 147\,\text{GeV}$ have to be treated as independent observables and correlations between these observables can safely be neglected.

Note, however, that the information from *all* invariant mass bins is encoded in our χ^2 function. So, for example, the χ^2 value at $m_H = 147\,\text{GeV}$ has a contribution due to the fact that there is a signal at $m_H = 126.5\,\text{GeV}$. If the model under consideration had a Higgs boson outside the discovery reach of LHC (or no Higgs boson at all), the theory prediction for all signal strengths in all invariant mass bins would be zero. This leads to a constant contribution to the χ^2, which we are allowed to drop. Now assume that the model has a Higgs boson with some mass m_H and a predicted signal strength $\hat{\mu}_{\text{th}}(m_H)$. Let $\hat{\mu}_{\text{ex}}(m_H)$ and $\Delta\hat{\mu}(m_H)$ be the measured signal strength and experimental error for the corresponding invariant mass bin. After dropping the constant, the χ^2 function is

$$\chi^2(m_H) = \frac{[\hat{\mu}_{\text{th}}(m_H) - \hat{\mu}_{\text{ex}}(m_H)]^2 - [\hat{\mu}_{\text{ex}}(m_H)]^2}{[\Delta\hat{\mu}(m_H)]^2} \quad . \tag{2}$$

If there is a clear signal at the Higgs mass m_H, the second term gives a large negative contribution to the χ^2 function. This contribution is not present if m_H is in a region without a signal, so the minimum of the χ^2 function will usually be at a Higgs mass close to the signal.

In the present analysis, the following experimental inputs are used:

i) $\hat{\mu}(pp \to H \to WW^*)$ measured by ATLAS [15],

ii) $\hat{\mu}(pp \to H \to \gamma\gamma)$ measured by ATLAS [16],

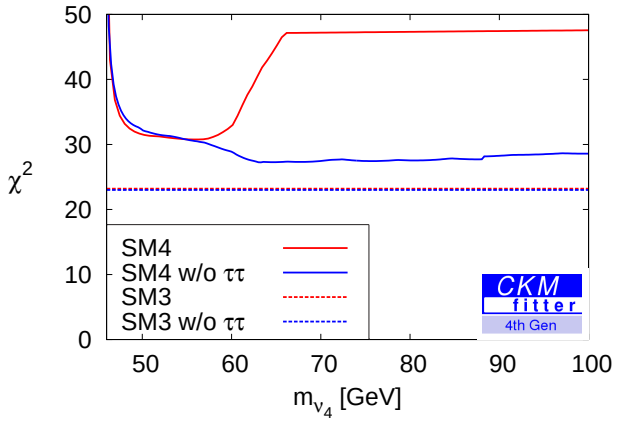

FIG. 1: Minimum χ^2 values for a fixed neutrino mass as a function of m_{ν_4}. The blue (red) lines show the results from the combined analysis of EWPOs and Higgs signal strengths with (without) the $H \to \tau\tau$ channel. The solid and dashed lines correspond to the SM4 and SM3, respectively.

iii) $\hat{\mu}(p\bar{p} \to HV \to Vb\bar{b})$ measured by CDF and D0 [17],

iv) $\hat{\mu}(pp \to H \to ZZ^*)$ and $\hat{\mu}(pp \to H \to \tau\bar{\tau})$ measured by ATLAS [13],

iv) the electroweak precision observables (EWPOs) M_Z, Γ_Z, $\sigma_{\rm had}$, $A^l_{\rm FB}$, $A^c_{\rm FB}$, $A^b_{\rm FB}$, A_l, A_c, A_b, $R_l = \Gamma_{l^+l^-}/\Gamma_{\rm had}$, R_c, R_b, $\sin^2\theta^{\rm eff}_l$ measured at LEP and SLC [18] as well as m_t, M_W, Γ_W and $\Delta\alpha^{(5)}_{\rm had}$ [19].

v) the lower bounds $m_{t',b'} \gtrsim 600$ GeV(from the LHC) [20–23] and $m_{l_4} > 101\,\text{GeV}$ (from LEP2) [19].

