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$R(s)$ and Z decay in order α_s^4: complete results

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We report on our calculation of the order α_s^4 axial singlet contributions for the decay rates of the Z-boson as well as the vector singlet contribution to the cross section for electron-positron annihilation into hadrons. Together with recently finished $\mathcal{O}(\alpha_s^4)$ calculations of the non-signlet corrections [1, 2], the new results directly lead us to the first *complete* $\mathcal{O}(\alpha_s^4)$ predictions for the total hadronic decay rate of the Z-boson and the ratio $R(s) = \frac{\sigma(e^+e^-\to\text{hadrons})}{\sigma(e^+e^-\to\mu^+\mu^-)}$.

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1. Introduction

Inclusive quark production through a decay of a heavy virtual photon, Z boson or τ is a process of importance for QCD as the theory of strong interactions. Perturbative QCD (pQCD) provides theoretically clean prediction for the process (see, e.g. [3, 4]).

Combined with the precise determination of the Z-boson decay rate into hadrons at LEP [5] this has led to one of the most precise determinations of the strong coupling constant $\alpha_s(M_Z)$. An alternative and also very precise determination of $\alpha_s(M_Z)$ as derived from $\alpha_s(M_\tau)$ has been recently obtained from the $\mathcal{O}(\alpha_s^4)$ prediction [1] for the ratio $R_\tau = \frac{\Gamma(\tau\to\text{hadrons})}{\Gamma(\tau\to l+\bar{\nu}_l+\nu_\tau)}$ and the experimental determinations of R_τ by ALEPH, CLEO and OPAL collaborations (see, e.g. [4]).

Note that while the $\mathcal{O}(\alpha_s^4)$ predictions for R_τ are complete this is not the case for $R(s)$ and the Z-decay rate. The missing pieces are related to so-called singlet diagrams (see Fig. 1 below). Note that while the top quarks can not be produced in Z-decays due to kinematical reasons, the (axial) singlet diagrams containing internal quark loops are *not* power suppressed (unlike similar loops for the vector singlet (and non-singlet) diagrams. This remarkable phenomenon first shows up at order α_s^2 and was first established and fully investigated in works [6, 7]. The full account of singlet diagrams at order α_s^3 was performed in papers [8, 9, 10] (vector case) and in [11, 12, 13, 10] (axial case).

In the present work we present the results of the calculations of the order α_s^4 axial singlet contributions for the decay rates of the Z-boson as well as the vector singlet contribution to the cross section for electron-positron annihilation into hadrons. Note that we will not dwell on any phenomenological applications of our calculations as they have been recently discussed in some detail in [14].

2. Preliminaries

The interaction of the Z boson to quarks is described (in the lowest order approximation in the weak coupling constant) by adding to the QCD Lagrangian an extra term of the form $M_Z\left(\frac{G_F}{2\sqrt{2}}\right)^{1/2} Z^\alpha J^0_\alpha$, with $J^0_\alpha = \sum_i \overline{\psi}_i\gamma_\alpha(g_i^V - g_i^A\gamma_5)\psi_i$ being the neutral quark current. As a result, the hadronic decay rate of the Z boson (Γ_Z^h) including all strong interaction corrections may be viewed as an incoherent sum of vector (Γ_Z^V) and axial (Γ_Z^A) contributions. By the optical theorem both contributions can be conveniently related to the correlators of vector and axial vector quark currents. The general definition for the latter reads:

$$\begin{aligned}\Pi^{V/A}_{\mu\nu;i,j}(q) &= i\int e^{iqx}\langle 0|\ T\ j^{V/A}_{\mu,i}(x) j^{V/A}_{\nu,j}(0)\ |0\rangle\ \mathrm{d}x \\ &= g^{\mu\nu}q^2\Pi^{V/A}_{1;i,j}(-q^2) + q^\mu q^\nu \Pi^{V/A}_{2;i,j}(-q^2)\end{aligned} \tag{2.1}$$

with $j^V_{\mu,i} = \overline{\psi}_i\ \gamma_\mu\ \psi_i = V^i_\mu$ and $j^A_{\mu,i} = \overline{\psi}_i\ \gamma_\mu\ \gamma_5\psi_i = A^i_\mu$. The corresponding absorptive parts are defined as follows:

$$R^{V/A}_{i,j}(s) = 12\pi\Im\ \Pi^{V/A}_{1;i,j}(-s - i\varepsilon). \tag{2.2}$$

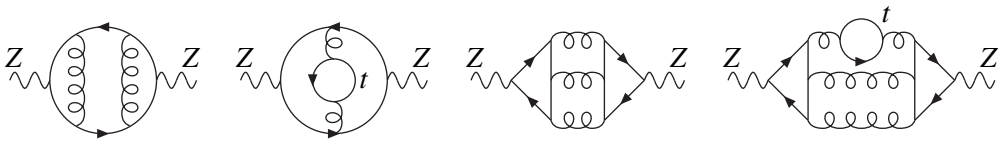

Figure 1: Examples of non-singlet diagrams (a), (b), where the two Z vertices are connected by a fermion line, and of singlet diagrams (c),(d), where the diagram can be split by only cutting gluon lines. The imaginary part of the non-singlet diagrams gives $R^{V/A,NS}$, while the imaginary part of the singlet diagrams is denoted by $R^{V/A,S}$.

The Z decay rate $\Gamma(Z \to \text{hadrons}) = \Gamma_0\big(R^V(M_Z^2) + R^A(M_Z^2)\big)$, where $\Gamma_0 = G_F M_Z^3/(24\pi\sqrt{2})$ and $R^{V/A}$ can be expressed in terms of $R_{i,j}^{V/A}$ defined in eq. (2.2), namely

$$R^V = \sum_{i,j} g_i^V\, g_j^V\, R_{i,j}^V\,, \qquad\qquad R^A = \sum_{i,j} g_i^A\, g_j^A\, R_{i,j}^A\,. \tag{2.3}$$

Similarly, the inclusive cross-section reaction of the reaction e^+e^- annihilation into hadrons through the photon is described by the current correlation function

$$\Pi_{\mu\nu}(q) = \int \mathrm{d}x e^{iqx} \langle 0|T[\ j_\mu^{\rm em}(x) j_\nu^{\rm em}(0)\ ]|0\rangle = (-g_{\mu\nu}q^2 + q_\mu q_\nu)\Pi^{EM}(-q^2)\,, \tag{2.4}$$

with the hadronic EM current

$$j_\mu^{\rm em} = \sum_f q_f \overline{\psi}_f \gamma_\mu \psi_f \quad \text{and} \quad R(s) = 12\pi\, \Im\Pi^{EM}(-s - i\varepsilon),$$

with q_f being the EM charge of the quark f. As a result, we arrive to the following representation for the ratio R(s) valid in massless approximation (precise definitions of R^{NS} and $R^{V,S}$ will be given below)

$$R(s) = \sum_{i,j} q_i\, q_j\, R_{i,j}^V(s) = \Big(\sum_i q_i^2\Big) R^{NS}(s) + \Big(\sum_i q_i\Big)^2 R^{VS}(s). \tag{2.5}$$

As the Z-boson is much heavier than all known quarks but the top one, it is natural[1] to neglect all power suppressed light quark mass corrections when dealing with $\mathcal{O}(\alpha_s^4)$ contribution to Γ_Z^h. It is customary to split $R_{i,j}^{V/A}$ into two contributions as described in Fig. 1

$$R_{i,j}^{V/A}(M_Z^2) = \delta_{ij}^\ell R^{NS}(M_Z^2) + R_{ij}^{V/A,S}(M_Z^2)\,, \tag{2.6}$$

with the delta function $\delta_{ij}^\ell \equiv \delta_{ij}$ if both flavours i and j are light and $\delta_{ij}^\ell = 0$ if either i or/and j refer to the top quark. In the non-singlet diagrams there is no top quark present in the fermion loop connecting the two external currents, because these diagrams have no physical cut and therefore have no imaginary part contributing to $R^{NS}(s = M_Z^2)$. This, together with the assumed masslessness

[1]Mass corrections to both vector and axial vector correlator due to other massive quarks are dominated by the bottom quark and can be classified by orders in m_b^2/M_Z^2 and α_s. Up to $\mathcal{O}(\alpha_s^2 m_b^2/M_Z^2)$ and $\mathcal{O}(\alpha_s^2 m_b^4/M_Z^4)$ they can be found in [3], as well terms of order $\alpha_s^2 m_b^2/M_Z^2$ (const + log m_b^2/M_Z^2) and $\alpha_s^2 m_b^2/M_t^2$ (const + log m_b^2/M_Z^2) that arise from axial vector singlet contributions. Terms of order $\alpha_s^3 m_b^4/M_Z^4$ and $\alpha_s^4 m_b^2/M_Z^2$ can be found in [15] and [16] respectively. Corrections of order $\alpha_s^2 m_Z^2/m_t^2$ and $\alpha_s^3 m_Z^2/m_t^2$ from singlet and non-singlet terms are known from [7, 6, 17] and [10] respectively.

