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Three-loop anomalous dimensions for squarks in supersymmetric QCD

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Abstract

In this paper we evaluate the renormalization constants and anomalous dimensions for the squark wave function and mass within supersymmetric QCD. These results complement the ones obtained in Ref. [1] and thus provide further confirmation on the applicability of dimensional reduction to supersymmetric QCD at three-loop order. The three-loop anomalous dimension constitute important input to precision predictions of the supersymmetric mass spectrum as obtained from the evolution from the GUT to the TeV energy scale.

1 Introduction

Supersymmetry (SUSY) (for a review see, e.g., Ref. [2]) has a number of appealing properties which classifies it as a promising extension of the Standard Model (SM). Among them are the possibility of gauge coupling unification, a dark matter candidate, and a solution to the hierarchy problem.

Although there is yet no clear evidence for the realization of SUSY in nature it is mandatory to be prepared both on the experimental and theoretical side. Currently there are several experimental groups who eagerly look for signatures of supersymmetry in the data provided by the CERN Large Hadron Collider (LHC). As far as theory is concerned it is on the one hand important to provide precise predictions for production cross sections involving SUSY particles. On the other hand there are a number of quantities which require higher order loop corrections. A prominent example is the prediction of the lightest Higgs boson mass which recently became available to three loops [3, 4, 5] resulting in an uncertainty which can nevertheless be of the order of about 1 GeV [5]. Another example where higher order corrections within a supersymmetric theory are very welcome are the renormalization group functions. They are crucial for the running from low to high energy scales and constitute an important input for the spectrum generators (see, e.g., Refs. [6, 7, 8]) which predict the SUSY spectrum on the basis of only a few assumptions at energies of about 10^{16} GeV.

The canonical choice for the regularization used for higher order loop calculations is dimensional regularization (DREG). However, it is known since about 30 years that DREG breaks SUSY. As a way out dimensional reduction (DRED) has been formulated [9, 10, 11] which takes over most of the convenient features from DREG and is thus a viable alternative for practical multi-loop calculations. It is worth mentioning that DRED is equivalent to DREG for non-SUSY theories as has been shown in Refs. [12, 13, 14, 15, 16, 17, 18]. Furthermore it has been demonstrated in a number of papers [19, 20, 21, 22, 1, 23] that DRED is consistent with SUSY QCD at the three-loop level. In this paper we provide as new ingredients a further contribution by computing three-loop renormalization constants for the mass and mixing angle of squarks in the minimal subtraction scheme, which in the context of DRED is called $\overline{\mathrm{DR}}$.

The renormalization constants and the corresponding anomalous dimensions up to two-loop order has been computed in Ref. [24, 25, 26, 27, 5]. Three-loop corrections have been considered in Refs. [19, 20, 21, 22, 28] using relations between the beta functions of the gauge and Yukawa couplings and the anomalous dimensions of the symmetry breaking parameters that can be established in a softly broken supersymmetric theory [29, 30, 31]. In Ref. [1] the wave function renormalization constants of quarks, squarks, gluons, gluinos, ghosts and ϵ scalars and the renormalization constants for the quark and gluino mass were calculated to three-loop order in the framework of SUSY QCD. In Ref. [1] also the β function for the strong coupling constant has been derived from all possible three-point functions. The fact that in each case the same expression has been obtained provides a check on the consistency of DRED with gauge invariance and supersymmetry. In this paper the squark renormalization constants are computed to three loops using the component field approach. The main difficulty of this calculation in contrast to the renormalization constants for the gluino and quark masses is that the squark mass renormalization constant depends on the masses of the occurring particles in the loops although a renormalization scheme based on minimal subtraction is adopted. Furthermore, there is an interplay of the renormalization of the ϵ scalar and the squark mass which will also be discussed in this paper.

The remainder of the paper is organized as follows: In the next Section we derive formulae for the squark renormalization constants and briefly outline the procedure used for the construction of the exact mass dependence. Furthermore, the renormalization of the ϵ scalars is discussed in detail. Our results are presented in Section 3 and Section 4 contains the conclusions.

2 Formalism

The calculations in this paper are performed in the framework of SUSY QCD with $n_q = 5$ massless quarks and a massive top quark (m_t). The scalar super partners of the latter has two mass eigenstates ($m_{\tilde{t}_1}$ and $m_{\tilde{t}_2}$) which may have different masses and thus a non-vanishing mixing angle occurs. The super partners of the n_q light quarks are assumed to have degenerate masses ($m_{\tilde{q}}$) and vanishing mixing angle. A generalization to a non-degenerate spectrum is possible in a straightforward way from the formalism for the top squark sector which is discussed in detail in the following. The gluino mass is denoted by $m_{\tilde{g}}$.

Most of the formulae which we are going to present in the following can already be found in Ref. [5]. For completeness we repeat the most important ones here and extend them to three loops. Unless stated otherwise all parameters in the following derivation are $\overline{\mathrm{DR}}$ quantities which depend on the renormalization scale μ. For the sake of compactness the latter is omitted. Bare quantities are marked by a superscript "(0)".

It is common to denote the left- and right-handed components of the top squark by $\tilde{t}_L$ and $\tilde{t}_R$, respectively. The corresponding mass matrix is given by

$$\mathcal{M}_{\tilde{t}}^2 = \begin{pmatrix} m_t^2 + M_Z^2\left(\frac{1}{2} - \frac{2}{3}\sin^2\vartheta_W\right)\cos 2\beta + M_{\tilde{Q}}^2 & m_t\left(A_t - \mu_{\mathrm{SUSY}}\cot\beta\right) \\ m_t\left(A_t - \mu_{\mathrm{SUSY}}\cot\beta\right) & m_t^2 + \frac{2}{3}M_Z^2\sin^2\vartheta_W\cos 2\beta + M_{\tilde{U}}^2 \end{pmatrix}$$

$$\equiv \begin{pmatrix} m_{\tilde{t}_L}^2 & m_t X_t \\ m_t X_t & m_{\tilde{t}_R}^2 \end{pmatrix} \tag{1}$$

with $X_t = A_t - \mu_{\mathrm{SUSY}}\ \cot\beta$. A_t is the soft SUSY breaking tri-linear coupling, and $M_{\tilde{U}}$ and $M_{\tilde{Q}}$ are the soft SUSY breaking masses. With the help of the unitary transformation

$$\begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix} = R_{\tilde{t}}^\dagger \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}, \tag{2}$$

it is possible to diagonalize $\mathcal{M}_{\tilde{t}}^2$

$$\begin{pmatrix} m_{\tilde{t}_1}^2 & 0 \\ 0 & m_{\tilde{t}_2}^2 \end{pmatrix} = R_{\tilde{t}}^\dagger\, \mathcal{M}_{\tilde{t}}^2\, R_{\tilde{t}}\,, \tag{3}$$

where the eigenvalues are the masses of the eigenstates $\tilde{t}_1$ and $\tilde{t}_2$. They read

$$m_{\tilde{t}_{1,2}}^2 = \frac{1}{2}\left[m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2 \mp \sqrt{\left(m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2\right)^2 + 4m_t^2 X_t^2}\,\right]. \tag{4}$$

The unitary transformation can be parametrized by the mixing angle

$$R_{\tilde{t}} = \begin{pmatrix} \cos\theta_t & -\sin\theta_t \\ \sin\theta_t & \cos\theta_t \end{pmatrix}, \tag{5}$$

with

$$\sin\left(2\theta_t\right) = \frac{2m_t\left(A_t - \mu_{\rm SUSY}\,\cot\beta\right)}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}\,. \tag{6}$$