Unfortunately, there is no data for signal strengths as a function of the Higgs mass from CMS.

On the theory side, the global fits with a variable Higgs mass were done with the CKMfitter software [24]. The EWPOs in the SM4 were calculated with the method described in [25], using FeynArts, FormCalc and LoopTools [26–28] to compute the SM4 corrections to the EWPOs. The EWPOs in the SM3 were calculated with the ZFitter software [29–31]. The Higgs width and branching ratios in the SM4 and SM3 were calculated with HDECAY v. 4.45 [32], which implements results of [33–36]. The SM3 Higgs production cross sections were taken from [37] (LHC) and [38, 39] (Tevatron). For the numerical integration required to compute the p-values we use the Dvegas code [40] which was developed in the context of [41, 42].

RESULTS

To show the impact of the $H \to \tau\tau$ signal strength we plot the minimal χ^2 value with and without the $H \to \tau\tau$ input as a function of the mass m_{ν_4} of the fourth-generation neutrino in Fig. 1. We see that for $m_{\nu_4} \lesssim 60\,\text{GeV}$ the minimum χ^2 values are almost the same with and without the $H \to \tau\tau$ input. For $m_{\nu_4} \gtrsim 65\,\text{GeV}$ the $H \to \tau\tau$ input increases the minimum χ^2 by more than 20. We also see that without the $H \to \tau\tau$ input the SM4 favours large values of m_{ν_4}.With the $H \to \tau\tau$ signal strengths included, the smallest χ^2 values are obtained for m_{ν_4} between 50 and 60 GeV.

This can be understood as follows: the production rate of Higgs bosons in gluon fusion is enhanced by a factor of 9 in the SM4 due to the contributions from additional heavy quark loops. On the other hand, the effective HWW, HZZ and $H\gamma\gamma$ couplings are suppressed by the higher order corrections discussed in [36]. No such suppression is possible for $H \to \tau\tau$, so we would expect a $H \to \tau\tau$ signal strength of 9. The only way to reduce this signal strength is to open the invisible $H \to \nu_4\bar{\nu}_4$ decay mode, which then suppresses all branching ratios by a common factor. Thus, for large values of m_{ν_4}, the fit gets considerably worse if the $H \to \tau\tau$ channel is included.

Figs. 2 and 3 show the minimum χ^2 value as a function of the Higgs mass m_H in the SM3 and SM4, respectively. The solid lines show the results of the combined analysis of signal strengths and EWPOs while for the dashed lines only the Higgs signal strengths (including $H \to \tau\tau$) were used as inputs. We see that the SM3 clearly prefers a Higgs mass near 126.5 GeV. This is in agreement with a similar analysis presented in [43]. There is another local minimum at $m_H = 145\,\text{GeV}$, but with a considerably larger χ^2 value. The χ^2 function of the SM4 in the combined analysis of signal strengths and EWPOs also has one minimum at $m_H = 126.5\,\text{GeV}$ and another one at $m_H = 147\,\text{GeV}$. Here,

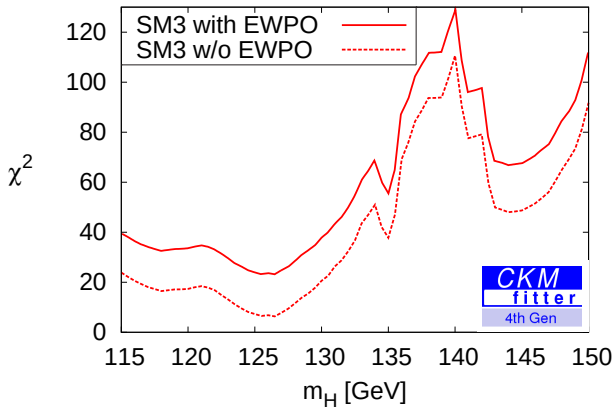

FIG. 2: The minimum χ^2 value of the SM3 as a function of the Higgs mass m_H. The solid line shows the results of the combined analysis of signal strengths and EWPOs. For the dashed line only the signal strengths were included in the fit.