of all quarks but top leads to the factorized form of the non-singlet term in eq. (2.6). Note that the *internal* top quark loops like in diagram (b) of Fig. 1 still contribute to $R^{NS}(s=M_Z^2)$ if the strong coupling is defined for 6 flavours. However, it is well known that such contributions could be naturally described (up to power suppressed terms) by transition from the full $n_f=6$ QCD to the effective massless $n_f=5$ one (see, e.g. [3] and references therein). In fact, the same is true for the vector singlet term (θ_{ij}^h below is defined as 1 if either i or/and j refer to the top quark and 0 in all other cases)

$$R_{ij}^{V,S}(M_Z) \equiv (1-\theta_{ij}^h)\,R^{V,S}(M_Z) + \mathcal{O}(M_Z^2/M_t^2).$$

The corresponding massless calculations of R^{NS} in order α_s^4 have been recently finished [1, 2]. In what follows we concentrate on the singlet terms $R^{V,S}$ and $R^{A,S}$.

3. γ_5-treatment

As is well-known the treatment of γ_5 within dimensional regularization is a non-trivial problem by itself (for an excellent review see [18]). Following works [11, 12] in all our calculations we employ, in fact, two different definitions of γ_5. First, for all non-singlet diagrams completely anticommuting naive γ_5 have been used. Second, for singlet diagrams we employ essentially the 't Hooft-Veltman definition [19]

$$A_\alpha^i = \overline{\psi}_i\gamma_\alpha\gamma_5\psi_i \equiv \frac{\xi_5^A(a_s)}{6}\,\mathrm{i}\,\varepsilon_{\alpha\beta\nu\rho}\,\overline{\psi_i}\gamma_\beta\gamma_\nu\gamma_\rho\psi_i.\,, \tag{3.1}$$

where the current $\overline{\psi_i}\gamma_\beta\gamma_\nu\gamma_\rho\psi_i$ is assumed to be minimally renormalized.

The finite normalization factor $\xi_5^A = 1-4a_s/3+O(a_s^2)$ on the rhs of (3.1) is necessary [20, 21] for the current (3.1) to obey the usual (non-anomalous) Ward identities which in turn are crucial in renormalizing the Standard Model.

In principle, one could (and even have to!) use one and the same definition (3.1) also for non-singlet diagrams. This would result to much more complicated calculations due to significantly longer traces encountered. Fortunately, it is not necessary because the factor ξ_5^A is *chosen* in such a way to restore the anti-commutativity of the γ_5 (for a detailed discussion, see [21]).

4. Vector $\mathcal{O}(\alpha^4)$ singlet term $R^{V,S}$

From purely technical point of view the calculation of the massless five-loop diagrams contributing to $\Pi_{ij}^{V,S}$ is not much different from those contributing to $\Pi^{V,NS}$. Using the same methods as described in [1, 2] we have obtained (below $a_s=\alpha_s(\mu)/\pi$ and μ is the renormalization scale in the $\overline{\rm MS}$ scheme)

$$\begin{aligned} R^{V,S}(s) = \quad & a_s^3\left(\frac{55}{72}-\frac{5}{3}\zeta_3\right) \\ & + a_s^4\left(n_l\left[-\frac{745}{432}+\frac{65}{24}\zeta_3+\frac{5}{6}\zeta_3^2-\frac{25}{12}\zeta_5-\frac{55}{144}\ln\frac{\mu^2}{s}+\frac{5}{6}\zeta_3\ln\frac{\mu^2}{s}\right]\right. \\ & \left. + \frac{5795}{192}-\frac{8245}{144}\zeta_3-\frac{55}{4}\zeta_3^2+\frac{2825}{72}\zeta_5+\frac{605}{96}\ln\frac{\mu^2}{s}-\frac{55}{4}\zeta_3\ln\frac{\mu^2}{s}\right). \end{aligned} \tag{4.1}$$

5. Axial vector $\mathcal{O}(\alpha^4)$ singlet term $R_{i,j}^{A,S}$

Due to the obvious property[2] $R_{i,j}^{A,S} = R_{i',j'}^{A,S}$ if all 4 indexes refer to the massless quarks and the fact that $g_A^u + g_A^d = g_A^c + g_A^s = 0$, we can write the axial singlet part of the Z decay rate as follows:[3]

$$R^{A,S} = R_{tt}^{A,S} - 2R_{tb}^{A,S} + R_{bb}^{A,S}. \tag{5.1}$$

All diagrams contributing to the first two terms of (5.1) contain at least one top quark loop. The third term receives contributions by both the completely massless diagrams and those with top quark loop (the latter start from order α_s^3, an example is given by Fig. 1 (d)).

