SFB/CPP-12-30
TTP12-22
LPN12-068

Decoupling constant for α_s and the effective gluon-Higgs coupling to three loops in supersymmetric QCD

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Abstract

We compute the three-loop QCD corrections to the decoupling constant for α_s which relates the Minimal Supersymmetric Standard Model to Quantum Chromodynamics with five or six active flavours. The new results can be used to study the stability of α_s evaluated at a high scale from the knowledge of its value at M_Z. We furthermore derive a low-energy theorem which allows the calculation of the coefficient function of the effective Higgs boson-gluon operator from the decoupling constant. This constitutes the first independent check of the matching coefficient to three loops.

PACS numbers: 11.30.Pb 12.38.Bx

1 Introduction

The decoupling of particles with masses much heavier than the considered energy scale has a long history [1]. It is tightly connected to the construction of an effective theory containing only the light active degrees of freedom in the dynamical part of the Lagrange density. Within the framework of QCD decoupling constants for the strong coupling α_s are known at two- [2–4], three [4] and even four-loop order [5, 6]. Recently also the expression for the simultaneous decoupling of two heavy quarks has been computed at the three-loop level [7].

Decoupling relations are also important in the context of supersymmetry where the Standard Model constitutes the effective theory. Two-loop corrections for a degenerate supersymmetric mass spectrum are known from Ref. [8, 9] and the general result can be found in Ref. [10]. In this paper we compute the three-loop corrections for several different assumptions on the masses of the MSSM.

There is an interesting connection between the decoupling constants and the effective coupling of a CP neutral Higgs boson to gluons which is defined via the Lagrange density (the superscript 0 marks bare quantities)

$$\mathcal{L}_{Y,\mathrm{eff}} = -\frac{\phi^0}{v^0} C_1^0 \mathcal{O}_1^0 + \mathcal{L}_{QCD}^{(5)} , \tag{1}$$

with

$$\mathcal{O}_1^0 = \frac{1}{4} G_{\mu\nu}^0 G^{0,\mu\nu} . \tag{2}$$

where ϕ is the Higgs field v is the vacuum expectation value and $G_{\mu\nu}$ the field strength tensor in QCD. $\mathcal{L}_{QCD}^{(5)}$ is the QCD Lagrange density with five active flavours. The first term in Eq. (1) describes the coupling of the Higgs boson to two, three and four gluons.

In Ref. [4] a low-energy theorem (LET) has been derived which connects C_1 to the derivative of the decoupling constant for α_s with respect to the top quark mass. As far as supersymmetry is concerned a next-to-leading order (NLO) version of the LET has been derived in Ref. [11] (see also Ref. [12]). In this way the NLO supersymmetric QCD (SQCD) corrections to C_1 obtained in Ref. [13] could be confirmed. We re-derive the LET, apply it at three loops and thus obtain the coefficient function C_1 which is needed for NNLO prediction of Higgs boson production and decay within the MSSM. With our calculation we confirm the result for C_1 obtained in Ref. [14, 15] by an explicit calculation of the vertex diagrams.

The outline of this paper is as follows: In the next Section we describe the calculation of the decoupling constant for α_s to three loops and discuss the numerical influence in the computation of $\alpha_s(M_{\mathrm{GUT}})$. Afterwards we derive in Section 3 an all-order low-energy-theorem which we use to compute C_1 to NNLO accuracy. We summarize and conclude the paper in Section 4. In the Appendix we present a compact expression of the exact two-loop result for the decoupling coefficient.

2 Decoupling of heavy supersymmetric particles

In order to compute the decoupling effects of heavy particles from the running of α_s one can use the well-established formalism derived in Ref. [4]. It has been applied to supersymmetry in Refs. [8–10] where two-loop corrections have been computed.

The starting point is the relation between the strong coupling in the full theory, which is in our case the MSSM, respectively, SQCD, and the effective theory, QCD

$$\alpha_s^{(\mathrm{QCD})}(\mu) = \zeta_{\alpha_s}(\mu)\alpha_s^{(\mathrm{SQCD})}(\mu)\,. \tag{3}$$

At that point some comments are in order:

- $\alpha_s^{(\mathrm{QCD})}(\mu)$ is defined in the five or six flavour theory, depending on whether the top quark is integrated out together with the supersymmetric particles or not.
- $\alpha_s^{(\mathrm{QCD})}(\mu)$ is defined in the $\overline{\mathrm{MS}}$ scheme based on Dimensional Regularization (DREG). $\alpha_s^{(\mathrm{SQCD})}(\mu)$ is defined in the $\overline{\mathrm{DR}}$ scheme since the supersymmetric theory is regularized using Dimensional Reduction (DRED). DRED is implemented with ε scalars, where the details can be found in Refs. [15, 16].
- $\zeta_{\alpha_s}(\mu)$ as introduced in Eq. (3) has two tasks: (i) it has to decouple the heavy particles not present in the effective theory, and (ii) $\zeta_{\alpha_s}(\mu)$ has to ensure the change of regularization from DRED to DREG. In principle the two tasks can be performed in two steps as it has been proposed in Refs. [8–10]. However, it is more convenient to choose the same renormalization scale for the decoupling and the change of scheme. Calculations along these lines have also been performed in Ref. [14, 17].
- In principle each vertex containing α_s can be used in order to compute ζ_{α_s}. It is, however, convenient to use the gluon-ghost vertex in order to compute the decoupling constant via [4]

$$\zeta_{\alpha_s}^0 = \left(\frac{\tilde{\zeta}_1^0}{\tilde{\zeta}_3^0\sqrt{\zeta_3^0}}\right)^2, \tag{4}$$

  where the superscript "0" marks bare quantities. $\tilde{\zeta}_1$, $\tilde{\zeta}_3$ and ζ_3 are the decoupling constants of the gluon-ghost vertex, ghost and gluon propagator, respectively. They are obtained from the hard part of the corresponding Green's function. The corresponding formulae can be found in Ref. [4] where a derivation has been performed in the framework of QCD. It can be taken over to SQCD without modifications. The renormalized decoupling constant is obtained from

$$\zeta_{\alpha_s} = \frac{Z_{\alpha_s}}{Z_{\alpha_s'}}\zeta_{\alpha_s}^0\,. \tag{5}$$

  where Z_{α_s} and $Z_{\alpha_s'}$ are the renormalization constants for α_s in the full and effective theory, respectively.

- All occurring parameters are renormalized in the $\overline{\text{DR}}$ scheme, except the ε scalar mass which is renormalized on-shell with the condition $M_\varepsilon = 0$. The corresponding counterterms can, e.g., be found in Ref. [18].

Assuming a strong hierarchy among the quarks one encounters in the case of QCD vacuum diagrams which contain only one mass scale. The occurring integrals can even be computed up to four-loop order [5,6]. Two scales appear if two quarks are integrated out simultaneously. This has been done in Ref. [7] to three-loop accuracy.

In the case of supersymmetry significantly more mass scales have to be considered. In our approach we have the gluino and top squark masses ($m_{\tilde{g}}$, $m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$) and a generic squark mass $m_{\tilde{q}}$ which we take as the average of the up, down, strange, charm and bottom squarks. In addition there is the ε scalar (M_ε) and the top quark (m_t) mass. The latter only appears if we match to five-flavour QCD since $m_t = 0$ is chosen for the matching to six-flavour QCD. Up to two loops ζ_{α_s} can nevertheless be computed exactly [10] taking into account the dependence on all mass parameters. The analytical result can be found in the Appendix. At three-loop order, however, approximations have to be adopted in order to be able to compute the integrals. Motivated by scenarios which are currently discussed in the literature we have chosen

$$
\begin{aligned}
&(\text{h1}) \qquad m_{\tilde{q}} \approx m_{\tilde{t}_1} \approx m_{\tilde{t}_2} \approx m_{\tilde{g}} \gg m_t \,, \\
&(\text{h2}) \qquad m_{\tilde{q}} \approx m_{\tilde{t}_2} \approx m_{\tilde{g}} \gg m_{\tilde{t}_1} \gg m_t \,, \\
&(\text{h3}) \qquad m_{\tilde{q}} \approx m_{\tilde{t}_2} \approx m_{\tilde{g}} \gg m_{\tilde{t}_1} \approx m_t \,,
\end{aligned}
\tag{6}
$$

where in the case of "$\gg$" an asymptotic expansion in the corresponding hierarchy is performed. In the case of "$\approx$" a naive Taylor expansion in the difference of the particle masses is sufficient. For all hierarchies we assume that M_ε is not zero but much smaller than all other masses. In this way we ensure that the ε scalar is integrated out and not present in the effective theory. Thus, in the latter dimensional regularization can be used. In what follows the heavy mass scales for each hierarchy are also denoted by m_{SUSY} in case they are identified.