The renormalization constants connected to the top squark are extracted from the top squark propagator. At tree-level it is a diagonal 2×2 matrix which receives non-diagonal entries at loop-level. It is convenient to absorb the corresponding counterterms into a renormalization constant for the mixing angle which we introduce via

$$\theta_t^{(0)} = \theta_t + \delta\theta_t\,. \tag{7}$$

In order to be able to write down the renormalized top squark propagator we define the renormalization constants as follows: The wave function renormalization constant defined through

$$\begin{pmatrix} \tilde{t}_1^{(0)} \\ \tilde{t}_2^{(0)} \end{pmatrix} = \mathcal{Z}_{\tilde{t}}^{1/2} \begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix} \tag{8}$$

can be parametrized by a universal factor $\tilde{Z}_2^{1/2}$ and the renormalization constant for the mixing angle

$$\mathcal{Z}_{\tilde{t}}^{1/2} = \tilde{Z}_2^{1/2} \begin{pmatrix} \cos\delta\theta_t & \sin\delta\theta_t \\ -\sin\delta\theta_t & \cos\delta\theta_t \end{pmatrix}. \tag{9}$$

This equation follows from Eq. (2) and $(\tilde{t}_L^{(0)}, \tilde{t}_R^{(0)})^T = \tilde{Z}_2^{1/2}(\tilde{t}_L, \tilde{t}_R)^T$. Furthermore, the renormalized mass matrix can be parametrized as follows

$$\begin{pmatrix} (m_{\tilde{t}_1}^{(0)})^2 & 0 \\ 0 & (m_{\tilde{t}_2}^{(0)})^2 \end{pmatrix} \to \begin{pmatrix} m_{11}^2 Z_{m_{11}} & m_{12}^2 Z_{m_{12}} \\ m_{21}^2 Z_{m_{21}} & m_{22}^2 Z_{m_{22}} \end{pmatrix} \equiv \mathcal{M}\,, \tag{10}$$

where we require that the off-diagonal elements in the renormalized mass matrix vanish. As a consequence, the counterterm $\delta\theta_t$ takes care of the divergences in the self-energy contribution where a $\tilde{t}_1$ transforms into a $\tilde{t}_2$ or vice versa. This can be seen in the explicit formulae given below. The diagonal elements of Eq. (10) can be identified with the renormalization of the masses

$$(m_{\tilde{t}_i}^{(0)})^2 = m_{ii}^2 Z_{m_{ii}} = m_{\tilde{t}_i}^2 Z_{m_{\tilde{t}_i}}\,. \tag{11}$$

In order to formulate the renormalization conditions it is convenient to consider the renormalized inverse top squark propagator given by

$$i\mathcal{S}^{-1}(p^2) = p^2\left(\mathcal{Z}_{\tilde{t}}^{1/2}\right)^\dagger \mathcal{Z}_{\tilde{t}}^{1/2} - \left(\mathcal{Z}_{\tilde{t}}^{1/2}\right)^\dagger \left[\mathcal{M} - \Sigma(p^2)\right]\mathcal{Z}_{\tilde{t}}^{1/2} \tag{12}$$

where

$$\Sigma(p^2) = \begin{pmatrix} \Sigma_{11}(p^2) & \Sigma_{12}(p^2) \\ \Sigma_{21}(p^2) & \Sigma_{22}(p^2) \end{pmatrix}, \tag{13}$$

stands for the matrix of the squark self energy. In the $\overline{\mathrm{DR}}$ scheme the renormalization conditions read

$$\mathcal{S}_{ij}^{-1}(p^2)\Big|_{\mathrm{pp}} = 0\,, \tag{14}$$

where "pp" stands for the "pole part".

In order to obtain explicit formulae for the evaluation of the renormalization constants it is convenient to define perturbative expansions of the quantities entering Eq. (14). Up to three-loop order we have

$$\begin{aligned} Z_k &= 1 + \left(\frac{\alpha_s}{4\pi}\right)\delta Z_k^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2\delta Z_k^{(2)} + \left(\frac{\alpha_s}{4\pi}\right)^3\delta Z_k^{(3)} + \mathcal{O}(\alpha_s^4)\,, \\ \delta\theta_t &= \left(\frac{\alpha_s}{4\pi}\right)\delta\theta_t^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2\delta\theta_t^{(2)} + \left(\frac{\alpha_s}{4\pi}\right)^3\delta\theta_t^{(3)} + \mathcal{O}(\alpha_s^4)\,, \\ \Sigma_{ij} &= \left(\frac{\alpha_s}{4\pi}\right)\Sigma_{ij}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2\Sigma_{ij}^{(2)} + \left(\frac{\alpha_s}{4\pi}\right)^3\Sigma_{ij}^{(3)} + \mathcal{O}(\alpha_s^4)\,, \end{aligned} \tag{15}$$

where $i,j \in \{1,2\}$ and $k \in \{2, m_{\tilde{t}_1}, m_{\tilde{t}_2}\}$. Inserting these equations into (12) one can solve Eq. (14) iteratively order-by-order in α_s. At one-loop order one gets

$$\begin{aligned} \left\{\Sigma_{ii}^{(1)} - m_{\tilde{t}_i}^2\left(\delta\tilde{Z}_2^{(1)} + \delta Z_{m_{\tilde{t}_i}}^{(1)}\right) + p^2\delta\tilde{Z}_2^{(1)}\right\}\Bigg|_{\mathrm{pp}} &= 0\,, \quad i = 1,2\,, \\ \left\{\Sigma_{12}^{(1)} - \delta\theta_t^{(1)}\left(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2\right)\right\}\Bigg|_{\mathrm{pp}} &= 0\,. \end{aligned} \tag{16}$$

The terms proportional to p^2 in the first equation of (16) are used to compute the wave function renormalization constant which is independent of all occurring masses. Thus they can be set to zero and one obtains

$$\delta\tilde{Z}_2^{(1)} = -\frac{1}{p^2}\Sigma_{11}^{(1)}(p^2)\Big|_{\mathrm{pp}} = -\frac{1}{p^2}\Sigma_{22}^{(1)}(p^2)\Big|_{\mathrm{pp}}\,. \tag{17}$$

Once $\delta\tilde{Z}_2^{(1)}$ is known Eq. (16) is used to obtain $\delta Z_{m_{\tilde{t}_i}}^{(1)}$ keeping the mass dependence in $\Sigma_{ii}^{(1)}$ (see below for more details). The second equation of (16) is used to obtain the renormalization constant of the mixing angle via

$$\delta\theta_t^{(1)} = \frac{\Sigma_{12}^{(1)}}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}\Bigg|_{\mathrm{pp}}\,. \tag{18}$$