As $M_Z \ll 2M_t$, one can use the effective theory methods to compute top-mass-dependent diagrams as a series in the ratio $\frac{M_Z^2}{4M_t^2}$. The procedure was elaborated long ago and successfully employed (see works [22, 11, 12]) to get all ingredients of eq. (5.1) at order α_s^3 at leading order in $1/M_t$ expansion (still keeping *all* power non-suppressed terms, including those which depends on $\ln(\mu^2/M_t^2)$). From purely technical point of view the evaluation at order α_s^4 involves absorptive parts of five-loop diagrams with massless propagators and, in addition, absorptive parts of four-loop diagrams combined with one-loop massive tadpoles, etc. down to one-loop massless diagrams together with four-loop massive tadpoles. The latter have been computed with the help of the Laporta's algorithm [23] implemented in Crusher [24]. The massive tadpoles with number of loops less or equal three have been independently recalculated with the help of the FORM program MATAD [25]. Our results for $R_{tt}^{A,S}$, $R_{tb}^{A,S}$, $R_{bb}^{A,S}$ and $R^{A,S}$ read

$$R_{tt}^{A,S} = (a_s^5)^4 \left[\frac{15}{64} - \frac{15}{8}\ell_{\mu t} + \frac{15}{4}\ell_{\mu t}^2\right], \tag{5.2}$$

$$\begin{aligned}
R_{tb}^{A,S} = {} & (a_s^5)^2 \left[\frac{3}{8} - \frac{3}{2}\ell_{\mu t}\right] + (a_s^5)^3 \left[-\frac{3869}{288} + \frac{55}{8}\zeta_3 - \frac{45}{8}\ell_{\mu t} - \frac{25}{8}\ell_{\mu t}^2\right] \\
& + (a_s^5)^4 \left[-\frac{370478273}{14515200} - \zeta_2 + \frac{1309601}{16800}\zeta_3 - \frac{4225817}{34560}\zeta_4 - \frac{10453}{288}\zeta_5 \right. \\
& \quad - 2\zeta_2 \ln(2) - \frac{89}{48}\zeta_4 \ln(2) - \frac{5861}{1080}\zeta_2 (\ln(2))^2 + \frac{2}{9}\zeta_2 (\ln(2))^3 + \frac{5861}{6480}(\ln(2))^4 \\
& \quad - \frac{1}{45}(\ln(2))^5 + \frac{5861}{270}a_4 + \frac{8}{3}a_5 - \frac{37}{32}\ell_{\mu Z} - \frac{47015}{576}\ell_{\mu t} \\
& \quad \left. + \frac{709}{8}\zeta_3\ell_{\mu t} + \frac{37}{8}\ell_{\mu Z}\ell_{\mu t} - \frac{363}{16}\ell_{\mu t}^2 - \frac{193}{32}\ell_{\mu t}^3\right],
\end{aligned} \tag{5.3}$$

[2] Obvious, thanks to the existence of the unitary $SU(n_l)$ symmetry in the flavour subspace of the first $1 \ldots n_l = 5$ massless quarks.

[3] Note that separate terms on the rhs of (5.1) are *not scale-invariant*, while their sum is [11, 12].

$$
\begin{aligned}
R^{A,S}_{bb} &= (a_s^5)^2\left[-\frac{17}{2}-3\ell_{\mu Z}\right]\\
&+ (a_s^5)^3\left[-\frac{4673}{48}+\frac{23}{2}\zeta_2+\frac{67}{4}\zeta_3-\frac{373}{8}\ell_{\mu Z}-\frac{23}{4}\ell_{\mu Z}^2\right.\\
&\qquad\left.-\frac{1}{12}\ell_{\mu t}-\frac{1}{2}\ell_{\mu t}^2\right]\\
&+ (a_s^5)^4\left[-\frac{79017683}{82944}+\frac{8747}{32}\zeta_2+\frac{54179}{128}\zeta_3+\frac{1481}{128}\zeta_4-\frac{6455}{96}\zeta_5\right.\\
&\qquad-\zeta_2(\ln(2))^2+\frac{1}{6}(\ln(2))^4+4a_4-\frac{174767}{288}\ell_{\mu Z}+\frac{529}{8}\zeta_2\ell_{\mu Z}\\
&\qquad+\frac{1519}{8}\zeta_3\ell_{\mu Z}-\frac{8747}{64}\ell_{\mu Z}^2-\frac{529}{48}\ell_{\mu Z}^3-\frac{1975}{288}\ell_{\mu t}+\frac{37}{8}\zeta_3\ell_{\mu t}\\
&\qquad\left.-\frac{247}{48}\ell_{\mu t}^2-\frac{25}{24}\ell_{\mu t}^3\right],
\end{aligned}
\tag{5.4}
$$