At three-loop order terms up to $\mathcal{O}(1/m_{\text{SUSY}}^{10})$ have been computed for (h1) and (h3) and up to $\mathcal{O}(1/m_{\tilde{t}_1}^6)$ and $\mathcal{O}(1/m_{\text{SUSY}}^6)$ for (h2). For each mass difference at least four expansion terms (i.e. terms including $(m_i^2 - m_j^2)^3$) could be evaluated. It is either possible to expand in the linear or the quadratic mass difference. Formally both choices are equivalent, however, in practice it turns out that depending on the actual numerical values of the parameters one can be significantly better behaved than the other. Similarly there is a freedom to choose a mass parameter, m_R, around which the expansion is performed. m_R should be of the order of the involved masses. Note that for (h1) and (h2) only one reference mass m_R is required whereas for (h3) one needs two as can be seen from Eq. (6). Again there may be significant numerical differences and thus we adopt the following choices when evaluating the three-loop corrections to the decoupling coefficient

$$
(\text{h1}) \qquad m_R = m_{\tilde{t}_1} \,, m_R = m_{\tilde{t}_2} \,, m_R = m_{\tilde{g}} \,, m_R = m_{\tilde{q}} \,, m_R = \frac{m_{\tilde{t}_1} + m_{\tilde{t}_2} + 10 m_{\tilde{q}} + m_{\tilde{g}}}{13} \,,
$$

$$
\begin{aligned}
&\text{(h2)} \qquad m_R = m_{\tilde{t}_2}\,, m_R = m_{\tilde{g}}\,, m_R = m_{\tilde{q}}\,, m_R = \frac{m_{\tilde{t}_2} + 10 m_{\tilde{q}} + m_{\tilde{g}}}{12}\,,\\
&\text{(h3)} \qquad m_{R_1} = m_{\tilde{t}_2}\,, m_{R_1} = m_{\tilde{g}}\,, m_{R_1} = m_{\tilde{q}}\,, m_{R_1} = \frac{m_{\tilde{t}_2} + 10 m_{\tilde{q}} + m_{\tilde{g}}}{12}\,,\\
&\qquad\qquad m_{R_2} = m_t\,, m_{R_2} = m_{\tilde{t}_1}\,, m_{R_2} = \frac{m_t + m_{\tilde{t}_1}}{2}\,.
\end{aligned}
\tag{7}
$$

In the following it is convenient to consider the perturbative expansion of ζ_{α_s} which we define as

$$
\zeta_{\alpha_s}(\mu) \;=\; 1 + \frac{\alpha_s^{(\mathrm{SQCD})}}{\pi}\zeta_{\alpha_s}^{(1)} + \left(\frac{\alpha_s^{(\mathrm{SQCD})}}{\pi}\right)^2 \zeta_{\alpha_s}^{(2)} + \left(\frac{\alpha_s^{(\mathrm{SQCD})}}{\pi}\right)^3 \zeta_{\alpha_s}^{(3)} + \ldots\,, \tag{8}
$$

where the μ dependence of $\alpha_s^{(\mathrm{SQCD})}$ and $\zeta_{\alpha_s}^{(i)}$ is suppressed on the right-hand side.

The general results are quite lengthy and will not be presented in this paper. However, in order to get an impression of the results we present ζ_{α_s} for the hierarchy (h1) with a degenerate supersymmetric mass spectrum which reads

$$
\begin{aligned}
\zeta_{\alpha_s}^{(1)} &= -\frac{1}{4} - l_S - \frac{l_t}{6}\,,\\
\zeta_{\alpha_s}^{(2)} &= \frac{307}{288} + \left(-\frac{77}{72} + \frac{7}{3} l_x\right) l_t + \frac{49}{36} l_t^2 - \frac{25}{36} l_x + l_x^2 + x_{tS}\left(\frac{1}{432} + \frac{1}{9} l_t + \frac{13}{72} l_x\right)\\
&\quad + x_{tS}^2\left(-\frac{1597}{21600} + \frac{61}{720} l_x\right) + \ldots\,,\\
\zeta_{\alpha_s}^{(3)} &= \frac{162443}{62208} - \frac{8509}{3456}\zeta(3) + \left(-\frac{27013}{5184} + \frac{2581}{432} l_x - \frac{7}{2} l_x^2\right) l_t + \left(\frac{6361}{1728} - \frac{49}{12} l_x\right) l_t^2\\
&\quad - \frac{343}{216} l_t^3 - \frac{21583}{5184} l_x + \frac{641}{288} l_x^2 - l_x^3 + x_{tS}\left(-\frac{90481643}{3888000} + \frac{47429}{2304}\zeta(3)\right.\\
&\quad \left. + \left(\frac{12163}{21600} - \frac{122}{135} l_x\right) l_t - \frac{79}{216} l_t^2 + \frac{51353}{86400} l_x - \frac{69}{128} l_x^2\right) + x_{tS}^2\left(\frac{1542497350769}{64012032000}\right.\\
&\quad \left. - \frac{2330095}{110592}\zeta(3) + \left(\frac{585083}{12700800} - \frac{26807}{60480} l_x\right) l_t - \frac{2}{27} l_t^2 + \frac{3208403}{3386880} l_x - \frac{104479}{181440} l_x^2\right)\\
&\quad + \ldots\,,
\end{aligned}
\tag{9}
$$

where $x_{tS} = m_t^2/m_{\mathrm{SUSY}}^2$, $l_t = \ln(\mu^2/m_t^2)$, $l_S = \ln(\mu^2/m_{\mathrm{SUSY}}^2)$ and $l_x = \ln(x_{tS})$. The ellipses denote terms of order x_{tS}^3. The corresponding results where the matching is performed to six-flavour QCD, i.e. where the top quark is not integrated out and thus treated as massless in the loop integrals, reads

$$
\begin{aligned}
\zeta_{\alpha_s}^{(1)} &= -\frac{1}{4} - l_S\,,\\
\zeta_{\alpha_s}^{(2)} &= \frac{77}{96} - \frac{7}{12} l_S + l_S^2\,,
\end{aligned}
$$

$$\zeta_{\alpha_s}^{(3)} = -\frac{11203}{4608} - \frac{1495}{576} l_S + \frac{541}{288} l_S^2 - l_S^3 + \frac{4657}{9216}\zeta(3)\,. \tag{10}$$

All analytical expressions corresponding to the hierarchies of Eq. (6) can be found in the file `decsusy3l.m` obtained from [19].

Let us in the following test our approximation at two loops by comparing to the exact result. For this purpose we adopt the following values for the input parameters

$$\begin{aligned} m_t = 150 \text{ GeV}\,, \quad A_t = 100 \text{ GeV}\,, \quad M_{\tilde{Q}_3} = 500 \text{ GeV}\,, \quad \mu_{\text{SUSY}} = 100 \text{ GeV}\,, \\ \tan\beta = 10\,, \quad M_Z = 91.2 \text{ GeV}\,, \quad \sin^2\theta_W = 0.2233\,. \end{aligned} \tag{11}$$

where A_t is the trilinear coupling, μ_{SUSY} is the Higgs-Higgsino bilinear coupling from the super potential, $\tan\beta$ is the ratio of the vacuum expectation values of the two Higgs doublets, M_Z is the Z boson mass, θ_W the weak mixing angle and $M_{\tilde{Q}_3}$, a soft SUSY breaking parameter for the squark doublet of the third family. Furthermore we set the renormalization scale to $\mu = 500$ GeV. These parameters can be used to compute $m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$ and θ_t as a function of the singlet soft SUSY breaking parameter of the top squark, $M_{\tilde{u}_3,R}$ (see, e.g., Ref. [20]) by diagonalizing the corresponding mass matrix. The result is shown in Fig. 1(a). Furthermore we choose for simplicity $m_{\tilde{t}_2} = m_{\tilde{q}} = m_{\tilde{g}}$. This allows us to consider in Fig. 1(b) both the exact result for $\zeta_{\alpha_s}^{(2)}$ (solid line) and the approximations (dahed lines) based on the hierarchies (h1), (h2) and (h3). The latter are obtained from the (naive) averages over the various representations, i.e., the different choices of m_R according to Eq. (7). One observes that in the whole range of $M_{\tilde{u}_3,R}$ at least one of the hierarchies approximates the exact to a high degree, which provides the motivation to proceed in a similar way at three loops.