Proceeding to two loops we obtain the equations

$$\Bigg[\Sigma_{ii}^{(2)} + \delta\tilde{Z}_2^{(1)}\Sigma_{ii}^{(1)} - m_{\tilde{t}_i}^2\left(\delta\tilde{Z}_2^{(2)} + \delta\tilde{Z}_2^{(1)}\delta Z_{m_{\tilde{t}_i}}^{(1)} + \delta Z_{m_{\tilde{t}_i}}^{(2)}\right) + \delta\tilde{Z}_2^{(2)}p^2$$

$$
+ (-1)^{(i+1)} \delta\theta_t^{(1)} \Big(- 2\Sigma_{12}^{(1)} + \delta\theta_t^{(1)} \big(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2 \big) \Big) \Bigg] \Bigg|_{\rm pp} = 0 \,, \quad i = 1, 2 \,, \tag{19}
$$

$$
\begin{aligned}
&\Bigg[- \delta\theta_t^{(2)} \Big(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2 \Big) - \delta\theta_t^{(1)} \delta\tilde{Z}_2^{(1)} \Big(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2 \Big) - \delta\theta_t^{(1)} \delta Z_{m_{\tilde{t}_1}}^{(1)} m_{\tilde{t}_1}^2 + \delta\theta_t^{(1)} \delta Z_{m_{\tilde{t}_2}}^{(1)} m_{\tilde{t}_2}^2 \\
&+ \delta\theta_t^{(1)} \Sigma_{11}^{(1)} - \delta\theta_t^{(1)} \Sigma_{22}^{(1)} + \delta\tilde{Z}_2^{(1)} \Sigma_{12}^{(1)} + \Sigma_{12}^{(2)} \Bigg] \Bigg|_{\rm pp} = 0 \,,
\end{aligned} \tag{20}
$$

which are solved for $\tilde{Z}_2^{(2)}$, $\delta Z_{m_{\tilde{t}_i}}^{(2)}$ and $\delta\theta_t^{(2)}$ using the same strategy as at one-loop level. Similarly, at three-loop order we have

$$
\begin{aligned}
&\Bigg[(-1)^{i+1} \Bigg\{ \Big(\delta\theta_t^{(1)} \Big)^2 \Big(\delta\tilde{Z}_2^{(1)} \big(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2 \big) + \delta Z_{m_{\tilde{t}_1}}^{(1)} m_{\tilde{t}_1}^2 - \delta Z_{m_{\tilde{t}_2}}^{(1)} m_{\tilde{t}_2}^2 - \Sigma_{11}^{(1)} + \Sigma_{22}^{(1)} \Big) \\
&\qquad + \delta\theta_t^{(1)} \Big(2\, \delta\theta_t^{(2)} \big(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2 \big) - 2\delta\tilde{Z}_2^{(1)} \Sigma_{12}^{(1)} - 2\Sigma_{12}^{(2)} \Big) - 2\delta\theta_t^{(2)} \Sigma_{12}^{(1)} \Bigg\} \\
&+ \delta\tilde{Z}_2^{(1)} \Big(\Sigma_{ii}^{(2)} - \delta Z_{m_{\tilde{t}_i}}^{(2)} m_{\tilde{t}_i}^2 \Big) - \delta\tilde{Z}_2^{(2)} \delta Z_{m_{\tilde{t}_i}}^{(1)} m_{\tilde{t}_i}^2 + \delta\tilde{Z}_2^{(2)} \Sigma_{ii}^{(1)} - \delta\tilde{Z}_2^{(3)} m_{\tilde{t}_i}^2 + \delta\tilde{Z}_2^{(3)} p^2 \\
&- \delta Z_{m_{\tilde{t}_i}}^{(3)} m_{\tilde{t}_i}^2 + \Sigma_{ii}^{(3)} \Bigg] \Bigg|_{\rm pp} = 0 \,, \quad i = 1, 2 \,,
\end{aligned} \tag{21}
$$

$$
\begin{aligned}
&\Bigg[\delta\theta_t^{(1)} \Bigg(- \delta\tilde{Z}_2^{(1)} \delta Z_{m_{\tilde{t}_1}}^{(1)} m_{\tilde{t}_1}^2 + \delta\tilde{Z}_2^{(1)} \delta Z_{m_{\tilde{t}_2}}^{(1)} m_{\tilde{t}_2}^2 + \delta\tilde{Z}_2^{(1)} \Sigma_{11}^{(1)} - \delta\tilde{Z}_2^{(1)} \Sigma_{22}^{(1)} \\
&\qquad - \delta\tilde{Z}_2^{(2)} \big(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2 \big) - \delta Z_{m_{\tilde{t}_1}}^{(2)} m_{\tilde{t}_1}^2 + \delta Z_{m_{\tilde{t}_2}}^{(2)} m_{\tilde{t}_2}^2 + \Sigma_{11}^{(2)} - \Sigma_{22}^{(2)} \Bigg) \\
&+ \delta\theta_t^{(2)} \Big(- \delta\tilde{Z}_2^{(1)} \big(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2 \big) - \delta Z_{m_{\tilde{t}_1}}^{(1)} m_{\tilde{t}_1}^2 + \delta Z_{m_{\tilde{t}_2}}^{(1)} m_{\tilde{t}_2}^2 + \Sigma_{11}^{(1)} - \Sigma_{22}^{(1)} \Big) \\
&- \delta\theta_t^{(3)} \big(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2 \big) + \delta\tilde{Z}_2^{(1)} \Sigma_{12}^{(2)} + \delta\tilde{Z}_2^{(2)} \Sigma_{12}^{(1)} + \Sigma_{12}^{(3)} + \frac{2}{3} \Big(\delta\theta_t^{(1)} \Big)^3 \big(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2 \big) \\
&- 2 \Big(\delta\theta_t^{(1)} \Big)^2 \Sigma_{12}^{(1)} \Bigg] \Bigg|_{\rm pp} = 0 \,.
\end{aligned} \tag{22}
$$

Sample diagrams contributing to Σ_{11} up to three loops can be found in Fig. 1; the contributions to Σ_{12} and Σ_{22} look very similar. Once the quantities Σ_{11}, Σ_{12} and Σ_{22} are known to three-loop order it is possible to extract the renormalization constants for the squark wave function and mass and the mixing angle from Eqs. (21) and (22).

As compared to the corresponding self-energy contributions for fermions or gauge bosons, which after proper projection only lead to logarithmically divergent integrals, the quantities in the above equations have mass dimension two. As a consequence the renormalization constants of the squark masses and the mixing angles depend on the occurring masses, even in a minimal