$$
\begin{aligned}
R^{A,S} &= (a_s^5)^2\left[-\frac{37}{4}-3\ell_{\mu Z}+3\ell_{\mu t}\right]\\
&+ (a_s^5)^3\left[-\frac{5075}{72}+\frac{23}{2}+\zeta_2+3\zeta_3-\frac{373}{8}\ell_{\mu Z}-\frac{23}{4}\ell_{\mu Z}^2\right.\\
&\qquad\left.+\frac{67}{6}\ell_{\mu t}+\frac{23}{4}\ell_{\mu t}^2\right]\\
&+ (a_s^5)^4\left[-\frac{13083735979}{14515200}+\frac{8811}{32}\zeta_2+\frac{17967167}{67200}\zeta_3+\frac{553219}{2160}\zeta_4+\frac{1541}{288}\zeta_5\right.\\
&\qquad+4\zeta_2\ln(2)+\frac{89}{24}\zeta_4\ln(2)+\frac{5321}{540}\zeta_2(\ln(2))^2-\frac{4}{9}\zeta_2(\ln(2))^3-\frac{5321}{3240}(\ln(2))^4\\
&\qquad+\frac{2}{45}(\ln(2))^5-\frac{5321}{135}a_4-\frac{16}{3}a_5-\frac{174101}{288}\ell_{\mu Z}+\frac{529}{8}\zeta_2\ell_{\mu Z}\\
&\qquad+\frac{1519}{8}\zeta_3\ell_{\mu Z}-\frac{8747}{64}\ell_{\mu Z}^2-\frac{529}{48}\ell_{\mu Z}^3+\frac{11125}{72}\ell_{\mu t}-\frac{1381}{8}\zeta_3\ell_{\mu t}\\
&\qquad\left.-\frac{37}{4}\ell_{\mu Z}\ell_{\mu t}+\frac{2111}{48}\ell_{\mu t}^2+\frac{529}{48}\ell_{\mu t}^3\right].
\end{aligned}
\tag{5.5}
$$

Here $a_s^5=\alpha_s(\mu)/\pi$ in the effective (topless) $n_f=5$ QCD, $\ell_{\mu Z}=\ln\frac{\mu^2}{M_Z^2}$, $\ell_{\mu t}=\ln\frac{\mu^2}{M_t^2}$, and M_t is the pole top quark mass. In addition, $\zeta_n=\zeta(n)$ is Riemann's zeta function and $a_n=\mathrm{Li}_n(1/2)=\sum_{i=1}^{\infty}1/(2^i i^n)$.

Finally, setting $\mu=M_Z$, we arrive at the following numerical form of (5.5)

$$
\begin{aligned}
R^{A,S} &= (a_s^5)^2\left[-9.25+3.\ln\frac{M_Z^2}{M_t}\right]\\
&+ (a_s^5)^3\left[-47.9632+11.1667\ln\frac{M_Z^2}{M_t}+5.75\ln^2\frac{M_Z^2}{M_t}\right]\\
&+ (a_s^5)^4\left[147.093-52.9912\ln\frac{M_Z^2}{M_t}+43.9792\ln^2\frac{M_Z^2}{M_t}+11.0208\ln^3\frac{M_Z^2}{M_t}\right].
\end{aligned}
\tag{5.6}
$$

6. Conclusion

All our calculations have been performed on a SGI ALTIX 24-node IB-interconnected cluster of 8-cores Xeon computers and on the HP XC4000 supercomputer of the federal state Baden-Württemberg using parallel MPI-based [26] as well as thread-based [27] versions of FORM [28]. For evaluation of color factors we have used the FORM program COLOR [29]. The diagrams have been generated with QGRAF [30]. This work was supported by the Deutsche Forschungsgemeinschaft in the Sonderforschungsbereich/Transregio SFB/TR-9 "Computational Particle Physics" and by RFBR grants 11-02-01196 and 10-02-00525.

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