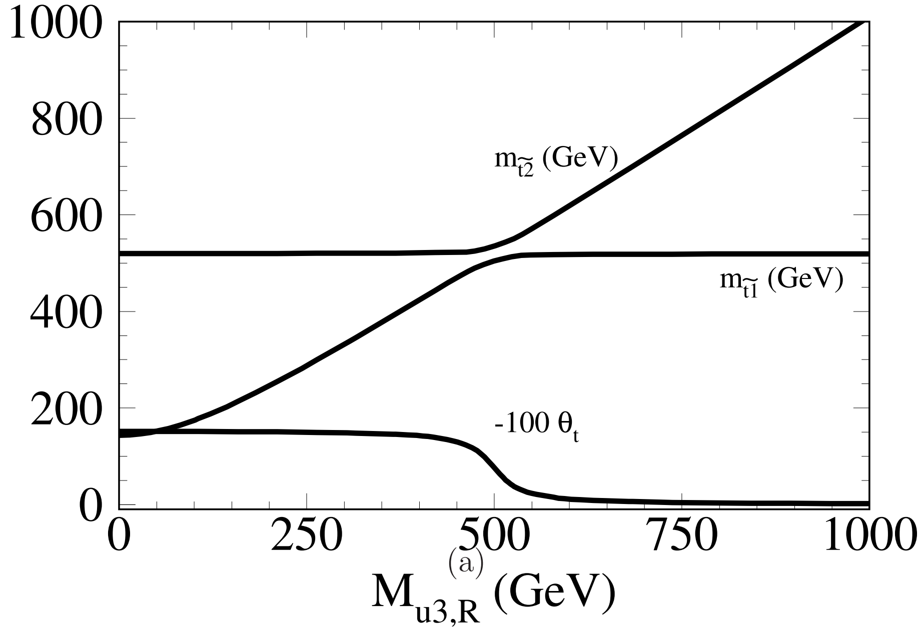
Since at three-loop order the exact result is not known a criterion is needed in order to select the best approximation among the various choices at hand. For this reason we define

$$\delta_{\text{app}} = \left|\frac{\zeta_{\text{app}}^{(2)} - \zeta_{\text{exact}}^{(2)}}{\zeta_{\text{exact}}^{(2)}}\right| + \left|\frac{\zeta_{\text{app}}^{(3)c} - \zeta_{\text{app}}^{(3)}}{\zeta_{\text{app}}^{(3)}}\right|\,, \tag{12}$$

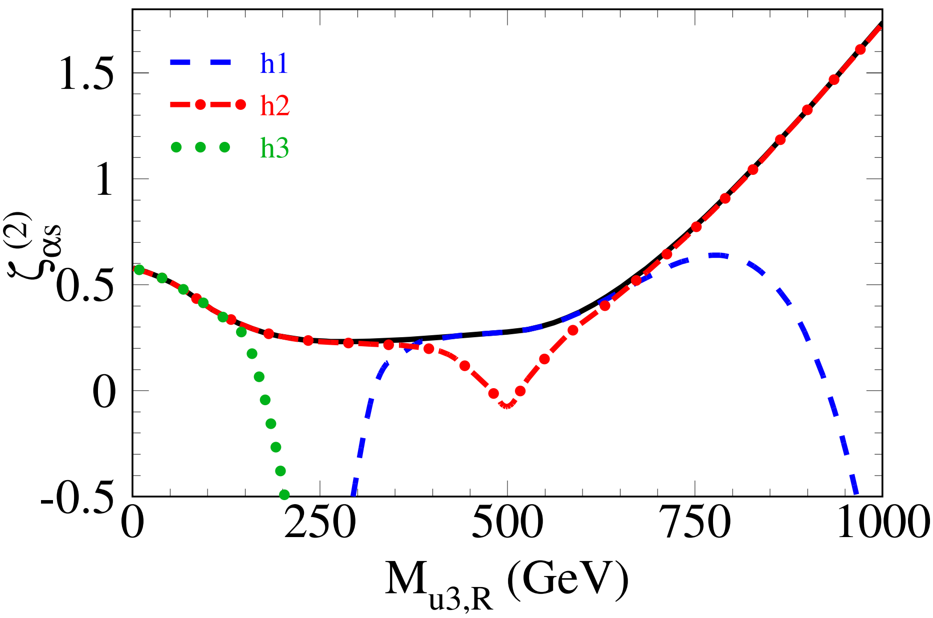
where "app" marks an approximation result and the superscript "c" indicates that the highest terms in the expansions are cut. For each set of input parameters we choose the representation which leads to the minimal value of δ_{app}. The first term on the right-hand side of Eq. (12) guarantees that the approximation works well at two-loop order whereas the second term assures the convergence of the expansion.

The three-loop result $\zeta_{\alpha_s}^{(3)}$ is shown in Fig. 2 as a function of $M_{\tilde{u}_3,R}$. The notation for the three hierarchies is as in Fig. 1. The thick lines are obtained using all available expansion terms whereas for the thin curves the highest order is set to zero. Thus the difference between the thick and the corresponding thin lines is a measure for the quality of the convergence.

One observes a smiliar behaviour as at two-loop order: For small values of $M_{\tilde{u}_3,R}$, which correspond to small values of $m_{\tilde{t}_1}$, both (h2) and (h3) provide good approximations. With



(a)



(b)

Figure 1: (a) $m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$ and θ_t obtained from the diagonalization of the top squark mass matrix as a function of the soft SUSY breaking parameter $M_{\tilde{u}_3,R}$. (b) $\zeta^{(2)}_{\alpha_s}$ as a function of $M_{\tilde{u}_3,R}$ using the parameters of Eq. (11). The exact result is shown as solid black line.

increasing $M_{\tilde{u}_3,R}$ (h3) becomes worse whereas $\zeta^{(3)}_{\rm app}$ and $\zeta^{(3)c}_{\rm app}$ for (h2) are still practically on top of each other. For values 300 GeV $\lesssim M_{\tilde{u}_3,R} \lesssim$ 800 GeV the top squark masses are

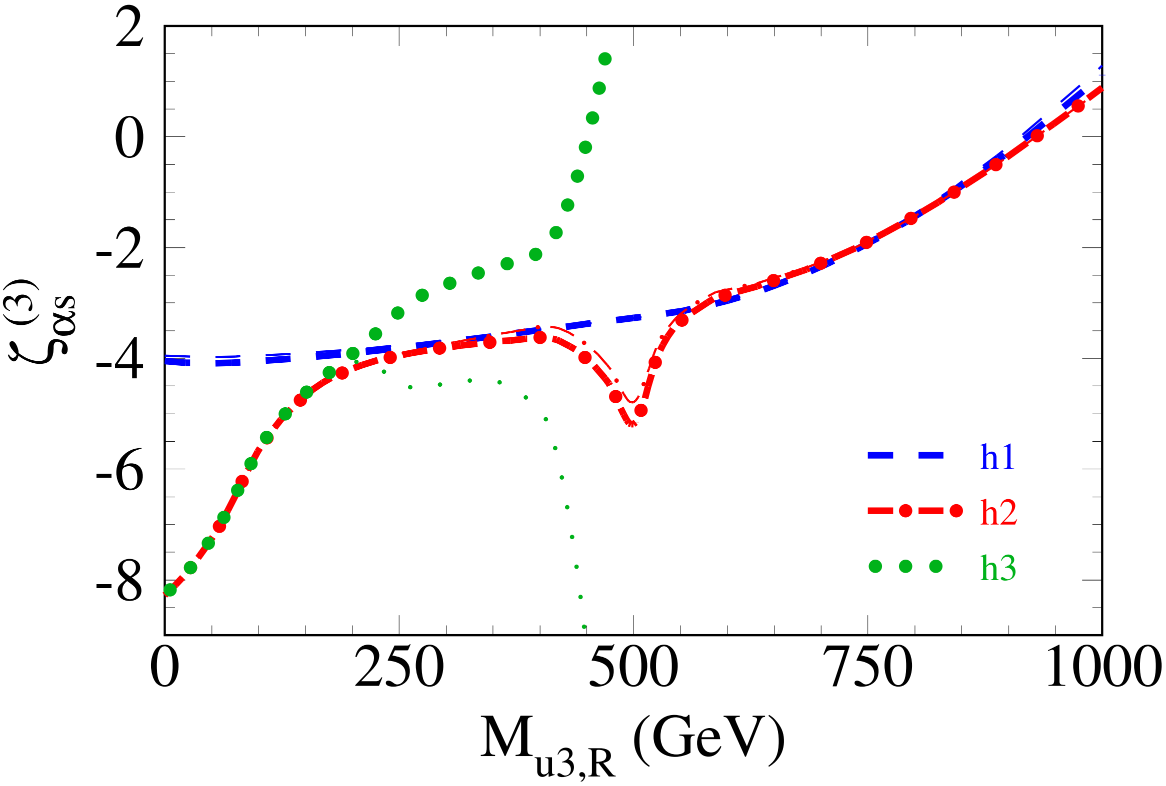


Figure 2: $\zeta_{\alpha_s}^{(3)}$ as a function of $M_{\tilde{u}_3,R}$ using the parameters of Eq. (11). Thick lines include all available terms whereas for the thin lines the highest terms are cancelled.

relatively close to each other which is the region of validity for (h1). For higher values one observes again a strong hierarchy between $m_{\tilde{t}_1}$ and $m_{\tilde{t}_2}$ and thus (h2) takes over. It is interesting to note that for each value of $M_{\tilde{u}_3,R}$ there is at least one hierarchy with a small value of δ_{app} and thus an expected good approximation to the unknown exact result. Furthermore, the approximations show a significant overlap so that the whole range of $M_{\tilde{u}_3,R}$ is covered.