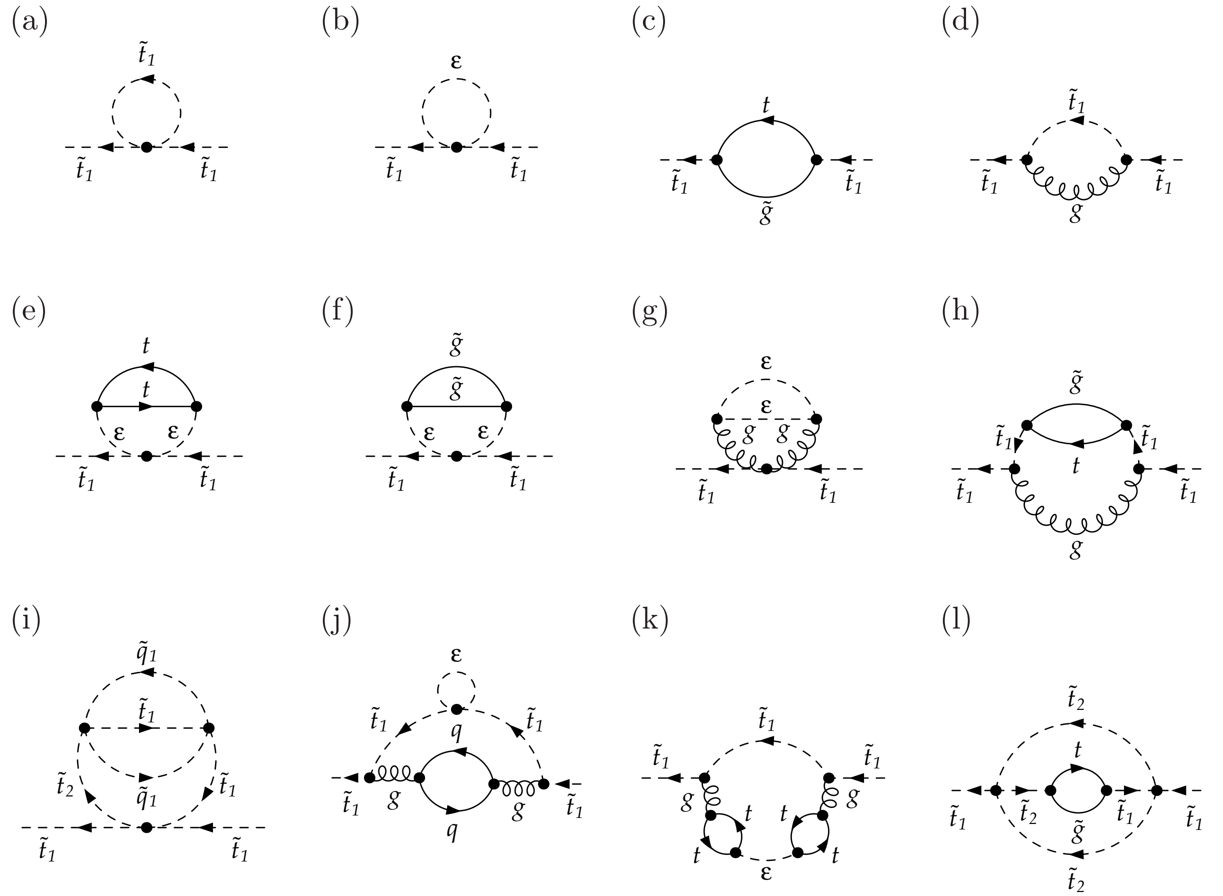

Figure 1: Sample diagrams contributing to Σ_{11} at one, two and three loops. The symbols t, $\tilde{t}_i$, g, $\tilde{g}$ and ϵ denote top quarks, top squarks, gluons, gluinos, and ϵ scalars, respectively.

subtraction scheme like $\overline{\mathrm{DR}}$. At three-loop order an exact evaluation of the corresponding integrals is not possible. It is nevertheless possible to reconstruct the complete dependence on the occurring masses using repeated asymptotic expansions and in addition some knowledge about the structure of the final result. The latter can be induced from the known results at one- and two-loop order. Besides the polynomial dependence inverse powers of first (second) order in $m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}$ occur in the two-loop contributions to $Z_{m_{\tilde{t}_i}}$ $(\delta\theta_t)$. Thus we expect that in $\delta Z^{(3)}_{m_{\tilde{t}_i}}$ at most $1/(m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2})^2$ and in $\delta\theta_t^{(3)}$ at most $1/(m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2})^3$ terms appear. Asymptotic expansion leads to results where these denominators are expanded in a geometric series. If sufficient terms are evaluated it is straightforward to properly reconstruct the inverse mass differences.

Using asymptotic expansion for several different hierarchies it is possible to check that the final result is independent of the actual choice. In our calculation we have chosen the external momentum as the largest scale in order to avoid infrared divergences[1] and the ϵ-scalar mass as the smallest. As far as the squark masses, the gluino and the top quark mass is concerned any

[1]Note that there are still massless gluons and light quarks in the theory.

hierarchy can be chosen. We decided to consider the three choices

$$\begin{aligned}
&q^2 \gg m_{\tilde{t}_2}^2 \gg m_{\tilde{q}}^2 \gg m_{\tilde{t}_1}^2 \gg m_{\tilde{g}}^2 \gg m_t^2 \gg m_\epsilon^2\,,\\
&q^2 \gg m_{\tilde{g}}^2 \gg m_{\tilde{q}}^2 \gg m_{\tilde{t}_2}^2 \gg m_{\tilde{t}_1}^2 \gg m_t^2 \gg m_\epsilon^2\,,\\
&q^2 \gg m_{\tilde{g}}^2 \gg m_{\tilde{q}}^2 \gg m_{\tilde{t}_2}^2 \gg m_t^2 \gg m_{\tilde{t}_1}^2 \gg m_\epsilon^2\,.
\end{aligned} \tag{23}$$

We have checked that in all cases we obtain the same results for $Z_{m_{\tilde{t}_i}}$ and $\delta\theta_t$. Note that in the last hierarchy the top quark mass is even larger than the corresponding squark mass which is allowed since the mass dependence in the $\overline{\rm DR}$ counterterms has no physical meaning.

In each hierarchy of Eq. (23) six mass ratios appear. Some of the expansions are simple and can be truncated after a few terms. E.g., all terms with inverse contributions in q^2 can immediately be set to zero. Similarly, all mass ratios where one has a top squark mass in the denominator and m_t, $m_{\tilde{g}}$ or $m_{\tilde{q}}$ in the numerator only low-order expansion terms appear in the final result. This has been checked by increasing the expansion depth and verifying that the higher order terms are zero. Due to the occurrence of $1/(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)$ terms in the exact result several terms in $m_{\tilde{t}_1}/m_{\tilde{t}_2}$ have to be kept in the expressions for the self energies in order to be able to reconstruct the geometric series. In practice we compute terms up to $(m_{\tilde{t}_1}/m_{\tilde{t}_2})^8$ and check that after including two more powers in the top squark mass ratio the final result does not change.

At this point some comments on the treatment of the ϵ scalar mass, m_ϵ, are in order. In practice there are two renormalization schemes for m_ϵ which are frequently used, the $\overline{\rm DR}$ and on-shell scheme. In the latter one requires that the renormalized mass vanishes in each order in perturbation theory whereas in the $\overline{\rm DR}$ prescription only the pole parts are subtracted by the renormalization constant. We will present our results in a first step for $\overline{\rm DR}$ ϵ scalar masses and afterwards discuss the difference to the on-shell scheme.

In the $\overline{\rm DR}$ scheme it is important to keep m_ϵ different from zero since the renormalization group equations for the squark masses and m_ϵ are coupled. A non-vanishing ϵ-scalar mass in intermediate steps is also required for the computation of the anomalous dimensions in the $\overline{\rm DR}'$ scheme [32] (see below) which was constructed in order to disentangle the running of m_ϵ from the one of the squark parameters.

After the calculation of the bare self energies we renormalize all occurring parameters in the $\overline{\rm DR}$ scheme. For our three-loop calculation we need the counterterms for α_s, m_t, $m_{\tilde{g}}$, $m_{\tilde{t}_i}$, θ_t and m_ϵ to two-loop order and the one for $m_{\tilde{q}}$ to one-loop approximation. Furthermore, also the QCD gauge parameter has to be renormalized to two loops since it appears in the results for the wave function anomalous dimensions. All relevant counterterms can be found in the `Mathematica` file provided together with Ref. [1] and in Ref. [5]. The two-loop corrections for the ϵ-scalar mass renormalization is provided in Ref. [33].

For the calculation of the three-loop integrals we make use of several computer programs which work hand-in-hand in order to reduce the manual interaction to a minimum. All Feynman diagrams are generated with the program `qgraf` [34]. The generated files are manipulated by a `perl` program [1], which implements the prescriptions of Ref. [35], in order to obtain the correct prefactors due to the Majorana character of the gluino. Afterwards the output is transformed to `FORM` [36] notation with the help of `q2e` and `exp` [37, 38]. `exp` furthermore applies the asymptotic expansion (see, e.g., Ref. [39]) in the hierarchies specified in Eq. (23). As a result only one-scale integrals up to three loops appear which can be evaluated with the packages `MINCER` [40] and

MATAD [41]. Let us mention that we implemented the DRED Feynman rules for SUSY QCD as given in Ref. [42, 1].