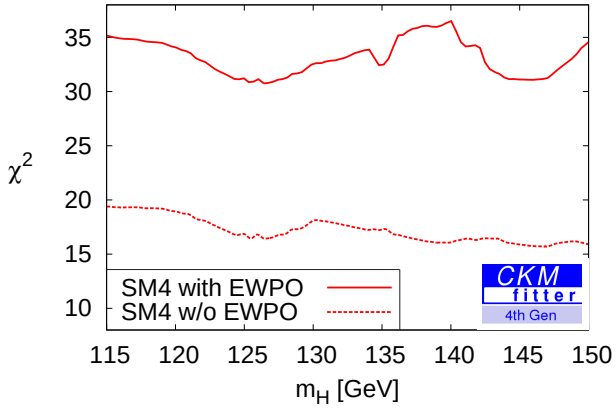

FIG. 3: The minimum χ^2 value of the SM4 as a function of the Higgs mass m_H. The solid line shows the results of the combined analysis of signal strengths and EWPOs. For the dashed line only the signal strengths were included in the fit.

the χ^2 values are almost the same, but still larger than the minimal χ^2 value of the SM3, $\chi^2 = 23.3$, by about 8 units. Note that for non-nested models or models with bounded parameters the relation between χ^2 values and p-values is no longer given by Wilks' theorem. Thus, in the case of the SM4, the number of degrees of freedom is an ill-defined concept and the p-values have to be calculated by numerical simulation. For the simulations we used the *my*Fitter package [11]. Further details on the statistical issues and the *my*Fitter simulation method can be found in [12]. For $m_H = 147\,\text{GeV}$ the signals at invariant masses near 126.5 GeV would be interpreted as statistical fluctuations. Then the data would be better described by the SM4 because it has more mechanisms for suppressing its Higgs signals. These mechanisms were discussed in [7].

Fig. 4 shows the pulls of the Higgs signal strengths for the SM3 with a Higgs mass of 126.5 GeV and the SM4 with a Higgs mass of 126.5 GeV or 147 GeV. We see that in the SM4 with $m_H = 126.5\,\text{GeV}$ the measured $H \to \tau\tau$ signal strength deviates by more than 4σ from its predicted value. This is due to the effect mentioned in the discussion of Fig. 1. For the SM4 with $m_H = 147\,\text{GeV}$ the measured signal strengths for the invariant mass bin at 147 GeV are in good agreement with their theory predictions. However, in that case the χ^2 receives a large contribution due to the fact that the measured values of the signal strengths in the invariant mass bin at 126.5 GeV deviate from their predicted values of zero.

Table I shows the p-values obtained from the likelihood ratio tests for the two SM4 Higgs masses. We see that, based on the Higgs signal strengths alone, the SM4 scenario with $m_H = 126.5\,\text{GeV}$ is ruled out at almost 4σ while the scenario with $m_H = 147\,\text{GeV}$ is only excluded at 3σ. At a fixed Higgs mass of 126.5 GeV the electroweak fit is actually better in the SM4 than in the SM3. Thus, if the EWPOs are included in the fit, the p-value increases to 2 permille, which corresponds to 3.1σ. The lower bound $m_{t',b'} \gtrsim 600$ GeV is not essential for this result, relaxing this bound to $m_{t',b'} \gtrsim 400$ GeV decreases the minimum-χ^2 by 0.6. For the SM4 scenario with $m_H = 147\,\text{GeV}$ the p-value