Let us in the following briefly discuss the numerical impact of the three-loop corrections computed in this paper. In Figs. 3 we show the strong coupling at the GUT scale, $\alpha_s^{(\text{SQCD})}(M_{\text{GUT}})$ with $M_{\text{GUT}} = 2 \cdot 10^{16}$ GeV as a function of the decoupling scale μ_{dec} which is obtained by the following procedure. The starting point is $\alpha_s^{(5),\overline{\text{MS}}}(M_Z)$. In a first step we run in the SM from $\mu = M_Z$ to $\mu = \mu_{\text{dec}}$ where the decoupling of the top quark and the SUSY particles is performed simultaneously and $\alpha_s^{(5)}(\mu_{\text{dec}})$ is transformed to $\alpha_s^{(\text{SQCD})}(\mu_{\text{dec}})$. The use of the SQCD β function finally leads to $\alpha_s^{(\text{SQCD})}(M_{\text{GUT}})$. The thick lines in Fig. 3 correspond to this procedure, i.e., we use the following chain in order

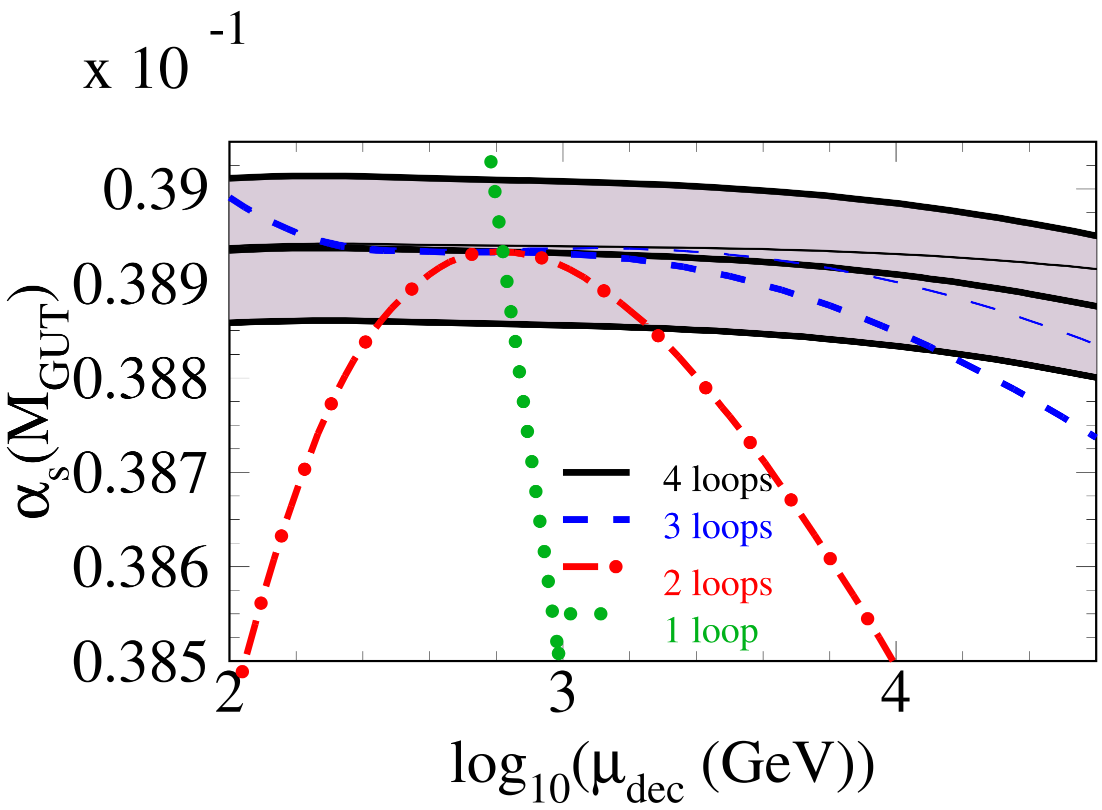


Figure 3: $\alpha_s^{(\mathrm{SQCD})}(M_{\mathrm{GUT}})$ as a function of μ_{dec}. Thick and thin lines correspond to the one- and two-step scenario, respectively. Thin lines are only shown for three- and four-loop running.

to arrive at $\alpha_s^{(\mathrm{SQCD})}(M_{\mathrm{GUT}})$

$$\alpha_s^{(5),\overline{\mathrm{MS}}}(M_Z) \stackrel{\mathrm{run.}}{\rightarrow} \alpha_s^{(5),\overline{\mathrm{MS}}}(\mu_{\mathrm{dec}}) \stackrel{\mathrm{dec.}}{\rightarrow} \alpha_s^{(\mathrm{SQCD})}(\mu_{\mathrm{dec}}) \stackrel{\mathrm{run.}}{\rightarrow} \alpha_s^{(\mathrm{SQCD})}(M_{\mathrm{GUT}})\,. \tag{13}$$

For a degenerate supersymmetric mass spectrum the decoupling constant can be found in Eq. (9).

Alternatively, in order to obtain the thin lines we integrate out the top quark in a separate step with $\mu = M_t$ (M_t is the on-shell top quark mass) and transform afterwards $\alpha_s^{(6),\overline{\mathrm{MS}}}$ to $\alpha_s^{(\mathrm{SQCD})}(M_{\mathrm{GUT}})$ in analogy to Eq. (13). Thus we have

$$\begin{aligned} &\alpha_s^{(5),\overline{\mathrm{MS}}}(M_Z) \stackrel{\mathrm{run.}}{\rightarrow} \alpha_s^{(5),\overline{\mathrm{MS}}}(M_t) \stackrel{\mathrm{dec.}}{\rightarrow} \alpha_s^{(6),\overline{\mathrm{MS}}}(M_t) \\ &\stackrel{\mathrm{run.}}{\rightarrow} \alpha_s^{(6),\overline{\mathrm{MS}}}(\mu_{\mathrm{dec}}) \stackrel{\mathrm{dec.}}{\rightarrow} \alpha_s^{(\mathrm{SQCD})}(\mu_{\mathrm{dec}}) \stackrel{\mathrm{run.}}{\rightarrow} \alpha_s^{(\mathrm{SQCD})}(M_{\mathrm{GUT}})\,. \end{aligned} \tag{14}$$

The decoupling constant needed for the transition from $\alpha_s^{(6),\overline{\mathrm{MS}}}$ to $\alpha_s^{(\mathrm{SQCD})}$ in the limit of degenerate SUSY masses is given in Eq. (10).

In order to obtain the numerical results in Fig. 3 we have used the measured result for $\alpha_s^{(5)}(M_Z)$ which reads [21]

$$\alpha_s^{(5)}(M_Z) \;=\; 0.1184 \pm 0.0007\,, \tag{15}$$

Furthermore we have adopted a `mSUGRA` scenario with

$$m_0 = 700 \text{ GeV}\,, \quad m_{1/2} = 600 \text{ GeV}\,, \quad \tan\beta = 10\,, \quad A_0 = 0\,, \quad \mu_{\rm SUSY} > 0\,. \tag{16}$$

as input for `softsusy` [22] in order to compute the supersymmetric mass spectrum. Note that there is only a weak dependence of the general features of our numerical result on the particular spectrum. However, it is convenient to make use of a spectrum generator in order to obtain directly the $\overline{\rm DR}$ values for the masses at the scale $\mu_{\rm dec}$. To our knowledge the running of the $\overline{\rm DR}$ parameters is only implemented to two-loop accuracy which poses a slight inconsistency in our analysis. However, this is only an minor effect and does not influence the main conclusions. In order to get an impression about the numerical values for the physical masses we show the $\overline{\rm DR}$ results for a typical scale $\mu_{\rm dec} = 1000$ GeV

$$\begin{array}{lll} m_t = 146.7 \text{ GeV}\,, & m_{\tilde{t}_1} = 1022 \text{ GeV}\,, & m_{\tilde{t}_2} = 1271 \text{ GeV}\,, \\ m_{\tilde{q}} = 1348 \text{ GeV}\,, & m_{\tilde{g}} = 1326 \text{ GeV}\,, & \theta_t = 1.26\,. \end{array} \tag{17}$$

At three-loop order the best appoximation is provided by the hierarchy (h1). In fact the quantity $\delta_{\rm app}$ in Eq. (12) takes the value $\delta_{\rm app} = 0.002$.

For consistency N-loop running has to be accompanied with $N-1$-loop decoupling relations. Thus, we can show curves for $N = 1, 2, 3$ and 4 which corresponds to the (thick) dotted, dash-dotted, dashed and solid line, respectively. Within QCD the beta function is known to four-loop accuracy [23, 24], however, the supersymmetric analogue only to three loops [16, 25, 26].[1] As a consequence for the four-loop curve in Fig. 3 we only use three-loop running above $\mu_{\rm dec}$.