Once the renormalization constants are available we compute the corresponding anomalous dimension with the help of

$$\gamma_X = \frac{\mu^2}{X}\frac{\mathrm{d}X}{\mathrm{d}\mu^2}\,, \tag{24}$$

where the quantity X is either a mass parameter or the mixing angle

$$X \in \left\{ m_{\tilde{t}_1}^2,\, m_{\tilde{t}_2}^2,\, m_{\tilde{q}}^2,\, m_{\tilde{g}},\, m_t,\, m_\epsilon^2,\, \theta_t \right\}. \tag{25}$$

In practice the derivation in Eq. (24) is taken after exploiting the relation between the bare and the renormalized quantity. Since bare parameters do not depend on μ the derivative acts only on the renormalization constant. In the case of the top quark and the gluino the latter are mass independent and thus the derivative w.r.t. μ can be rewritten into a derivative w.r.t. α_s. For the other parameters, however, one has to take into account the mass dependence of the Z factors. Let us as an example consider the anomalous dimension of $m_{\tilde{t}_i}$ which leads to the following chain of equations[2]

$$\begin{aligned}
\gamma_{m_{\tilde{t}_i}} &= -\frac{\mu^2}{Z_{m_{\tilde{t}_i}}}\,\frac{d}{d\mu^2}\,Z_{m_{\tilde{t}_i}} \\
&= -\frac{\mu^2}{Z_{m_{\tilde{t}_i}}}\left[\frac{dZ_{m_{\tilde{t}_i}}}{d\alpha_s}\,\frac{d\alpha_s}{d\mu^2} + \sum_X \frac{dZ_{m_{\tilde{t}_i}}}{dX}\,\frac{dX}{d\mu^2}\right] \\
&= -\left[\pi\,\beta\,\frac{d}{d\alpha_s}\left(\log Z_{m_{\tilde{t}_i}}\right) + \sum_X X\,\gamma_X\,\frac{d}{dX}\left(\log Z_{m_{\tilde{t}_i}}\right)\right],
\end{aligned} \tag{26}$$

where $\beta(\alpha_s)$ is the anomalous dimension of the strong coupling and X runs over the parameters listed in Eq. (25).

In the next Section we provide results for various anomalous dimensions. For this purpose it is convenient to introduce the following expansion

$$\gamma_X \;=\; -\frac{\alpha_s}{\pi}\sum_{n\geq 0}\left(\frac{\alpha_s}{\pi}\right)^n \gamma_X^{(n)}\,. \tag{27}$$

3 Results

In a first step we have computed the three-loop corrections to the squark wave function renormalization constant $\tilde{Z}_2$ (which is mass independent). In the following we present results for the anomalous dimensions $\gamma_{m_{\tilde{t}_1}}$, $\gamma_{m_{\tilde{t}_2}}$, $\gamma_{m_{\tilde{q}}}$ and γ_{θ_t} up to three-loop order which all have a non-trivial mass dependence. The corresponding results for the renormalization constants can be found in Mathematica format on the internet page [33].

[2] In the subscript for the anomalous dimensions we write $m_{\tilde{t}_1}$ instead of $m_{\tilde{t}_1}^2$, etc..

At one-loop order we obtain the following results

$$m_{\tilde{t}_1}^2\,\gamma_{m_{\tilde{t}_1}}^{(0)} = C_F\left[m_{\tilde{g}}^2 + \frac{1}{8}(1-c_{4t})\,\left(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2\right) + m_t^2 - m_{\tilde{g}}\,m_t\,s_{2t}\right]\,, \tag{28}$$

$$\theta_t\gamma_{\theta_t}^{(0)} = C_F c_{2t}\left[-\frac{m_{\tilde{g}}\,m_t}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} + \frac{s_{2t}}{4}\right]\,, \tag{29}$$

where the abbreviations $c_{nt} = \cos(n\theta_t)$ and $s_{nt} = \sin(n\theta_t)$ have been introduced and $C_F = (N_C^2-1)/(2N_C)$ is the Casimir operator of the fundamental representation of $SU(N_C)$. In Eq. (28) we have given the result for $\gamma_{m_{\tilde{t}_1}}$. The one for $\gamma_{m_{\tilde{t}_2}}$ is obtained by interchanging $m_{\tilde{t}_1}$ and $m_{\tilde{t}_2}$ and replacing θ_t by $-\theta_t$.

The two-loop coefficients read

$$\begin{aligned}
m_{\tilde{t}_1}^2\,\gamma_{m_{\tilde{t}_1}}^{(1)} &= C_A\,C_F\left\{\frac{3}{4}\,m_\epsilon^2 + \frac{11}{4}\,m_{\tilde{g}}^2 + \frac{3}{32}\,(1-c_{4t})\,\left(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2\right) + \frac{3}{4}\,m_t^2 - \frac{3}{2}\,m_{\tilde{g}}\,m_t\,s_{2t}\right\}\\
&+ C_F^2\left\{-\frac{3}{2}\,m_{\tilde{g}}^2 - \frac{1}{16}\,(1-c_{4t})\,\left(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2\right) - \frac{1}{2}\,m_t^2 + m_{\tilde{g}}\,m_t\,s_{2t}\right\}\\
&- C_F\,T_f\left\{n_q\left[\frac{1}{2}\,m_\epsilon^2 + \frac{3}{2}\,m_{\tilde{g}}^2 + m_{\tilde{q}}^2 + \frac{1}{16}\,(1-c_{4t})\,\left(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2\right) + \frac{1}{2}\,m_t^2 - m_{\tilde{g}}\,m_t\,s_{2t}\right]\right.\\
&\left.+ n_t\left[\frac{1}{2}\,m_\epsilon^2 + \frac{3}{2}\,m_{\tilde{g}}^2 + \frac{1}{16}\,(9-c_{4t})\,m_{\tilde{t}_1}^2 + \frac{1}{16}\,(7+c_{4t})\,m_{\tilde{t}_2}^2 - \frac{1}{2}\,m_t^2 - m_{\tilde{g}}\,m_t\,s_{2t}\right]\right\}\,,
\end{aligned} \tag{30}$$

$$\begin{aligned}
\theta_t\gamma_{\theta_t}^{(1)} &= C_F\,T_f\left\{n_q\left[\frac{m_{\tilde{g}}\,m_t}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}\,c_{2t} - \frac{1}{8}\,c_{2t}\,s_{2t}\right] + n_t\left[\frac{m_{\tilde{g}}\,m_t}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}\,c_{2t} - \frac{1}{8}\,c_{2t}\,s_{2t}\right]\right\}\\
&+ C_A\,C_F\left\{\frac{3}{16}\,c_{2t}\,s_{2t} - \frac{3}{2}\,\frac{m_{\tilde{g}}\,m_t}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}\,c_{2t}\right\} + C_F^2\left\{\frac{m_{\tilde{g}}\,m_t}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}\,c_{2t} - \frac{1}{8}\,c_{2t}\,s_{2t}\right\}\,,
\end{aligned} \tag{31}$$

where C_A is the Casimir operator of the adjoint representation of $SU(N_C)$ and $T_F = 1/2$ the trace normalization. n_t counts the top squark flavours and n_q counts the mass-degenerate squark flavours and at the same time the massless quarks. In practice we have $n_t = 1$ and $n_q = 5$, however, it is nevertheless convenient to keep the labels arbitrary.