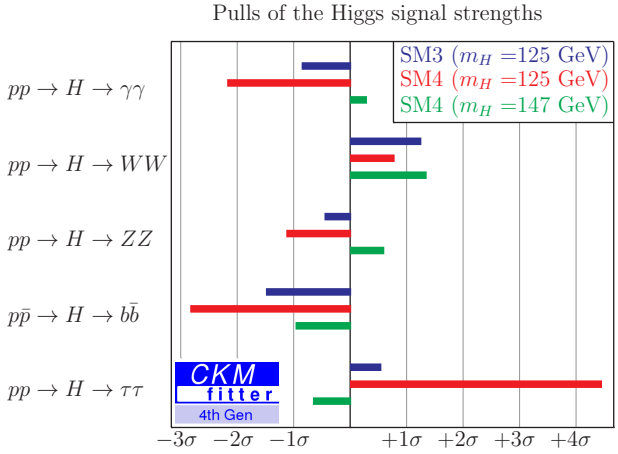

FIG. 4: Pulls of the Higgs signal strengths for the SM3 with $m_H = 126.5\,\text{GeV}$ and the SM4 with $m_H = 126.5\,\text{GeV}$ and $m_H = 147\,\text{GeV}$.

	SM4 @ 126.5 GeV	SM4 @ 147 GeV
Higgs only	$0.088 \cdot 10^{-3}$ (3.9σ)	$2.4 \cdot 10^{-3}$ (3.0σ)
Higgs+EWPOs	$2.0 \cdot 10^{-3}$ (3.1σ)	$0.74 \cdot 10^{-3}$ (3.4σ)

TABLE I: p-values obtained from the likelihood ratio tests for fixed Higgs mass. Both, the SM4 with $m_H = 126.5\,\text{GeV}$ and $m_H = 147\,\text{GeV}$ are compared to the SM3 with a fixed Higgs mass of 126.5 GeV. In the first row, only the Higgs signal strengths were used as inputs. The second row contains the results of the combined analysis of signal strengths and EWPOs. The number of standard deviations corresponding to each p-value are shown in parentheses.

drops to 0.74 permille (3.4σ). In any case, the SM4 is excluded at more than 3σ.

CONCLUSIONS

We presented a combined analysis of Higgs signal strengths and EWPOs in the context of the Standard Model with three or four fermion generations. The SM3 is in good agreement with the experimental data and the best-fit Higgs mass is 126.5 GeV. The SM4, on the other hand, struggles to describe the Higgs signal strengths measured at Tevatron and the LHC. The χ^2 function of the SM4 has two minima at $m_H = 126.5\,\text{GeV}$ and $m_H = 147\,\text{GeV}$ with essentially the same χ^2 value, which is larger than the minimal χ^2 value of the SM3 by 8 units. The second minimum of the SM4 χ^2 function occurs because the SM4 cannot reproduce the signal strengths measured at 126.5 GeV very well, so that an SM4 with a Higgs mass nowhere near the observed signals describes the data equally well as an SM4 with $m_H = 126.5\,\text{GeV}$. To quantitatively compare the performance of the SM3 and SM4 at describing the data we performed likelihood ratio tests for fixed Higgs masses m_H of 126.5 GeV in the SM3 and $m_H = 126.5\,\text{GeV},\ 147\,\text{GeV}$ in the SM4. The p-values were computed with a new numerical method [12] for likelihood ratio tests of non-nested models. If EWPOs and signal strengths are included in the fit we find p-values of $2.0 \cdot 10^{-3}$ and $0.74 \cdot 10^{-3}$, respectively, which means that the SM4 is excluded at the 3σ level. While this result is obtained for Dirac neutrinos, it will change only marginally for the case of Majorana neutrinos with two fourth-generation mass eigenstates ν_4, ν_5: the fit to the signal sthrengths will return the same invisible Higgs width, now corresponding to the sum of the four decay rates $\Gamma(H \to \nu_{4,5}\nu_{4,5})$. A marginal difference occurs once the EWPOs are included: choosing the ν_4–ν_5 mass splitting such that the eigenstate with the larger SU(2) doublet component becomes heavier, one can slightly improve the quality of the electroweak fit. The improvement is negligible, as indicated by the shallowness of the minimum of the SM4 χ^2 function in Fig. 1. While the SM4 is under severe pressure, a sequential fourth generation may still be viable in conjunction with an extended Higgs sector [44–47].

Acknowledgements

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