$\mu_{\rm dec}$ is an unphysical scale not predicted by theory. Thus, on general grounds, the dependence on $\mu_{\rm dec}$ has to diminish if higher order corrections are included. This is clearly visible in Fig. 3 where the dotted, dash-dotted, dashed and solid lines correspond to one-, two-, three- and four-loop running, respectively. Around the central scale of approximately 1000 GeV all loop orders lead to predictions which are quite close. However, a variation of $\mu_{\rm dec}$ leads to a relatively strong variation of the two-loop result which gets stabilized at three-loops and which furthermore gets to a large extend $\mu_{\rm dec}$ independent at four loops. Actually, varying $\mu_{\rm dec}$ between 100 GeV and 10 000 GeV changes $\alpha_s^{\rm (SQCD)}(M_{\rm GUT})$ by only 0.07%.

It is interesting to compare the variation of the individual curves with respect to $\mu_{\rm dec}$ with the experimental uncertainty induced from $\alpha_s^{(5),\overline{\rm MS}}(M_Z)$ which is indicated by the band around the four-loop curve. The two-loop prediction is inside the band for 300 GeV $\lesssim \mu_{\rm dec} \lesssim$ 1800 GeV whereas the three-loop curve leaves the band only for

[1]The four-loop SQCD β function is not yet complete [27].

$\mu_{\rm dec} \gtrsim 13\,000$ GeV. It is also interesting to mention that all higher order corrections are very small for $\mu_{\rm dec} \approx 650$ GeV.

Note that often $\mu_{\rm dec} = M_Z$ is chosen for the matching between the SM and the MSSM. This choice leads to strong deviation at two-loops. At three-loop order the results are already quite stable which is further supported at four loops.

Let us finally remark on the step-by-step decoupling of the top quark and the supersymmetric particles. The corresponding three- and four-loop results are shown as thin lines in Fig. 3. One observes even flatter curves than for the one-step scenario, however, the difference is numerically small and well within the uncertainty band. In this context we want to stress the wide range of $\mu_{\rm dec}$ which is considered in Fig. 3.

3 Low-energy theorem and Higgs-gluon coupling in supersymmetric QCD

In Ref. [4] the following formula valid for all orders in perturbation theory has been derived in the framework of QCD[2]

$$C_1 = D_h^{\rm QCD} \ln \zeta_{\alpha_s} \tag{18}$$

where

$$D_h^{\rm QCD} = -m_h \frac{\partial}{\partial m_h} \tag{19}$$

describes the derivative with respect to the heavy mass m_h. Thus the N-loop corrections to ζ_{α_s} immediately leads to N-loop corrections to C_1. Since in Eq. (18) a logarithmic derivative is taken and furthermore the dependence of ζ_{α_s} on m_h only occurs via $\ln(\mu^2/m_h^2)$ even the $(N+1)$ corrections of C_1 can be computed once the renormalization scale dependence of ζ_{α_s} at $(N+1)$-loop order is re-constructed with the help of the renormalization group equations.

The LET of Eq. (18) can easily be extended to the case where more than one heavy quark is present. This version has been used in Ref. [7] in order to derive C_1 for theories with several heavy quarks which couple in a Yukawa-like way to the Higgs boson.

The extension of Eq. (18) to NLO corrections in the framework of the MSSM has been considered in Ref. [11]. Because of the different setup of our calculation, which is mainly due to the ε scalars, we cannot take over the derivation of Ref. [11]. However, following the same line of reasoning as in Ref. [4] we obtain a version of the LET which is appropriate for the decoupling constants computed in the previous chapter. For this purpose it is convenient to consider the bare decoupling constant (see Eq. (4)) expressed in terms of

[2]Note the different normalization of the operator $\mathcal{O}_1$ in Ref. [4].

bare parameters. This leads to the LET in the form

$$C_1^0 \;=\; D_h^0 \ln \zeta_{\alpha_s}^0 \,. \tag{20}$$

D_h^0 contains derivatives with respect to bare parameters (indicated by the superscript "0") and can be written as

$$D_h^0 \;=\; D_{\tilde{t}}^0 + D_{\tilde{q}}^0 + V_t^0 \frac{\partial}{\partial m_t^0} + \left(\Lambda_\varepsilon^0\right)^2 \frac{\partial}{\partial \left(m_\varepsilon^0\right)^2} \,. \tag{21}$$

Λ_ε is the evanescent Higgs boson-ε scalar coupling which is best defined through the corresponding part of the Lagrange density [14]

$$\mathcal{L}_\varepsilon \;=\; -\frac{1}{2}\left(M_\varepsilon^0\right)^2 \varepsilon_\sigma^{0,a}\varepsilon_\sigma^{0,a} - \frac{\phi^0}{v^0}\left(\Lambda_\varepsilon^0\right)^2 \varepsilon_\sigma^{0,a}\varepsilon_\sigma^{0,a} \,. \tag{22}$$

For convenience we have also displayed the mass term for the ε scalar.

The derivative operators in Eq. (21) are defined through[3]

$$\begin{aligned}
D_{\tilde{t}} &= V_{11}^{\tilde{t}} \frac{\partial}{\partial m_{\tilde{t}_1}^2} + V_{22}^{\tilde{t}} \frac{\partial}{\partial m_{\tilde{t}_2}^2} + \frac{V_{12}^{\tilde{t}} + V_{21}^{\tilde{t}}}{2(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)} \frac{\partial}{\partial \theta_t} \,, \\
D_{\tilde{q}} &= V_{11}^{\tilde{q}} \frac{\partial}{\partial m_{\tilde{q}_1}^2} + V_{22}^{\tilde{q}} \frac{\partial}{\partial m_{\tilde{q}_2}^2} + \frac{V_{12}^{\tilde{q}} + V_{21}^{\tilde{q}}}{2(m_{\tilde{q}_1}^2 - m_{\tilde{q}_2}^2)} \frac{\partial}{\partial \theta_q} \,,
\end{aligned}$$

where the prefactors in the top quark sector are obtained from the relations

$$\begin{aligned}
V_t &= -m_t \frac{\cos\alpha}{\sin\beta} \,, \\
V_{\rm LL}^{\tilde{t}} &= -2m_t^2 \frac{\cos\alpha}{\sin\beta} + M_Z^2 \cos^2\theta_W \left(1 - \frac{1}{3}\tan^2\theta_W\right) \sin(\alpha+\beta) \,, \\
V_{\rm RR}^{\tilde{t}} &= -2m_t^2 \frac{\cos\alpha}{\sin\beta} + \frac{4}{3} M_Z^2 \sin^2\theta_W \sin(\alpha+\beta) \,, \\
V_{\rm LR}^{\tilde{t}} &= V_{\rm RL}^{\tilde{t}} = \frac{m_t}{\sin\beta}(-\mu_{\rm SUSY}\sin\alpha - A_t\cos\alpha) \,, \\
\begin{pmatrix} V_{11}^{\tilde{t}} & V_{12}^{\tilde{t}} \\ V_{21}^{\tilde{t}} & V_{22}^{\tilde{t}} \end{pmatrix} &= R(\theta_t)^\dagger \begin{pmatrix} V_{\rm LL}^{\tilde{t}} & V_{\rm LR}^{\tilde{t}} \\ V_{\rm RL}^{\tilde{t}} & V_{\rm RR}^{\tilde{t}} \end{pmatrix} R(\theta_t) \,,
\end{aligned}$$

with

$$R(\theta_t) = \begin{pmatrix} \cos\theta_t & -\sin\theta_t \\ \sin\theta_t & \cos\theta_t \end{pmatrix} \,.$$

All light quark masses are set to zero. Their averaged contribution is denoted by q and thus we have

$$V_{\rm LL}^{\tilde{q}} = \frac{1}{n_l}\left(\frac{n_l+n_t}{2} V_{\rm LL}^{\tilde{d}} + \frac{n_l-n_t}{2} V_{\rm LL}^{\tilde{u}}\right) \,,$$

[3]In order to keep the notation simple we omit the superscript "0" in these expressions.

$$
\begin{aligned}
V^{\tilde{q}}_{\rm RR} &= \frac{1}{n_l}\left(\frac{n_l+n_t}{2}V^{\tilde{d}}_{\rm RR} + \frac{n_l-n_t}{2}V^{\tilde{u}}_{\rm RR}\right)\,,\\
V^{\tilde{q}}_{\rm LR} &= V^{\tilde{q}}_{\rm RL} = 0\,,\\
V^{\tilde{u}}_{\rm LL} &= M_Z^2\cos^2\theta_W\left(1-\frac{1}{3}\tan^2\theta_W\right)\sin(\alpha+\beta)\,,\\
V^{\tilde{u}}_{\rm RR} &= \frac{4}{3}M_Z^2\sin^2\theta_W\sin(\alpha+\beta)\,,\\
V^{\tilde{d}}_{\rm LL} &= M_Z^2\cos^2\theta_W\left(-1-\frac{1}{3}\tan^2\theta_W\right)\sin(\alpha+\beta)\,,\\
V^{\tilde{d}}_{\rm RR} &= -\frac{2}{3}M_Z^2\sin^2\theta_W\sin(\alpha+\beta)\,,
\end{aligned}
$$

where "u" and "d" denote generic up- and down-type squarks, respectively, and the labels $n_l = 5$ and $n_t = 1$ are kept arbitrary for convenience. $V^{\tilde{q}}_{ij}$ with $i,j = 1,2$ are obtained in analogy to $V^{\tilde{t}}_{ij}$.