Let us now come to the three-loop results. The anomalous dimensions for the top squark masses are given by

$$\begin{aligned}
m_{\tilde{t}_1}^2\,\gamma_{m_{\tilde{t}_1}}^{(2)} &= C_F^3\left\{3\,m_{\tilde{g}}^2 + \frac{1}{2}\,m_t^2 - \frac{3}{2}\,m_{\tilde{g}}\,m_t\,s_{2t} + \frac{1}{16}\,(1-c_{4t})\,\left(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2\right)\right\}\\
&+ C_A^2\,C_F\left\{\frac{45}{32}\,m_\epsilon^2 + \frac{15}{4}\,m_{\tilde{g}}^2 + \frac{3}{8}\,m_t^2 - \frac{9}{8}\,m_{\tilde{g}}\,m_t\,s_{2t} + \frac{3}{64}\,(1-c_{4t})\,\left(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2\right)\right\}\\
&+ C_F^2\,C_A\left\{-\frac{9}{16}\,m_\epsilon^2 - \frac{21}{8}\,m_{\tilde{g}}^2 - \frac{3}{8}\,m_t^2 + \frac{9}{8}\,m_{\tilde{g}}\,m_t\,s_{2t} - \frac{3}{64}\,(1-c_{4t})\,\left(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2\right)\right\}
\end{aligned}$$

$$
\begin{aligned}
&+ C_F\, T_f^2 \Bigg\{ n_t^2 \left[\frac{3}{8}\, m_\epsilon^2 - \frac{3}{2}\, m_{\tilde g}^2 + \frac{3}{4}\, m_{\tilde t_1}^2 - m_t^2 + \frac{3}{4}\, m_{\tilde g}\, m_t\, s_{2t} - \frac{1}{32}\,(13 - c_{4t}) \left(m_{\tilde t_1}^2 - m_{\tilde t_2}^2 \right) \right] \\
&+ n_q^2 \left[\frac{3}{8}\, m_\epsilon^2 - \frac{3}{2}\, m_{\tilde g}^2 + \frac{3}{4}\, m_{\tilde q}^2 - \frac{1}{4}\, m_t^2 + \frac{3}{4}\, m_{\tilde g}\, m_t\, s_{2t} - \frac{1}{32}\,(1 - c_{4t}) \left(m_{\tilde t_1}^2 - m_{\tilde t_2}^2 \right) \right] \\
&+ n_q\, n_t \left[\frac{3}{4}\, m_\epsilon^2 - 3\, m_{\tilde g}^2 + \frac{3}{4}\, m_{\tilde q}^2 + \frac{3}{4}\, m_{\tilde t_1}^2 - \frac{5}{4}\, m_t^2 + \frac{3}{2}\, m_{\tilde g}\, m_t\, s_{2t} \right. \\
&\left. - \frac{1}{16}\,(7 - c_{4t}) \left(m_{\tilde t_1}^2 - m_{\tilde t_2}^2 \right) \right] \Bigg\} \\
&+ C_F^2\, T_f \Bigg\{ n_t \left[\frac{3}{8}\, m_\epsilon^2 - \frac{27}{4}\, m_{\tilde g}^2 + \frac{3}{4}\, m_{\tilde t_1}^2 - \frac{7}{4}\, m_t^2 + 3\, m_{\tilde g}\, m_t\, s_{2t} + 9\, m_{\tilde g}^2\, \zeta_3 + \frac{3}{2}\, m_t^2\, \zeta_3 \right. \\
&\left. - \frac{9}{2}\, m_{\tilde g}\, m_t\, s_{2t}\, \zeta_3 + \left(m_{\tilde t_1}^2 - m_{\tilde t_2}^2 \right) \left(\frac{1}{8}\, c_{4t} - \frac{1}{2} + \frac{3}{16}\, \zeta_3 - \frac{3}{16}\, c_{4t}\, \zeta_3 \right) \right] \\
&+ n_q \left[\frac{3}{8}\, m_\epsilon^2 - \frac{27}{4}\, m_{\tilde g}^2 + \frac{3}{4}\, m_{\tilde q}^2 - m_t^2 + 3\, m_{\tilde g}\, m_t\, s_{2t} + 9\, m_{\tilde g}^2\, \zeta_3 + \frac{3}{2}\, m_t^2\, \zeta_3 - \frac{9}{2}\, m_{\tilde g}\, m_t\, s_{2t}\, \zeta_3 \right. \\
&\left. + \left(m_{\tilde t_1}^2 - m_{\tilde t_2}^2 \right) \left(\frac{1}{8}\, c_{4t} - \frac{1}{8} + \frac{3}{16}\, \zeta_3 - \frac{3}{16}\, c_{4t}\, \zeta_3 \right) \right] \Bigg\} \\
&+ C_A\, C_F\, T_f \Bigg\{ n_q \left[\frac{1}{8}\, m_t^2 - \frac{3}{2}\, m_\epsilon^2 - \frac{15}{8}\, m_{\tilde q}^2 - \frac{3}{8}\, m_{\tilde g}\, m_t\, s_{2t} - 9\, m_{\tilde g}^2\, \zeta_3 - \frac{3}{2}\, m_t^2\, \zeta_3 \right. \\
&\left. + \frac{9}{2}\, m_{\tilde g}\, m_t\, s_{2t}\, \zeta_3 + \left(m_{\tilde t_1}^2 - m_{\tilde t_2}^2 \right) \left(\frac{1}{64} - \frac{1}{64}\, c_{4t} - \frac{3}{16}\, \zeta_3 + \frac{3}{16}\, c_{4t}\, \zeta_3 \right) \right] \\
&+ n_t \left[2\, m_t^2 - \frac{3}{2}\, m_\epsilon^2 - \frac{15}{8}\, m_{\tilde t_1}^2 - \frac{3}{8}\, m_{\tilde g}\, m_t\, s_{2t} - 9\, m_{\tilde g}^2\, \zeta_3 - \frac{3}{2}\, m_t^2\, \zeta_3 + \frac{9}{2}\, m_{\tilde g}\, m_t\, s_{2t}\, \zeta_3 \right. \\
&\left. + \left(m_{\tilde t_1}^2 - m_{\tilde t_2}^2 \right) \left(\frac{61}{64} - \frac{1}{64}\, c_{4t} - \frac{3}{16}\, \zeta_3 + \frac{3}{16}\, c_{4t}\, \zeta_3 \right) \right] \Bigg\}, \qquad (32)
\end{aligned}
$$

where ζ_3 is Riemann's zeta function with the value $\zeta_3 = 1.2020569\ldots$. The three-loop expression for γ_{θ_t} reads

$$
\begin{aligned}
\theta_t \gamma_{\theta_t}^{(2)} = {} & C_F\, T_f^2 \Bigg\{ n_q^2 \left[\frac{m_{\tilde g}\, m_t}{m_{\tilde t_1}^2 - m_{\tilde t_2}^2}\, \frac{3}{4}\, c_{2t} - \frac{1}{16}\, c_{2t}\, s_{2t} \right] + n_t^2 \left[\frac{m_{\tilde g}\, m_t}{m_{\tilde t_1}^2 - m_{\tilde t_2}^2}\, \frac{3}{4}\, c_{2t} - \frac{1}{16}\, c_{2t}\, s_{2t} \right] \\
& + n_q\, n_t \left[\frac{m_{\tilde g}\, m_t}{m_{\tilde t_1}^2 - m_{\tilde t_2}^2}\, \frac{3}{2}\, c_{2t} - \frac{1}{8}\, c_{2t}\, s_{2t} \right] \Bigg\} \\
& + C_F^3 \left\{ \frac{1}{8}\, c_{2t}\, s_{2t} - \frac{m_{\tilde g}\, m_t}{m_{\tilde t_1}^2 - m_{\tilde t_2}^2}\, \frac{3}{2}\, c_{2t} \right\} + C_A^2\, C_F \left\{ \frac{3}{32}\, c_{2t}\, s_{2t} - \frac{m_{\tilde g}\, m_t}{m_{\tilde t_1}^2 - m_{\tilde t_2}^2}\, \frac{9}{8}\, c_{2t} \right\} \\
& + C_A\, C_F^2 \left\{ \frac{m_{\tilde g}\, m_t}{m_{\tilde t_1}^2 - m_{\tilde t_2}^2}\, \frac{9}{8}\, c_{2t} - \frac{3}{32}\, c_{2t}\, s_{2t} \right\}
\end{aligned}
$$