After applying D_h^0 to $\zeta^0_{\alpha_s}$ of Section 2 we obtain the coefficient function C_1 expressed in terms of bare parameters. Thus, in a next step one has to perform the parameter renormalization.[4] Furthermore, it is necessary to take into account the operator renormalization constant, often denoted by Z_{11} [14], to obtain a finite result for the coefficient function which can then be compared to [14, 15].

An alternative version of the LET (compared to Eq. (20)) is obtained by exploiting the fact that Z_{α_s} and $Z_{\alpha_s'}$ are independent of the parameters occurring in D_h^0. Thus we can write

$$
C_1 \;=\; D_h^0 \ln\zeta_{\alpha_s}\,, \tag{23}
$$

where it is still understood that C_1 and ζ_{α_s} are expressed in terms of unrenormalized parameters. After computing C_1 with the help of Eq. (23) the parameters have to be renormalized as before, however, the operator renormalization constant is not necessary anymore.

A third version of the LET reads

$$
C_1 \;=\; D_h \ln\zeta_{\alpha_s}\,. \tag{24}
$$

In this equation all quantities are expressed in terms of $\overline{\rm DR}$ renormalized quantities and $\alpha_s^{\rm SQCD}$, except the evanescent couplings (M_ε and Λ_ε) which are renormalized to zero. It is very convenient to use Eq. (24) since it directly leads to a finite result for C_1. It is worth noting that the computation of C_1 from Eq. (24) avoids the introduction of the evanescent coupling Λ_ε. This can be understood by considering the renormalized version of D_h in Eq. (21) where the last term vanishes due to the condition $\Lambda_\varepsilon^2 = (\Lambda_\varepsilon^0)^2 - \delta\Lambda_\varepsilon^2 = 0$.

[4] The details are described in Ref. [14, 15].

Due to the derivatives in Eq. (21) the expansion depth available for ζ_{α_s} is reduced. Nevertheless we can compare the results to the findings of Ref. [14, 15] where C_1 has been computed from vertex diagrams. For all three hierarchies we found complete agreement for the first three terms in the mass difference, i.e. up to order $(m_i^2 - m_j^2)^2$. Furthermore, for (h1) [(h3)] terms up to $1/m_{\rm SUSY}^6$ [$1/m_{\rm SUSY}^4$] could be compared successfully and for (h2) all terms including $\mathcal{O}(1/m_{\tilde{t}_1}^4)$ and $\mathcal{O}(1/m_{\rm SUSY}^4)$ agree. Thus the calculation of the decoupling constant together with the application of the LET provides an independent confirmation of the Higgs-gluon coupling at three-loop order.

The LET in Eq. (20) differs from the one presented in [11] by the term involving Λ_ε^0 (see Eq. (21)). Up to NLO it is possible to avoid such a contribution [11, 13], at three-loop order, however, a renormalization of the Higgs boson-ε scalar coupling is mandatory (see Ref. [14, 15] for a detailed discussion) in case derivatives with respect to bare parameters are taken.

4 Conclusions

In this paper we have computed the three-loop SQCD corrections to the decoupling constant relating α_s defined in full MSSM to the one defined in QCD. The occurring three-loop integrals have been evaluated by applying expansions in various hierarchies and thus results are obtained which are valid in a large part of the parameter space. The decoupling constant constitutes an important ingredient in the relation of $\alpha_s(M_Z)$ and $\alpha_s(M_{\rm GUT})$. We have shown that the inclusion three-loop terms to the decoupling constant in combination with four-loop corrections to the β function leads to results for $\alpha_s(M_{\rm GUT})$ which are practically independent of the decoupling scale $\mu_{\rm dec}$, where the effective theory is matched to the full one, even when considering a variation of $\mu_{\rm dec}$ by more than two orders of magnitude.

A further interesting application of the decoupling constant is its relation to the effective Higgs-gluon coupling C_1 which is obtained by simple derivatives with respect to the involved parameters. This calculation constitutes an independent check of the results obtaines in Ref. [14,15] by an explicit calculation. In this paper we provide the corresponding LET which contains all features also present at higher orders in perturbation theory. It is valid to all orders in perturbation theory. We have checked that the renormalized version (cf. Eq. (24)) works including three-loop SQCD corrections.

Acknowledgements

This work was supported by the DFG through the SFB/TR 9 “Computational Particle Physics” and by the European Commission through the contract PITN-GA-2010-264564 (LHCPhenoNet). We would like to thank Konstantin Chetyrkin and Luminita Mihaila

for useful discussions and Luminita Mihaila for carefully reading the manuscript.

Appendix: Exact one- and two-loop result for ζ_{α_s}

In this Section we present the results for ζ_{α_s} up to two loops taking into account the exact dependence on the occurring masses. All parameters are renormalized in the $\overline{\mathrm{DR}}$ scheme except M_ε which is renormalized on-shell.

In contrast to Eq. (8) the coefficients of $\alpha_s^{(5)}$ defined through

$$\zeta_{\alpha_s}(\mu) \quad = \quad 1 + \frac{\alpha_s^{(5)}}{\pi}\tilde{\zeta}^{(1)}_{\alpha_s} + \left(\frac{\alpha_s^{(5)}}{\pi}\right)^2 \tilde{\zeta}^{(2)}_{\alpha_s} + \dots \,, \tag{25}$$

is presented. The results read

$$\begin{aligned}
\tilde{\zeta}^{(1)}_{\alpha_s} = -\frac{1}{4}\Bigg\{ & C_A\left[\frac{1}{3}+\frac{2}{3}l_{\tilde{g}}\right] + T_F\left[N_t\left(\frac{1}{3}l_{\tilde{t}_1}+\frac{1}{3}l_{\tilde{t}_2}+\frac{4}{3}l_t\right)+\frac{2N_q}{3}l_{\tilde{q}}\right] \\
& + \epsilon\Bigg[T_F\left(N_t\left\{\frac{1}{6}l^2_{\tilde{t}_1}+\frac{1}{6}l^2_{\tilde{t}_2}+\frac{2}{3}l_t^2+\zeta_2\right\}+N_q\left\{+\frac{1}{3}l^2_{\tilde{q}}+\frac{1}{3}\zeta_2\right\}\right) \\
& + C_A\left(\frac{1}{3}L_\epsilon+\frac{1}{3}l^2_{\tilde{g}}+\frac{1}{3}\zeta_2\right)\Bigg]\Bigg\}\,,
\end{aligned}$$

$$\begin{aligned}
\tilde{\zeta}^{(2)}_{\alpha_s} = \frac{1}{16}\Bigg\{ & C_A^2\left[-\frac{7}{36}-\frac{2}{3}l_{\tilde{g}}\right] + C_AT_F\Bigg[N_q\left(\frac{5}{9}+\frac{2m^2_{\tilde{g}}}{3\mathcal{D}_{\tilde{q}\tilde{g}}}l_{\tilde{g}}-\frac{2m^2_{\tilde{g}}}{3\mathcal{D}_{\tilde{q}\tilde{g}}}l_{\tilde{q}}\right) \\
& + N_t\Bigg(1+\frac{4\mathcal{N}_{11\,\tilde{t}_1}}{3\mathcal{D}_{\tilde{t}_1}}+\frac{4m^2_{\tilde{g}}m^2_{\tilde{t}_1}m_t^2\mathcal{N}_{5\,\tilde{t}_1}}{\mathcal{D}^3_{\tilde{t}_1}}\Phi(m_t,m_{\tilde{t}_1},m_{\tilde{g}}) \\
& - \frac{2\mathcal{N}_{3\,\tilde{t}_1}}{3\mathcal{D}^2_{\tilde{t}_1}}l_{\tilde{t}_1}+\left[-\frac{8}{3}+\frac{16m^2_{\tilde{g}}m^2_{\tilde{t}_1}\mathcal{N}_{15\,\tilde{t}_1}}{\mathcal{D}^2_{\tilde{t}_1}}-\frac{2\mathcal{N}_{21\,\tilde{t}_1}}{3\mathcal{D}_{\tilde{t}_1}}\right]l_t+\left[\frac{2m^2_{\tilde{g}}\mathcal{N}_{19\,\tilde{t}_1}}{3\mathcal{D}_{\tilde{t}_1}}-\frac{8m^2_{\tilde{g}}m^2_{\tilde{t}_1}\mathcal{N}_{6\,\tilde{t}_1}}{\mathcal{D}^2_{\tilde{t}_1}}\right]l_{\tilde{g}} \\
& + c_{\theta_t}s_{\theta_t}\Bigg[-\frac{4m_{\tilde{g}}m_t\mathcal{N}_{1\,\tilde{t}}\mathcal{N}_{2\,\tilde{t}}}{3\mathcal{D}_{\tilde{t}_1}\mathcal{D}_{\tilde{t}_2}}-\frac{8m_{\tilde{g}}m^2_{\tilde{t}_1}m_t\mathcal{N}_{1\,\tilde{t}_1}}{\mathcal{D}^3_{\tilde{t}_1}}\Phi(m_t,m_{\tilde{t}_1},m_{\tilde{g}})+\frac{8m^2_{\tilde{t}_1}m_t\mathcal{N}_{7\,\tilde{t}_1}}{3\mathcal{D}^2_{\tilde{t}_1}m_{\tilde{g}}}l_{\tilde{t}_1} \\
& + \left(\frac{8m_t\mathcal{N}_{8\,\tilde{t}_1}}{3\mathcal{D}_{\tilde{t}_1}m_{\tilde{g}}}-\frac{16m_{\tilde{g}}m^2_{\tilde{t}_1}m_t\mathcal{N}_{6\,\tilde{t}_1}}{\mathcal{D}^2_{\tilde{t}_1}}\right)l_t-\left[\frac{8m^3_{\tilde{g}}m_t}{3\mathcal{D}_{\tilde{t}_1}}+\frac{16m_{\tilde{g}}m^2_{\tilde{t}_1}m_t}{\mathcal{D}_{\tilde{t}_1}}+\frac{32m^3_{\tilde{g}}m^2_{\tilde{t}_1}m_t^3}{\mathcal{D}^2_{\tilde{t}_1}}\right]l_{\tilde{g}}\Bigg]\Bigg)\Bigg] \\
& + C_FT_F\Bigg[N_q\left(\frac{13}{6}-\frac{2M_\epsilon^2}{3m^2_{\tilde{q}}}+\frac{4m^2_{\tilde{g}}}{3m^2_{\tilde{q}}}+\frac{4m^4_{\tilde{g}}l_{\tilde{g}}}{3m^2_{\tilde{q}}\mathcal{D}_{\tilde{q}\tilde{g}}}-2l_{\tilde{q}}-\frac{4m^2_{\tilde{g}}l_{\tilde{q}}}{3\mathcal{D}_{\tilde{q}\tilde{g}}}\right) \\
& + N_t\Bigg(\frac{2}{3}-\frac{2M_\epsilon^2}{3m^2_{\tilde{t}_1}}+\frac{4m^2_{\tilde{g}}}{3m^2_{\tilde{t}_1}}+\frac{4m_t^2}{3m^2_{\tilde{t}_1}}-\frac{4m^2_{\tilde{g}}\mathcal{N}_{13\,\tilde{t}_1}}{3\mathcal{D}_{\tilde{t}_1}}-\frac{8m^4_{\tilde{g}}m^2_{\tilde{t}_1}m_t^2\mathcal{N}_{13\,\tilde{t}_1}}{\mathcal{D}^3_{\tilde{t}_1}}\Phi(m_t,m_{\tilde{t}_1},m_{\tilde{g}})
\end{aligned}$$

$$
\begin{aligned}
&+\left[\frac{4m_{\tilde g}^2 m_{\tilde t_1}^2}{3\mathcal{D}_{\tilde t_1}}+\frac{16m_{\tilde g}^4 m_{\tilde t_1}^2 m_t^2}{\mathcal{D}_{\tilde t_1}^2}-\frac{5}{3}\right]l_{\tilde t_1}\\
&-\frac{5}{3}l_{\tilde t_2}+\left[2+\frac{4m_{\tilde g}^2}{3m_{\tilde t_1}^2}+\frac{4m_t^2}{3m_{\tilde t_1}^2}+\frac{8m_{\tilde g}^4\mathcal{N}_{12\,\tilde t_1}}{\mathcal{D}_{\tilde t_1}^2}-\frac{4m_{\tilde g}^2\mathcal{N}_{9\,\tilde t_1}}{3\mathcal{D}_{\tilde t_1}m_{\tilde t_1}^2}\right]l_t\\
&+\left[\frac{4m_{\tilde g}^4\mathcal{N}_{20\,\tilde t_1}}{3\mathcal{D}_{\tilde t_1}m_{\tilde t_1}^2}-\frac{8m_{\tilde g}^4\mathcal{N}_{11\,\tilde t_1}}{\mathcal{D}_{\tilde t_1}^2}\right]l_{\tilde g}\\
&+s_{\theta_t}c_{\theta_t}\Bigg[\frac{16m_{\tilde g}^3 m_t}{3\mathcal{D}_{\tilde t_1}}-\frac{8m_{\tilde g}m_t}{3m_{\tilde t_1}^2}+\frac{16m_{\tilde g}^3 m_{\tilde t_1}^2 m_t\mathcal{N}_{14\,\tilde t_1}}{\mathcal{D}_{\tilde t_1}^3}\Phi(m_t,m_{\tilde t_1},m_{\tilde g})\\
&-\frac{16m_{\tilde g}m_{\tilde t_1}^2\mathcal{N}_{10\,\tilde t_1}}{3\mathcal{D}_{\tilde t_1}^2 m_t}l_{\tilde t_1}+\left(\frac{8m_{\tilde g}m_t\mathcal{N}_{2\,\tilde t_1}}{3\mathcal{D}_{\tilde t_1}m_{\tilde t_1}^2}-\frac{8m_{\tilde g}m_t}{3m_{\tilde t_1}^2}-\frac{16m_{\tilde g}^3 m_t\mathcal{N}_{11\,\tilde t_1}}{\mathcal{D}_{\tilde t_1}^2}\right)l_t\\
&+\left(\frac{16m_{\tilde g}^5 m_t\mathcal{N}_{16\,\tilde t_1}}{\mathcal{D}_{\tilde t_1}^2}+\frac{8m_{\tilde g}^3\mathcal{N}_{1\,\tilde t}}{3m_t\mathcal{N}_{18\,\tilde t_1}\mathcal{N}_{18\,\tilde t_2}}-\frac{8m_{\tilde g}^3\mathcal{N}_{4\,\tilde t_1}}{3\mathcal{D}_{\tilde t_1}m_{\tilde t_1}^2\mathcal{N}_{18\,\tilde t_1}}\right)l_{\tilde g}\Bigg]\\
&+\left(s_{\theta_t}^2-s_{\theta_t}^4\right)\left[-\frac{2\mathcal{N}_{1\,\tilde t}^2}{3m_{\tilde t_1}^2 m_{\tilde t_2}^2}+\left(\frac{4}{3}-\frac{4m_{\tilde t_1}^2}{3m_{\tilde t_2}^2}\right)l_{\tilde t_1}\right]\Bigg)\Bigg]\Bigg\}+\left\{\begin{array}{c} m_{\tilde t_2}\leftrightarrow m_{\tilde t_1}\\ \theta_t\to-\theta_t\end{array}\right\}, \qquad (26)
\end{aligned}
$$

where $C_F=4/3$, $C_A=3$, $T_F=1/2$, $N_t=1$, $N_q=5$, ζ_n is the Riemann zeta function, $l_x=\ln(\mu^2/m_x^2)$, $L_\epsilon=\ln(\mu^2/M_\epsilon^2)$ and M_ϵ is the ϵ scalar mass. Furthermore we have