$$
\begin{aligned}
&+ C_F^2\, T_f \left\{ n_q \left[\frac{m_{\tilde g}\, m_t}{m_{\tilde t_1}^2 - m_{\tilde t_2}^2}\, c_{2t} \left(3 - \frac{9}{2}\,\zeta_3 \right) + c_{2t}\, s_{2t} \left(\frac{3}{8}\,\zeta_3 - \frac{1}{4} \right) \right] \right.\\
&+ n_t \left[\frac{m_{\tilde g}\, m_t}{m_{\tilde t_1}^2 - m_{\tilde t_2}^2}\, c_{2t} \left(3 - \frac{9}{2}\,\zeta_3 \right) + c_{2t}\, s_{2t} \left(\frac{3}{8}\,\zeta_3 - \frac{1}{4} \right) \right]\\
&+ C_A\, C_F\, T_f \left\{ n_q \left[\frac{m_{\tilde g}\, m_t}{m_{\tilde t_1}^2 - m_{\tilde t_2}^2}\, c_{2t} \left(\frac{9}{2}\,\zeta_3 - \frac{3}{8} \right) + c_{2t}\, s_{2t} \left(\frac{1}{32} - \frac{3}{8}\,\zeta_3 \right) \right] \right.\\
&\left. + n_t \left[\frac{m_{\tilde g}\, m_t}{m_{\tilde t_1}^2 - m_{\tilde t_2}^2}\, c_{2t} \left(\frac{9}{2}\,\zeta_3 - \frac{3}{8} \right) + c_{2t}\, s_{2t} \left(\frac{1}{32} - \frac{3}{8}\,\zeta_3 \right) \right] \right\}. \qquad (33)
\end{aligned}
$$

At that point a brief comment on degenerate top squarks is in order. In the expressions for $\gamma_{m_{\tilde t_i}}$ the limit $m_{\tilde t_2} \to m_{\tilde t_1}$ can be taken naively. Furthermore one has to nullify the mixing angle. The quantity γ_{θ_t} is not defined in the mass-degenerate case which is reflected by the fact that the limit $m_{\tilde t_2} \to m_{\tilde t_1}$ does not exist in Eqs. (29), (31) and (33).

In order to compare with the results in the literature we have to transform our results to the anomalous dimensions for the quantities $M_{\tilde Q}$, $M_{\tilde U}$ and A_t as given in Eq. (1). This is conveniently achieved with the help of Eq. (3) which is differentiated w.r.t. μ^2. The resulting equations are then solved for the $\gamma_{M_{\tilde Q}}$, $\gamma_{M_{\tilde U}}$ and γ_{A_t}. We have compared the resulting one-, two- and three-loop expressions with the results in the literature [24, 26, 22] and found complete agreement. Note that the method used in Ref. [22] is based on a relation of the anomalous dimensions to an all-order expression in the so-called NSVZ scheme [43] whereas in this work a diagrammatic approach has been used to evaluate the three-loop corrections. We refrain from providing explicit results for $\gamma_{M_{\tilde Q}}$ and $\gamma_{M_{\tilde U}}$ which, however, can be found in the `Mathematica` file [33]. Note that we have $\gamma_{M_{\tilde Q}} = \gamma_{M_{\tilde U}}$ which is expected since electroweak effects are neglected [29]. This serves as a welcome check for our calculation. The result for γ_{A_t} is proportional to the gluino mass and is thus quite compact. Up to three-loop order it is given by

$$
\begin{aligned}
\frac{\mu^2}{A_t}\frac{d}{d\mu^2} A_t = \gamma_{A_t} = \frac{m_{\tilde g}}{A_t} &\left\{ \frac{\alpha_s}{\pi}\, C_F + \left(\frac{\alpha_s}{\pi}\right)^2 \left[\frac{3}{2}\, C_A\, C_F - C_F^2 - C_F\,(n_q + n_t)\, T_f \right] \right.\\
&+ \left(\frac{\alpha_s}{\pi}\right)^3 \left[\frac{9}{8}\, C_A^2\, C_F - \frac{9}{8}\, C_A\, C_F^2 + \frac{3}{2}\, C_F^3 - \frac{3}{4}\, C_F\,(n_q+n_t)^2\, T_f^2 \right.\\
&\left.\left. + C_A\, C_F\,(n_q+n_t)\, T_f \left(\frac{3}{8} - \frac{9}{2}\,\zeta_3 \right) + C_F^2\,(n_q+n_t)\, T_f \left(-3 + \frac{9}{2}\,\zeta_3 \right) \right] \right\}. \qquad (34)
\end{aligned}
$$

For completeness let us also provide the result for mass-degenerate squarks which is given by

$$
m_{\tilde q}^2 \gamma_{m_{\tilde q}}^{(0)} = C_F\, m_{\tilde g}^2\,, \qquad (35)
$$

$$
\begin{aligned}
m_{\tilde q}^2 \gamma_{m_{\tilde q}}^{(1)} = {}& C_A\, C_F \left\{ \frac{3}{4}\, m_\epsilon^2 + \frac{11}{4}\, m_{\tilde g}^2 \right\} - C_F^2\, \frac{3}{2}\, m_{\tilde g}^2\\
&- C_F\; T_f \left\{ n_q \left[\frac{1}{2}\, m_\epsilon^2 + \frac{3}{2}\, m_{\tilde g}^2 + m_{\tilde q}^2 \right] + n_t \left[\frac{1}{2}\, m_\epsilon^2 + \frac{3}{2}\, m_{\tilde g}^2 + \frac{1}{2}\, m_{\tilde t_1}^2 + \frac{1}{2}\, m_{\tilde t_2}^2 - m_t^2 \right] \right\}, \qquad (36)
\end{aligned}
$$