$$
\begin{aligned}
\mathcal{D}_{\tilde t_i}=&m_{\tilde g}^4+\left(m_{\tilde t_i}^2-m_t^2\right)^2-2m_{\tilde g}^2\left(m_{\tilde t_i}^2+m_t^2\right),\\
\mathcal{D}_{\tilde q\tilde g}=&m_{\tilde g}^2-m_{\tilde q}^2,\\
\mathcal{N}_{1\,\tilde t}=&m_{\tilde t_1}^2-m_{\tilde t_2}^2,\\
\mathcal{N}_{2\,\tilde t}=&m_{\tilde g}^4+m_t^2\left(m_{\tilde t_2}^2-3m_t^2\right)-m_{\tilde g}^2\left(m_{\tilde t_1}^2+m_{\tilde t_2}^2-2m_t^2\right)+m_{\tilde t_1}^2\left(m_{\tilde t_2}^2+m_t^2\right),\\
\mathcal{N}_{1\,\tilde t_i}=&m_{\tilde g}^6-\left(m_{\tilde t_i}^2-m_t^2\right)^2\left(m_{\tilde t_i}^2+m_t^2\right)-m_{\tilde g}^4\left(3m_{\tilde t_i}^2+m_t^2\right)+m_{\tilde g}^2\left(3m_{\tilde t_i}^4+m_t^4\right),\\
\mathcal{N}_{2\,\tilde t_i}=&m_{\tilde g}^4-2m_{\tilde t_i}^4+2m_{\tilde t_i}^2m_t^2+m_{\tilde g}^2\left(m_{\tilde t_i}^2-m_t^2\right),\\
\mathcal{N}_{3\,\tilde t_i}=&m_{\tilde g}^8-m_{\tilde g}^6\left(3m_{\tilde t_i}^2+4m_t^2\right)+\left(-3m_{\tilde t_i}^2+m_t^2\right)\left(-m_{\tilde t_i}^2m_t+m_t^3\right)^2\\
&+m_{\tilde g}^4\left(3m_{\tilde t_i}^4+13m_{\tilde t_i}^2m_t^2+6m_t^4\right)-m_{\tilde g}^2\left(m_{\tilde t_i}^6+6m_{\tilde t_i}^4m_t^2+5m_{\tilde t_i}^2m_t^4+4m_t^6\right),\\
\mathcal{N}_{4\,\tilde t_i}=&\left(m_{\tilde g}^2-3m_{\tilde t_i}^2\right)m_t\left(m_{\tilde g}^2+3m_{\tilde t_i}^2-m_t^2\right),\\
\mathcal{N}_{5\,\tilde t_i}=&m_{\tilde g}^4-2m_{\tilde g}^2m_{\tilde t_i}^2+m_{\tilde t_i}^4-m_t^4,\\
\mathcal{N}_{6\,\tilde t_i}=&m_{\tilde g}^4+m_{\tilde t_i}^4-m_{\tilde t_i}^2m_t^2-m_{\tilde g}^2\left(2m_{\tilde t_i}^2+3m_t^2\right),\\
\mathcal{N}_{7\,\tilde t_i}=&5m_{\tilde g}^6+2\left(m_{\tilde t_i}^2-m_t^2\right)^3-2m_{\tilde g}^4\left(4m_{\tilde t_i}^2+3m_t^2\right)+m_{\tilde g}^2\left(m_{\tilde t_i}^4-4m_{\tilde t_i}^2m_t^2+3m_t^4\right),\\
\mathcal{N}_{8\,\tilde t_i}=&m_{\tilde g}^4+7m_{\tilde g}^2m_{\tilde t_i}^2-2m_{\tilde t_i}^4+2m_{\tilde t_i}^2m_t^2,\\
\mathcal{N}_{9\,\tilde t_i}=&m_{\tilde g}^4+m_{\tilde t_i}^4+m_{\tilde g}^2\left(4m_{\tilde t_i}^2-m_t^2\right),
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{N}_{10\,\tilde{t}_i} =& m_{\tilde{g}}^6 - \left(m_{\tilde{t}_i}^2 - m_t^2\right)^3 + m_{\tilde{g}}^4\left(-3m_{\tilde{t}_i}^2 + 2m_t^2\right) + m_{\tilde{g}}^2\left(3m_{\tilde{t}_i}^4 - 5m_{\tilde{t}_i}^2 m_t^2 + 2m_t^4\right)\,,\\
\mathcal{N}_{11\,\tilde{t}_i} =& m_{\tilde{g}}^4 + m_{\tilde{t}_i}^4 - m_{\tilde{t}_i}^2 m_t^2 - m_{\tilde{g}}^2\left(2m_{\tilde{t}_i}^2 + m_t^2\right)\,,\\
\mathcal{N}_{12\,\tilde{t}_i} =& m_{\tilde{g}}^4 + m_{\tilde{t}_i}^4 - 3m_{\tilde{t}_i}^2 m_t^2 - m_{\tilde{g}}^2\left(2m_{\tilde{t}_i}^2 + m_t^2\right)\,,\\
\mathcal{N}_{13\,\tilde{t}_i} =& m_{\tilde{g}}^2 - m_{\tilde{t}_i}^2 + m_t^2\,,\\
\mathcal{N}_{14\,\tilde{t}_i} =& m_{\tilde{g}}^4 - 2m_{\tilde{g}}^2 m_{\tilde{t}_i}^2 + \left(m_{\tilde{t}_i}^2 - m_t^2\right)^2\,,\\
\mathcal{N}_{15\,\tilde{t}_i} =& m_{\tilde{g}}^4 + m_{\tilde{t}_i}^4 - 2m_{\tilde{t}_i}^2 m_t^2 - 2m_{\tilde{g}}^2\left(m_{\tilde{t}_i}^2 + m_t^2\right)\,,\\
\mathcal{N}_{16\,\tilde{t}_i} =& m_{\tilde{g}}^2 - m_{\tilde{t}_i}^2 - m_t^2\,,\\
\mathcal{N}_{18\,\tilde{t}_i} =& m_{\tilde{g}}^2 - m_{\tilde{t}_i}^2\,,\\
\mathcal{N}_{19\,\tilde{t}_i} =& m_{\tilde{g}}^2 + 11m_{\tilde{t}_i}^2 + m_t^2\,,\\
\mathcal{N}_{20\,\tilde{t}_i} =& m_{\tilde{g}}^2 + 4m_{\tilde{t}_i}^2 - m_t^2\,,\\
\mathcal{N}_{21\,\tilde{t}_i} =& m_{\tilde{g}}^4 + m_{\tilde{t}_i}^2\left(m_{\tilde{t}_i}^2 + m_t^2\right) + m_{\tilde{g}}^2\left(22m_{\tilde{t}_i}^2 + m_t^2\right)\,.
\end{aligned}
$$

where following abbreviations have been introduced

$$
\begin{aligned}
\lambda(x,\,y) =& \sqrt{(1-x-y)^2 - 4xy}\,,\\
\mathrm{Cl}_2(x) =& \mathrm{Im}\Big[\mathrm{Li}_2(e^{ix})\Big]\,,\\
\Phi_1(x,\,y) =& \lambda^{-1}(x,\,y)\Big\{2\ln\left[\tfrac{1}{2}(1+x-y-\lambda(x,\,y))\right]\ln\left[\tfrac{1}{2}(1-x+y-\lambda(x,\,y))\right] + \tfrac{1}{3}\pi^2\\
&- \ln x\ln y - 2\mathrm{Li}_2[\tfrac{1}{2}(1+x-y-\lambda(x,\,y))] - 2\mathrm{Li}_2[\tfrac{1}{2}(1-x+y-\lambda(x,\,y))]\Big\}\,,\\
\Phi_2(x,\,y) =& \frac{2}{\sqrt{-\lambda^2(x,\,y)}}\Big\{\mathrm{Cl}_2\big(2\arccos\frac{-1+x+y}{2\sqrt{xy}}\big)\\
&+ \mathrm{Cl}_2\big(2\arccos\frac{1+x-y}{2\sqrt{x}}\big) + \mathrm{Cl}_2\big(2\arccos\frac{1-x+y}{2\sqrt{y}}\big)\Big\}\,,
\end{aligned}
$$

$$
\Phi(m_1,m_2,m_3) = \begin{cases}
m_3^2\lambda^2\Big(\frac{m_1^2}{m_3^2},\,\frac{m_2^2}{m_3^2}\Big)\Phi_2\Big(\frac{m_1^2}{m_3^2},\,\frac{m_2^2}{m_3^2}\Big) & \mathrm{Re}\Big[\lambda^2\Big(\frac{m_1^2}{m_3^2},\,\frac{m_2^2}{m_3^2}\Big)\Big] < 0\\
m_3^2\lambda^2\Big(\frac{m_1^2}{m_3^2},\,\frac{m_2^2}{m_3^2}\Big)\Phi_1\Big(\frac{m_1^2}{m_3^2},\,\frac{m_2^2}{m_3^2}\Big) & m_1+m_2 \le m_3\\
m_1^2\lambda^2\Big(\frac{m_2^2}{m_1^2},\,\frac{m_3^2}{m_1^2}\Big)\Phi_1\Big(\frac{m_2^2}{m_1^2},\,\frac{m_3^2}{m_1^2}\Big) & m_2+m_3 \le m_1\\
m_2^2\lambda^2\Big(\frac{m_1^2}{m_2^2},\,\frac{m_3^2}{m_2^2}\Big)\Phi_1\Big(\frac{m_1^2}{m_2^2},\,\frac{m_3^2}{m_2^2}\Big) & m_1+m_3 \le m_2\,.
\end{cases}
$$

The one-loop result agrees with Ref. [8]. The two-loop result has also been considered in Ref. [10], however, no compact result has been presented. Furthermore, the decoupling has only been considered within DRED, i.e., the transition from DREG to DRED has been performed in a separate step.

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