$$
\begin{aligned}
m_{\tilde{q}}^2 \gamma_{m_{\tilde{q}}}^{(2)} = {} & C_F^3\, 3\, m_{\tilde{g}}^2 - C_A\, C_F^2 \left\{ \frac{9}{16}\, m_\epsilon^2 + \frac{21}{8}\, m_{\tilde{g}}^2 \right\} + C_A^2\, C_F \left\{ \frac{45}{32}\, m_\epsilon^2 + \frac{15}{4}\, m_{\tilde{g}}^2 \right\} \\
& + C_F\, T_f^2 \left\{ n_q^2 \left[\frac{3}{8}\, m_\epsilon^2 - \frac{3}{2}\, m_{\tilde{g}}^2 + \frac{3}{4}\, m_{\tilde{q}}^2 \right] \right. \\
& + n_q\, n_t \left[\frac{3}{4}\, m_\epsilon^2 - 3\, m_{\tilde{g}}^2 + \frac{3}{4}\, m_{\tilde{q}}^2 + \frac{3}{8}\, m_{\tilde{t}_1}^2 + \frac{3}{8}\, m_{\tilde{t}_2}^2 - \frac{3}{4}\, m_t^2 \right] \\
& \left. + n_t^2 \left[\frac{3}{8}\, m_\epsilon^2 - \frac{3}{2}\, m_{\tilde{g}}^2 + \frac{3}{8}\, m_{\tilde{t}_1}^2 + \frac{3}{8}\, m_{\tilde{t}_2}^2 - \frac{3}{4}\, m_t^2 \right] \right\} \\
& - C_A\, C_F\, T_f \left\{ n_q \left[\frac{3}{2}\, m_\epsilon^2 + \frac{15}{8}\, m_{\tilde{q}}^2 + 9\, m_{\tilde{g}}^2\, \zeta_3 \right] \right. \\
& \left. + n_t \left[\frac{3}{2}\, m_\epsilon^2 + \frac{15}{16}\, m_{\tilde{t}_1}^2 + \frac{15}{16}\, m_{\tilde{t}_2}^2 - \frac{15}{8}\, m_t^2 + 9\, m_{\tilde{g}}^2\, \zeta_3 \right] \right\} \\
& + C_F^2\, T_f \left\{ n_q \left[\frac{3}{8}\, m_\epsilon^2 - \frac{27}{4}\, m_{\tilde{g}}^2 + \frac{3}{4}\, m_{\tilde{q}}^2 + 9\, m_{\tilde{g}}^2\, \zeta_3 \right] \right. \\
& \left. + n_t \left[\frac{3}{8}\, m_\epsilon^2 - \frac{27}{4}\, m_{\tilde{g}}^2 + \frac{3}{8}\, m_{\tilde{t}_1}^2 + \frac{3}{8}\, m_{\tilde{t}_2}^2 - \frac{3}{4}\, m_t^2 + 9\, m_{\tilde{g}}^2\, \zeta_3 \right] \right\}. \qquad (37)
\end{aligned}
$$

One observes that all terms which do not involve n_t can be obtained from $\gamma_{m_{\tilde{t}_1}}$ by setting $m_{\tilde{t}_2} = m_{\tilde{t}_1}$, $m_t = 0$ and $\theta_t = 0$.

When applying the anomalous dimensions derived in this paper one has to consider the combined set of differential equations of all $\overline{\mathrm{DR}}$ parameters appearing on the r.h.s. of the above results. This concerns in particular the unphysical ϵ-scalar mass which means that although m_ϵ is set to zero at one scale it is different from zero once this scale is changed. A way out from this situation is to renormalize the ϵ scalar mass on-shell. We have computed the resulting anomalous dimensions and provide the results in Ref. [33]. Alternatively one could shift the squark masses by a finite term which is chosen such that the ϵ scalar decouples from the system of differential equations. The resulting renormalization scheme is called $\overline{\mathrm{DR}}'$ scheme and has been suggested in Ref. [32]. In our approximation the finite shift is needed up to two loops which is given by [32, 44]

$$
m_{\tilde{f}}^2 \to m_{\tilde{f}}^2 - \frac{\alpha_s}{\pi}\, \frac{1}{2}\, C_F\, m_\epsilon^2 + \left(\frac{\alpha_s}{\pi} \right)^2 C_F\, m_\epsilon^2 \left(\frac{1}{4}\, T_f\, \left(n_q + n_t \right) + \frac{1}{4}\, C_F - \frac{3}{8}\, C_A \right), \qquad (38)
$$

where $f = t$ or $f = q$.[3] We have checked that after inserting this shift in $\gamma_{m_{\tilde{t}_1}}$, $\gamma_{m_{\tilde{t}_2}}$ and $\gamma_{m_{\tilde{q}}}$ the parameter m_ϵ drops out from the resulting anomalous dimension. Again we refrain from listing explicit results, however, provide the analytic expressions in [33].

All results presented above can be found in `Mathematica` format on the webpage [33]. In addition we provide the results for the anomalous dimensions $\gamma_{M_{\tilde{Q}}}$, $\gamma_{M_{\tilde{U}}}$ and γ_{A_t} and the renormalization constants for the squark masses and the mixing angle in the top squark system. The

[3] Of course, T_f is not altered.

	1 loop	2 loops	3 loops
$m_{\tilde{t}_1}$ (GeV)	1425	1416	1378
$m_{\tilde{t}_2}$ (GeV)	1677	1670	1632
θ_t	0.658	0.659	0.656
$m_{\tilde{q}}$ (GeV)	1580	1573	1535

Table 1: Numerical values for the $\overline{\rm DR}'$ parameters for $\mu = M_Z$ using the numbers in Eq. (39) as input and solving the system of differential equations with one-, two- or three-loop anomalous dimensions in the squark sector.

`Mathematica` file contains furthermore the result for $\gamma_{m_{\tilde{t}_1}}$, $\gamma_{m_{\tilde{t}_2}}$ and $\gamma_{m_{\tilde{q}}}$ for on-shell ϵ scalar masses and in the $\overline{\rm DR}'$ scheme.

Let us finally perform a simplified analysis in order to exemplify the numerical impact of the three-loop corrections. In our example we fix the following values of the $\overline{\rm DR}'$ parameters at the scale $\mu = \mu_G = 10^{16}$ GeV

$$\begin{aligned}
&m_{\tilde{t}_1} = 400\text{ GeV}\,, \quad m_t = 67\text{ GeV}\,, \quad \theta_t = 0.1\,, \quad \alpha_s = 0.0425\,,\\
&m_{\tilde{t}_2} = m_{\tilde{g}} = m_{\tilde{q}} = 600\text{ GeV}\,,
\end{aligned} \tag{39}$$

and use the anomalous dimensions obtained in this paper and in Ref. [1] to compute the corresponding values for $\mu = M_Z$. Since our aim is to study the numerical importance of the three-loop anomalous dimensions in the squark sector we neglect all threshold effects. Furthermore, we use for the running of α_s, $m_{\tilde{g}}$ and m_t always the three-loop approximation whereas in the case of the squark masses and θ_t the loop-order is varied from one to three.

The values of $m_t = m_t(\mu_G)$ and $\alpha_s = \alpha_s(\mu_G)$ in Eq. (39) are chosen such that three-loop running leads to $m_t(M_Z) = 170$ GeV and $\alpha_s(M_Z) = 0.118$. The results for $m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$, θ_t and $m_{\tilde{q}}$ at the scale $\mu = M_Z$ can be found in Tab. 1.

We observe a small change in the mixing angle by about 0.4%. As far as the squark masses are concerned one observes a moderate shift of a few GeV when going from one to two loops. After switching on the three-loop terms, however, the squark masses are decreased by about 40 GeV which is approximately an order of magnitude larger than the two-loop corrections. Nevertheless it corresponds to a shift in the masses of about 3% which is a reasonable amount for a three-loop SUSY QCD term. Our observation coincides with the findings of Ref. [22] where also relatively large three-loop corrections for the squarks have been identified.

4 Conclusions

In this paper the renormalization constants for the squarks and the corresponding mixing angle have been computed to three-loop order within supersymmetric QCD. Thus, all anomalous dimensions of the physical parameters are now available to order α_s^3 and can thus be used to relate their mass values at the GUT and electroweak or TeV scale.

Our calculation has been performed using dimensional reduction for the regularization of the divergent loop integrals which is realized with the help of massive ϵ scalars. As far as the renormalization of the ϵ scalar mass is concerned we have evaluated our results for three different schemes: $\overline{\text{DR}}$, $\overline{\text{DR}}'$ and on-shell. Our results agree with Ref. [22] which supports the consistency of DRED with SUSY QCD since in Ref. [22] the results have been obtained without a diagrammatic calculation.

A simplified numerical analysis shows that the three-loop corrections to the squark masses are numerically important (see also [22]) and thus should be included in the spectrum generators which incorporate the running from the GUT to the electroweak scale.

All renormalization constants and anomalous dimensions computed in this paper can be downloaded from the URL [33] in `Mathematica` format.

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